

## Polarized parton distribution functions reexamined

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The hard-gluonic contribution to the first moment of the polarized proton structure function  $g_1^p(x)$  is dependent on the factorization convention chosen in defining the quark spin density and the hard cross section for photon-gluon scattering. Two extremes of interest, namely, gauge-invariant and chiral-invariant factorization schemes, are considered. We show that in order to satisfy the positivity constraint for sea and gluon polarizations, the polarized valence quark distributions should fully account for the observed  $g_1^p(x)$  at  $x \geq 0.2$ . This together with the first-moment and perturbative QCD constraints puts a pertinent restriction on the shape of  $\Delta u_v(x)$  and  $\Delta d_v(x)$ . The spin-dependent sea distribution in the gauge-invariant factorization scheme is extracted from the data of  $g_1^p(x)$ . It is shown in the chiral-invariant scheme that it is possible to interpret the  $g_1^p(x)$  data with anomalous gluonic contributions, yet a best least  $\chi^2$  fit to the data implies a gluon spin distribution which violates the positivity condition  $|\Delta G(x)| \leq G(x)$ . We then propose a more realistic set of parton spin distributions with sea polarization and with a moderate value of  $\Delta G$ . The polarized parton distributions in this paper are presented in the next-to-leading order of QCD at the scale  $Q^2 = 10 \text{ GeV}^2$ . Predictions for the polarized structure functions  $g_1^n(x)$  of the neutron and  $g_1^d(x)$  of the deuteron are given.

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### I. INTRODUCTION

In the past few years we have witnessed a remarkable progress in the study of polarized hadron structure functions and the related proton spin issue. Experimentally, new measurements of the longitudinal spin-dependent structure functions on various targets in polarized deep inelastic lepton-hadron scattering became available. The polarized structure functions  $g_1^p(x)$  of the proton [1,2],  $g_1^n(x)$  of the neutron [3], and  $g_1^d(x)$  of the deuteron [4,5] have been measured recently. The original European Muon Collaboration (EMC) experiment on  $g_1^p(x)$  [6], which has triggered a great deal of interest in the proton spin structure, is confirmed by the new high-statistics experimental data. Theoretically, a direct first-principles lattice QCD calculation of the proton matrix elements of the axial vector current, which is free of the  $\eta'$  and related problems encountered before [7], also became available very recently [8,9]. The calculated quark spin is consistent with experiment. It is also evident from the lattice calculation that it is the disconnected diagram, which is presumably dominated by the axial anomaly, that explains why the total spin carried by the quarks in a polarized proton is smaller than naively expected.

In spite of the aforementioned progress, the extraction of spin-dependent parton distribution functions, especially for sea quarks and gluons, from the measured polarized hadron structure functions remains largely ambiguous and controversial. One main issue has to do with the debate of whether or not gluons contribute to  $\Gamma_1^p$ , the first moment of  $g_1^p(x)$ . Depending on the interpretation on the discrepancy between experiment and the naive expectation for  $\Gamma_1^p$  (i.e., the Ellis-Jaffe sum rule [10]), two different sets of polarized parton distributions are often presented in the literature in the fol-

lowing way. First, one makes some parametrizations for spin-dependent parton densities based on some plausible (model) constraints. Then fitting these parametrizations to the data of  $g_1^p(x)$ , etc., one obtains (i) a best fit of  $\Delta u(x)$ ,  $\Delta d(x)$ , and  $\Delta s(x)$  at fixed  $Q_0^2$  by assuming  $\Delta G(x, Q_0^2) = 0$  or (ii) a best fit of  $\Delta G(x)$  and the polarized valence distributions  $\Delta u_v(x)$ ,  $\Delta d_v(x)$  with no sea polarization.

However, most of the parton spin densities presented in the literature are problematic. First, model-independent QCD constraints on the valence spin densities  $\Delta u_v(x)$  and  $\Delta d_v(x)$  at  $x \rightarrow 1$  are not respected in many existing parametrizations. Second, most authors fail to employ a correct kernel  $\Delta \sigma^{\gamma G}(x)$ , the hard cross section for photon-gluon scattering, to evaluate the gluonic contribution to the proton structure function  $g_1^p(x)$ . As we are going to stress in Sec. II, whether or not gluons contribute to  $\Gamma_1^p$  is purely factorization dependent [11]. Once a factorization scheme is chosen, the "hard" kernel is completely fixed up to the factorization scale  $\mu_{\text{fact}}$ . A determination of parton spin distributions using any other kernels, for instance, the  $\delta$  kernel, is certainly not trustworthy.

In this paper we shall give a critical analysis of the polarized parton distributions. We first give a brief overview in Sec. II on the role of the hard-gluonic contribution to the first moment of the polarized proton structure function. Based on the gauge-invariant and chiral-invariant factorization schemes, we then proceed to extract parton spin distributions from the  $g_1^p(x)$  data in Secs. III and IV, respectively. Sections V and VI contain discussions and conclusions.

### II. FRAMEWORK

The sea-quark or anomalous gluonic interpretation for the violation of the Ellis-Jaffe sum rule depends on the factor-

ization scheme defined for the quark spin density and the cross section for photon-gluon scattering. Much of the factorization scheme dependence and the related issues are already addressed by Bodwin and Qiu [11]. To set up the notation and the results necessary for our purposes we will recapitulate the main points in Ref. [11].

The general expression of the proton structure function in the presence of QCD corrections to order  $\alpha_s$  is

$$g_1^p(x, Q^2) = \frac{1}{2} \sum_{i=1}^{n_f} e_i^2 \int_x^1 \frac{dy}{y} \left\{ \Delta q_i(y, Q^2) \left[ \delta \left( 1 - \frac{x}{y} \right) + \frac{\alpha_s(Q^2)}{2\pi} \Delta f_q \left( \frac{x}{y} \right) \right] - \frac{\alpha_s(Q^2)}{2\pi} \Delta \sigma^{\text{hard}} \left( \frac{x}{y} \right) \Delta G(y, Q^2) \right\}, \quad (1)$$

where  $\Delta f_q$  depends on the regularization scheme chosen. Since the unpolarized parton distributions are usually parametrized and fitted to data in the modified minimal subtraction (MS) scheme, it is natural to adopt the same regularization scheme for polarized parton distributions in which  $\Delta f_q(x) = f_q(x) - 4/3(1+x)$  and (see, e.g., [12])

$$f_q(x) = \frac{4}{3} \left[ (1+x^2) \left( \frac{\ln(1-x)}{1-x} \right)_+ - \frac{3}{2} \frac{1}{(1-x)_+} - \left( \frac{1+x^2}{1-x} \right) \ln x + 3 + 2x - \left( \frac{9}{2} + \frac{\pi^2}{3} \right) \delta(1-x) \right], \quad (2)$$

where the “+” distribution is given by

$$\int_0^1 g(x) \left( \frac{f(x)}{1-x} \right)_+ dx = \int_0^1 f(x) \frac{g(x) - g(1)}{1-x} dx. \quad (3)$$

The first moment of  $f_q(x)$  and  $\Delta f_q(x)$  is 0 and  $-2$ , respectively. The parton spin densities in Eq. (1) are defined by  $\Delta q(x) = q^\uparrow(x) + \bar{q}^\uparrow(x) - q^\downarrow(x) - \bar{q}^\downarrow(x)$  and  $\Delta G(x) = G^\uparrow(x) - G^\downarrow(x)$ . It is known that a direct calculation of the polarized photon-gluon scattering box diagram indicates that  $\Delta \sigma(x)$  has collinear and infrared singularities when  $m^2 = p^2 = 0$ , where  $m$  is the quark mass and  $p^2$  is the four-momentum squared of the gluon. Depending on the choice of the soft cutoff, one obtains<sup>1</sup> (i)  $m^2 = 0$  and  $p^2 \neq 0$  [13],

$$\Delta \sigma_{\text{CCM}}(x) = (1-2x) \left( \ln \frac{Q^2}{-p^2} + \ln \frac{1}{x^2} - 2 \right), \quad (4)$$

(ii)  $m^2 \neq 0$  and  $p^2 = 0$  [14],

<sup>1</sup>It is known that  $\int_0^1 \Delta \sigma(x) dx = 0$  in the MS scheme [11]. However, the “hard” part of  $\Delta \sigma(x)$  is dependent of the factorization scheme chosen, as elucidated below. For this reason we shall discuss various soft cutoff schemes.

$$\Delta \sigma_{\text{AR}}(x) = (1-2x) \left( \ln \frac{Q^2}{m^2} + \ln \frac{1-x}{x} - 1 \right) - 2(1-x), \quad (5)$$

and (iii) dimensional regularization<sup>2</sup> [12],

$$\Delta \sigma_{\text{R}}(x) = (1-2x) \left( \frac{1}{\epsilon} + \gamma_E + \ln \frac{Q^2}{4\pi\mu_{\text{MS}}^2} + \ln \frac{1-x}{x} - 1 \right) - 2(1-x), \quad (6)$$

where  $\mu_{\text{MS}}$  is a regulated scale in the minimal-subtraction scheme. For the first moment of  $\Delta \sigma(x)$ , it is easily seen that

$$\int_0^1 \Delta \sigma_{\text{CCM}}(x) dx = 1, \quad \int_0^1 \Delta \sigma_{\text{AR}}(x) dx = \int_0^1 \Delta \sigma_{\text{R}}(x) dx = 0. \quad (7)$$

The result (7) can be understood as follows. Any term which is antisymmetric under  $x \rightarrow 1-x$ , for instance, terms proportional to  $\text{const} \times (1-2x)$ , makes no contribution to  $\int_0^1 \Delta \sigma(x) dx$ , a consequence of chiral symmetry or helicity conservation, recalling that the gluon splitting function is of the form  $\Delta P_{qG}(x) = (2x-1)/2$ . However, there is a chiral-symmetry-breaking term proportional to  $(1-x)$  in the mass-regulator and dimensional regularization schemes, which compensates the hard contribution arising from the region  $k_\perp^2 \sim Q^2$ , where  $k_\perp$  is the transverse momentum of the quark in the photon-gluon box diagram.

Now, in order to consider hard-gluonic contributions to  $g_1^p(x)$  (by “hard,” we mean contributions with  $k_\perp^2 \gtrsim \mu_{\text{fact}}^2$ ), one has to introduce a factorization scale  $\mu_{\text{fact}}$  to subtract the unwanted soft contribution, i.e., the contribution arising from the distribution of quarks and antiquarks in a gluon:

$$\Delta \sigma^{\text{hard}}(x, Q^2/\mu_{\text{fact}}^2) = \Delta \sigma(x, Q^2) - \Delta \sigma^{\text{soft}}(x, \mu_{\text{fact}}^2). \quad (8)$$

In practice, one makes an approximate expression for the box diagrams that is valid for  $k_\perp^2 \ll Q^2$  and then introduces an ultraviolet cutoff on the integration variable  $k_\perp$  to ensure that only the region  $k_\perp^2 \leq \mu_{\text{fact}}^2$  contributes to the soft part [11]. The choice of the regulator specifies the factorization convention. When the ultraviolet cutoff is gauge invariant, it breaks chiral symmetry due to the presence of the axial anomaly and hence makes a contribution to  $\Delta \sigma^{\text{soft}}$ . Using the dimensional regulator for the ultraviolet cutoff it follows that [11]

<sup>2</sup>The last term  $-2(1-x)$  in Eqs. (5) and (6) was neglected in the original work of Altarelli and Ross [14] and of Ratcliffe [12], respectively. It arises from chiral symmetry breaking due to the  $m^2 \neq 0$  cutoff in the mass-regulator scheme and the violation of the identity  $\{\gamma_\mu, \gamma_5\} = 0$  in the dimensional regularization scheme when  $\epsilon \neq 0$ . One may argue that this contribution is soft, for example, in the mass-regulator scheme if  $m^2 \ll \mu_{\text{fact}}^2$  and hence it does not contribute to “hard”  $\Delta \sigma$ . However, the cancellation of the  $\ln(Q^2/m^2)$  term, which depends logarithmically on the soft cutoff, from different  $x$  regions is not reliable because chiral symmetry may be broken at some hadronic scale.

$$\begin{aligned}\Delta\sigma_{\text{CCM}}^{\text{soft}}(x) &= (1-2x)\left(-\frac{1}{\epsilon}-\gamma_E+\ln\frac{4\pi\mu_{\text{MS}}^2}{-p^2}\right. \\ &\quad \left.+\ln\frac{1}{x(1-x)}-1\right)+2(1-x), \\ \Delta\sigma_{\text{AR}}^{\text{soft}}(x) &= (1-2x)\left(-\frac{1}{\epsilon}-\gamma_E+\ln\frac{4\pi\mu_{\text{MS}}^2}{m^2}\right), \\ \Delta\sigma_{\text{R}}^{\text{soft}}(x) &= 0,\end{aligned}\quad (9)$$

for various soft cutoffs, and hence

$$\int_0^1\Delta\sigma_{\text{CCM}}^{\text{soft}}(x)dx=1,\int_0^1\Delta\sigma_{\text{AR}}^{\text{soft}}(x)dx=\int_0^1\Delta\sigma_{\text{R}}^{\text{soft}}(x)dx=0.\quad (10)$$

In the mass-regulator and dimensional-regulator schemes, the original soft contributions in (5) and (6) are canceled by the contribution from chiral symmetry breaking introduced by the ultraviolet cutoff. Therefore, in the gauge-invariant factorization scheme

$$\Delta\sigma^{\text{hard}}(x)=(1-2x)\left(\ln\frac{Q^2}{\mu_{\text{fact}}^2}+\ln\frac{1-x}{x}-1\right)-2(1-x),\quad (11)$$

where  $\mu_{\text{fact}}^2=4\pi\mu_{\text{MS}}^2\exp(-\gamma_E-1/\epsilon)$ . It follows that  $\int_0^1\Delta\sigma^{\text{hard}}(x)dx=0$ . Note that  $\Delta\sigma^{\text{hard}}(x)$  is independent of the choice of soft and ultraviolet regulators. In this scheme, the quark spin has a gauge-invariant local operator definition:

$$s_\mu\Delta q=\langle p|\bar{q}\gamma_\mu\gamma_5q|p\rangle,\quad (12)$$

where  $s_\mu$  is the proton spin vector; it is  $Q^2$  dependent because of the nonvanishing two-loop anomalous dimension associated with the flavor-singlet quark operator. The fact that gluons do not contribute to  $\Gamma_1^p$  is in accordance with the operator product expansion (OPE) analysis in which only the quark operator contributes to  $\Gamma_1^p$  at the twist-2, spin-1 level [15]:

$$\begin{aligned}\int_0^1g_1^p(x)dx &= -\frac{1}{2}\left(1-\frac{\alpha_s}{\pi}\right)\langle p^\uparrow|\sum e_q^2\bar{q}\gamma_\mu\gamma_5q|p^\uparrow\rangle s^\mu \\ &= \frac{1}{2}\left(1-\frac{\alpha_s}{\pi}\right)\left(\frac{4}{9}\Delta u+\frac{1}{9}\Delta d+\frac{1}{9}\Delta s\right),\end{aligned}\quad (13)$$

where  $\Delta q\equiv\int_0^1\Delta q(x)dx$ .

By contrast, it is also possible to choose a chiral-invariant but gauge-variant ultraviolet cutoff, so that [11]

$$\begin{aligned}\Delta\tilde{\sigma}^{\text{hard}}(x) &= (1-2x)\left(\ln\frac{Q^2}{\mu_{\text{fact}}^2+m^2-p^2x(1-x)}\right. \\ &\quad \left.+\ln\frac{1-x}{x}-1\right)-(1-x)\frac{2m^2-p^2x(1-2x)}{\mu_{\text{fact}}^2+m^2-p^2x(1-x)}\end{aligned}\quad (14)$$

and  $\int_0^1\Delta\tilde{\sigma}^{\text{hard}}(x)dx=1$  for  $\mu^2\gg p^2$ ,  $m^2$ . In this chiral-invariant factorization scheme, the quark spin distributions in a gluon are obtained by a direct cutoff on the  $k_\perp$  integration:

$$\Delta q'^G(x,\mu_{\text{fact}}^2)=\int_0^{\mu_{\text{fact}}^2}d^2k_\perp\Delta q^G(x,k_\perp); \quad (15)$$

that is, all the quarks with  $k_\perp^2\lesssim\mu_{\text{fact}}^2$  in the gluon distribution are factored into the quark spin distribution. Contrary to the first scheme,  $\Delta q'\equiv\int_0^1\Delta q'(x)dx$  cannot be written as a matrix element of a gauge-invariant local operator,<sup>3</sup> but it is  $Q^2$  independent as the gauge-variant ultraviolet cutoff in this scheme does not flip helicity; it is thus close and parallel to the naive intuition in the parton model that the quark helicity is not affected by gluon emissions. Replacing  $\Delta q(x)$  by  $\Delta q'(x)$  and  $\Delta\sigma^{\text{hard}}(x)$  by  $\Delta\tilde{\sigma}^{\text{hard}}(x)$  in Eq. (1), we find

$$\int_0^1g_1^p(x)dx=\frac{1}{2}\left(1-\frac{\alpha_s}{\pi}\right)\sum e_q^2\left(\Delta q'-\frac{\alpha_s}{2\pi}\Delta G\right).\quad (16)$$

Consequently,  $\Delta q$  and  $\Delta q'$  are related by

$$\Delta q=\Delta q'-\frac{\alpha_s}{2\pi}\Delta G.\quad (17)$$

Finally, we notice that it is also possible to choose an intermediate ultraviolet cutoff scheme which is neither gauge nor chiral invariant, so in general  $\Delta q=\Delta q'-\lambda(\alpha_s/2\pi)\Delta G$  for arbitrary  $\lambda$  ( $\lambda=0$  and  $\lambda=1$  corresponding to gauge- and chiral-invariant factorization schemes, respectively) [17]. It is clear that the issue of whether or not gluons contribute to  $\Gamma_1^p$  is purely a matter of the factorization scheme chosen in defining the quark spin density and the hard gluon-photon scattering cross section; a change of the factorization convention merely shifts the contribution of  $\Delta q(x)$  and  $\Delta\sigma^{\text{hard}}(x)$  in such a way that the physical proton-photon cross section remains unchanged. Although this controversy was resolved sometime ago by Bodwin and Qiu [11] (see also Manohar [18], Bass and Thomas [16]), it is considerably unfortunate that many of recent articles are still biased on the anomalous-gluonic interpretation of the  $g_1^p(x)$  data and that the work of Bodwin and Qiu is either overlooked or not widely recognized and well appreciated in the literature.

### III. POLARIZED PARTON DISTRIBUTIONS IN THE GAUGE-INVARIANT FACTORIZATION SCHEME

This section is devoted to studying the spin-dependent valence and sea distributions based on the  $g_1^p(x)$  data. We shall see that the positivity condition  $|\Delta s(x)|\leq s(x)$  due to the positivity of unpolarized parton distribution puts a very

<sup>3</sup>Other main disparities between  $\Delta q$  and  $\Delta q'$  are as follows. (i) It is perhaps less known that [16] the spin-dependent Altarelli-Parisi evolution equations apply directly only to the *gauge-invariant* parton spin distributions. To evaluate the  $Q^2$  evolution of  $\Delta q'(x)$  and  $\Delta q'$ , one has to first apply Eq. (36) for example. (ii) In principle,  $\Delta q'$  and  $\Delta G$  have a simple partonic definition: the former (latter) can be identified in one-jet (two-jet) events in polarized deep inelastic scattering [13].

useful constraint on the shape of the polarized valence quark distributions. The presence of the gluon polarization will affect the shape of  $\Delta s(x)$ , but not its first moment.

To begin with, the combination of all EMC, Spin Muon Collaboration (SMC), E142, and E143 data for  $\Gamma_1^p$  together with the SU(3) parameters [19]  $F+D=1.2573\pm 0.0028$  and  $3F-D=0.579\pm 0.026$  yields, at  $\langle Q^2 \rangle = 10 \text{ GeV}^2$  [20],

$$\begin{aligned} \Delta u &= 0.83 \pm 0.03, & \Delta d &= -0.43 \pm 0.03, \\ \Delta s &= -0.10 \pm 0.03, \end{aligned} \quad (18)$$

and, hence,

$$\Delta \Sigma \equiv \Delta u + \Delta d + \Delta s = 0.31 \pm 0.07. \quad (19)$$

Decomposing  $\Delta q$  into its valence and sea components  $\Delta q = \Delta q_v + \Delta q_s$ , we shall follow Ref. [21] to assume that sea polarization is SU(3) invariant, i.e.,  $\Delta u_s = \Delta d_s = \Delta s$ . This assumption is justified since it leads to  $\Delta u_v + \Delta d_v = 0.60$  from Eq. (18), which is very close to the naive expectation that  $\Delta \Sigma = 3F - D = 0.579$  in the absence of sea polarization. The sea polarization is also found to be SU(3) symmetric within errors in the lattice calculation [8,9]. This is understandable since the disconnected insertions (for a definition of connected and disconnected insertions, see [8,9]), from which the sea-quark polarization originates, are presumably dominated by the triangle diagram and hence are independent of the light quark masses in the loop. (This effect is absent in unpolarized distributions.) Therefore, for SU(3) symmetric sea polarization, we obtain from (18) that

$$\Delta u_v = 0.93, \quad \Delta d_v = -0.33. \quad (20)$$

The Monte Carlo computation [8,9] shows that the magnitude of valence quark polarizations arising from the connected diagram is close to that given by (20).

In terms of valence and sea spin distributions, Eq. (1) can be recast to the form

$$\begin{aligned} g_1^p(x, Q^2) &= \frac{1}{2} \int_x^1 \frac{dy}{y} \left\{ \left[ \frac{4}{9} \Delta u_v(y, Q^2) + \frac{1}{9} \Delta d_v(y, Q^2) \right. \right. \\ &\quad \left. \left. + \frac{2}{3} \Delta s(y, Q^2) \right] \left[ \delta \left( 1 - \frac{x}{y} \right) + \frac{\alpha_s(Q^2)}{2\pi} \Delta f_q \left( \frac{x}{y} \right) \right] \right. \\ &\quad \left. - \frac{\alpha_s(Q^2)}{6\pi} \Delta \sigma^{\text{hard}} \left( \frac{x}{y}, \frac{Q^2}{\mu_{\text{fact}}^2} \right) \Delta G(y, Q^2) \right\}. \quad (21) \end{aligned}$$

Recall that in the gauge-invariant factorization scheme, gluons contribute to  $g_1^p(x)$ , but not to  $\Gamma_1^p$ . In general, both sea quarks and gluons contribute to the polarized structure function, but we will begin with the extreme case (i)  $\Delta s(x) \neq 0$ ,  $\Delta G(x) = 0$ . Since the unpolarized sea distribution is small at  $x > 0.2$ , the positivity constraint  $|\Delta s(x)| \leq s(x)$  implies that the data of  $g_1^p(x)$  at  $x > 0.2$  should be almost accounted for by  $\Delta u_v(x)$  and  $\Delta d_v(x)$ . Therefore, the shape of the spin-dependent valence quark densities is nicely restricted by the measured  $g_1^p(x)$  at  $x > 0.2$  together with the first-moment constraint (20) and the perturbative QCD requirement [22] that valence quarks at  $x=1$  remember the spin of the parent proton, i.e.,  $\Delta u_v(x)/u_v(x)$ ,  $\Delta d_v(x)/d_v(x) \rightarrow 1$  as  $x \rightarrow 1$ . In

order to ensure the validity of the positivity condition  $|\Delta q_v(x)| \leq q_v(x)$ , we choose the Martin-Roberts-Stirling set  $A'[\text{MRS}(A')]$  [23] parametrized in the  $\overline{\text{MS}}$  scheme at  $Q^2 = 4 \text{ GeV}^2$  as unpolarized valence parton distributions:

$$\begin{aligned} u_v(x, Q^2 = 4 \text{ GeV}^2) &= 2.26x^{-0.441}(1-x)^{3.96}(1-0.54\sqrt{x} \\ &\quad + 4.65x), \\ d_v(x, Q^2 = 4 \text{ GeV}^2) &= 0.279x^{-0.665}(1-x)^{4.46}(1+6.80\sqrt{x} \\ &\quad + 1.93x). \end{aligned} \quad (22)$$

Accordingly, we must employ the same  $\overline{\text{MS}}$  scheme for polarized parton distributions [see Eq. (1)] in order to apply the positivity constraint. For the spin-dependent valence distributions we assume that they have the form

$$\Delta q_v(x) = x^\alpha (1-x)^\beta (a + b\sqrt{x} + cx + dx^{1.5}), \quad (23)$$

with  $\alpha$  and  $\beta$  given by Eq. (22). We find that an additional term proportional to  $x^{1.5}$  is needed in (23) in order to satisfy the above three constraints.

For the data of  $g_1^p(x)$ , we will use the SMC [1] and EMC [6] results, both being measured at the mean value of  $Q_0^2 = 10 \text{ GeV}^2$ . Following the SMC analysis we have used the new  $F_2(x)$  structure function measured by NMC [24], which has a better accuracy at low  $x$ , to update the EMC data (see Fig. 1). The best least  $\chi^2$  fit to  $g_1^p(x)$  at  $x \geq 0.2$  by (23) is found to be<sup>4</sup>

$$\begin{aligned} \Delta u_v(x, Q_0^2) &= x^{-0.441}(1-x)^{3.96}(0.928 + 0.149\sqrt{x} - 1.141x \\ &\quad + 11.612x^{1.5}), \\ \Delta d_v(x, Q_0^2) &= x^{-0.665}(1-x)^{4.46}(-0.038 - 0.43\sqrt{x} - 5.260x \\ &\quad + 8.443x^{1.5}), \end{aligned} \quad (24)$$

at  $Q_0^2 = 10 \text{ GeV}^2$ , which satisfies all aforementioned constraints.<sup>5</sup> Since  $\Delta d_v$  is negative while  $\Delta d_v(x)$  is positive as  $x \rightarrow 1$ , it means that the sign of  $\Delta d_v(x)$  flips somewhere at  $x = x_0$  [25]. We find that  $x_0 = 0.496$  in our case.

It is evident from Fig. 1 that a negative sea polarization is required to explain the observed  $g_1^p(x)$  at small  $x$ . Assuming  $\Delta G(x, Q_0^2) = 0$  at this moment, we find from (21), (24), and the data of  $g_1^p(x)$  that the polarized strange quark distribution is determined to be

$$\begin{aligned} \Delta s(x, Q_0^2) &= -x^{-1.17}(1-x)^{9.63}(0.013\sqrt{x} + 0.862x \\ &\quad - 1.186x^{1.5}), \end{aligned} \quad (25)$$

<sup>4</sup>It was assumed in Ref. [21] that  $\Delta u_v(x) = \alpha(x)u_v(x)$ ,  $\Delta d_v(x) = \beta(x)d_v(x)$  with  $\alpha(x), \beta(x) \rightarrow 1$  as  $x \rightarrow 1$  and  $\alpha(x), \beta(x) \rightarrow 0$  as  $x \rightarrow 0$ . However, the constraint at  $x=0$  is not a consequence of QCD. In the present work we find that  $\Delta u_v(x)/u_v(x) = 0.41$  and  $\Delta d_v(x)/d_v(x) = -0.136$  at  $x=0$ . As a result,  $|\Delta q_v(x)|$  is usually larger than  $|\Delta s(x)|$  even at very small  $x$  (see Fig. 5 below).

<sup>5</sup>We have evolved  $q_v(x, Q^2)$  from  $Q^2 = 4 \text{ GeV}^2$  to  $10 \text{ GeV}^2$  in order to compare with  $\Delta q_v(x, Q_0^2)$ .

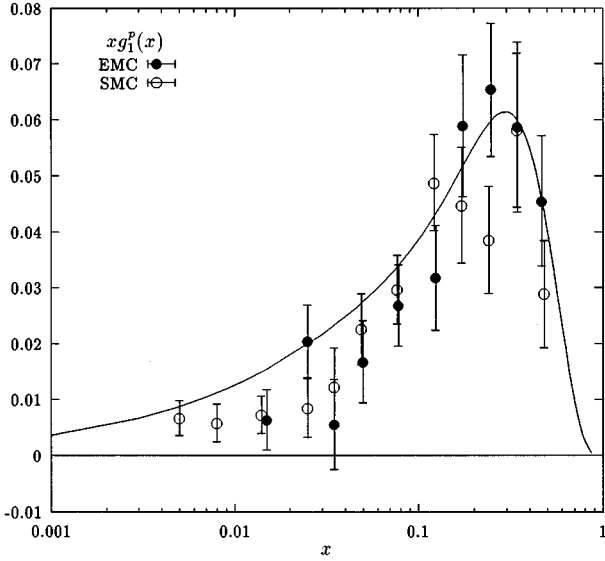


FIG. 1. The theoretical curve of  $xg_1^p(x)$  fitted to the EMC and SMC data at  $x \geq 0.2$  with the polarized valence quark distributions given by (24) and without sea and gluon polarizations.

with  $\Delta s = -0.109$  and  $\chi^2/N_{\text{DF}} = 12.24/22$ , where uses of  $\mu_{\text{fact}} \sim 1 \text{ GeV}$  and  $Q^2 = Q_0^2 = 10 \text{ GeV}^2$  have been made. It is easily seen from Fig. 2 that the positivity condition  $|\Delta s(x)/s(x)| \leq 1$  is respected.

To illustrate the importance of having a least  $\chi^2$  fit of  $g_1^p(x)$  at  $x \geq 0.2$  by  $\Delta u_v(x)$  and  $\Delta d_v(x)$ , let us consider another parametrization as an example<sup>6</sup>:

$$\begin{aligned} \Delta u_v(x) &= 0.3588x^{-0.54}(1-x)^{3.64}(1+18.36x), \\ \Delta d_v(x) &= -0.1559x^{-0.54}(1-x)^{4.64}(1+18.36x), \end{aligned} \quad (26)$$

with  $\Delta u_v = 0.93$  and  $\Delta d_v = -0.33$ . It is evident that, contrary to Fig. 1, this parametrization gives a reasonable eye-fit to the data (though  $\chi^2/N_{\text{DF}} = 30/22$ ) even at small  $x$ , as depicted in Fig. 3. One cannot tell if there is truly a discrepancy between theory and experiment unless the first moment of  $g_1^p(x)$  is calculated and compared with data, i.e.,  $(\Gamma_1^p)_{\text{theory}} = 0.176 \pm 0.006$  versus  $(\Gamma_1^p)_{\text{expt}} = 0.142 \pm 0.008 \pm 0.011$  [1]. Following the same procedure as before, we find that the sea polarization necessary to fit the data violates the positivity condition when  $x > 0.2$ . This example gives a nice demonstration that an eye-fit to the data can be quite misleading. Therefore, we conclude that in order to satisfy the positivity constraint due to sea polarization, valence quark spin densities should fully account for the observed  $g_1^p(x)$  at  $x \geq 0.2$ . As a consequence, a deviation of theory from experiment for the polarized structure function should manifest at small  $x$ .

It has been argued that a bound on  $\Delta s$ , namely,  $|\Delta s| \leq 0.052_{-0.052}^{+0.023}$  [27], can be derived based on the information of the behavior of  $s(x)$  measured in deep inelastic

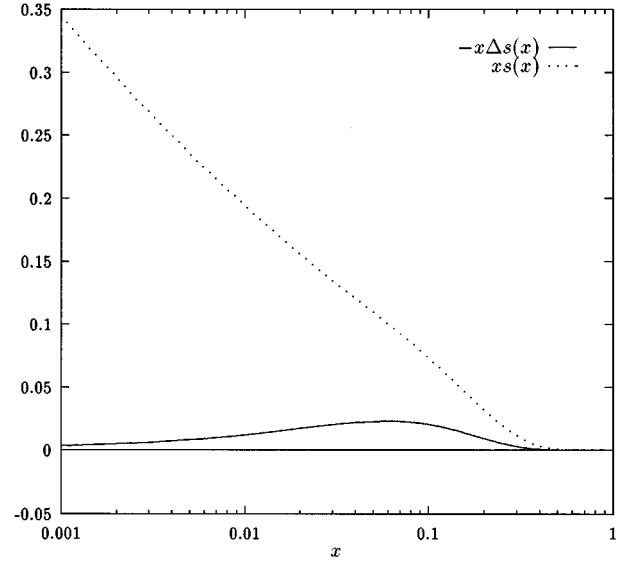


FIG. 2. The polarized strange quark distribution  $-\Delta s(x)$  fitted to the data of  $g_1^p(x)$ . Also shown is the unpolarized strange quark distribution evaluated at  $Q^2 = 10 \text{ GeV}^2$  using the MRS(A') parametrization [23].

neutrino experiments and on the positivity constraint. However, this argument is quite controversial [28]. We note that since the strange quark distribution parametrized by MRS(A') [23] yields  $\int_0^1 xs(x)dx = 0.0182$  for the strange sea momentum, it is consistent with the bound  $\int_0^1 xs(x)dx \leq 0.048 \pm 0.022$  extracted from the neutrino-nucleon experiment [29]. Hence, our  $\Delta s(x)$  does satisfy all known constraints. More importantly, a sea polarization of order  $-0.11$  in the polarized proton is confirmed by lattice calculations [8,9].

In a realistic case, it is very unlikely that  $\Delta G(x, Q^2)$  vanishes at some scale  $Q_0^2$  for all  $x$ . Even if  $\Delta G(x, Q_0^2) = 0$  at  $Q^2 = Q_0^2$ , it can be radiatively generated at  $Q^2 > Q_0^2$ . In the absence of any information on the shape of the magnitude of gluon polarization except for the restriction  $|\Delta G(x)/G(x)| \leq 1$ , we first take

$$\Delta G(x, Q_0^2) = 2.5A_G(1-x)^{7.44}, \quad (27)$$

with  $A_G = 8.44$  and  $\Delta G = 2.5$ , as an illustration. This parametrization is taken from the set A of gluon distribution in Ref. [25] but with a different normalization for our purpose. We see from Fig. 4 that the effect of polarized gluons is to suppress  $g_1^p(x)$  at  $x \leq 0.01$  and enhance  $g_1^p(x)$  at  $0.01 < x < 0.15$  so that the net contribution to  $\Gamma_1^p$  vanishes; that is, hard gluons contribute to  $g_1^p(x)$  but not to  $\Gamma_1^p$  in the gauge-invariant factorization scheme. Since a realistic polarized gluon distribution ought to have its first moment lie somewhere between 0 and 2.5, we take

$$\begin{aligned} \Delta G(x, Q_0^2) &= 0.199x^{-1.17}(1-x)^{5.33}(0.03 - 1.71\sqrt{x} + 3.01x \\ &\quad + 43.5x^{1.5}), \end{aligned} \quad (28)$$

as determined below for case (iv). The first moment of this gluon spin density is  $\Delta G = 0.5$ . The presence of  $\Delta G(x)$  will

<sup>6</sup>This parametrization is taken from Ref. [26] expect that we have made a different normalization in order to satisfy the first-moment constraint (20).

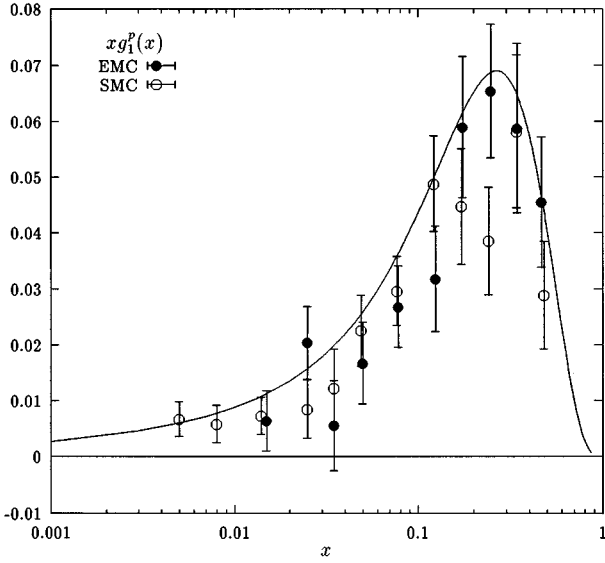


FIG. 3. The predicted curve of  $xg_1^p(x)$  arising from the spin-dependent valence quark spin distributions given by (26) without sea and gluon contributions. At first sight, it appears to give a reasonable eye-fit to the data.

affect the shape of  $\Delta s(x)$  but not its first moment. Following the same extracting procedure as before for  $\Delta s(x)$ , we find

$$\Delta s(x, Q_0^2) = -x^{-1.17}(1-x)^{9.63}(0.014\sqrt{x} + 0.865x - 1.189x^{1.5}), \quad (29)$$

with  $\Delta s = -0.11$  and  $\chi^2/N_{\text{DF}} = 11.75/22$ . The parametrizations (28) and (29) are regarded as the representative spin-dependent parton distributions for case (ii), as exhibited in Fig. 5.

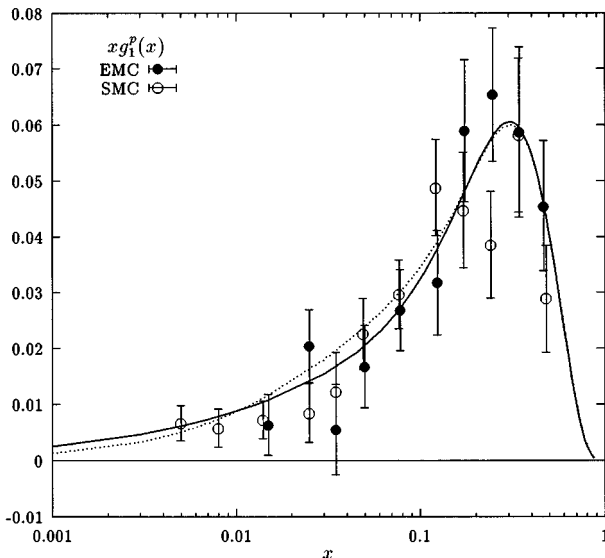


FIG. 4. Two theoretical curves for  $xg_1^p(x)$ . The solid line is the predicted curve for case (i) with  $\chi^2/N_{\text{DF}} = 12.24/22$  and the dotted curve with  $\chi^2/N_{\text{DF}} = 14.95/22$  is for case (i) plus the polarized gluon distribution given by (27).

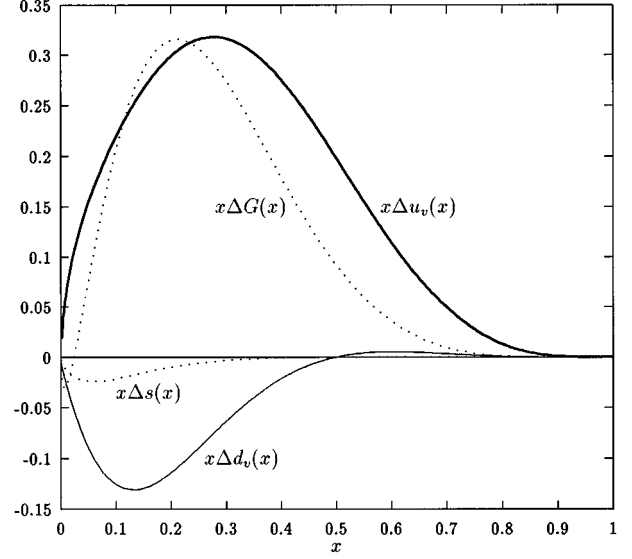


FIG. 5. Parton spin distributions for case (ii) parametrized at  $Q^2 = 10 \text{ GeV}^2$ .

#### IV. POLARIZED PARTON DISTRIBUTIONS IN THE CHIRAL-INVARIANT FACTORIZATION SCHEME

As elaborated on in Sec. II, in the chiral-invariant factorization scheme the quark spin  $\Delta q'$  is  $Q^2$  independent, and gluons contribute to the first moment of the polarized proton structure function. Since  $\Delta q' = \Delta q + (\alpha_s/2\pi)\Delta G$  and  $\Delta s_v = 0$ , it is obvious that  $\Delta q'_v = \Delta q_v$  for SU(3) symmetric sea polarization. We shall follow Ref. [21] to assume that this is also true for their  $x$  dependence, i.e.,  $\Delta q'_v(x) = \Delta q_v(x)$ . Since  $\Delta G(x)$  is also independent of the factorization chosen, we thus have

$$g_1^p(x, Q^2) = \frac{1}{2} \int_x^1 \frac{dy}{y} \left\{ \left[ \frac{4}{9} \Delta u_v(y, Q^2) + \frac{1}{9} \Delta d_v(y, Q^2) + \frac{2}{3} \Delta s'(y, Q^2) \right] \left[ \delta \left( 1 - \frac{x}{y} \right) + \frac{\alpha_s(Q^2)}{2\pi} \Delta f_q \left( \frac{x}{y} \right) \right] - \frac{\alpha_s(Q^2)}{6\pi} \Delta \tilde{\sigma}^{\text{hard}} \left( \frac{x}{y}, \frac{Q^2}{\mu_{\text{fact}}^2} \right) \Delta G(y, Q^2) \right\}, \quad (30)$$

with  $\Delta \tilde{\sigma}^{\text{hard}}$  being given by (14). We note that several different expressions for the kernel have been employed in the literature. For example,  $\Delta \tilde{\sigma}(z) = \delta(1-z)$  was used by Altarelli and Stirling [30],  $\Delta \tilde{\sigma}(z) = (1-2z)\ln[(1-z)/z]$  by Ellis *et al.* [31], and by Ross and Roberts [32]. However, as we have stressed in Sec. II, a correct procedure of subtracting the soft contribution from  $\Delta \tilde{\sigma}(z)$  will yield a unique  $\Delta \tilde{\sigma}^{\text{hard}}(z)$  up to the factorization scale  $\mu_{\text{fact}}$ , which is independent of the choice of soft and ultraviolet regulators.

We first discuss the extreme case, namely, (iii)  $\Delta s'(x) = 0$  and  $\Delta G(x) \neq 0$ , which is just opposite to the other extreme case (i). It has been advocated that [33,13] a total absence of sea polarization and an anomalous gluonic contribution might offer an attractive and plausible solution to the so-called ‘‘proton spin crisis’’ by accounting for the discrepancy between the Ellis-Jaffe sum rule and experiment

for  $\Gamma_1^p$ . It follows from (18) that  $\Delta\Sigma' = 0.58$  with  $\Delta s' = 0$ , consistent with what expected from the relativistic quark model. To implement a large  $\Delta\Sigma'$  and a vanishing  $\Delta s'$  demands a large gluon polarization:  $\Delta G = -(2\pi/\alpha_s)\Delta s = 2.5$  at  $Q^2 = 10 \text{ GeV}^2$ . Can the data of  $g_1^p(x)$  be explained solely by  $\Delta u_v(x)$ ,  $\Delta d_v(x)$ , and  $\Delta G(x)$  without sea polarization? To examine this issue we note that the gluon polarization is subject to the constraint

$$J(x, Q^2) = -\frac{\alpha_s}{6\pi} \int_x^1 \frac{dy}{y} \Delta\tilde{\sigma}^{\text{hard}}\left(\frac{x}{y}, \frac{Q^2}{\mu_{\text{fact}}^2}\right) \Delta G(y, Q^2), \quad (31)$$

where

$$\begin{aligned} J(x, Q^2) = & g_1^p(x, Q^2) - \frac{1}{2} \int_x^1 \frac{dy}{y} \\ & \times \left[ \frac{4}{9} \Delta u_v(y, Q^2) + \frac{1}{9} \Delta d_v(y, Q^2) \right] \\ & \times \left[ \delta\left(1 - \frac{x}{y}\right) + \frac{\alpha_s(Q^2)}{2\pi} \Delta f_q\left(\frac{x}{y}\right) \right]. \quad (32) \end{aligned}$$

Apart from the positivity constraints, can one treat  $\Delta u_v(x)$ ,  $\Delta d_v(x)$ , and  $\Delta G(x)$  as free parameters and fit them to the measured  $g_1^p(x)$ ? The point is that when one works in the gauge-invariant factorization scheme, the shape of the polarized valence quark distributions, which is factorization scheme independent, is constrained by the positivity condition  $|\Delta s(x)| \leq s(x)$ , in particular in the region  $x \geq 0.2$ . Therefore, the left-hand side of (31) is basically fixed by the data of  $g_1^p(x)$  and the phenomenological  $\Delta u_v(x)$  and  $\Delta d_v(x)$  given by (24). The polarized gluon distribution can be extracted from the Mellin transformation of (31) (for a detail of the procedure, see Ref. [21]).

The best least squares fit we found (see Fig. 6) for  $\mu_{\text{fact}} \sim 1 \text{ GeV}$  and  $Q^2 = Q_0^2 = 10 \text{ GeV}^2$  is

$$\begin{aligned} \Delta G(x, Q_0^2) = & x^{-1.17}(1-x)^{5.33}(0.03 - 1.71\sqrt{x} + 3.01x \\ & + 43.5x^{1.5}) \quad (33) \end{aligned}$$

with  $\chi^2/N_{\text{DF}} = 10.4/22$  and  $\Delta G = 2.51$ . There are two salient features with this spin-dependent gluon density: (i)  $\Delta G(x)$  is negative at very small  $x$ ,  $x < 0.025$ . This is because the best  $\chi^2$  fit to  $J(x)$  is positive at small  $x$ . We find that  $\Delta G(x) = 0$  corresponds to a maximum  $xJ(x)$  occurred at  $x \sim 0.025$ . Consequently, a negative behavior of  $\Delta G(x)$  at very small  $x$  is natural. (ii) the positivity constraint  $|\Delta G(x)| \leq G(x)$  is violated at  $x > 0.15$  (see Fig. 6). Hence, this  $\Delta G(x)$  is physically unacceptable. However, we note that a fit to the  $g_1^p(x)$  data with the polarized gluon distribution (27), which does respect the positivity condition, is equally acceptable with  $\chi^2/N_{\text{DF}} = 14.13/22$  (see the thick solid curve in Fig. 7). We thus conclude that it is still possible to reproduce the data of  $g_1^p(x)$  with anomalous gluonic contributions (of course, the shape of the gluon spin distribution is basically arbitrary), yet a best least  $\chi^2$  fit to data with  $\chi^2/N_{\text{DF}} = 10.4/22$  demands a polarized gluon distribu-

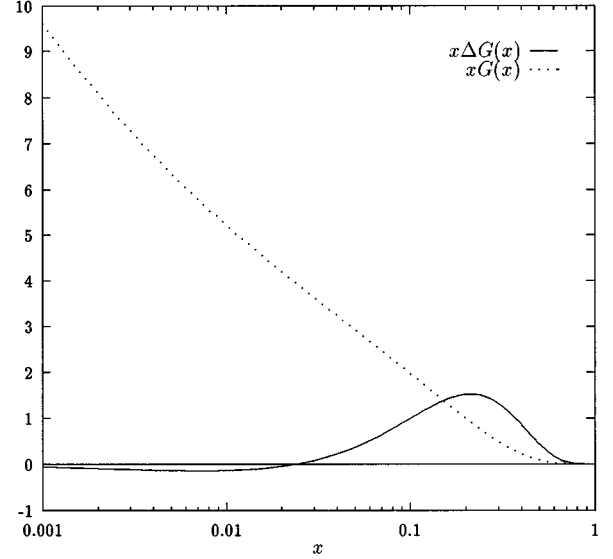


FIG. 6. The polarized gluon distribution extracted from a best least  $\chi^2$  fit to the data of  $g_1^p(x)$  by assuming  $\Delta s'(x) = 0$ . Also shown is the unpolarized gluon distribution evaluated at  $Q^2 = 10 \text{ GeV}^2$  using the MRS(A') parametrization [23].

tion violating the positivity constraint. Needless to say, we have to await high-quality data in the future to pin down the issue.

There exist in the literature various parametrizations for polarized parton distribution functions fitted to the data within the framework of the chiral-invariant factorization scheme. However, most of them are not reliable or trustworthy owing to the incorrect use of the hard cross section  $\Delta\tilde{\sigma}^{\text{hard}}$  for photon-gluon scattering, among other things. For

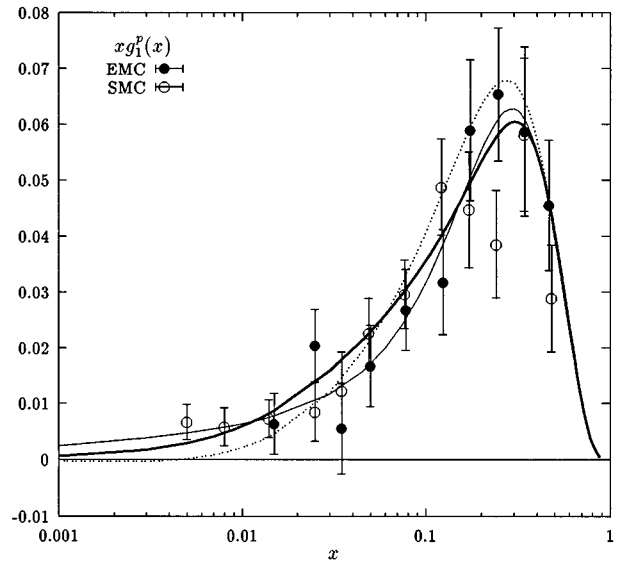


FIG. 7. Three theoretical curves for  $xg_1^p(x)$ . With (27) for the polarized gluon distribution, the thick solid curve is calculated using (24) for valence quark spin distributions and (14) for the kernel, while the solid and dotted curves are based on (26) for  $\Delta q_v(x)$  and the  $\Delta$  kernel  $\Delta\sigma(x) = \delta(1-z)$  for the former curve and the kernel (14) for the latter.

example, the predicted  $g_1^p(x)$  using (26) for  $\Delta q_v(x)$ , (27) for  $\Delta G(x)$  together with the  $\delta$  kernel  $\Delta\tilde{\sigma}^{\text{hard}}(z) = \delta(1-z)$  fits the data very well with  $\chi^2/N_{\text{DF}} = 11.9/22$  (see the solid curve in Fig. 7). But the same set of parton spin distributions fails to fit the data at small  $x$ ,  $x < 0.01$ , when the correct kernel (14) is employed (shown by the dotted curve in Fig. 7 with  $\chi^2/N_{\text{DF}} = 18.3/22$ ). This is because gluon contributions at small  $x$  gain more weight via the convolution with the non- $\delta$  kernel. Recall that the same set of polarized valence quark distributions also leads to an unacceptable sea polarization when fitted to the data (see Sec. III). Therefore, we believe that our valence quark spin distributions parametrized by (24) are more sensible than any others.

Since in a realistic case it is likely that sea polarization is nonvanishing and the value of  $\Delta G$  is between 0 and 2.5, this leads to the more realistic case (iv)  $\Delta s'(x) \neq 0$  and  $\Delta G(x) \neq 0$ . If we assume that the shape of  $\Delta G(x)$  remains the same as that of (33), the positivity condition of the gluon distribution requires that

$$\Delta G(x, Q_0^2) = 0.199x^{-1.17}(1-x)^{5.33}(0.03 - 1.71\sqrt{x} + 3.01x + 43.5x^{1.5}), \quad (34)$$

corresponding to  $\Delta G = 0.5$ . Substituting this into Eq. (30) determines  $\Delta s'(x)$ , which we find can be parametrized as

$$\Delta s'(x, Q_0^2) = -x^{-1.17}(1-x)^{9.63}(0.01\sqrt{x} + 0.69x - 0.949x^{1.5}) \quad (35)$$

with  $\Delta s' = -0.087$  and  $\chi^2/N_{\text{DF}} = 11.8/22$ .

## V. DISCUSSIONS

In Secs. III and IV we have considered four different cases for polarized parton distributions in the gauge-invariant and chiral-invariant factorization schemes: (i)  $\Delta s(x) \neq 0$ ,  $\Delta G(x) = 0$ , (ii)  $\Delta s(x) \neq 0$ ,  $\Delta G(x) \neq 0$ , (iii)  $\Delta s'(x) = 0$ ,  $\Delta G(x) \neq 0$ , and (iv)  $\Delta s'(x) \neq 0$ ,  $\Delta G(x) \neq 0$ . Since the value of  $\Delta G$  ought to lie somewhere between 0 and 2.5, it appears to us that case (ii) or case (iv) is more realistic. Note that cases (ii) and (iv) are not totally independent. This is because for a given  $\Delta G(x)$ , which is factorization scheme independent, the sea quark spin distributions  $\Delta q_s'(x)$  and  $\Delta q_s(x)$  are not independent and they are related via [21] (see also Eq. (57) of [16])

$$\Delta q_s'(x) = \Delta q_s(x) + \frac{\alpha_s}{\pi} \int_x^1 \frac{dy}{y} \left(1 - \frac{x}{y}\right) \Delta G(y), \quad (36)$$

derived from Eqs. (1), (11), and (14), where the assumption  $\Delta q_v'(x) = \Delta q_v(x)$  has been made (see Sec. IV). Clearly, its first moment is precisely Eq. (17), as it should be. We have explicitly checked that  $\Delta s(x)$  of (29) and  $\Delta s'(x)$  of (35) do satisfy the relation (36). The spin-dependent parton distributions in this work are presented in the  $\overline{\text{MS}}$  scheme in the next-to-leading order of QCD (for a similar work, see [34]).

It is straightforward to compute the polarized structure functions  $g_1^n(x)$  of the neutron,  $g_1^d(x)$  of the deuteron, and their first moments  $\Gamma_1^n$  and  $\Gamma_1^d$ , respectively. The various polarized distributions satisfy the relation

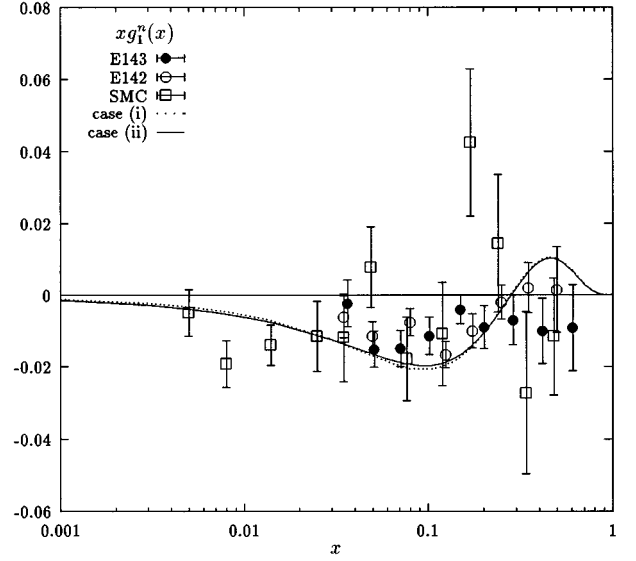


FIG. 8. The predicted polarized structure function  $g_1^n(x)$  of the neutron at  $Q^2 = 10 \text{ GeV}^2$  using the parton spin distributions in cases (i) and (ii). Also shown are the E142, E143, and SMC data at the average  $Q^2$  of each  $x$  bin.

$$g_1^p(x) + g_1^n(x) = \frac{2}{1 - 1.5\omega_D} g_1^d(x), \quad (37)$$

with  $\omega_D = 0.058$  being the probability that the deuteron is in a  $D$  state. We find

$$\Gamma_1^n = -0.053, \quad \Gamma_1^d = 0.040, \quad \text{at } Q^2 = 10 \text{ GeV}^2, \quad (38)$$

while, experimentally [1–6],

$$\Gamma_1^n = \begin{cases} -0.022 \pm 0.007 \pm 0.009, & \text{E142 at } \langle Q^2 \rangle = 2 \text{ GeV}^2, \\ -0.037 \pm 0.008 \pm 0.011, & \text{E143 at } \langle Q^2 \rangle = 3 \text{ GeV}^2, \\ -0.063 \pm 0.024 \pm 0.013, & \text{SMC at } \langle Q^2 \rangle = 10 \text{ GeV}^2, \end{cases} \quad (39)$$

and

$$\Gamma_1^d = \begin{cases} 0.034 \pm 0.009 \pm 0.006, & \text{SMC at } \langle Q^2 \rangle = 10 \text{ GeV}^2, \\ 0.042 \pm 0.003 \pm 0.004, & \text{E143 at } \langle Q^2 \rangle = 3 \text{ GeV}^2. \end{cases} \quad (40)$$

Shown in Figs. 8 and 9 are the predicted polarized structure functions  $xg_1^n(x)$  and  $xg_1^d(x)$ , respectively, at  $Q^2 = 10 \text{ GeV}^2$  using the parton spin distributions in cases (i) and (ii). We see that although our predictions are consistent in gross with experiments, new measurements of  $g_1^n(x)$  and  $g_1^d(x)$  with refined accuracy are certainly needed. Note that the  $Q^2$  dependence of polarized structure functions is not discussed here since our parton spin distributions are parametrized at  $Q_0^2 = 10 \text{ GeV}^2$  and it is known that only  $Q^2 > Q_0^2$  evolution is governed by the Altarelli-Parisi equations.

## VI. CONCLUSIONS

The fact that the size of the hard-gluonic contribution to  $\Gamma_1^p \equiv \int_0^1 g_1^p dx$  is purely a matter of the factorization conven-



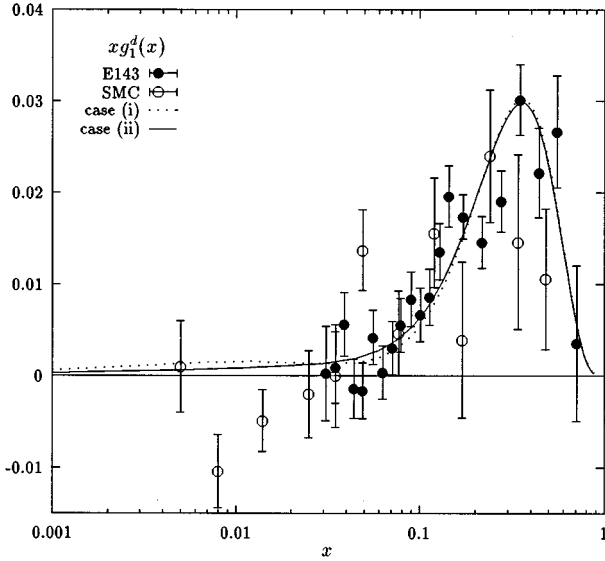


FIG. 9. Same as Fig. 8 except for the deuteron. The SMC data of  $g_1^d(x)$  are evaluated at  $Q^2 = 10 \text{ GeV}^2$ , while E143 data at the average  $Q^2$  of each  $x$  bin.

tion chosen in defining the quark spin distribution promotes us to consider four different possibilities of polarized parton distribution functions in two extreme factorization schemes: gauge-invariant and chiral-invariant ones. One cannot tell experimentally whether or not gluons contribute to  $\Gamma_1^p$ . We stressed that the hard cross section for photon-gluon scattering is unique up to the factorization scale  $\mu_{\text{fact}}$  and is independent of the choice of the soft and ultraviolet regulators.

Owing to the positivity constraints for sea and gluon polarizations,  $g_1^p(x)$  at  $x \geq 0.2$  should receive almost all contributions from polarized valence quark distributions. This together with the first moment and perturbative QCD constraints puts a very nice restriction on the shape of  $\Delta u_v(x)$  and  $\Delta d_v(x)$ . Equation (24) is our best result for valence quark spin distributions at average  $\langle Q^2 \rangle = 10 \text{ GeV}^2$ . Working in the gauge-invariant factorization scheme, we have extracted the polarized sea distribution function from the EMC and SMC data of  $g_1^p(x)$  with the results (25) and (29) for cases (i) and (ii), respectively. All polarized parton distributions in this work are presented in the next-

leading order of QCD at the scale  $Q^2 = 10 \text{ GeV}^2$ .

Based on the chiral invariant scheme and the aforementioned valence quark spin densities, we have found that it is possible to explain the measurements of  $g_1^p(x)$  with anomalous gluonic contributions, yet a least  $\chi^2$  fit to the data indicates that the best fitted gluon spin distribution violates the positivity condition  $|\Delta G(x)| \leq G(x)$ . We have considered a more realistic set of parton spin distributions with a moderate value of  $\Delta G = 0.5$  and with a nonvanishing sea polarization. Many parametrizations of polarized parton distributions presented in the literature are not trustworthy due mainly to the use of an incorrect hard cross section for photon-gluon scattering.

In principle, the choice of the set of  $\Delta q(x)$ ,  $\Delta G(x)$ ,  $\Delta \sigma^{\text{hard}}(z)$  or of  $\Delta q'(x)$ ,  $\Delta G(x)$ ,  $\Delta \bar{\sigma}^{\text{hard}}(z)$  to describe the polarized hadron structure function is a matter of convention. In fact, for a given  $\Delta G(x)$ ,  $\Delta q'(x)$  and  $\Delta q(x)$  are related via Eq. (36). In practice, the gauge-invariant quantity  $\Delta q$  is probably more convenient and natural to use since it can be expressed as a nucleon matrix element of a local gauge-invariant operator. It is calculable in lattice QCD and, more importantly, its  $Q^2$  evolution is directly governed by the polarized Altarelli-Parisi equations, which is not the case for  $\Delta q'$  and  $\Delta q'(x)$  [see footnote 3 after Eq. (15)].

Of course, inclusive polarized deep inelastic scattering experiments alone cannot reveal the magnitude and shape of the gluon spin distribution, and one has to await measurements of  $\Delta G$  in independent processes in order to fully understand the proton spin structure. Nevertheless, there does exist a truly theoretical progress since the EMC measurement of  $g_1^p$ , namely, the lattice calculation of the proton matrix elements of the axial current [8,9]. The empirical SU(3) invariance observed by lattice QCD for the sea polarization manifested in the disconnected insertion strongly suggests that it is the axial anomaly which is responsible for the negative sea polarization and which explains the smallness of the quark spin content of the proton.

#### ACKNOWLEDGMENTS

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