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Perturbative QCD analysis of B meson decays

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The resummation of large QCD radiative corrections, including leading and next-to-leading logarithms, in the pion electromagnetic form factor is reviewed. A similar formalism is applied to exclusive processes involving heavy mesons, and leads to Sudakov suppression for the semileptonic decay $B\to\pi l\nu$. It is found that, with the inclusion of Sudakov effects, a perturbative QCD analysis of this decay is reliable for the energy fraction of the pion above 0.4. Combining predictions from soft pion theorems we estimate that the upper limit of the matrix element $|V_{ub}|$ is 2.8–4.8$\times 10^{-3}$ from different models of the B meson wave function.

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I. INTRODUCTION

It has been shown that perturbative QCD (PQCD) is applicable to exclusive processes such as elastic hadron form factors for an energy scale higher than a few GeV [1]. The enlargement of the range of applicability from much higher energies [2] down to this low scale is due to the inclusion of the transverse momentum dependence into factorization theorems. This dependence appears in an exponential factor, which arises from the resummation of large radiative corrections and gives Sudakov suppression for elastic scattering of isolated colored quarks. The detailed derivation of the Sudakov factors for hadron-hadron Landshoff scattering is given in [3]. Similar expressions have been obtained and employed in the PQCD analysis of the pion and proton form factors [1,4–6], pion Compton scattering [7], and other exclusive processes [8]. Predictions from this modified version of factorization theorems have been examined and found to be dominated by perturbative contributions.

All of the above analyses of large corrections involve only light hadrons. In this paper we shall extend the resummation technique to exclusive processes containing both light and heavy mesons, such as B meson decays, and organize the Sudakov corrections to all orders.

An important work in the study of the standard model is to determine the mixing angles in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The decay $K\to\pi l\nu$ contains the information of the matrix element $|V_{us}|$, and chiral symmetry provides a precise method to study this process [9]. $|V_{cb}|$ is determined by exploring the $B\to Dl\nu$ decay, for which heavy quark symmetry is an appropriate tool [10]. As to $|V_{ub}|$, which can be measured reliably from the $B\to\pi l\nu$ decay [11], neither of the theories is proper.

Recently, an analysis of the semileptonic decay $B\to\pi l\nu$ based on the heavy quark effective theory (HQET) has been performed by Burdman *et al.* [12]. They determined the normalization of relevant form factors in terms of soft pion relations, and gave the ratio of these form factors to the corresponding ones in the $D\to\pi l\nu$ decay. However, an explicit evaluation of the $B\to\pi$ transition form factors is not yet successful.

We shall show that by incorporating Sudakov effects, PQCD is applicable to the semileptonic decay $B\to\pi l\nu$ as the pion recoils sufficiently fast. By combining our predictions with those from soft pion theorems, which were derived in the framework of HQET [12], the total decay rate and the branching ratio are estimated. Comparing this estimation with experimental data, we extract the upper limit of the CKM matrix element $|V_{ub}|$. Hence, PQCD complements HQET in the study of heavy meson decays. The formalism developed in this work can be easily generalized to other heavy-to-light transitions.

The first attempt to apply the PQCD formalism including Sudakov suppression to the $B\to\pi l\nu$ decay is made by Akhoury *et al.* [13]. However, they did not consider the transverse momentum dependence, and their predictions are much smaller than ours. As claimed in [13], the hard gluon involved in the decay process is off shell at most by an amount of $1.4\Lambda_{\rm QCD}m_b$, m_b being the b quark mass. In our approach the hard gluon is off shell roughly by $8\Lambda_{\rm QCD}m_b$. Therefore, the perturbative analysis presented here is more reliable.

The resummation of Sudakov logarithms for the pion form factor is reviewed in Sec. II. The resummation technique for the semileptonic B meson decay is developed in Sec. III, where the full expression for the Sudakov factor including leading and next-to-leading logarithms is given. Section IV contains the numerical analysis of the factorization formulas for relevant form factors. Section V is our conclusion.

II. PION FORM FACTOR

We review factorization theorems including Sudakov effects for a simple light-to-light process, the pion electromagnetic form factor, which is expressed as the convolution of a hard scattering amplitude with wave functions. We investigate radiative corrections to the factorization formula, and explain how they are absorbed into the convolution factors. The first step is to find the leading momentum regions of radiative corrections, from which important contributions to loop integrals arise. There are two types of important contri-

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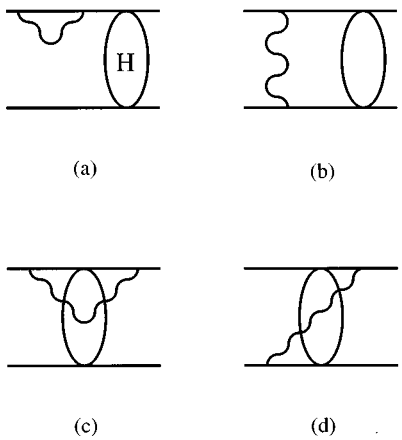

FIG. 1. $O(\alpha_s)$ corrections to the basic factorization of the pion form factor.

butions: collinear, when the loop momentum is parallel to the incoming or outgoing pion momentum, and soft, when the loop momentum is much smaller than the momentum transfer Q^2 of the process. Here Q^2 is assumed to be large and serves as an ultraviolet cutoff of loop integrals. We associate small transverse momentum $\mathbf{k}_T$ with the partons of the pion, which is taken as an infrared cutoff.

Each type of important contributions gives large single logarithms. They may overlap to give double (leading) logarithms in some cases. These large logarithms, appearing in a product with α_s, must be organized in order not to spoil the perturbative expansion. It is known that single logarithms can be summed to all orders using renormalization group (RG) methods, and double logarithms must be treated by the resummation technique developed in [14]. This technique is most easily explained in axial gauge $n\cdot A=0$, n being a gauge vector and A the gauge field.

The diagrams shown in Fig. 1 represent $O(\alpha_s)$ corrections to the basic factorization of the pion form factor, which contain the large logarithms mentioned above. In axial gauge the two-particle reducible diagrams, such as Figs. 1(a) and 1(b), have double logarithms from the overlap of collinear and soft divergences, while the two-particle irreducible corrections, such as Figs. 1(c) and 1(d), contain only single soft logarithms. This distinction is consistent with the physical picture: Two partons moving in the same direction can interact with each other through collinear or soft gluons, while those moving apart from each other can interact only through soft gluons. Below we shall concentrate on reducible corrections, and demonstrate how they are summed into a Sudakov factor.

A careful analysis shows that soft divergences cancel between Figs. 1(a) and 1(b), as well as between Figs. 1(c) and 1(d), in the asymptotic region with $b\to 0$, b being the conjugate variable to $\mathbf{k}_T$. Therefore, reducible corrections are dominated by collinear divergences, and can be absorbed into the pion wave function $\mathcal{P}$, which involves similar dynamics. Irreducible corrections, due to the cancellation of soft divergences, are then absorbed into the hard scattering amplitude H. Hence, the factorization picture holds at least asymptotically after radiative corrections are included. The cancellation of soft divergences is closely related to the universality of wave functions. For a large b, double logarithms are present and the resummation technique must be implemented.

Based on the above reasoning, the factorization formula for the pion form factor in the b space is written as [1]

$$F_\pi(Q^2)=\int_0^1 dx_1 dx_2\int\frac{d^2\mathbf{b}}{(4\pi)^2}\mathcal{P}(x_2,b,P_2,\mu)\times\tilde{H}(x_1,x_2,b,Q,\mu)\mathcal{P}(x_1,b,P_1,\mu), \quad (1)$$

where μ is the factorization and renormalization scale, and b can be regarded as the separation between two valence quarks. $\tilde{H}$ is the Fourier transform of H. P_1 and P_2 are the momenta of the incoming and outgoing pions, respectively. We choose the Breit frame, in which $P_1^+=P_2^-=Q/\sqrt{2}$ and all other components of P's vanish, $Q^2=-(P_1-P_2)^2$ being the momentum transfer mentioned before. Equation (1) depends only on a single b, because the virtual quark line involved in H is thought of as far from mass shell, and shrunk to a point [1]. The wave function $\mathcal{P}$ includes all leading logarithmic enhancements at large b.

The basic idea of the resummation technique is as follows. If the double logarithms appear in an exponential form $\mathcal{P}\sim\exp(-\text{const}\times\ln Q\ln\ln Q)$, the task will be simplified by studying the derivative of $\mathcal{P}$, $d\mathcal{P}/d\ln Q=C\mathcal{P}$. The coefficient C contains only large single logarithms, and can be treated by RG methods. Therefore, working with C one reduces the double-logarithm problem to a single-logarithm problem.

The two invariants appearing in $\mathcal{P}$ are $P\cdot n$ and n^2. Because of the scale invariance in n of the gluon propagator,

$$N^{\mu\nu}(q)=\frac{-i}{q^2}\left(g^{\mu\nu}-\frac{n^\mu q^\nu+q^\mu n^\nu}{n\cdot q}+n^2\frac{q^\mu q^\nu}{(n\cdot q)^2}\right), \quad (2)$$

$\mathcal{P}$ depends only on a single large scale $\nu^2=(P\cdot n)^2/n^2$. It is then easy to show that the differential operator $d/d\ln Q$ can be replaced by d/dn:

$$\frac{d}{d\ln Q}\mathcal{P}=-\frac{n^2}{P\cdot n}P^\alpha\frac{d}{dn^\alpha}\mathcal{P}. \quad (3)$$

The motivation for this replacement is that the momentum P flows through both quark and gluon lines, but n appears only on gluon lines. The analysis then becomes easier by studying the n, instead of P, dependence.

Applying d/dn_α to the gluon propagator, we get

$$\frac{d}{dn_\alpha}N^{\mu\nu}=-\frac{1}{q\cdot n}(N^{\mu\alpha}q^\nu+N^{\nu\alpha}q^\mu). \quad (4)$$

The momentum q that locates at both ends of the differentiated gluon line is contracted with a vertex, where the gluon attaches. After adding together all diagrams with different differentiated gluon lines and using the Ward identity, we arrive at the differential equation of $\mathcal{P}$ as shown in Fig. 2(a), in which the square vertex represents

$$gT^a\frac{n^2}{P\cdot nq\cdot n}P_\alpha,$$

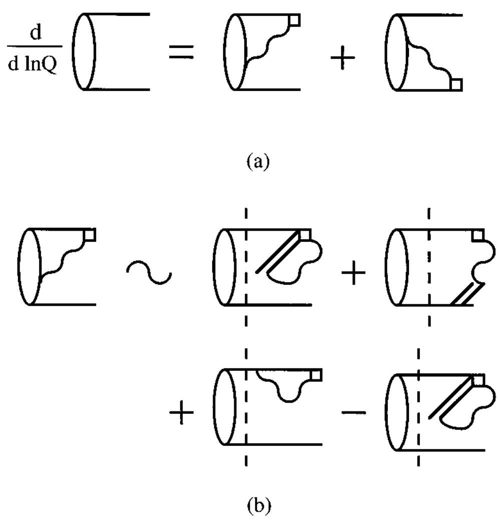

FIG. 2. Graphic representation of Eq. (5).

T^a being a color matrix. An important feature of the square vertex is that the gluon momentum q does not lead to collinear divergences because of the nonvanishing n^2. The leading regions of q are then soft and ultraviolet, in which Fig. 2(a) can be factorized according to Fig. 2(b) at lowest order. The part on the left-hand side of the dashed line is exactly $\mathscr{P}$, and that on the right-hand side is assigned to the coefficient C.

Therefore, we need a function $\mathscr{K}$ for the absorption of the soft divergences from the first two diagrams in Fig. 2(b), and a function $\mathscr{G}$ for the ultraviolet divergences from the other two diagrams. The soft subtraction employed in $\mathscr{G}$ is to avoid double counting. Generalizing the two functions to all orders, we derive the differential equation of $\mathscr{P}$:

$$\frac{d}{d\ln Q}\mathscr{P}(x,b,P,\mu)=[2\mathscr{K}(b\mu)+\mathscr{G}(x\nu/\mu)+\mathscr{G}((1-x)\nu/\mu)]\mathscr{P}(x,b,P,\mu). \tag{5}$$

$\mathscr{K}$ and $\mathscr{G}$ have been calculated to one loop, and their single logarithms have been organized to give the evolutions in b and Q, respectively [3]. They possess individual ultraviolet poles, but their sum $\mathscr{K}+\mathscr{G}$ is finite such that Sudakov logarithms are RG invariant.

Substituting the expressions for $\mathscr{K}$ and $\mathscr{G}$ into Eq. (5), we obtain the solution

$$\mathscr{P}(x,b,P,\mu)=\exp\left[-\sum_{\xi=x,\,1-x}s(\xi,b,Q)\right]\bar{\mathscr{P}}(x,b,\mu). \tag{6}$$

The exponent s, grouping the double logarithms in $\mathscr{P}$, is expressed in terms of the variables

$$\hat{q}\equiv\ln[\xi Q/(\sqrt{2}\Lambda)],$$

$$\hat{b}\equiv\ln(1/b\Lambda) \tag{7}$$

as [1]

$$s(\xi,b,Q)=\frac{A^{(1)}}{2\beta_1}\hat{q}\ln\left(\frac{\hat{q}}{\hat{b}}\right)+\frac{A^{(2)}}{4\beta_1^2}\left(\frac{\hat{q}}{\hat{b}}-1\right)-\frac{A^{(1)}}{2\beta_1}(\hat{q}-\hat{b})-\frac{A^{(1)}\beta_2}{4\beta_1^3}\hat{q}\left[\frac{\ln(2\hat{b})+1}{\hat{b}}-\frac{\ln(2\hat{q})+1}{\hat{q}}\right]-\left[\frac{A^{(2)}}{4\beta_1^2}-\frac{A^{(1)}}{4\beta_1}\ln\left(\frac{e^{2\gamma-1}}{2}\right)\right]\ln\left(\frac{\hat{q}}{\hat{b}}\right)+\frac{A^{(1)}\beta_2}{8\beta_1^3}[\ln^2(2\hat{q})-\ln^2(2\hat{b})], \tag{8}$$

with $\Lambda\equiv\Lambda_{\rm QCD}$. The coefficients $A^{(i)}$ and β_i are

$$\beta_1=\frac{33-2n_f}{12},\quad \beta_2=\frac{153-19n_f}{24},$$

$$A^{(1)}=\frac{4}{3},\quad A^{(2)}=\frac{67}{9}-\frac{\pi^2}{3}-\frac{10}{27}n_f+\frac{8}{3}\beta_1\ln\left(\frac{e^{\gamma}}{2}\right), \tag{9}$$

where $n_f=3$ in this case is the number of quark flavors and γ is the Euler constant. To derive Eq. (8), a spacelike gauge vector $n\propto(1,-1,\ \mathbf{0})$ has been chosen.

The functions $\mathscr{P}$ and $\tilde{H}$ still contain single logarithms from ultraviolet divergences, which need to be summed using their RG equations [3]:

$$\mathscr{D}\bar{\mathscr{P}}(x,b,\mu)=-2\gamma_q\bar{\mathscr{P}}(x,b,\mu), \tag{10}$$

$$\mathscr{D}\tilde{H}(x_i,b,Q,\mu)=4\gamma_q\tilde{H}(x_i,b,Q,\mu), \tag{11}$$

with

$$\mathscr{D}=\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}. \tag{12}$$

$\gamma_q=-\alpha_s/\pi$ is the quark anomalous dimension in axial gauge. Solving Eq. (10), the large-b behavior of $\mathscr{P}$ is summarized as

$$\mathscr{P}(x,b,P,\mu)=\exp\left[-\sum_{\xi=x,\,1-x}s(\xi,b,Q)-2\int_{1/b}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(g(\bar{\mu}))\right]\bar{\mathscr{P}}(x,b,1/b). \tag{13}$$

We express the initial condition $\bar{\mathscr{P}}$ in Eq. (13) as

$$\bar{\mathscr{P}}(x,b,1/b)=\tilde{\phi}(x,b)+O(\alpha_s(1/b)), \tag{14}$$

where the evolution of $\bar{\mathscr{P}}$ in b is denoted by $O(\alpha_s)$, and the argument b in $\tilde{\phi}$ corresponds to the intrinsic transverse momentum dependence of the pion wave function [6].

Similarly, the RG analysis applied to $\tilde{H}$ gives

$$\tilde{H}(x_i,b,Q,\mu)=\exp\left[-4\int_{\mu}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(g(\bar{\mu}))\right]\tilde{H}(x_i,b,Q,t), \tag{15}$$

where t is the largest mass scale involved in the hard scattering:

$$t=\max(\sqrt{x_1x_2}Q,1/b). \tag{16}$$

The scale $\sqrt{x_1x_2}Q$ is associated with the longitudinal momentum of the hard gluon and $1/b$ with the transverse momentum. Combining all the above exponents, we derive the factorization formula for the pion form factor,

$$F_\pi(Q^2)=\int_0^1 dx_1dx_2\int\frac{d^2\mathbf{b}}{(4\pi)^2}\tilde{\phi}(x_1,b)\tilde{\phi}(x_2,b)\times\tilde{H}(x_1,x_2,b,Q,t)\exp[-S(x_1,x_2,b,Q)], \tag{17}$$

where the complete Sudakov exponent is given by

$$S(x_1,x_2,b,Q)=\sum_{i=1}^{2}[s(x_i,b,Q)+s(1-x_i,b,Q)]-\frac{2}{\beta_1}\ln\frac{\hat{t}}{\hat{b}}, \tag{18}$$

with $\hat{t}=\ln(t/\Lambda)$.

Variation of e^{-S} with b has been displayed in [1]: It shows a strong falloff in the large b region, vanishing for $b>1/\Lambda$, and approaches unity in the small-b region, where the Sudakov logarithms diminish. If b is small, the running coupling constant α_s with its argument set to t will be small, regardless of the values of x's. When b is large and $x_1x_2Q^2$ is small, α_s is still large. However, the Sudakov factor in Eq. (17) strongly suppresses this region. Since the main contributions to the factorization formula come from the small-b region, the perturbation theory becomes relatively self-consistent.

III. DECAY $B\to\pi l\nu$

We extend the PQCD formalism for the pion form factor to exclusive processes involving both light and heavy mesons. In particular, we concentrate on the semileptonic decay $B\to\pi l\nu$. We shall show that PQCD is appropriate to this process when the pion is energetic enough. We first analyze the leading regions of radiative corrections, and derive the Sudakov factor including both leading and next-to-leading logarithms.

The amplitude of the decay $B\to\pi l\nu$ is written as

$$A(P_1,P_2)=\frac{G_F}{\sqrt{2}}V_{ub}\bar{\nu}\gamma_\mu(1-\gamma_5)l\langle\pi(P_2)|\bar{u}\gamma^\mu b|B(P_1)\rangle, \tag{19}$$

where the four-fermion interaction with the Fermi coupling constant $G_F=1.166\times10^{-5}$ GeV^{-2} has been inserted. P_1 and P_2 are the momenta of the B meson and of the pion, respectively. We start with the lowest-order factorization for the matrix element $M^\mu=\langle\pi(P_2)|\bar{u}\gamma^\mu b|B(P_1)\rangle$ with a hard exchanged gluon as shown in Fig. 3, the left-hand side being the B meson at rest and the right-hand side a fast-recoiling pion. The heavy b quark is represented by a bold line. The symbol $\times$ denotes the electroweak vertex with the CKM matrix element V_{ub}, from which a lepton pair emerges.

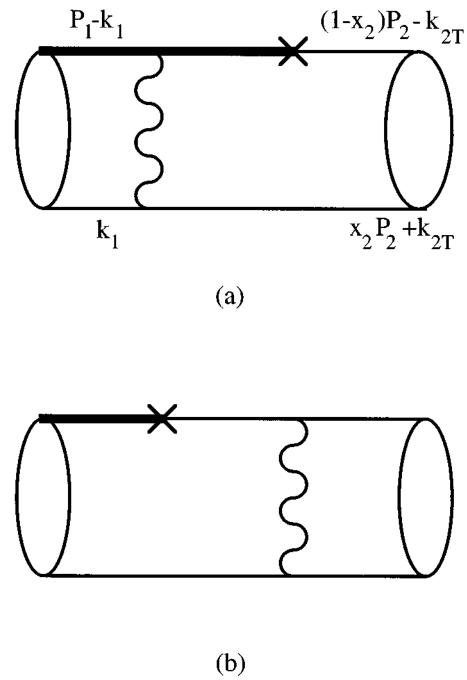

FIG. 3. Lowest-order factorization for the decay $B\to\pi l\nu$.

Parton momenta are assigned as in Fig. 3: The b quark carries P_1-k_1, and the accompanying light quark carries k_1. They satisfy the on-shell conditions $P_1^2=m_B^2$, $(P_1-k_1)^2\approx m_b^2$ and $k_1^2\approx0$, m_B being the B meson mass. In the Breit frame P_1 has the components $P_1^+=P_1^-=m_B/\sqrt{2}$ and vanishing transverse components. k_1 may have a large minus component k_1^- and small transverse components $\mathbf{k}_{1T}$, which will serve as the infrared cutoff of loop corrections below. The assignment of parton momenta on the pion side is similar to that for the pion form factor, as shown in Fig. 3. The large component of P_2 is $P_2^+=\eta m_B/\sqrt{2}$, η being related to the energy fraction of the pion by $P_2^0=\eta m_B/2$. The physical range of η is $0\leq\eta\leq1$, since the pion carries away at most half of the rest energy of the B meson. The transverse momentum associated with the valence quarks of the pion is denoted by $\mathbf{k}_{2T}$. The invariant mass of the lepton-neutrino pair is given by $m_l^2=(P_1-P_2)^2=(1-\eta)m_B^2$.

We now consider radiative corrections to the above basic factorization. The essential step is again to locate the leading regions of radiative corrections in axial gauge. For reducible corrections on the pion side, the conclusion is the same as before: They produce double logarithms with soft ones canceled in the asymptotic region, and can be absorbed into the pion wave function, giving the evolution of the wave function. Irreducible corrections, with an extra gluon connecting a quark in the pion and a quark in the B meson, contain only soft divergences, which also cancel asymptotically. Hence, they are absorbed into a hard scattering amplitude.

On the left-hand side, three diagrams showing $O(\alpha_s)$ corrections are displayed in Fig. 4. Figure 4(a), giving a self-energy correction to the massive b quark, produces only soft

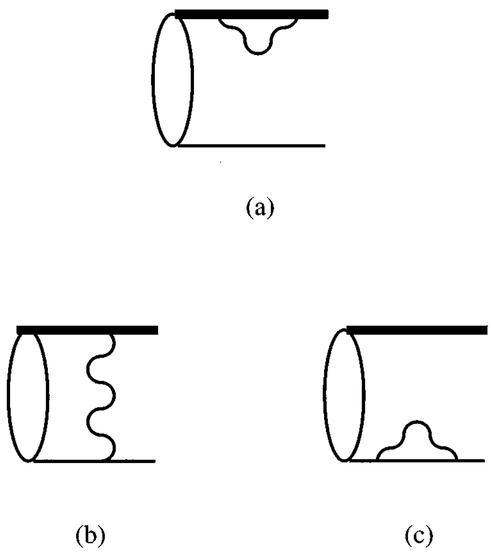

FIG. 4. $O(\alpha_s)$ corrections to the B meson wave function.

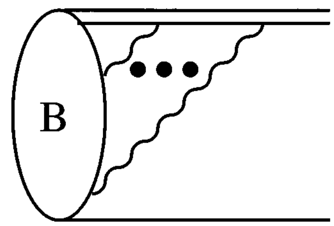

FIG. 5. Eikonal approximation for the b quark line.

divergences, and is thus not leading. If k_1^- is small, collinear divergences in Figs. 4(b) and 4(c), which arise from the loop momentum with a large component parallel to k_1, will not be pinched. Figures 4(b) and 4(c) then also give only soft divergences. This is consistent with the physical picture that a soft light valence quark cannot interact with the heavy quark through a fast moving gluon. However, the B meson wave functions employed in Sec. IV exhibit substantial probability of finding the light quark with k_1^- of order m_B, though they peak at small k_1^-. Therefore, Figs. 4(b) and 4(c) contribute collinear divergences. Note that Fig. 4(b) contains soft divergences which are not completely canceled by those from Figs. 4(a) and 4(c) even in the asymptotic region. In conclusion, Figs. 4(b) and 4(c) indeed contain double logarithms, which must be organized by the resummation technique.

Since the collinear divergences on the B meson side are less important due to suppression from the wave function, reducible corrections are basically dominated by soft divergences, and can be absorbed into the B meson wave function, which is also dominated by soft dynamics. This absorption should be compared to that on the pion side, where reducible corrections are dominated by collinear divergences.

We then write down the factorization formula for the decay $B\to\pi l\nu$ in transverse configuration space:

$$M^\mu=\int_0^1 dx_1 dx_2\int\frac{d^2\mathbf{b}_1}{4\pi}\frac{d^2\mathbf{b}_2}{4\pi}\mathcal{P}_\pi(x_2,\mathbf{b}_2,P_2,\mu)\times\tilde{H}^\mu(x_1,x_2,\mathbf{b}_1,\mathbf{b}_2,m,\mu)\mathcal{P}_B(x_1,\mathbf{b}_1,P_1,\mu),\qquad(20)$$

where both the pion and B meson wave functions $\mathcal{P}_\pi$ and $\mathcal{P}_B$ contain leading double logarithms. Here two b's are introduced, because the virtual quark line in the hard scattering may not be far from mass shell, and cannot be shrunk to a point as explained later. Hence, we need b_1 (b_2) to denote the separation between the two valence quarks of the B meson (pion). The approximation $m_b\approx m_B=m$ has been made for simplicity. The momentum fraction x_1 is defined by k_1^-/P_1^-. $\tilde{H}^\mu$ is the Fourier transform of the hard scattering amplitude derived from Fig. 3, whose explicit expression will be given in Sec. IV. The resummation of the double logarithms in $\mathcal{P}_\pi$ has been performed in the previous section. We quote the results directly with Q set to ηm and n_f set to 4 [13]. Below we shall study $\mathcal{P}_B$.

There are two major difficulties in summing up the double logarithms in Figs. 4(b) and 4(c). First, Fig. 4(a), giving only single soft logarithms, must be excluded. Second, there are many invariants involved in $\mathcal{P}_B$ that are constructed from P_1, k_1, and n, such as P_1^2, $P_1\cdot k_1$, $P_1\cdot n$, $k_1\cdot n$, and n^2. The fact that $\mathcal{P}_B$ contains many invariants renders the technique of replacing d/dm by d/dn inapplicable, m being the large scale of this process, because some large invariants such as P_1^2 cannot be related to n.

However, the difficulties can be overcome by applying the eikonal approximation to the heavy quark line as shown in Fig. 5. In the collinear region with the loop momentum q parallel to k_1 and in the soft region of q, it is possible to replace the b quark line by an eikonal line:

$$\frac{(\not{P}_1-\not{k}_1+\not{q}+m)\gamma^\alpha}{(P_1-k_1+q)^2-m^2}\approx\frac{P_1^\alpha}{P_1\cdot q}+R,\qquad(21)$$

where the remaining part R either vanishes as contracted with the matrix structure of the B meson wave function or is less leading. The factor $1/(P_1\cdot q)$ is associated with the eikonal propagator, and the numerator P_1^α is absorbed into the vertex, where a gluon attaches to the eikonal line. The physics involved in this approximation is that a soft gluon or a gluon moving parallel to k_1 cannot explore the details of the b quark, and its dynamics can be decoupled from the b quark. This idea is similar to the flavor symmetry employed in HQET. An explicit evaluation of radiative corrections confirms this approximation. The first difficulty is then resolved, because self-energy diagrams of an eikonal line are excluded by definition [15].

The eikonal approximation also reduces the number of large invariants involved in $\mathcal{P}_B$. We have the scale invariance in P_1 as shown by the Feynman rule for an eikonal line in Eq. (21), in addition to the scale invariance in n. Hence, P_1 does not lead to a large scale, and the only large scale is k_1^-, which must appear in the ratios $(k_1\cdot n)^2/n^2$ and $(k_1\cdot P_1)^2/P_1^2$.

A direct lowest-order investigation on the diagrams in Fig. 4 indicates that the second scale $(k_1\cdot P_1)^2/P_1^2$ does not exist. A lowest-order analysis helps to find out where and what important logarithms are. These important logarithms will certainly be summed to all orders as shown below. It suffices to examine Fig. 4(b), since Fig. 4(a) has been excluded, and Fig. 4(c) gives a contribution that depends only on k_1 and n. The loop integral associated with Fig. 4(b) is proportional to

$$\int \frac{d^4q}{(2\pi)^4}\frac{P_1^\alpha\gamma^\beta(\not k_1+\not q)}{P_1\cdot q(k_1+q)^2}N_{\alpha\beta}e^{i\mathbf{q}_T\cdot\mathbf{b}}, \tag{22}$$

in which the eikonal approximation for the b quark has been employed. It is apparent that only the first term $g_{\alpha\beta}$ in $N_{\alpha\beta}$ leads to the P_1 dependence for the specific choice of the gauge vector $n\propto(1,-1,\ \mathbf{0})$ as in the pion case. Because the large logarithms arise from the leading regions, we concentrate on the soft and collinear regions of q. In the soft region Eq. (22) becomes

$$\int \frac{d^4q}{(2\pi)^4}\frac{P_1\cdot k_1}{P_1\cdot q k_1\cdot q q^2}e^{i\mathbf{q}_T\cdot\mathbf{b}}, \tag{23}$$

which possesses scale invariance in k_1, and thus the ratio $(k_1\cdot P_1)^2/P_1^2$ cannot exist. In a similar way, we can show that this ratio does not appear in the collinear region either.

Therefore, with the eikonal approximation the problem is simplified to one in analogy with the pion case. Now $\mathscr{P}_B$ depends only on the single large scale $\nu'^2=(k_1\cdot n)^2/n^2$, and d/dm can be replaced by d/dn. Following the same procedures as in Sec. II, the differential equation of $\mathscr{P}_B$ is derived as

$$\frac{d}{d\ln m}\mathscr{P}_B=\frac{d}{d\ln k_1^-}\mathscr{P}_B=[\mathscr{K}(b\mu)+\mathscr{G}(\nu'/\mu)]\mathscr{P}_B, \tag{24}$$

where the lowest-order $\mathscr{K}$ is obtained from Fig. 6(a), and $\mathscr{G}$ from Fig. 6(b), with the square vertex representing

$$gT^a\frac{n^2}{k_1\cdot nq\cdot n}k_1^\alpha.$$

Note the absence of the diagram corresponding to the self-energy correction to the eikonal line. Since the B meson mass m is a Lorentz invariant, differentiation with respect to m in Eq. (24) should be regarded as a mathematical tool, and what really varies is the parton momentum $k_1^-=x_1m/\sqrt{2}$.

Comparing Fig. 6(a) with Fig. 2(a), we find that the evaluation of $\mathscr{K}$ for the B meson is similar to that for the pion except the third diagram. The contribution from this extra diagram is proportional to

$$\int \frac{d^4q}{(2\pi)^4}\frac{n^2k_1^\alpha P_1^\beta}{k_1\cdot nq\cdot nP_1\cdot q}N_{\alpha\beta}e^{i\mathbf{q}_T\cdot\mathbf{b}}, \tag{25}$$

which vanishes for $n\propto(1,-1,\ \mathbf{0})$. Because no new ultraviolet and infrared divergences are introduced, this diagram does not spoil the RG invariance of the Sudakov logarithms. The functions $\mathscr{K}$ and $\mathscr{G}$ for the B meson are then the same as those for the pion.

Substituting the expressions for $\mathscr{K}$ and $\mathscr{G}$ into Eq. (24), we obtain the solution

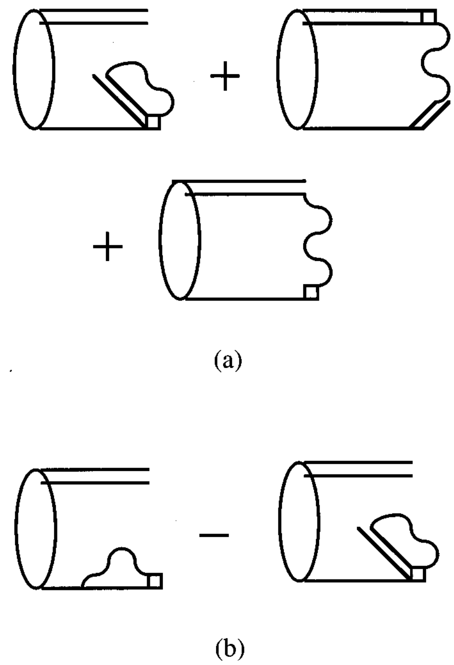

FIG. 6. Lowest-order diagrams for (a) the function $\mathscr{K}$ and for (b) the function $\mathscr{G}$ associated with the B meson.

$$\mathscr{P}_B(x_1,b_1,P_1,\mu)=\exp[-s(x_1,b_1,m)]\bar{\mathscr{P}}_B(x_1,b_1,\mu), \tag{26}$$

where the exponent s is given by Eq. (8) but with $n_f=4$ [13]. Summing up the single logarithms in $\mathscr{P}_B$, Eq. (26) becomes

$$\mathscr{P}_B(x_1,b_1,P_1,\mu)=\exp\left[-s(x_1,b_1,m)-2\int_{1/b_1}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(g(\bar{\mu}))\right]\tilde{\phi}_B(x_1,b_1)+O(\alpha_s(1/b_1)), \tag{27}$$

where the anomalous dimension γ_q is the same as before. Combining the summation of the single logarithms in $\tilde{H}^\mu$ and the results from $\mathscr{P}_\pi$, we derive the complete Sudakov exponent

$$S(x_i,b_i,\eta,m)=s(x_1,b_1,m)+s(x_2,b_2,\eta m)+s(1-x_2,b_2,\eta m)-\frac{1}{\beta_1}\left(\ln\frac{\hat{t}}{\hat{b}_1}+\ln\frac{\hat{t}}{\hat{b}_2}\right). \tag{28}$$

t is the largest scale involved in the hard scattering, which will be defined in Sec. IV.

IV. NUMERICAL RESULTS

Having derived the Sudakov exponent for the semileptonic decay $B\to\pi l\nu$, we evaluate the form factors, and examine how much of a contribution comes from the perturbative region with small b_i. The expression for lowest-order $H^{\mu(a)}$ from Fig. 3(a) is written as

$$H^{\mu(a)}=\mathrm{tr}\left[\gamma_\alpha\frac{\gamma_5 \not P_2}{\sqrt{2N_c}}\gamma^\mu\frac{\not P_1-x_2\not P_2+\not k_{2T}+m}{(P_1-x_2P_2+\mathbf{k}_{2T})^2-m^2}\gamma^\alpha\frac{(\not P_1+m)\gamma_5}{\sqrt{2N_c}}\right]\frac{-g^2N_c\mathscr{C}_F}{(x_2P_2-k_1+\mathbf{k}_{2T})^2}$$
$$=\frac{4(1+x_2\eta)g^2\mathscr{C}_Fm^2}{[x_2\eta m^2+\mathbf{k}_{2T}^2][x_1x_2\eta m^2+(\mathbf{k}_{1T}-\mathbf{k}_{2T})^2]}P_2^\mu, \tag{29}$$

where the factors $\gamma_5\not P_2/\sqrt{2N_c}$ and $(\not P_1+m)\gamma_5/\sqrt{2N_c}$ are the matrix structures of the pion and B meson wave functions, respectively [13], $\mathscr{C}_F=4/3$ is a color factor, and N_c is the number of colors. The relation $k_1^-=x_1m/\sqrt{2}$ has been inserted. In the second expression $\mathbf{k}_{2T}$ in the fermion propagator is not neglected. For the pion form factor, the corresponding transverse momentum dependence is negligible, because there is not the constant 1, but only x_2, in the numerator, which cancels the singularity from $x_2\rightarrow 0$ in the denominator. However, in the present case, due to the massiveness of the b quark, the constant 1 exists and such a cancellation does not occur as shown in Eq. (29). To ensure that the virtual quark is part of the hard scattering, $\mathbf{k}_{2T}$ must be retained.

Similarly, the expression for lowest-order $H^{\mu(b)}$ is given by

$$H^{\mu(b)}=\mathrm{tr}\left[\gamma_\alpha\frac{\gamma_5\not P_2}{\sqrt{2N_c}}\gamma^\alpha\frac{\not P_2+\not k_1}{(P_2+k_1)^2}\gamma^\mu\frac{(\not P_1+m)\gamma_5}{\sqrt{2N_c}}\right]\frac{-g^2N_c\mathscr{C}_F}{(x_2P_2-k_1+\mathbf{k}_{2T})^2}=\frac{4g^2\mathscr{C}_Fx_1\eta m^2}{[x_1\eta m^2+\mathbf{k}_{1T}^2][x_1x_2\eta m^2+(\mathbf{k}_{1T}-\mathbf{k}_{2T})^2]}P_1^\mu$$
$$-\frac{4g^2\mathscr{C}_Fx_1m^2}{[x_1\eta m^2+\mathbf{k}_{1T}^2][x_1x_2\eta m^2+(\mathbf{k}_{1T}-\mathbf{k}_{2T})^2]}P_2^\mu. \tag{30}$$

To derive the second formula, we have replaced k_1^μ by

$$\frac{P_2\cdot k_1}{P_1\cdot P_2}P_1^\mu+\left(\frac{P_1\cdot k_1}{P_1\cdot P_2}-\frac{2}{\eta}\frac{P_2\cdot k_1}{P_1\cdot P_2}\right)P_2^\mu. \tag{31}$$

Here $\mathbf{k}_{1T}$ in the fermion propagator is negligible, because the singularity from $x_1\rightarrow 0$ is removed by the numerator. However, we keep it for consistency.

Performing the Fourier transform of Eqs. (29) and (30), the matrix element M^μ is written as

$$M^\mu=f_1P_1^\mu+f_2P_2^\mu. \tag{32}$$

The factorization formulas for the $B\rightarrow\pi$ transition form factors f_1 and f_2 are given by

$$f_1=16\pi\mathscr{C}_Fm^2\int_0^1dx_1dx_2\int_0^\infty b_1db_1b_2db_2\phi_B(x_1)\phi_\pi(x_2)x_1\eta h(x_1,x_2,b_1,b_2,\eta,m)\exp[-S(x_i,b_i,\eta,m)] \tag{33}$$

and

$$f_2=16\pi\mathscr{C}_Fm^2\int_0^1dx_1dx_2\int_0^\infty b_1db_1b_2db_2\phi_B(x_1)\phi_\pi(x_2)[-x_1h(x_1,x_2,b_1,b_2,\eta,m)$$
$$+(1+x_2\eta)h(x_2,x_1,b_2,b_1,\eta,m)]\exp[-S(x_i,b_i,\eta,m)], \tag{34}$$

respectively, with

$$h(x_1,x_2,b_1,b_2,\eta,m)=\alpha_s(t)K_0(\sqrt{x_1x_2\eta}mb_2)[\theta(b_1-b_2)K_0(\sqrt{x_1\eta}mb_1)I_0(\sqrt{x_1\eta}mb_2)$$
$$+\theta(b_2-b_1)K_0(\sqrt{x_1\eta}mb_2)I_0(\sqrt{x_1\eta}mb_1)]. \tag{35}$$

K_0 and I_0 are the modified Bessel functions of order zero. To derive Eqs. (33) and (34), we have employed the relation $\phi(x)=\tilde\phi(x,b=0)/4\pi$. Here we neglect the evolution in b and the intrinsic b dependence of the wave functions, since these two effects cancel partially: The former gives an enhancement [4], but the latter leads to a suppression [6]. Similar to Eq. (16), we choose the largest scale t associated with the hard gluon as

$$t=\max(\sqrt{x_1x_2\eta}m,1/b_1,1/b_2). \tag{36}$$

The running coupling constant is given by

$$\frac{\alpha_s(t)}{\pi}=\frac{1}{2\beta_1\hat t}-\frac{\beta_2}{4\beta_1^3}\frac{\ln 2\hat t}{\hat t^2}. \tag{37}$$

We consider the following two models of the B meson wave function, which have been adopted in [13]. They are the oscillator wave function [16]

$$\Phi_B^{(\mathrm{I})}(x,\mathbf{k}_T)=\frac{N}{\omega^2}\sqrt{x(1-x)}\exp\left[-\frac{m_B^2}{2\omega^2}\left(\frac{1}{2}-x-\frac{m_b^2}{2m_B^2}\right)^2\right]\exp\left(-\frac{\mathbf{k}_T^2}{2\omega^2}\right)\approx\frac{N}{\omega^2}\sqrt{x(1-x)}\exp\left(-\frac{m^2}{2\omega^2}x^2\right)\exp\left(-\frac{\mathbf{k}_T^2}{2\omega^2}\right), \tag{38}$$

in our approximation $m_b\approx m_B=m$, and [17]

$$\Phi_B^{(\mathrm{II})}(x,\mathbf{k}_T)=N'\left[C+\frac{m_b^2}{1-x}+\frac{\mathbf{k}_T^2}{x(1-x)}\right]^{-2}, \tag{39}$$

where x is the momentum fraction of the light quark in the B meson. The parameters in $\Phi_B^{(\mathrm{I})}$ are $\omega=0.4$ GeV and $m=5.28$ GeV. The constant N is determined by the normalization of the wave function,

$$\int_0^1 dx\int\frac{d^2\mathbf{k}_T}{16\pi^3}\Phi_B^{(\mathrm{I})}(x,\mathbf{k}_T)\equiv\int_0^1 dx\,\phi_B^{(\mathrm{I})}(x)=\frac{f_B}{2\sqrt{3}}, \tag{40}$$

$f_B=160$ MeV being the B meson decay constant [18], which leads to $N=176.9$ GeV. The constants N' and C in $\Phi_B^{(\mathrm{II})}$ are determined by the normalizations [17]

$$\int_0^1 dx\int\frac{d^2\mathbf{k}_T}{16\pi^3}\Phi_B^{(\mathrm{II})}(x,\mathbf{k}_T)=\frac{f_B}{2\sqrt{3}},$$

$$\int_0^1 dx\int\frac{d^2\mathbf{k}_T}{16\pi^3}[\Phi_B^{(\mathrm{II})}(x,\mathbf{k}_T)]^2=\frac{1}{2}, \tag{41}$$

from which $N'=760.66\ \mathrm{GeV}^3$ and $C=-26.888\,973\ \mathrm{GeV}^2$ are obtained.

The Fourier transform of Φ_B gives

$$\tilde{\phi}_B^{(\mathrm{I})}(x,b)=\int\frac{d^2\mathbf{k}_T}{(2\pi)^2}\Phi_B^{(\mathrm{I})}(x,\mathbf{k}_T)e^{i\mathbf{k}_T\cdot\mathbf{b}}=\frac{N}{2\pi}\sqrt{x(1-x)}\exp\left(-\frac{m^2}{2\omega^2}x^2\right)\exp\left(-\frac{\omega^2b^2}{2}\right), \tag{42}$$

$$\tilde{\phi}_B^{(\mathrm{II})}(x,b)=\frac{N'bx^2(1-x)^2}{4\pi\sqrt{m^2x+Cx(1-x)}}K_1(\sqrt{m^2x+Cx(1-x)}\,b), \tag{43}$$

with K_1 the modified Bessel function of order one. As stated before, we neglect the intrinsic b dependence of the wave functions, and obtain

$$\phi_B^{(\mathrm{I})}(x)=\frac{1}{4\pi}\lim_{b\to 0}\tilde{\phi}_B^{(\mathrm{I})}(x,b)=\frac{N}{8\pi^2}\sqrt{x(1-x)}\exp\left(-\frac{m^2}{2\omega^2}x^2\right), \tag{44}$$

$$\phi_B^{(\mathrm{II})}(x)=\frac{N'x(1-x)^2}{16\pi^2[m^2+C(1-x)]}. \tag{45}$$

Obviously, both models peak at small x, which characterizes the soft dynamics in the B meson. However, the probability at intermediate x is indeed comparable at least in model II, and the resummation of double logarithms performed in Sec. III is essential. At last, we employ the Chernyak-Zhitnitsky model [19] for the pion wave function [20]:

$$\tilde{\phi}_\pi^{\mathrm{CZ}}(x,b)=5\sqrt{3}f_\pi x(1-x)(1-2x)^2\times\left(4\pi\exp\left[-\frac{x(1-x)b^2}{4a_\pi^2}\right]\right), \tag{46}$$

$$\phi_\pi^{\mathrm{CZ}}(x)=5\sqrt{3}f_\pi x(1-x)(1-2x)^2, \tag{47}$$

with $f_\pi=93$ MeV the pion decay constant and $a_\pi=2.02\ \mathrm{GeV}^{-1}$.

We are now ready to compute f_1 and f_2 in Eqs. (33) and (34) numerically. Setting $\Lambda=0.2$ GeV, the results of f_i from the two models of $\phi_B(x)$, with b_1 and b_2 integrated up to the same cutoff $b_{1c}=b_{2c}=b_c$, are shown in Fig. 7. For $\phi_B^{(\mathrm{II})}$, it is found that at $\eta=0.4$ approximately 50% of the contribution to f_i comes from the region with $\alpha_s(1/b_c)/\pi<0.5$ or, equivalently, $b_c<0.6\Lambda$. At $\eta=0.5$, 55% of the contribution is accumulated in this perturbative region. As $\eta=1$, the perturbative contribution has reached 70%. It implies that the PQCD analysis of the decay $B\to\pi l\nu$ in the range of $\eta>0.4$ is relatively self-consistent [1,4]. For $\phi_B^{(\mathrm{I})}$, the perturbative analysis becomes reliable for $\eta>0.5$. This is because $\phi_B^{(\mathrm{I})}$ peaks at a smaller $x=0.05$ ($\phi_B^{(\mathrm{II})}$ peaks at $x=0.11$), which enhances the soft contribution. This is also the reason predictions from $\phi_B^{(\mathrm{I})}$ are about twice larger than those from $\phi_B^{(\mathrm{II})}$.

In the approach of [13] the transverse momentum dependence was not considered. Instead, the energies of the virtual

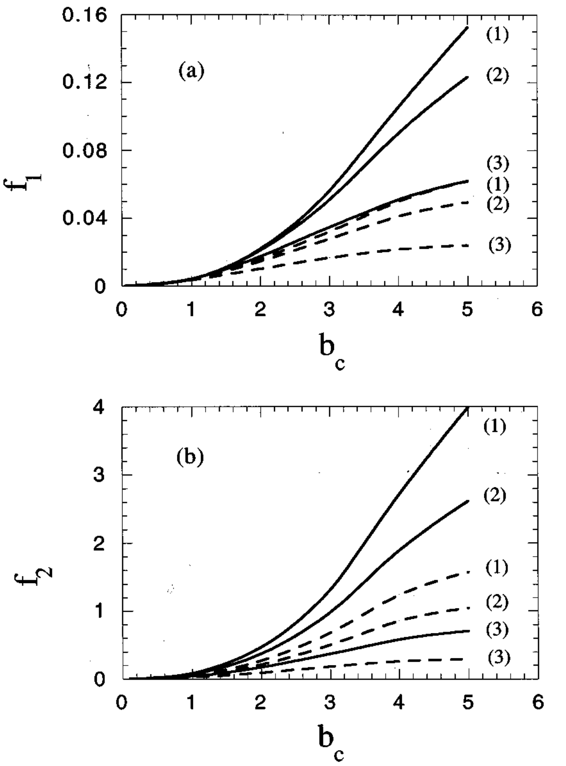

FIG. 7. Dependence of (a) f_1 and (b) f_2 on the cutoff b_c derived from $\phi_B^{(\mathrm{I})}$ (solid lines) and from $\phi_B^{(\mathrm{II})}$ (dashed lines) for (1) $\eta=0.4$, (2) $\eta=0.5$, and (3) $\eta=1.0$.

quark and of the virtual gluon involved in the hard scattering were taken as the ultraviolet and infrared cutoffs of radiative corrections, respectively. The resulting Sudakov logarithms, which are proportional to ln m/k_1^-, then give weaker suppression. From the steepest descent approximation of their Sudakov factor, the saddle point was found at $k_1^-\approx 1.4\Lambda$ for $\eta=1$. Similarly, we determine the saddle point of our Sudakov factor in Eq. (28) by

$$\frac{\partial S}{\partial b_1}=\frac{\partial S}{\partial b_2}=0\,, \qquad (48)$$

from which a larger scale $1/b_1=10\Lambda$ for $\Lambda=0.1$ GeV or $1/b_1=6\Lambda$ for $\Lambda=0.2$ GeV is obtained. Hence, the perturbative expansion in our formalism is more reliable. At such a large scale, the radiative corrections to the hard scattering with a triple gluon vertex, which were responsible for Sudakov suppression in [13], are in fact of higher twist and unimportant.

The magnitude of f_2 is much larger than that of f_1, especially in the small-η region. This fact is consistent with their behaviors in the soft pion limit as derived in HQET [12], where f_2 is found to have a pole at $\eta\to 0$,

$$\lim_{\eta\to 0} f_2 \approx \frac{2f_{B*}}{\eta f_\pi} g_{BB*\pi}\,, \qquad (49)$$

for $m_{B*}\approx m_B$, m_{B*} being the B^* meson mass. f_1 in the soft pion limit vanishes like $1-\sqrt{m_{B*}/m_B}$. In the above expression $g_{BB*\pi}\approx 0.75$ [21] is the $BB^*\pi$ coupling constant, and $f_{B*}\approx 1.1f_B$ [22] is the decay constant of the B^* meson.

Compared to the approaches [13,23] in the literature that are based on the Brodsky-Lepage QCD exclusive theory [24], our predictions for the $B\to\pi$ transition form factors are much larger. The reason is as follows. Since the previous approaches did not include the transverse momentum dependence, the virtual quark in the hard scattering may go on shell as $x_2\to 0$ as shown in Eq. (29). In order that factorization theorems make sense, this quark propagator must be subtracted, and thus the constant 1 in the numerator of Eq. (29) is removed. This subtraction then leads to much smaller results. In our formalism the factorization is preserved by adding transverse momenta and Sudakov suppression that control the magnitude of transverse momenta, instead of by subtraction. Hence, the large difference between our predictions and those in [13,23] is basically attributed to the inclusion of the transverse momentum dependence and the Sudakov effects, not to the particular choice of wave functions.

With the results of f_1 and f_2, we compute the differential decay rate of $B^0\to\pi^- l^+\nu$ for massless leptons [13],

$$\frac{d\Gamma}{d\eta}=|V_{ub}|^2\frac{G_F^2 m^5\eta^3}{768\pi^3}|f_1+f_2|^2\equiv|V_{ub}|^2R(\eta), \qquad (50)$$

where the second formula defines the function $R(\eta)$. Predictions of $R(\eta)$ are shown in Fig. 8, from which the decrease of $d\Gamma/d\eta$ with η is observed. This behavior is opposite to that given in [13], which shows an increase with η starting from zero at $\eta=0.5$. Such a dip at the middle value of η is again due to the subtraction of the on-shell fermion propagator from hard scattering [13]. Predictions in [13] for the differential decay rate are almost 10^3 times smaller than ours.

The differential decay rate in the soft pion limit is obtained from Eq. (49):

$$\lim_{\eta\to 0} R(\eta)=\frac{G_F^2 m^5\eta}{192\pi^3}\frac{f_{B*}^2}{f_\pi^2}g_{BB*\pi}^2\,, \qquad (51)$$

which exhibits a linear relation with η. Extrapolating Eq. (51) and our factorization formulas to each other, we observe a fair match around $\eta=0.2$ as shown in Fig. 8. Certainly, this extrapolation may not be reliable, but it is interesting to observe the match of these two different approaches at intermediate η. It implies that our PQCD formalism is successful at large η, but becomes worse quickly in the soft pion limit. On the contrary, the soft pion technique is appropriate at small η, but gives an overestimation in the perturbative region. The overlap indicates the transition [7,25] of the B meson decays to PQCD at middle values of η and the complementarity between soft pion theorems and the perturbative formalism.

We then estimate the total decay rate Γ by integrating $d\Gamma/d\eta$ using Eq. (51) for $\eta<0.2$ and using our predictions for $\eta>0.2$. We obtain $0.4\times 10^{-11}|V_{ub}|^2$ GeV from soft pion theorems, and $1.8\times 10^{-11}|V_{ub}|^2$ and $0.3\times 10^{-11}|V_{ub}|^2$ GeV

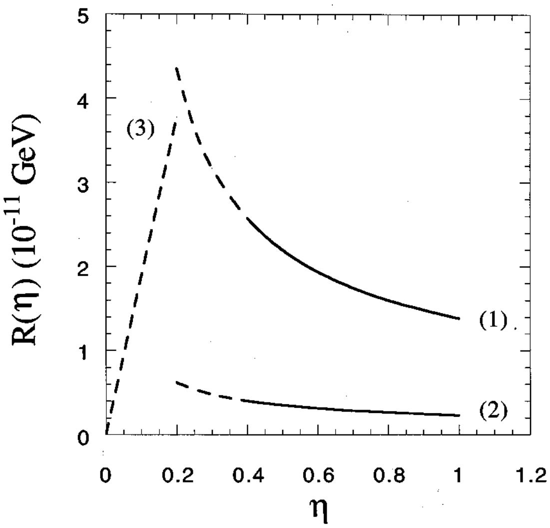

FIG. 8. Dependence of $R(\eta)$ on η derived from (1) $\phi_B^{(\mathrm{I})}$, from (2) $\phi_B^{(\mathrm{II})}$, and from (3) soft pion theorems. Dashed lines represent the extrapolation.

for the use of $\phi_B^{(\mathrm{I})}$ and $\phi_B^{(\mathrm{II})}$, respectively, from the PQCD formalism. Their sum gives $\Gamma \approx 2.2\times 10^{-11}|V_{ub}|^2$ GeV for model I and $0.7\times 10^{-11}|V_{ub}|^2$ for model II. They correspond to branching ratios $43|V_{ub}|^2$ and $14|V_{ub}|^2$, respectively, for the total width of the B^0 meson is $(0.51\pm 0.02)\times 10^{-9}$ MeV [26]. The current experimental limit on the branching ratio of $B^0\to\pi^- l^+ \nu$ is 3.3×10^{-4} [27]. We then extract the matrix element $|V_{ub}|<2.8\times 10^{-3}$ from model I and $|V_{ub}|<4.8\times 10^{-3}$ from model II. The value 0.003 given in the literature [26] is located in the above range.

It is also interesting to investigate the effects from the variation of parameters and wave functions involved in this work. Decreasing the QCD scale from $\Lambda = 0.2$ to 0.1 GeV, we obtain form factors which are roughly 30% larger. When including the intrinsic b dependence of the wave functions, the perturbative region will be extended down to $\eta > 0.3$, but the magnitudes of the form factors will decrease. When employing the asymptotic pion wave function [20]

$$\tilde{\phi}_\pi^{\mathrm{as}}(x,b) = \sqrt{3} f_\pi x(1-x)\left(4\pi \exp\left[-\frac{x(1-x)b^2}{4a_\pi^2}\right]\right), \tag{52}$$

$$\phi_\pi^{\mathrm{as}}(x) = \sqrt{3} f_\pi x(1-x), \tag{53}$$

which is less end-point concentrated, predictions for the form factors will become about 40%. The outcomes of the study on the intrinsic b dependence are listed in Table I, where the results of f_1+f_2 at $\eta=1$ presented in Fig. 7 are taken as unity for simplicity, and those from various wave functions in Eqs. (42), (43), (46), and (52) are expressed in terms of percentage. The outcomes for other η are close to those listed in Table I.

It is observed that the intrinsic b dependence in the B meson wave functions is more important, which gives rise to a 50% difference. That in the pion wave functions gives only 1–3 %. Note that if ϕ_π^{as} is used, and the intrinsic b dependence of both the B meson and pion wave functions is considered, the result will decrease to 20%. It implies that the form of the wave functions still needs to be determined precisely. However, we emphasize that the complete b dependence of wave functions includes the evolution proportional to $\alpha_s(1/b)$. Taking into account the evolution will moderate the drastic difference due to the intrinsic b dependence.

TABLE I. Percentage for results of f_1+f_2 at $\eta=1$ from various wave functions.

	$\phi_\pi^{\mathrm{CZ}}(x)$	$\phi_\pi^{\mathrm{CZ}}(x,b)$	$\phi_\pi^{\mathrm{as}}(x)$	$\phi_\pi^{\mathrm{as}}(x,b)$
$\phi_B^{(\mathrm{I})}(x)$	1	97%	40%	38%
$\phi_B^{(\mathrm{I})}(x,b)$	58%	56%	25%	24%
$\phi_B^{(\mathrm{II})}(x)$	1	98%	36%	35%
$\phi_B^{(\mathrm{II})}(x,b)$	49%	48%	21%	20%

V. CONCLUSION

In this paper we have applied the resummation technique to the semileptonic decay $B\to\pi l\nu$, and derived the Sudakov factor up to next-to-leading logarithms in this heavy-to-light transition process. The idea is to employ the eikonal approximation for the heavy b quark line such that its nonleading self-energy diagram is excluded, and the number of large scales involved in the B meson wave function is reduced. The resummation of double logarithms in the heavy meson is then simplified to one in analogy with the light meson case. The PQCD calculation of the differential decay rate including Sudakov effects has been examined and found to be reliable for η above 0.4. By combining our predictions with soft pion results and comparing them with experimental data, we have estimated the total decay rate, and extracted the upper limit 2.8–4.8×10^{-3} for the CKM matrix element $|V_{ub}|$.

We do not observe the dip at $\eta=0.5$ for the differential decay rate as predicted in [13], which arises from the subtraction of an on-shell fermion propagator from the hard scattering. This subtraction is not necessary in our analysis because of the inclusion of the transverse momentum dependence. The behavior of the differential decay rate in η is also opposite to that in [13].

We have investigated the effects from the intrinsic b dependence of wave functions. Another source of the b dependence, the evolution, still needs further consideration. Our formalism can be easily applied to a similar semileptonic decay $B\to\rho l\nu$ and other nonleptonic B meson decays, which will be published elsewhere.

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