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CP violation and $\Delta I = 1/2$ enhancement in *K* decays

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We study *CP*-conserving and *CP*-violating $K^0 \to \pi\pi$ and $K^0 \to \pi\pi\gamma$ decays, using the same techniques which explain the $\Delta I = 1/2$ enhancement of the former to also explore *CP* violation of the latter transitions. If *CP* violation is driven by the $WW\gamma$ vertex, we show that direct *CP* violation in $K_L \to \pi\pi\gamma$ is scaled to the $s \to d\gamma$ $E1$ quark transition and the latter is *suppressed* by the GIM mechanism (compatible with recent experiments). In the same spirit, the dominant $\Delta I = 1/2$ enhancement of *CP*-conserving kaon weak decays can be scaled to an $s \to d$ quark transition which is *enhanced* by the GIM mechanism.

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I. INTRODUCTION

It is an observed fact [1] that the weak decays of neutral kaons have two striking features: $\Delta I = 1/2$ enhancement and also *CP* violation. Within the past few years, experiments [1–3] have begun to suggest that *direct CP* violation (CPV) in $K^0 \to \pi\pi$ and also in $K^0 \to \pi\pi\gamma$ decays may be (insignificantly) small. In that case CPV may still occur in kaon first order weak (FOW) transitions, and then extended without further CPV via second order weak (SOW) K^0-$\bar{K}^0$ mixing. Since in fact FOW neutral kaon decays are observed to be $\Delta I = 1/2$ enhanced whether or not *CP* is conserved, it may be possible to construct a theory enhancing FOW *CP*-conserving $\Delta I = 1/2$ $K_S \to \pi\pi$ and $\pi\pi\gamma$ decays while suppressing FOW $K_L \to \pi\pi$ and $\pi\pi\gamma$ CPV transitions, yet allowing SOW K^0-$\bar{K}^0$ mixing to proceed in a normal *CP*-conserving manner. In this paper we develop such a (hadronic) program, but always expressed in a quark model chiral framework.

We begin in Sec. II by reviewing recent experimental CPV findings and then couching them in the language of unitarity and also chiral Lagrangians. Next in Sec. III we discuss a *CP*-conserving quark model (based on an $s \to d$ transition) which not only recovers the observed kaon $\Delta I = 1/2$ FOW amplitudes due to Glashow-Iliopoulos-Maiani (GIM) *enhancement*, but also explains SOW K^0-$\bar{K}^0$ mixing and correctly predicts the K_L-K_S mass difference Δm_{LS}. This includes an analysis linking the FOW decay rate of K_S to the SOW Δm_{LS} via unitarity with Δm_{LS} linked to the square of the quark $s \to d$ weak transition.

Then in Sec. IV we return to *CP* violation (CPV) for the neutral kaon system. The same quark model of Sec. III but now coupled to a nonstandard CPV vertex $WW\gamma$ in turn is shown to give an unobservably small $s \to d\gamma$ direct CPV amplitude due to GIM *suppression* as one expects from the CPV small $K_L \to \pi\pi\gamma$ decay of Sec. II. Finally in Sec. V we suggest how such a quark picture can generate a CKM matrix which roughly builds in the observed CPV phase parameter $\delta \approx 0.0033$ via a photon in a loop coupled to the CPV $WW\gamma$ vertex. In Appendix A we summarize why the $s \to d$ quark transition cannot be transformed away, while in Appendix B we provide further details supporting the $\Delta I = 1/2$ rule via tadpole dominance for K^0 decays.

II. *CP* VIOLATION DATA AND PHENOMENOLOGY

The most recent Particle Data Group (PDG) compilation [1] of *CP* violation experiments now finds the $K^0 \to \pi\pi$ amplitude ratios

$$|\eta_{+-}| = \left| \frac{M(K_L \to \pi^+\pi^-)}{M(K_S \to \pi^+\pi^-)} \right| = (2.269 \pm 0.023) \times 10^{-3},$$

$$|\eta_{00}| = \left| \frac{M(K_L \to \pi^0\pi^0)}{M(K_S \to \pi^0\pi^0)} \right| = (2.259 \pm 0.023) \times 10^{-3}, \tag{1}$$

along with phase angles (defined relative to $K_S \sim K^0 - \bar{K}^0$ and $K_L \sim K^0 + \bar{K}^0$ states in the *CP*-conserving limit)

$$\phi_{+-} = 44.3° \pm 0.8°, \quad \phi_{00} = 43.3° \pm 1.3°. \tag{2}$$

Since η_{+-} and η_{00} in (1) are very close in magnitude, the reasonable consequence

$$|\eta_{+-}| = |\eta_{00}| = |\varepsilon| \approx 2.26 \times 10^{-3} \tag{3}$$

then suggests that a possible direct CPV parameter ε' must be extremely small, if not zero [2]. This is also compatible with the most recent measurements of ε'/ε reported in [1].

Semileptonic weak CPV has also been detected as

$$\delta = \frac{\Gamma(K_L^0 \to \pi^- l^+ \nu) - \Gamma(K_L^0 \to \pi^+ l^- \nu)}{\Gamma(K_L^0 \to \pi^- l^+ \nu) + \Gamma(K_L^0 \to \pi^+ l^- \nu)} = (3.27 \pm 0.12) \times 10^{-3}. \tag{4}$$

Since this δ in (4) is related to the CPV parameter ε in (3) as $\delta \approx 2\,\mathrm{Re}\varepsilon$ or [4]

$$\mathrm{Re}\varepsilon = (1.635 \pm 0.060) \times 10^{-3} = |\varepsilon| \cos\phi,$$

assuming the magnitude $|\varepsilon|$ from (3) along with $\varepsilon'=0$ leads to the above phase angle ϕ as being

$$\phi=\arccos[\mathrm{Re}\varepsilon/|\varepsilon|]=43.7°. \tag{5}$$

Then because ϕ in (5) is compatible with ϕ_{+-},ϕ_{00} in (2), one can characterize this CPV phase ϕ by $\delta=2|\varepsilon|\cos\phi$ in (4) above. We shall return to this CPV phase δ in Sec. V.

However, the most recent CPV data concerns $K_L\to\pi\pi\gamma$ decay. Since the dominant CPV amplitude presumably is due to K^0-$\bar{K}^0$ mixing, Ramberg *et al.* [3] measured its effect via the inner bremsstrahlung $E1$ amplitude, looking for any significant deviation from (1) or (3) as a signal of direct emission CP violation. Their result [3],

$$|\eta_{+-\gamma}|=\left|\frac{M(K_L\to\pi^+\pi^-\gamma,E1)}{M(K_S\to\pi^+\pi^-\gamma,E1)}\right|=(2.15\pm0.26)\times10^{-3}, \tag{6}$$

indicates that direct CP violation (beyond K^0-$\bar{K}^0$ mixing) is indeed small, bounded by

$$|\varepsilon'_{+-\gamma}|/\varepsilon<0.3$$

at the 90% confidence level.

Unitarity has been used [5] to link the FOW decay rate of K_S to the SOW K_L-K_S mass difference Δm_{LS}. The latter measured value [1]

$$\Delta m_{LS}/m_K=(0.705\pm0.003)\times10^{-14}, \tag{7}$$

and ignoring CPV then suggests keeping only the overwhelmingly dominant $\pi\pi$ intermediate states in the saturation of unitarity:

$$2\Delta m_{LS}\approx\Gamma_S \quad \text{or} \quad \phi=\arctan 2\Delta m_{LS}/\Gamma_S\approx45°, \tag{8}$$

as expected. But for the exact measured lifetime $\tau_S=(0.8926\pm0.0012)\times10^{-10}$ sec, Eq. (8) gives a more accurate (CPV) phase angle determined by unitarity (while ignoring any ε'):

$$\phi\approx\arctan 2\Delta m_{LS}\tau_S=43.7°, \tag{9}$$

which is in very good agreement with (2) and (5).

The calculation of the inner bremsstrahlung (IB) CPV amplitude was given long ago [6,7] in the context of an effective weak Lagrangian characterizing CP violation:

$$\mathscr{L}_w\sim\mathrm{Tr}(\lambda_7 D_\mu M D^\mu M^\dagger), \tag{10}$$

where $M=\exp(i\phi_a\gamma_a/f_\pi)$, $f_\pi\approx93$ MeV, $D_\mu M=\partial_\mu M+ieA_\mu[Q,M]$, and Q is quark charge matrix diag $(2/3,-1/3,-1/3)$. Normalizing the overall amplitude to $M(K_S\to\pi^+\pi^-)=M$ and using (10), the IB $E1$ squared amplitude integrating over phase space for photon energies 20 MeV leads to the CPV branching ratio [6,8],

$$\Gamma_{\mathrm{IB}}(K_L\to\pi^+\pi^-\gamma)/\Gamma_{K_L}=1.4\times10^{-5}, \tag{11}$$

very near the observed ratio $(1.52\pm0.16)\times10^{-5}$.

Using (10) and (11) as a starting point and employing a fourth-order derivative chiral Lagrangian [similar to (10)], Ref. [8] estimated the CP-violating interference between the IB $E1$ amplitude giving (11) and a possible direct emission (DE) $E1$ amplitude [only recently measured [3] to be small in (6)]. In fact Ref. [8] modeled little, if any, direct CP violation in $K_L\to\pi^+\pi^-\gamma$ decay beyond (IB) K^0-$\bar{K}^0$ mixing.

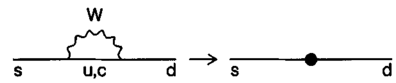

FIG. 1. Single quark line $s\to d$ transition.

III. QUARK MODEL GIM ENHANCEMENT AND THE $\Delta I=1/2$ RULE

The above signals of CPV are always seen in neutral kaon weak decays. However, the latter kaon decays also obey another unique characteristic—$\Delta I=1/2$ enhancement with $\pi\pi$ decay rates $\Gamma_{+-}/\Gamma_{00}\sim2$. To show that nature always generates this $\Delta I=1/2$ enhancement regardless of the CP properties of the neutral kaon, we form the PDG branching ratios in Ref. [1] for CP-conserving and CP-violating $K^0\to2\pi$ decays, respectively:

$$\frac{\Gamma^S_{+-}}{\Gamma^S_{00}}=2.19\pm0.02, \quad \frac{\Gamma^L_{+-}}{\Gamma^L_{00}}=2.22\pm0.09. \tag{12}$$

This dual $\Delta I=1/2$ enhancement in (12) can be generated from a large $\Delta I=1/2$ tadpole transition $\langle0|H_w^{\mathrm{PV}}|K^0\rangle$, making both the CP-conserving $\langle0|H_w^{\mathrm{PV}}|K_S\rangle$ and CP-violating $\langle0|H_w^{\mathrm{PV}}|K_L\rangle$ $\Delta I=1/2$ tadpoles also enhanced.

This being said, we now review a quark model version of (CP-conserving) $\Delta I=1/2$ enhancement for $K^0\to2\pi$ decays. First recall the GIM quark chiral weak current in a (low energy) SU(4) symmetry scheme [9] with $\gamma^L_\mu=\gamma_\mu(1-\gamma_5)$:

$$j^w_\mu=c_1(\bar{u}\gamma^L_\mu d+\bar{c}\gamma^L_\mu s)+s_1(\bar{u}\gamma^L_\mu s-\bar{c}\gamma^L_\mu d). \tag{13}$$

Using (13), the manifest $\Delta I=1/2$ weak self-energy Σ_{sd} depicted in Fig. 1 is [10,11] (scaled to a charmed quark mass $m_c\approx1.6$ GeV)

$$\Sigma_{sd}=b\bar{d}\not{p}(1-\gamma_5)s,$$

$$-b=(G_F s_1 c_1/8\pi^2\sqrt{2})(m_c^2-m_u^2)\approx5.6\times10^{-8}. \tag{14}$$

Note that the usual log-divergent self-energy is cancelled by the GIM mechanism, signaled by the $m_c^2-m_u^2$ factor in (14). In effect this GIM factor $m_c^2-m_u^2$ enhances the $(\Delta I=1/2)$ scale of b and generates the observed $\Delta I=1/2$ rule. Since many physicists believe that such an off-diagonal self-energy as (14) can be transformed away, we give explicit details of why this is not the case in Appendix A. In the context of the standard SU(2)×U(1) theory, Weinberg [12] has shown that "truly weak" (tadpole) graphs of $O(\alpha m_q^2/M_W^2)$ [such as the $s\to d$ tadpole in (14)] *cannot* be rotated away.

While (14) was computed in 't Hooft–Feynman gauge [13], corrections to (14) are very small $O([m_s-m_d]^2/M_W^2)$ in other covariant gauges [14]. But given (14) and the associated weak current of Fig. 2, its divergence coupled with current algebra and working on the kaon-mass shell then gives [14]

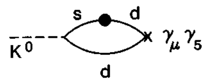

FIG. 2. First-order weak K^0 to vacuum quark axial current.

$$q^{\mu}M_{\mu}=-\frac{1}{2}\langle 0|H_w|K^0\rangle \approx i\sqrt{2}bf_K m_K^2 \tag{15}$$

(recalling that the strong axial current matrix element is $\langle 0|A_{\mu}^{6-i7}|K^0\rangle = i\sqrt{2}f_K q_{\mu}$). Here the strong interaction kaon decay constant $f_K\approx 113$ MeV is weighted by the $s\rightarrow d$ weak scale b from (14) in the weak axial current in (15). The implied weak tadpole scale in (15) is then

$$|\langle 0|H_w|K^0\rangle|\approx 2\sqrt{2}|bf_K m_K^2|\approx 4.4\times 10^{-9}\ \ \text{GeV}^3, \tag{16}$$

from which Fig. 3 for $f_{\pi}\approx 93$ MeV predicts the $K_S\rightarrow \pi\pi$ amplitude as [11,15] (we review the link between the K^0 tadpole and $K_S\rightarrow\pi\pi$ decays in Appendix B)

$$\begin{aligned}|M(K_S\rightarrow\pi\pi)| &= \frac{\sqrt{2}}{2f_{\pi}^2}|\langle 0H_w|K^0\rangle|\left(1-\frac{m_{\pi}^2}{m_K^2}\right)\\ &\approx 2|b|(f_K/f_{\pi}^2)(m_K^2-m_{\pi}^2)\\ &\approx 34\times 10^{-8}\ \ \text{GeV}.\end{aligned} \tag{17}$$

The latter is of course $\Delta I=1/2$ enhanced, and is in fact near the observed $K_S\rightarrow\pi\pi$ amplitude [1] 37×10^{-8} GeV. This $\Delta I=1/2$ tadpole transition $\langle 0|H_w|K^0\rangle$ in (16), giving the CP-conserving amplitude (17) as depicted in Fig. 3, also generates a CP-violating amplitude. Both are compatible with the observed branching ratios Eqs. (12).

Apart from this FOW agreement, one can also check the (CP-conserving) unitarity relation (8), becoming (assuming the dominance of the two pion intermediate states) when employing Eq. (17) for $q\approx 0.416m_K$ and $f_K/f_{\pi}\approx 1.22$:

$$2\Delta m_{LS}\approx \Gamma_S=\frac{3q}{16\pi m_K^2}|M(K_S\rightarrow\pi\pi)|^2\approx 4b^2 m_K. \tag{18}$$

For $|b|\approx 5.6\times 10^{-8}$ from (14), Eq. (18) predicts Δm_{LS} to be

$$\Delta m_{LS}/m_K\approx 2b^2\approx 0.63\times 10^{-14}, \tag{19}$$

reasonably near the observed value in (7).

Alternatively one can examine SOW K^0-$\bar{K}^0$ mixing in the CP-conserving limit. For $\phi=45°$, the K^0-$\bar{K}^0$ mass matrix becomes, for $\lambda=\langle\bar{K}^0|H_w^{(2)}|K^0\rangle$,

$$\begin{pmatrix} m_{K^0}^2 & \lambda\\ \lambda & m_{K^0}^2\end{pmatrix}\rightarrow\begin{pmatrix} m_S^2 & 0\\ 0 & m_L^2\end{pmatrix}, \tag{20}$$

which is diagonalized via

$$\sin 2\phi=\frac{2\lambda}{m_L^2-m_S^2}\approx\frac{\lambda}{\Delta m_{LS}m_K}=1. \tag{21}$$

In the spirit of the FOW current M_{μ} in Fig. 2, the SOW current N_{μ} of Fig. 4 has the divergence found from the soft kaon theorem of current algebra [11,16],

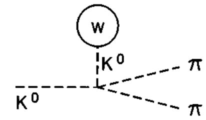

FIG. 3. $K^0\rightarrow 2\pi$ weak tadpole diagram.

$$q^{\mu}N_{\mu}=if_K\langle\bar{K}^0|H_w^{(2)}|K^0\rangle\approx i2b^2f_K m_K^2, \tag{22}$$

or cancelling out the strong interaction scale f_K in (22),

$$\lambda=\langle\bar{K}^0|H_w^{(2)}|K^0\rangle\approx 2b^2m_K^2. \tag{23}$$

Finally equating (21) and (23) one is led to [16]

$$\Delta m_{LS}/m_K\approx 2b^2\approx 0.63\times 10^{-14}, \tag{24}$$

which again recovers the quark model unitarity relation (19) and is also compatible with the measured Δm_{LS} in (7).

The fact that the GIM enhanced $\Delta I=1/2$ scale of b in (14) empirically matches the first order weak (FOW) $K_{2\pi}$ amplitude in (17), the unitarity-determined K_L-K_S mass difference Δm_{LS} in (19), and the (SOW) quantum mechanical-diagonalized Δm_{LS} in (24) lends strong support for the GIM quark model SU(4) picture. In the next section we show that the GIM mechanism also helps to resolve our central concern—that of CP violation.

IV. CP VIOLATION AND THE $WW\gamma$ VERTEX

In order to extend this quark model CP-conserving picture of Sec. III to CP violation (CPV), we consider a (nonstandard) CPV $WW\gamma$ vertex:

$$\langle\gamma_{\mu}(q)W_{\beta}|W_{\alpha}\rangle=ie\lambda_w\varepsilon_{\alpha\beta\mu\sigma}q^{\sigma}. \tag{25}$$

In the standard minimally coupled (CP-conserving) SU(2) $\times$U(1) theory, λ_w in (25) would vanish. But in Ref. [17], Marciano and Queijeiro employed the CPV coupling (25) together with the 1986 bound on the CPV neutron electric dipole moment (EDM) $|d_n|<6\times 10^{-25}$ e cm. The latter sets a scale of the proposed λ_w CPV strength in (25). They applied the (nonstandard) U gauge $g_{\alpha'\alpha}-k_{\alpha'}k_{\alpha}/M_W^2$ to two W boson propagators coupled to a photon as in (25) to compute CPV $d\rightarrow d\gamma$ and $u\rightarrow u\gamma$ constituent-quark radiative transitions. Then using the usual quark wave functions for a ddu neutron, they concluded that the CPV neutron EDM has the form [17]

$$|d_n|=\frac{5eG_Fm_N\lambda_w}{24\sqrt{2}\pi^2}X,$$

$$X=\ln\frac{\Lambda^2}{M_W^2}-3-\frac{M_W^2}{\Lambda^2}+\frac{4\ \ln\Lambda^2/M_W^2}{(\Lambda^2/M_W^2)-1}. \tag{26}$$

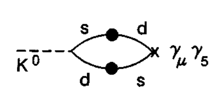

FIG. 4. Second-order weak K^0 to vacuum quark axial current.

Next Ref. [17] assumed a very high ultraviolet (UV) cutoff $\Lambda^2 \gg M_W^2$, in which case (26) would suggest $|\lambda_w| < 10^{-3}$, so that the CPV vertex (25) would play a minimal role in the observed CPV neutral K decays. More recently, however, other physicists (concerned with CP-conserving and CP-violating weak transitions) have argued [18,19] for a much lower weak cutoff Λ. From a (strong interaction) dynamical mass perspective, the vector meson ρ mass of 770 MeV can be taken as an UV cutoff mass in a $\bar{q}q$ quark model framework. For weak interactions this might correspond to an UV weak cutoff

$$\frac{\Lambda^2}{M_W^2} \sim 1. \tag{27}$$

Alternatively using the Veltman formula [20] to eliminate quadratically divergent SU(2)×U(1) (tadpole) graphs, the (Higgs boson mass $m_H = \Lambda$) UV cutoff becomes

$$\frac{\Lambda^2}{M_W^2} = 4(m_t^2/M_W^2) - (2 + M_Z^2/M_W^2) \sim 15, \tag{28}$$

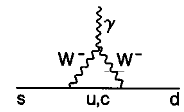

FIG. 5. CP-violating $s \to d\gamma$ graph.

using the recent observations [21] that the top quark mass is near $m_t \sim 170$–200 GeV. Thus from the estimates (27) or (28) we might expect an UV weak cutoff scale much less than $10^4 M_W$, but more like $\Lambda^2/M_W^2 \sim 1$–15. Indeed in Ref. [22], precision electroweak tests are shown to be best satisfied if $\ln\Lambda \approx \ln M_H$. In fact if we assume $\Lambda^2/M_W^2 \sim 5$ midway between (27) and (28), the factor X in (26) becomes 0.02. In this case (26) and the measured lower bound on the CPV neutron edm *still allows* λ_w *to be near unity*.

Given that $\lambda_w \approx 1$ in the nonstandard $WW\gamma$ coupling (25), we now study the $s \to d\gamma$ CPV transition depicted in Fig. 5. Again using the nonstandard U gauge as in Ref. [17], the $s \to d\gamma_\mu(q)$ amplitude magnitude of Fig. 5 is [setting $q=0$ once the $\varepsilon_{\alpha\beta\mu\sigma}q^\sigma$ factor in the $WW\gamma$ coupling (25) is taken into account]

$$|M_\mu| = \left| e\lambda_w \varepsilon_{\alpha\beta\mu\sigma} q^\sigma \int \frac{d^k k}{(2\pi)^4} \frac{g_w^2 s_1 c_1 \gamma_{\alpha'}(\not k - \not p)}{8(k^2 - M_W^2)^2} \left[\frac{1}{(k-p)^2 - m_u^2} - \frac{1}{(k-p)^2 - m_c^2} \right] \gamma_{\beta'} 2(1-\gamma_5) \left(g_{\alpha'\alpha} - \frac{k_{\alpha'} k_\alpha}{M_W^2} \right) \left(g_{\beta'\beta} - \frac{k_{\beta'} k_\beta}{M_W^2} \right) \right| = \left| \frac{e\lambda_w G_F s_1 c_1 5(m_c^2 - m_u^2)}{8\pi^2 \sqrt{2} 6 M_W^2} \right| (\gamma_\mu p \cdot q - p_\mu \not q)(1-\gamma_5). \tag{29}$$

As in the CP-conserving case of the $s \to d$ transition in (14), this CPV $s \to d\gamma$ amplitude has been rendered finite via the GIM mechanism. However, in (29) M^μ is GIM *suppressed* because $(m_c^2 - m_u^2) M_W^{-2} \sim 10^{-4}$, whereas in (14) the factor $b\bar{d}\not p(1-\gamma_5)s$ is GIM *enhanced* in this "truly weak" tadpole of order $G_F m_c^2$ (also see [19]).

Assuming then that the CPV $s \to d\gamma$ amplitude (29) is scaled to a "large" λ_w value ~ 1, this amplitude is still GIM *suppressed* and could not lead to measurable effects. Specifically this "very small" $s \to d\gamma$ CPV amplitude (29) generates an unobservable direct emission (DE)$K^0 \to \pi^+\pi^-\gamma\, E1$ transition which is CPV for K_L decay. In fact this is the conclusion of measurements in Ref. [3] resulting in Eq. (6) based on the interference between (negligible) CPV DE and nonzero IB $E1$ transitions.

Although the $WW\gamma$ vertex (25) does not lead to measurable direct emission CPV effects (even if λ_w is as large as unity) in the neutron edm in (26) or based on the $s \to d\gamma$ CPV transition in (29), we suggest that this $WW\gamma$ vertex (25) with $\lambda_w \approx 1$ could still generate the observed CPV in the $\Delta I = 1/2$ enhanced $K^0 \to 2\pi$ amplitude via the CPV radiatively dressed $s \to d$ transition depicted in Fig. 6. Here the "dot" again refers to the W-mediated u and c GIM loops of Fig. 1, and this radiative correction would then generate the amplitude ratio roughly of order [23]

$$|\varepsilon| = \left| \frac{M(K_L \to \pi\pi)}{M(K_S \to \pi\pi)} \right| \approx \frac{b\alpha/\pi}{b} = \alpha/\pi \approx 2.32 \times 10^{-3}. \tag{30}$$

Although the numerical coincidence between the observed $|\varepsilon| \approx 2.26 \times 10^{-3}$ in (3) and α/π was noted long ago [24,17], we suggest from (30) that Fig. 6 generated via the CPV vertex $WW\gamma$ in (25) with $\lambda_w \approx 1$ coupled with the GIM mechanism [enhanced in (14) but suppressed in (29)] may be the origin of CPV in $K^0 \to 2\pi$ decays. In (30) the $\Delta I = 1/2$ enhancement factor of b divides out in the construction of the CP-violating ratio $|\varepsilon|$ and all that remains is the photon self-energy factor of α/π.

V. EXTENSION TO CKM MIXING

Here we attempt qualitatively to construct in the quark model the flavor-mixing CKM matrix, but first focus on the SU(4) Cabibbo sector. In Sec. III we used the SU(4) GIM [9] weak current (13) expressed in the "mass basis" [4] with empirical Cabibbo angle $\theta_c \approx 13°$ [1,25]. Alternatively one could employ a weak quark current which is diagonal with respect to the "gauge basis" weak SU(4) flavor fields (u_0, d_0, s_0, c_0):

$$j_\mu^w = \bar{u}_0 \gamma_\mu^L d_0 + \bar{c}_0 \gamma_\mu^L s_0. \tag{31}$$

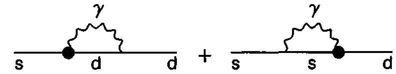

FIG. 6. CP-violating $s \to d$ transitions via photon self-energy.

Here the reference "diagonal in the gauge basis" refers to a charge-changing left-handed current which decouples the generations u_0,d_0 and c_0,s_0. The off-diagonal $Q=-1/3$ (weak flavor) quark mass matrix is then diagonalized in the quark mass basis according to

$$\begin{pmatrix} m_{d_0} & \lambda_{sd} \\ \lambda_{sd} & m_{s_0} \end{pmatrix} \rightarrow \begin{pmatrix} m_d & 0 \\ 0 & m_s \end{pmatrix}. \tag{32}$$

Such a quark flavor realignment is characterized by the mixing angle ϕ_{sd} defined through

$$\begin{gathered} d=d_0\cos\phi_{sd}-s_0\sin\phi_{sd}, \\ s=d_0\sin\phi_{sd}+s_0\cos\phi_{sd}, \\ \tan 2\phi_{sd}=2\lambda_{sd}/(m_{s_0}-m_{d_0}), \end{gathered} \tag{33}$$

in a manner similar to the SOW mixing of K^0 and $\bar{K}^0$ in (20).

Just as $2\lambda(m^2_{K_L}-m^2_{K_S})^{-1}=1$ in (21) because the K_L and K_S states have nearby masses (even in the CP-conserving limit), we argue by analogy that $2\lambda_{sd}(m_{s_0}-m_{d_0})^{-1}=1$ in (33) since s_0 and d_0 are presumably both light chiral quarks with nearby masses in this latter gauge basis. In that case (33) requires [26]

$$\tan 2\phi_{sd}=1 \quad \text{or} \quad \phi_{sd}=\pi/8. \tag{34}$$

Also the $Q=\frac{2}{3}$ $c-u$ quark sector can be diagonalized in a manner similar to (32) and (33):

$$\begin{pmatrix} m_{u_0} & \lambda_{cu} \\ \lambda_{cu} & m_{c_0} \end{pmatrix} \rightarrow \begin{pmatrix} m_u & 0 \\ 0 & m_c \end{pmatrix}, \tag{35}$$

$$\begin{gathered} u=u_0\cos\phi_{cu}-c_0\sin\phi_{cu}, \\ c=u_0\sin\phi_{cu}+c_0\cos\phi_{cu}, \\ \tan 2\phi_{cu}=2\lambda_{cu}/(m_{c_0}-m_{u_0}). \end{gathered} \tag{36}$$

Now, however, although the u quark is also a light chiral quark (as are d and s), the c quark is *not* a chiral quark but is much heavier with $m_c \gg m_u$. Thus $\tan 2\phi_{cu} \ll 1$ and the estimate in Ref. [26] suggests

$$\sin 2\phi_{cu} \sim \left[\frac{m_s-m_d}{m_c-m_u}\right]^{1/2} \approx 0.35 \quad \text{or} \quad \phi_{cu} \sim 10°, \tag{37}$$

for constituent quark mass differences $m_s-m_d \sim 170$ MeV and $m_c-m_u \sim 1400$ MeV.

In this SU(4) picture, the Cabibbo angle θ_c characterizes the *misalignment* of the weak quark current j^w_μ expressed in terms of the mass basis (13) versus the same j^w_μ being (generation) diagonal in the gauge basis (31). Reexpressing the latter back in terms of mass eigenstates,

$$\begin{aligned} j^w_\mu = & (\bar{u}\cos\phi_{cu}+\bar{c}\sin\phi_{su})\gamma^L_\mu(d\cos\phi_{sd}+s\sin\phi_{sd}) \\ & +(-\bar{u}\sin\phi_{cu}+\bar{c}\cos\phi_{cu})\gamma^L_\mu(-d\sin\phi_{sd}+s\cos\phi_{sd}), \end{aligned}$$

comparing this j^w_μ with (13) and using trigonometric identities, one deduces that the Cabibbo angle satisfies

$$\theta_C=\phi_{sd}-\phi_{cu}. \tag{38}$$

This fundamental relation has long been appreciated by Fritzsch [27], but we suggest that the minus sign in (38) is what is significant. Substituting the estimates (34) and (37) into this difference relation (38) one obtains the Cabibbo angle "mismatch"

$$\theta_C \approx 22.5° - 10° = 12.5°, \tag{39}$$

in qualitative agreement with the empirical Cabibbo angle.

If the c quark were chirally light like u, d, s or if all four quarks were heavy like b and t quarks, then the Cabibbo angle defined by (38) would vanish. In fact the latter is what happens for the Kobayashi-Maskawa (KM) mixing angles [26] θ_2 and θ_3 in the six-quark rather than the above four-quark picture. In effect, it is the (heavy) c quark which controls the size of the large Cabibbo angle in (39), in much the same manner that this same c quark drives the GIM-enhanced $\Delta I=\frac{1}{2}$ $K_{2\pi}$ amplitude in (14)–(17). However, there is no CPV in the SU(4) picture.

As is well known [1], the six-quark CKM picture also requires the CPV phase angle δ in (4) to enter the Cabibbo-KM (CKM) 3×3 mixing matrix [28]

$$V=\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3-s_2s_3e^{i\delta} & c_1c_2s_3+s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3+c_2s_3e^{i\delta} & c_1s_2s_3-c_2c_3e^{i\delta} \end{pmatrix}, \tag{40}$$

$$V \rightarrow \begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & -(1+i\delta) \end{pmatrix}, \tag{41}$$

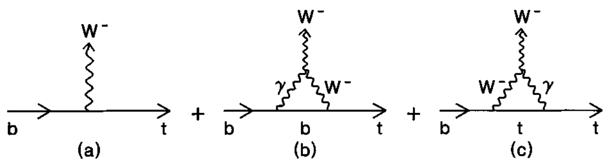

FIG. 7. CKM b to t graphs (a) lowest order and (b) one-loop order violating CP.

in the limit of a real SU(4) Cabibbo submatrix with $\theta_1 = -\theta_C$ and $\theta_2, \theta_3 \to 0$. Then the complex CPV phase appears only in V_{tb} (as originally suggested in Ref. [28]).

To proceed with CPV, we convert the GIM-suppressed $s \to d\gamma$ graph of Fig. 5 [driven by the CPV nonstandard $WW\gamma$ vertex of (25)] to the $b \to W^- t$ CKM mixing graphs for V_{tb} in Fig. 7 [the latter two diagrams again driven by the CPV nonstandard $WW\gamma$ vertex in (25)]. Just at the lowest order $b \to t$ current in Fig. 7(a) gives $V_{tb}^{(0)} = -1$ from $-g_w ie\gamma_\mu(1-\gamma_5)$, the first order correction of Fig. 7(c) generates the Feynman amplitude via (25) in the nonstandard U gauge:

$$g_w(ie\lambda_w)\int \frac{d^4k}{(2\pi)^4}\left[g^{\alpha\beta} - \frac{k^\alpha k^\beta}{M_W^2}\varepsilon_{\beta\mu\delta\sigma}k^\sigma g^{\delta\lambda}\right.$$
$$\left.\times\left[\frac{e_t c_1}{(k^2-M_W^2)k^2(k^2-m_t^2)}\right]\right[\gamma_\alpha(1-\gamma_5)(\not k - m_t)\gamma_\lambda]. \tag{42}$$

We have neglected Fig. 7(b) because it is suppressed relative to the larger propagator t mass in Fig. 7(c). Using the Dirac identity $\varepsilon_{\mu\alpha\lambda\sigma}\gamma^\alpha\gamma^\lambda\gamma^\sigma = 6i\gamma_\mu\gamma_5$ together with $e_t = (2/3)e$, the sum of $V_{tb}^{(0)} = -1$ and $V_{tb}^{(1)}$ from (42) generates V_{tb} in (40) as

$$-V_{tb} \approx 1 + i\lambda_w(\alpha/\pi)\ln(1+\Lambda^2/m_t^2). \tag{43}$$

In (43) we have neglected the W propagator mass occurring in (42) relative to the larger t quark mass. Taking the UV cutoff in (43) as in Sec. IV $\Lambda^2/m_t^2 \sim 3$ together with $\lambda_w \sim 1$ and comparing with the CKM matrix in (41), one obtains a rough estimate of the CPV phase

$$\delta \sim (\ln 4)\alpha/\pi \approx 0.003. \tag{44}$$

This qualitative CPV prediction (44) is not too distant from the measured phase $\delta \approx 0.0033$. Of course one could also consider other CKM representations such as that of Wolfenstein [29], where V_{tb} is taken as unity and the CPV phase appears elsewhere.

VI. SUMMARY

We have studied small CP-violating quark-mediated loop graphs primarily in the presence of large CP-conserving $\Delta I = 1/2$ $K_{2\pi}$ amplitudes, the latter driven by an $s \to d$ W-mediated loop diagram. While the latter CP-conserving loop amplitudes are GIM enhanced, the former CP-violating loop amplitudes are GIM suppressed. Such a GIM enhancement-suppression pattern may explain both the $\Delta I = 1/2$ rule for K^0 decays and also CP violation (which has so far only been detected in kaon weak decays).

In Sec. II we reviewed CP-violation data, where recent experiments suggest that the CP violation is characterized by $|\varepsilon| \approx 2.26\times 10^{-3}$ (and $\varepsilon' \approx 0$) and is realized only by K^0-$\bar{K}^0$ mixing with $\phi \approx 43.7°$ for $K_{S,L} \sim (1+\varepsilon)K^0 \mp (1-\varepsilon)\bar{K}^0$. Then the CPV phase is $\delta = 2|\varepsilon|\cos\phi \approx 0.0033$ in agreement with data [1]. In Sec. III we surveyed $\Delta I = 1/2$-enhanced CP-conserving $K_{2\pi}$ decays from the viewpoint of first-order-weak transitions and from the nonlinear unitarity link to the K_L-K_S mass difference Δm_{LS}. Then we diagonalized the second-order-weak kaon mass matrix to eliminate $\lambda = \langle \bar{K}^0|H^{(2)}|K^0\rangle$ and again recovered the empirical Δm_{LS}. These CP-conserving graphs are all GIM enhanced as $G_F(m_c^2 - m_u^2)$.

Then in Sec. IV we returned to CP violation via quark loops coupled to a nonstandard $WW\gamma$ vertex. We showed that such a CP-violating quark-mediated loop graph for an $s \to d\gamma$ transition is instead GIM suppressed as $G_F(m_c^2 - m_u^2)/M_W^2$, yet the CPV parameter $|\varepsilon|$ is roughly α/π, as anticipated.

Finally in Sec. V we again used the nonstandard $WW\gamma$ CPV vertex to construct one-loop order corrections to CKM matrix elements. These loop graphs for V_{33} are controlled by the very heavy top quark mass near [21] 200 GeV. Not surprisingly, with a photon in the loop we find the CPV phase $\delta \sim \alpha/\pi$. In the Appendixes we review arguments for why the $s \to d$ $\Delta I = 1/2$ truly weak tadpole cannot be transformed away as already anticipated by Weinberg [12].

With hindsight, we have assumed that CP violation (CPV) may be seen only in ε and that $\varepsilon' = 0$. Such was the original suggestion of Wolfenstein's superweak scheme [30]. More recent alternative treatments of $\Delta I = 1/2$ enhancement and CPV for K^0 decays fold in strong interaction QCD effects in the weak Hamiltonian or account for QCD "penguin" diagrams [31]. Early estimates of $\Delta I = 1/2$ penguin effects, however, were more than an order of magnitude shy of data [32]. Moreover, very recent improved treatments [33] of QCD penguin contributions go beyond the leading log approximation and find that at the cutoff scale $\mu = m_c$ the $K_{2\pi}^0$ penguin amplitude vanishes (in the high-voltage scheme) and is less than 20% of the measured amplitude when $\mu \approx 1$ GeV. Furthermore, Refs. [33] conclude that the improved treatment of QCD penguin diagrams likewise leads to a CPV parameter ratio ε'/ε which is also very small. Consequently, QCD effects do not significantly alter the results found in Secs. II–V of this paper concerning $\Delta I = 1/2$ enhancement and CP violation for K^0 weak decays.

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APPENDIX A

Here we spell out the reasons why the $s \to d$ quark $\Delta I = 1/2$ transition of Fig. 1 and Eq. (14) or the hadron

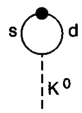

FIG. 8. $\Delta I=\frac{1}{2}$ K^0 to vacuum s-d quark loop tadpole.

$\Delta I=1/2$ tadpole transition $\langle 0|H_w|K^0\rangle$ of Fig. 3 and Eqs. (15)–(17) cannot be transformed away. The original null(tadpole) theorem of Feinberg, Kabir, and Weinberg [34] refers to (on-shell) lepton weak decays. There it is suggested (and reemphasized in Ref. [35]) that such a null (fermion) theorem does not apply to fermions *far off their mass shells*. The latter is the case for the weak tadpole in Fig. 3 depicted in $s\to d$ quark language in Fig. 8 for a (tightly bound) Nambu-Goldstone K^0-$\bar{s}d$ weak tadpole at zero invariant momentum transfer [35].

Then there is the null theorem of Coleman and Glashow [36], which rotates a $\bar{d}s$ term into the (semistrong) mass matrix. Such a $\bar{d}s$ term is *not* the form of the needed $\bar{d}\not{p}s$ self-energy in Eq. (14) and so the null theorem of Ref. [36] does *not* apply to the $s\to d$ transition in Sec. III.

Some physicists have suggested that the $\bar{d}\not{p}s$ self-energy in (14) can be Fourier-transformed to a kinetic energy term in an off-diagonal semistrong Lagrangian and thus rediagonalized away. However, the structure of $\bar{d}\not{p}s$ in (14) derives from the one-loop-order off-diagonal semistrong Lagrangian term. In fact the standard SU(2)×U(1) theory of Weinberg and Salam [37] generates tadpole terms of $O(\alpha)$ and of $O(\alpha m_q^2/M_W^2)$ [12]. While Weinberg shows that the $O(\alpha)$ tadpoles are "purely electromagnetic" and can indeed be removed, he also stresses that tadpoles of $O(\alpha m_q^2/M_W^2)$ are "truly weak" and *cannot* be transformed away [12]. Needless to say, the Σ_{sd} $s\to d$ tadpole in (14) is of order $G_F m_c^2=O(\alpha m_q^2/M_W^2)$; it is then one of Weinberg's "truly weak" tadpoles which *cannot* be so removed.

Lastly, in hadron language the (weak) kaon null tadpole theorems [38] claiming $\langle\tilde{0}|H_w|K^0\rangle=O(m_K^4)$ are generated by reducing in the heaviest kaon particle in $K\to 2\pi$ decays (as well as the light pions). However, this vacuum $|\tilde{0}\rangle$ is of all-orders strong *plus* first order weak [12]. On the other hand, the old-fashioned perturbation theory (OFPT) approach used throughout this paper always keeps the decaying kaon *on mass shell*, so that our OFPT vacuum $|0\rangle$ is clearly to all orders strong only [16]. So in our case $\langle 0|H_w|K^0\rangle=O(m_K^2)$, which circumvents the null theorems of Refs. [38] and $\langle 0|H_w|K^0\rangle$ cannot be so removed (suppressed).

While our OFPT quark mass matrix A can be rediagonalized to the form [39]

$$A=\begin{pmatrix} m_d & \Sigma_{sd}^{\mathrm{PC}} \\ \Sigma_{ds}^{\mathrm{PC}} & m_s \end{pmatrix}\to B=\begin{pmatrix} m_d' & 0 \\ 0 & m_s' \end{pmatrix}, \tag{A1}$$

defining B as in the all-orders-strong plus first-order-weak quark basis, the effect of the off-diagonal self-energy $\Sigma_{sd}^{\mathrm{PC}}=b\bar{d}\not{p}s$ is *not* completely abolished in B. In fact $\Sigma_{sd}^{\mathrm{PC}}$ reenters B via the transformation angle $\tilde{\phi}$ converting $A\to B$ in analogy with the (quantum-mechanical) construction of Δm_{LS} in (21) or ϕ_{sd} in (33):

$$\tan 2\tilde{\phi}=\frac{2\Sigma_{sd}^{\mathrm{PC}}}{m_s-m_d}, \tag{A2}$$

so it is clear that the effect of Σ_{sd} [i.e., in (14)] *cannot* be completely eradicated.

APPENDIX B

Here we summarize two methods by which partially conserved axial vector currents (PCAC) can be used to relate the K^0 tadpole to the $K^0\to\pi\pi$ amplitudes. Then we discuss $\Delta I=1/2$ single quark line transitions.

1. Strong interaction PCAC

The K^0 tadpole graph of Fig. 3 generates the amplitude magnitude

$$|\langle\pi\pi|H_w|K^0\rangle|=\frac{|\langle 0|H_w|K^0\rangle|}{m_K^2-0}M_{K^0\bar{K}^0(0)\to\pi\pi}, \tag{B1}$$

where the strong interaction amplitude M in (B1) is a Weinberg type [40] of PCAC low-energy $\pi\pi$ amplitude $(t-m_\pi^2)/f_\pi^2$ (in the t channel). The latter $\pi\pi\to\pi\pi$ result is extended to $K^0\bar{K}^0\to\pi\pi$ scattering [41] as $(t-m_\pi^2)/2f_\pi^2$, but now with $t=(p_K-0)^2=m_K^2$ on the decaying K^0 mass shell. Then the $K^0\to\pi\pi$ amplitude magnitude in (B1) becomes the anticipated result

$$|\langle\pi\pi|H_w|K^0\rangle|=\frac{|\langle 0|H_w|K^0\rangle|}{2f_\pi^2}(1-m_\pi^2/m_K^2), \tag{B2}$$

which is the first line of Eq. (17) in the text.

2. Weak interaction PCAC via weak chirality

Instead we derive the crucial PCAC-tadpole relation (B2) in a totally different way by also starting from a Weinberg-type [40] of strong interaction low-energy momentum (PCAC) expansion, but now combined with the standard weak chiral current algebra $[Q+Q_5,H_w]=0$ when H_w is driven by $V-A$ currents. More specifically, allowing the weak spurion to carry off no four-momentum, one can express the weak $K^0\to\pi^i\pi^j$ transition through second order in momentum as [40,14]

$$\langle\pi^i\pi^j|H_w|K^0\rangle=A(p_K^2+a_ip_i^2+a_jp_j^2). \tag{B3}$$

Keeping the K^0 meson always on mass shell $p_K^2=m_K^2$ while conserving four-momentum $p_K=p_i+p_j$, the Cabibbo–Gell-Mann theorem [42] requiring the $K_{2\pi}$ amplitudes (B3) to vanish in the SU(3) limit when $m_K^2\to m_\pi^2$, in turn forces $a_0=-1/2$ in (B3) for $K^0\to 2\pi^0$ decay. Also taking one π^0 four-momentum soft, Eq. (B3) becomes

$$\langle\pi^0\pi^0|H_w|K^0\rangle\to A\left(m_K^2-\frac{1}{2}m_K^2\right)=\frac{1}{2}Am_K^2.$$

However, PCAC applied to the left-hand-side (lhs) of the latter equation combined with weak chirality $[Q_5,H_w]=-[Q,H_w]$ requires

$$\langle \pi^0\pi^0|H_w|K^0\rangle \to (-i/f_\pi)\langle \pi^0|[Q_5^3,H_w]|K^0\rangle = (i/2f_\pi)\langle \pi^0|H_w|K^0\rangle.$$

Comparing the latter two equations uniquely determines the constant $A=(i/f_\pi m_K^2)\langle \pi^0|H_w|K^0\rangle$, so that the on-shell $K^0\to 2\pi^0$ amplitude in (B3) is

$$\langle \pi^0\pi^0|H_w|K^0\rangle = (i/f_\pi)\langle \pi^0|H_w|K^0\rangle(1-m_\pi^2/m_K^2). \tag{B4}$$

Finally, one can simply reduce in the π^0 on the rhs $K^0\to\pi^0$ weak transition again using PCAC and weak chirality,

$$\langle \pi^0|H_w|K^0\rangle \to (-i/f_\pi)\langle 0|[Q_5^3,H_w]|K^0\rangle = (i/2f_\pi)\langle 0|H_w|K^0\rangle,$$

so that (B4) becomes

$$\langle \pi^0\pi^0|H_w|K^0\rangle = -\frac{\langle 0|H_w|K^0\rangle}{2f_\pi^2}(1-m_\pi^2/m_K^2). \tag{B5}$$

Although slightly more tedious to derive, Eq. (B5) indeed matches the desired relation (B2).

An even more complicated rapidly varying pole approach [11,43] without directly employing PCAC also recovers (B5). Moreover, Ref. [44] demonstrates that the chiral Lagrangian of Cronin in Ref. [7] once again leads to (B4) or (B5). Alternatively one can bypass the K^0 tadpole in (15) and (16) but still recover the second line of (17) by invoking instead the soft pion theorem applied to the weak axial vector current

$$q^\mu M_\mu \approx i f_\pi \langle \pi^0|H_w|K^0\rangle,$$

equate it to $q^\mu M_\mu \approx i\sqrt{2} b f_K m_K^2$, and finally use (B4) above. Thus we take (B2) and (B5) or Eq. (17) as model-independent results.

3. Single quark line scale *b*

As the final step in understanding the $\Delta I=1/2$ rule for $K^0\to 2\pi$ decays, we review the $\Delta I=1/2 s\to d$ single quark line (SQL) weak transition scale b obtained in Eq. (14). Although there is still some controversy concerning rotating away this effect for $K^0_{2\pi}$ decays (hopefully resolved in Appendix A), there can be little or no other quark mechanism for three specific $\Delta S=1$ baryon transitions: $\Xi^-\to\Sigma^-\gamma$, $\Omega^-\to\Xi^-\pi^0$, and the p-wave $B\to B'\pi$ combination $B(\Sigma_0^+)+\sqrt{3}B(\Lambda_-^0)$. All of these three baryon transitions have empirical SQL scales averaging to [45]

$$b_{\rm avg} = -(5.5\pm 0.7)\times 10^{-8}, \tag{B6}$$

which is extremely close to the SQL $s\to d$ scale (14) computed as an off-diagonal W-mediated quark self-energy $b\approx -5.6\times 10^{-8}$.

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