# Challenging weak-scale supersymmetry at colliders

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Experimental searches for supersymmetry are entering a new era. As future experiments explore the mass range above the current lower bounds on superpartner masses, a failure to observe signals of superpartner production will begin to erode the central motivation for supersymmetry at the weak scale. In this article we present a detailed examination of which regions of supersymmetric parameter space are most natural and the extent to which weak-scale supersymmetry becomes unnatural if no superpartners are observed at CERN LEP II, the Fermilab Tevatron, possible upgrades of these machines, and the CERN LHC.

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## I. INTRODUCTION

Supersymmetry's continued growth in popularity as a candidate for physics beyond the standard model can be traced to two sources. First, there is suggestive, circumstantial evidence for weak-scale supersymmetry. The lightest superpartner (LSP) makes an attractive dark matter candidate, and the measured value of $\sin^2\theta$ coincides with the prediction of supersymmetric grand unification. A second, and no less influential, factor is the relative ease with which supersymmetry maintains consistency, both with precision tests of the standard model and with the failure to observe new particles in collider experiments, when compared to many of its competitors. This is because supersymmetric deviations from the standard model can be made arbitrarily small by simply raising the masses of superpartners.

However, the search for weak-scale supersymmetry has reached a milestone. Although previous experiments could raise the lower bound on superpartner masses without posing a serious challenge to supersymmetry, further increases in this bound will begin to erode a dominant motivation for supersymmetry at the weak scale. The central motivation for the appearance of supersymmetry at the weak scale is its ability to solve the naturalness problem [1]. In the standard model, the mass of the Higgs boson is quadratically divergent. If the standard model gives a complete description of nature below a scale $\Lambda$, quadratically divergent contributions to the square of the Higgs boson mass must cancel against bare terms to a part in $m_H^2/\Lambda^2$. If $\Lambda$ is large, precise cancellations must be engineered at every order in perturbation theory. Without additional structure or organizational principles beyond the standard model, it would be unnatural if the Higgs boson mass were much lighter than the cutoff, since the cancellations that ensure a light Higgs boson mass would be upset by a variety of minute changes in the fundamental parameters.

Supersymmetry allows scalar masses to remain light in comparison to a cutoff without delicate fine-tuning because the additional renormalization effects of superpartners cancel the quadratic divergences in a supersymmetric theory. However, because supersymmetry is broken, these cancellations are only achieved up to the mass splittings between the standard model particles and their superpartners. In this way, the scale of supersymmetry breaking assumes the role of the cutoff in the previous discussion. Accordingly, as superpartner masses increase, a point is reached where supersymmetry is no longer able to provide a complete explanation of why a light weak scale is natural.

Recently, we constructed a family of naturalness measures which reliably quantify how unnatural the supersymmetric standard model becomes as the masses of superpartners increase [2]. These naturalness measures provide a significant advance over popular characterizations of naturalness. Previous attempts to quantify naturalness [3] are known to overestimate fine-tuning by an order of magnitude or more [2]. Other less quantitative criteria are also problematic. For example, it is often loosely stated that superpartners must lie ''at or below 1 TeV'' if the weak scale is to arise naturally, or that superpartner masses should not lie ''too far above the weak scale.'' As defining rules for what constitutes naturalness, these compact statements are less than adequate. First, there are dramatic differences between the upper bounds on the masses of different superpartners. If we are willing to tolerate solutions with 10 (20) times more fine-tuning than ideally natural solutions require, squark masses may be as heavy as 750 (1200) GeV. By contrast, under this same criterion, the mass of the lightest neutralino cannot exceed 145 (250) GeV. Second, the often quoted range of superpartner masses ''at or below 1 TeV'' ignores the *progressive* worsening of the naturalness problem as superpartner masses are increased from the weak scale to 1 TeV. A scenario with any superpartner mass as heavy 1 TeV is significantly less natural than a variety of lower mass solutions. For example, a 1 TeV squark mass requires adjustments in the fundamental parameters with 16 times more precision than a 200 GeV squark mass. Moreover, for some types of superpartners, a mass of 1 TeV would be extremely unnatural. The mass of the lightest chargino can only reach 1 TeV at the expense of a factor of 45 in fine-tuning, and an LSP of mass 1 TeV requires fine-tuning to at least a part in 65.

In this paper we attempt to provide a detailed picture of what regions of supersymmetric (SUSY) parameter space are

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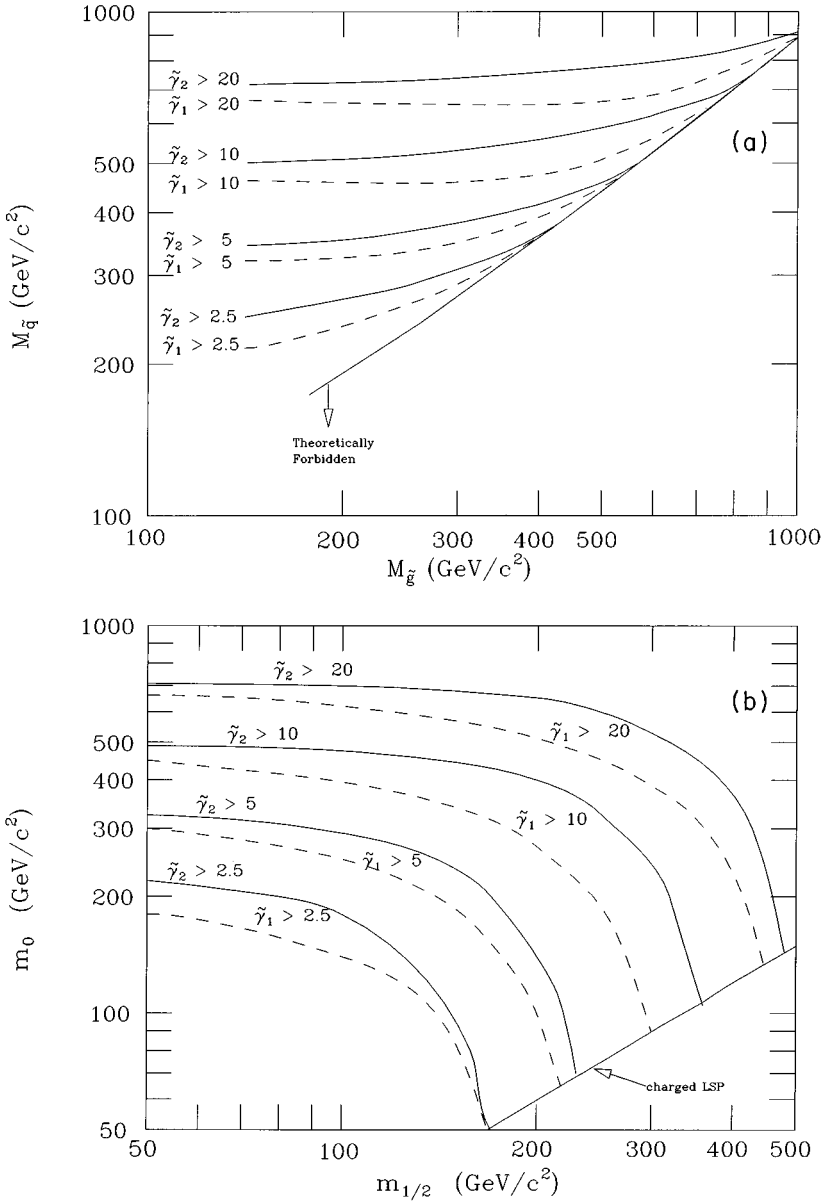


FIG. 1. (a) A comparison of the naturalness contours $\tilde{\gamma}_1$ and $\tilde{\gamma}_2$ in terms of the gluino mass and the mass of the lightest squark belonging to the first two generations. The dashed and solid curves represent contours above which the fine-tuning parameters exceed 2.5, 5, 10, and 20, respectively. (b) A comparison of the naturalness contours $\tilde{\gamma}_1$ and $\tilde{\gamma}_2$ in terms of the common gaugino mass $m_{1/2}$ and the common scalar mass $m_0$. The dashed and solid curves represent contours above which the fine-tuning parameters exceed 2.5, 5, 10, and 20, respectively.

most natural, where we should have the strongest expectations of observing signals of supersymmetry, and how supersymmetry's ability to accommodate a light weak scale naturally will be progressively challenged if signals of superpartner production are not observed in the future. After reviewing our quantitative naturalness measures, we construct naturalness contours and compare them to the SUSY discovery reach of current and future collider experiments.

## II. QUANTIFYING NATURALNESS

This section describes the quantitative methods we use to determine naturalness in the minimal supersymmetric standard model (MSSM). A detailed derivation of these naturalness measures and example applications can be found in Ref. [2]. If we parametrize our assumptions about the likelihood distribution of a theory's fundamental parameters, a fine-tuning measure can be constructed directly from probability arguments, and we can retain a functional parametrization of our assumptions that can be used to estimate the theoretical uncertainty in the naturalness measure. Consider an effective field theory with a set of measurable parameters $X$. Using the renormalization group, the $X$'s can be written in terms of the presumably more fundamental, high-energy boundary conditions of the effective theory. If we write the probability distribution for these ''fundamental parameters'' as

$$dP = f(a)da, \tag{2.1}$$

a naturalness measure $\gamma$ can be constructed from the following prescription: Write the observables in terms of the fundamental parameters $X = X(a)$, compute the probability that $X$ lies within a specified interval about $X$, average this probability over the likelihood distribution of the fundamental parameters, and then divide this average by the probability that $X$ lies within the specified interval about a particular value of $X$ [2]. With this prescription, ideally natural solutions will have $\gamma = 1$, while finely tuned solutions

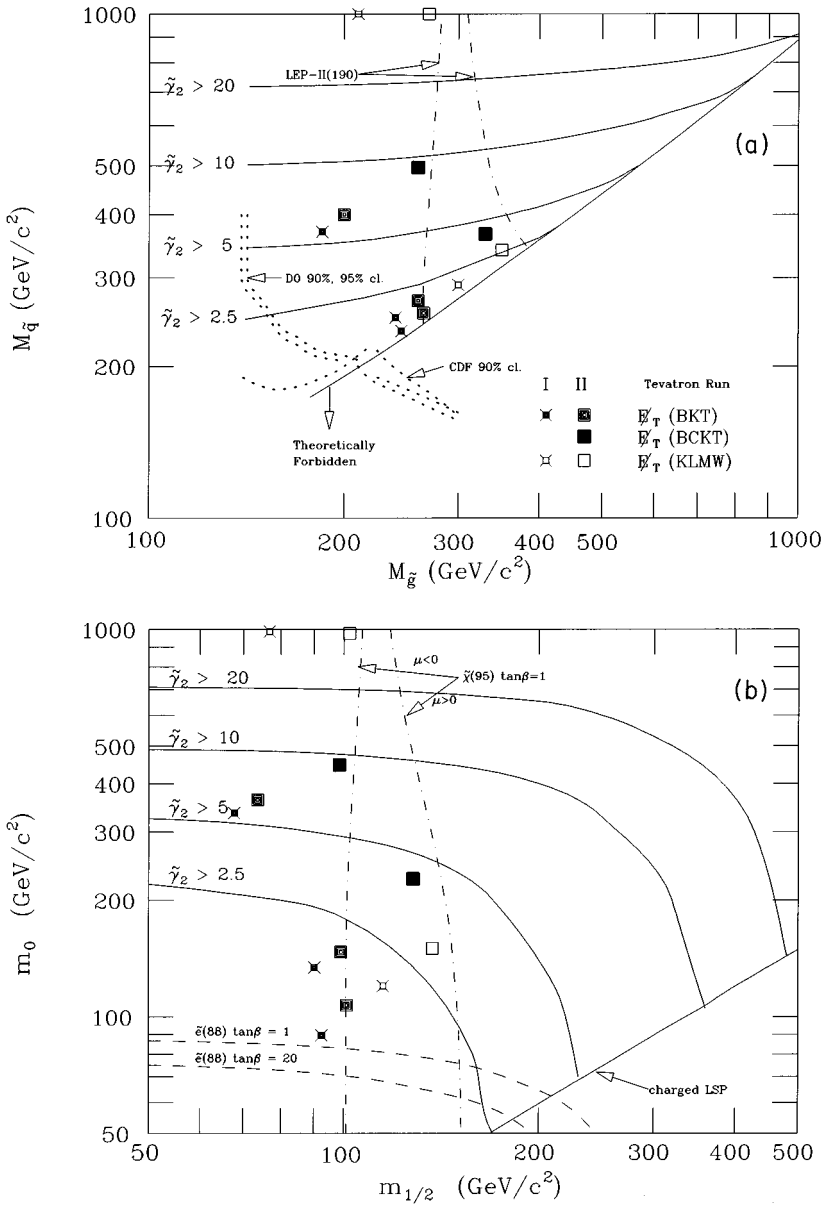


FIG. 2. (a) SUSY discovery prospects as a function of the gluino mass and the lightest squark mass of the first two generations. Estimates of the physics reach in missing energy and jet events ($\not\!\!E_T$) at Tevatron runs I and II are compared to naturalness contours. Individual data points represent various estimates of the gluino mass reach in the $\not\!\!E_T$channel for different squark masses. For comparison, the dot-dashed curves present the minimal and maximal physics reach of chargino production at LEP II. (b) SUSY discovery prospects as a function of the common gaugino mass $m_{1/2}$ and the common scalar mass $m_0$. Estimates of the physics reach in missing energy and jet events ($\not\!\!E_T$) at Tevatron runs I and II are compared to naturalness contours. Individual data points represent various estimates of the $m_{1/2}$ reach in the $\not\!\!E_T$ channel for different squark masses. For comparison, dot-dashed (dashed) curves approximate the minimal and maximal physics reach of chargino (selectron) production at LEP II.

yield $\gamma \gg 1$. If we define a sensitivity function $c(X,a) = |(a/X)(\partial X/\partial a)|$, and if we define an average sensitivity $\bar{c}$ by

$$1/\bar{c} = \frac{\int da\, a f(a) c(X;a)^{-1}}{a f(a) \int da}, \tag{2.2}$$

the naturalness measure can be written

$$\gamma = c/\bar{c}. \tag{2.3}$$

As suggested by Eq. (2.3), the naturalness measure $\gamma$ is equivalent to a refined version of Wilson's naturalness criteria [4]: Observable properties of a system, i.e., $X$, should not be *unusually unstable* with respect to minute variations in the fundamental parameters, $a$.

We will use this measure to study the naturalness of the weak scale as the masses of superpartners are increased. In the MSSM, electroweak symmetry breaking is induced by radiative effects. The $Z$-boson mass serves as a useful order parameter for electroweak symmetry breaking because it has a relatively simple dependence on the Lagrangian parameters:

$$\frac{1}{2} m_Z^2 = \frac{\bar{m}_{\Phi_d}^2 - \bar{m}_{\Phi_u}^2 \tan^2\beta}{\tan^2\beta - 1} - \mu^2. \tag{2.4}$$

The parameters $\bar{m}_{\Phi_d}^2$ and $\bar{m}_{\Phi_u}^2$ are the quadratic mass terms for the two Higgs doublets with additional terms from the one-loop corrections to the effective potential [2]. The $\mu$ parameter is a coupling between the two different Higgs doublets, and tan$\beta$ is the ratio of their vacuum expectation values.

In our study we focus on supersymmetric extensions of the standard model compatible with minimal supergravity and unification for which the soft supersymmetry breaking can be written in terms of four parameters $A$, $B$, $m_0$, and $m_{1/2}$. At low energies, the values of $\bar{m}_{\Phi_d}^2$, $\bar{m}_{\Phi_u}^2$, $\mu$, and tan$\beta$ can be written in terms of these four soft SUSY-

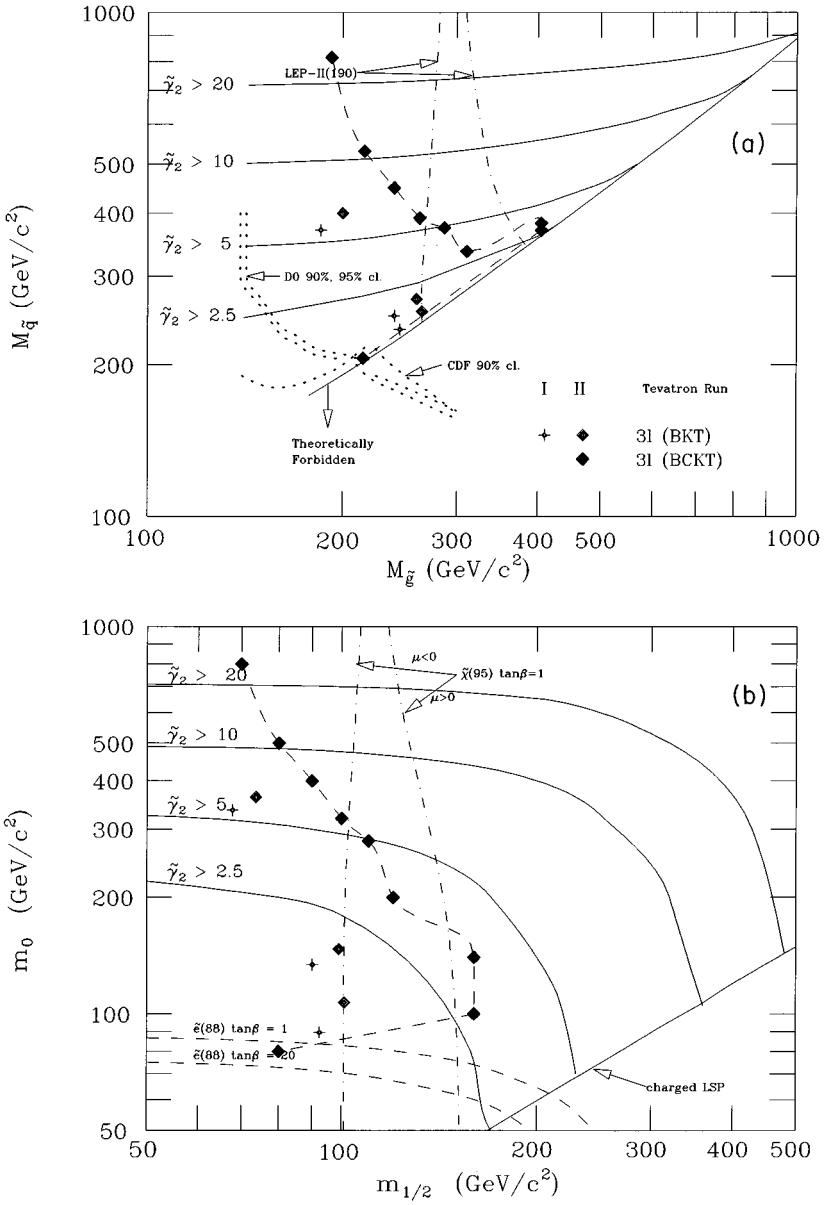


FIG. 3. (a) The SUSY discovery reach in the trilepton channel ($3l$) along with naturalness contours in the gluino squark mass plane. Individual data points show estimates of the gluino mass reach in the 3l channel for $\mu<0$ and $\tan\beta=2$. (b) The SUSY discovery reach in the trilepton channel $3l$ and naturalness contours in the $m_{1/2}$-$m_0$ plane. Individual data points show estimates of the gaugino mass reach in the $3l$ channel for $\mu<0$ and $\tan\beta=2$.

breaking parameters and other couplings at the high-energy scale such as the top quark Yukawa coupling $y_t$. We can evaluate the naturalness of the weak scale with respect to any of these parameters. To accurately asses the naturalness of a particular solution it is necessary to compute the fine-tuning with respect to several parameters.[1] Unnatural solutions are typically only fine-tuned with respect to a few of the fundamental parameters, and which parameters are fine-tuned will vary from solution to solution. For our purposes it is sufficient to consider the naturalness with respect to two dimensionful parameters and one dimensionless coupling. For each solution we define a cumulative naturalness measure by

$$\tilde{\gamma}=\max\{\gamma(y_t),\gamma(m_0),\gamma(m_{1/2})\}. \tag{2.5}$$

To evaluate $\gamma$ we must assume a distribution for the fundamental parameters $f(a)$ and a suitable range of integration. We will make two choices for the distribution $f_1(a)=1$ and $f_2(a)=1/a$. The difference between these respective $\gamma$'s is a measure of the theoretical uncertainty in naturalness contours. Inspection of Figs. 1(a) and 1(b) shows that this theoretical uncertainty is much less than order one, and typically lies in the range of a few to tens of a percent.

For each distribution, we average the sensitivity of the weak scale over a range of the fundamental parameters $a$ following the method of Ref. [2]. Since we employ two-loop renormalization group equations (RGE's) for all parameters (except the soft breaking ones), numerical methods are used to solve the RGE's and to integrate (2.2). The procedure is time consuming, especially for cases in which the range of $a$ is large. We evaluated the naturalness of approximately 4000 different solution sets from which we constructed the naturalness contours in Figs. 1–5. To efficiently determine these contours, we selectively explored the more pertinent regions of parameter space. Because the data points used to

[1]Computing the fine-tuning with respect to only one of the fundamental parameters leads to a significant underestimate of fine-tuning.

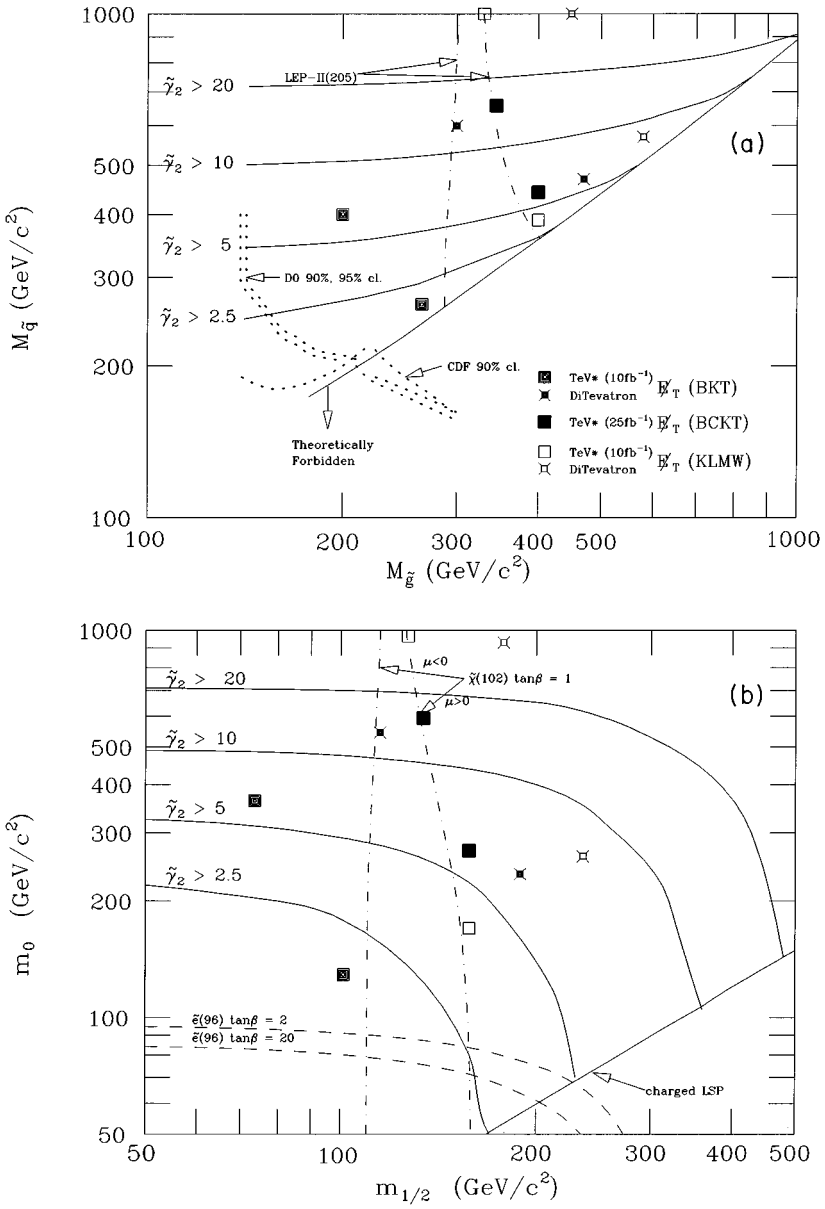


FIG. 4. (a) The SUSY discovery reach in the $\not\!\!E_T$ channel for two proposed Tevatron upgrades compared with naturalness contours in the gluino squark mass plane. (b) The SUSY discovery reach in the $\not\!\!E_T$ channel for two proposed Tevatron upgrades compared with naturalness contours in the $m_{1/2}$-$m_0$ plane.

construct our naturalness contours were scattered over a large range of the fundamental parameters, no particular choice of $A$ or tan$\beta$ is implicit in the naturalness contours found in Figs. 1–5. Particular choices for $A$ or tan$\beta$ would result in curves that would be more restrictive. Figures 2–5 compare the more conservative naturalness measure $\tilde{\gamma}_2$, to various estimates of SUSY discovery reaches. Each comparison is made both in terms of the gluino and squark masses [Figs. 2(a), 3(a), 4(a), and 5(a)], and in terms of the soft SUSY breaking parameters $m_{1/2}$ and $m_0$ [Figs. 2(b), 3(b), 4(b), and 5(b)].

### III. CHALLENGES TO SUSY AT COLLIDERS

At particle colliders, sparticles must be produced in pairs to conserve $R$ parity. Once produced, a superpartner decays through a cascade ending in a stable, unobservable LSP. Accordingly, signals of superpartner production at colliders would consist of events with missing energy and various combinations of leptons and jets. The CERN $e^+e^-$ collider LEP will soon be upgraded in energy to begin phase II. LEP II is expected to start a trial run just below the $WW$ threshold in late 1995, and by 1996 it is expected to achieve energies of $\sqrt{s}=175$–$190$ GeV. Potential searches for supersymmetry at LEP II have been studied by many groups [5–7]. Typically, chargino pair production and slepton pair production are the two most promising channels for discovering supersymmetry at LEP II, however in some cases neutralino pair production can provide the largest reach. The heaviest reach in slepton masses will come from selectron searches since, unlike staus and smuons, selectron pairs can be produced by $t$-channel neutralino exchange in addition to the $s$-channel production mediated through a $\gamma$ or Z. For small $m_0$, selectron searches should probe up to nearly the kinematic limit, reaching masses of 82 (88) [96] GeV for center of mass energies of 175 (190) [205] GeV [7]. For large $m_0$, searches for charginos should probe masses up to 87 (95) [102] GeV for center of mass energies of 175 (190) [205] GeV.

In Figs. 2 and 3 we compare the SUSY discovery reach in the selectron and chargino channels at LEP II with Fermilab

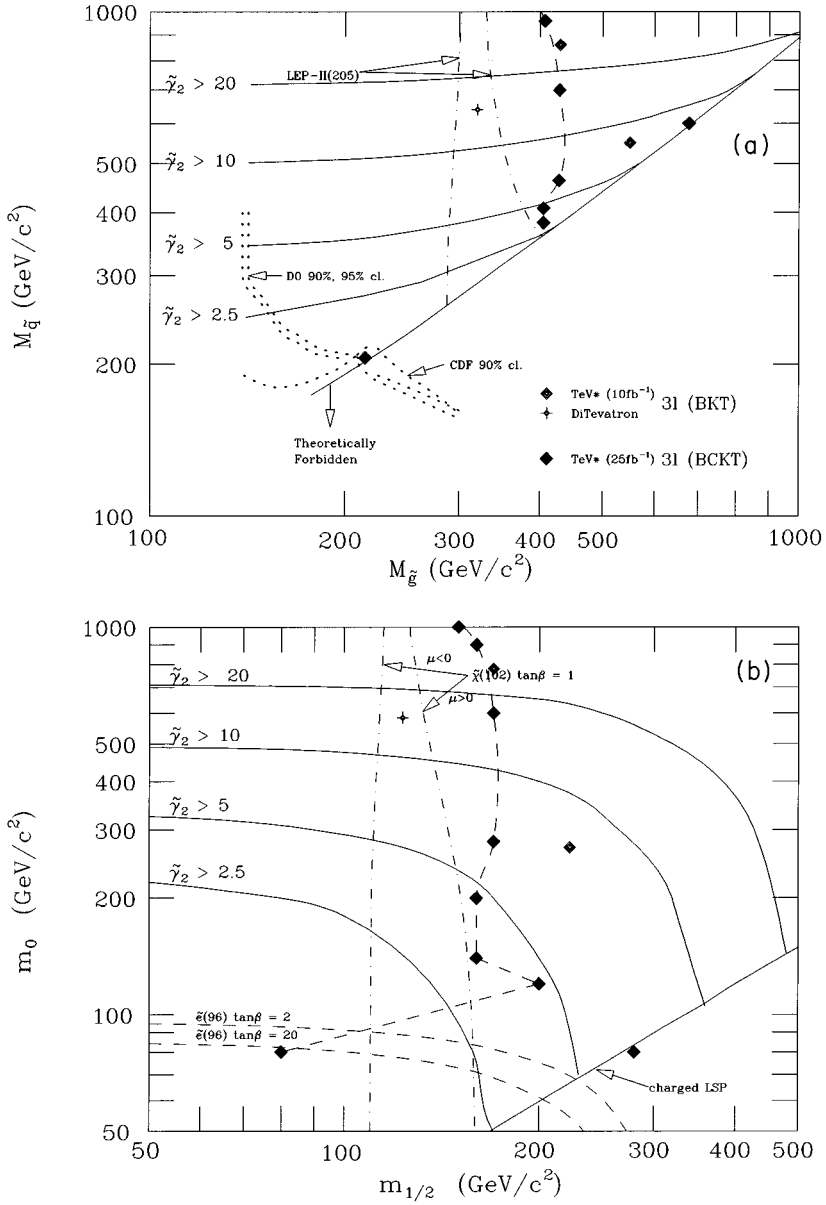


FIG. 5. (a) The SUSY discovery reach in the ($3l$) channel for two proposed Tevatron upgrades compared with naturalness contours in the gluino squark mass plane. Individual data points show estimates of the gluino mass reach in the $3l$ channel for $\mu<0$ and $\tan\beta=2$. (b) The SUSY discovery reach in the $3l$ channel for two proposed Tevatron upgrades compared with naturalness contours in the $m_{1/2}$-$m_0$ plane. Individual data points show estimates of the gaugino mass reach in the $3l$ channel for $\mu<0$ and $\tan\beta=2$.

Tevatron search reaches and naturalness contours.[2] The dashed curves $\tilde{e}(88)$ and $\tilde{\chi}^+(95)$ represent the approximate selectron and chargino mass reaches achievable with $\sqrt{s}=190$ GeV at LEP II. Because the reaches in these channels depend on $\tan\beta$ and the sign of $\mu m_{1/2}$, we have chosen values of $\tan\beta$ which represent larger and smaller reaches for each case. Our convention for the sign of $\mu$ agrees with Refs. [9,10].

Fermilab's $p\bar{p}$ collider, the Tevatron, will complete the second stage of run I at the end of this year. Through the end of run I, the classic missing transverse energy and jet events ($\not\!\!E_T$) will provide the largest reach into SUSY parameter space. The dotted lines in Figs. 2–5 display the mass limits on squarks and gluinos achieved by the Collider Detector at Fermilab (CDF) and D0 experiments during run IA [11]. By the end of the current Tevatron run, run IB, the CDF and D0 experiments can each be expected to have over 100 pb$^{-1}$ of data on tape. Run II, with the Tevatron's Main Injector (MI) upgrade, is anticipated for 1999. The MI run is expected to achieve ($\sqrt{s}=2$ TeV) and a yearly integrated luminosity of $\sim 1$ fb$^{-1}$ per experiment. Figures 2(a) and 2(b) compare naturalness contours to the discovery reach of missing energy and jet events during runs I and II, as estimated by three different analyses [9–13]. Although there is not always a clear consensus among these search reach estimates, several features are apparent. First, if the weak scale is to arise naturally, the fundamental soft SUSY-breaking parameters $m_0$ and $m_{1/2}$ should be significantly lighter than 1 TeV. Accordingly, it is very unlikely that $m_{\tilde{q}} \gg m_{\tilde{g}}$ unless the gluino is

[2]We caution the reader that estimates of search reaches should be interpreted carefully. A global determination of discovery reaches is a complicated undertaking. The Tevatron search reaches quoted here have only been calculated for a few selected points in SUSY parameter space (for recent work on a more global analysis of search reaches see Ref. [8]). Moreover, there is sometimes significant differences between the reach estimates of different groups, and the various background cuts in these analyses have not been optimized to yield the maximum reaches.

extremely light ($<1$ GeV) and has somehow managed to escape detection [14]. Second, to set the most significant bounds from missing energy and jet events, the CDF and D0 experiments should pay special attention to the regime $m_{\tilde{q}} \simeq m_{\tilde{g}}$. In addition, searches for chargino and selectron production at LEP II will cover a significant amount of the natural region of parameter space, particularly for $\mu > 0$.

Although missing energy and jet events will provide the Tevatron's strongest probe of supersymmetry during run I, searches in other channels will become important in the main injector era. During Tevatron run II, missing energy events with same sign dileptons and trileptons ($3l$) will provide a larger reach into parameter space[3] than the background limited $\not{E}_T$ channel for some values of $\tan\beta$ and $\mu$ [9,10]. Figures 3 and 5 compare the physics reach of the trilepton events, for $\mu < 0$ and $\tan\beta = 2$, to the discovery reach of LEP II and naturalness contours. While LEP II provides its largest reach into SUSY parameter space for $\mu > 0$, Tevatron searches in the $3l$ channel are most effective if $\mu < 0$. Moreover, for $\mu < 0$, the chances for observing trilepton signals at the Tevatron are promising for run II. Except for a small window where the branching ratio of the second heaviest neutralino into an invisible neutrino ($\tilde{\chi}_2^0 \to \nu\tilde{\nu}$) approaches 100%, the reach in the trilepton channel is largest for the most natural region of parameter space.

Further luminosity upgrades of the Tevatron, dubbed the TeV*, and the DiTevatron have been considered. The TeV* upgrade, achievable through antiproton recycling and storage, could reach integrated luminosities of 25 $\text{pb}^{-1}$, and the use of Superconducting Super Collider–(SSC-)type magnets could allow a DiTevatron upgrade to achieve $\sqrt{s} = 4$ TeV. In Figs. 4 and 5, we compare the SUSY discovery prospects of these proposed Tevatron upgrades with LEP II operating at $\sqrt{s} = 205$ GeV. The dashed curves in Figs. 4 and 5 labeled $\tilde{e}(96)$ and $\tilde{\chi}^+(102)$, represent the approximate mass reach of LEP II in these channels [7]. From Figs. 4(a) and 4(b) we see that searches in the $\not{E}_T$ channel at a DiTevatron would cover most of the natural region of SUSY parameter space, while searches in the $3l$ channel would be very promising at the TeV* if $\mu < 0$.

Finally we mention that the nonobservation of a significant amount of missing transverse energy and jet events at the CERN $\sqrt{s} = 14$ TeV $pp$ accelerator, the Large Hadron Collider (LHC), will eliminate the possibility of supersymmetry accommodating the weak scale naturally. The estimated search reaches of gluinos and squarks, which typically lie above 1 TeV, lie far outside the natural regions shown in these figures.

## IV. CONCLUSIONS

Searches for supersymmetry have reached a critical milestone. While the absence of observable signals of supersymmetry at past and present collider experiments has eliminated some regions of parameters space, experiments have not yet posed any significant challenge to the viability of the theory. It is still possible to find very natural solutions which satisfy all of the quoted, experimental bounds on superpartner masses. By contrast, as future searches extend beyond the mass ranges already explored, failure to observe signals of supersymmetry will begin to undermine a supersymmetric explanation of why the weak scale remains light. If LEP II and the Tevatron's Main Injector era do not produce superpartners, supersymmetry's explanation of weak-scale stability will be measurably, but not catastrophically, weakened. Conversely, if supersymmetry is relevant to the weak scale, the chance of observing a signal of superpartner production at these machines is promising. If signals of SUSY are not seen at the CERN Large Hadron Collider LHC, the stability of the weak scale can no longer be completely explained by low-energy supersymmetry.

## ACKNOWLEDGMENTS

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[3] We caution the reader that these signals, particularly the trilepton events, have a strong dependence on $\tan\beta$ and the sign of $\mu$. For this reason the discovery reaches in Figs. 3 and 5 cannot be taken as global reaches.

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