Magnetic moments of the 3/2 resonances and their quark spin structure

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We discuss magnetic moments of the $J=3/2$ baryons based on an earlier model for the baryon magnetic moments, allowing for flavor symmetry breaking in the quark magnetic moments as well as a general quark spin structure. From our earlier analysis of the nucleon-hyperon magnetic moments and the measured values of the magnetic moments of Δ^{++} and Ω^- we predict the other magnetic moments and deduce the spin structure of the resonance particles. We find from experiment that the total spin polarization of the decuplet baryons, $\Delta\Sigma(3/2)$, is considerably smaller than the nonrelativistic quark model value of 3, although the data are still not good enough to give a precise determination.

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I. INTRODUCTION

The recent precise measurement of the magnetic moment of Ω^-, $\mu(\Omega^-)=(-2.024\pm0.056)\mu_N$, [1] gives new possibilities for theoretical studies of the magnetic moments of the decuplet baryons. These magnetic moments constitute an important domain for investigating baryon structure and have earlier been studied, e.g., in quenched lattice gauge theory [2], quark models [3], the chiral bag model [4], and chiral perturbation theory [5].

In an earlier paper [6] we introduced a general parametrization of the baryon magnetic moments in order to account for necessary modifications of the nonrelativistic quark model (NQM) description. In this paper we extend this parametrization to the $J=3/2$ resonance particles in the decuplet.

Although only two of the magnetic moments of these states have been measured, it is still in principle possible to obtain interesting information on the spin structure of the resonance particles from these data. We analyze the magnetic moments of the spin 3/2 particles in this model and make predictions for the magnetic moments of the resonance particles using values for the quark magnetic moments extrapolated from the nucleon-hyperon data in our previous analysis.

II. ANALYSIS OF MAGNETIC MOMENTS FOR THE SPIN 3/2 RESONANCES

The magnetic moment of a hadron in an isomultiplet B can, in the quark model, be written as a linear sum of contributions from the various flavors:

$$\mu(B^i)=\mu_u\Delta u^{B^i}+\mu_d\Delta d^{B^i}+\mu_s\Delta s^{B^i}, \tag{1}$$

where μ_f is an effective magnetic moment of the quark of flavor f and Δf^{B^i} is the corresponding spin polarization for baryon B^i, i being the baryon charge state. By symmetry arguments the Δf^{B^i}'s in the octet baryons can be expressed as constant linear combinations of the three Δf's for the proton, which are the only spin polarizations needed to describe the octet:

$$\Delta f^{B^i}=\sum_{f'} M(B^i)_{ff'}\Delta f', \tag{2}$$

where f,f' runs over u,d,s, and the $M(B^i)$'s are matrices with constant elements. In particular, for the six mirror symmetric baryons of type $B(xyy)$, where x and y are different flavors, we have $\Delta y^{B^i}=\Delta u$, $\Delta x^{B^i}=\Delta d$, and $\Delta z^{B^i}=\Delta s$, where the flavor z is the nonvalence quark flavor. In the NQM the values of these spin polarizations are $\Delta u=\frac{4}{3}$, $\Delta d=-\frac{1}{3}$, and $\Delta s=0$.

In the model we consider in Ref. [6], we let the quark magnetic moments be different in different baryon isomultiplets, but the same within each isospin multiplet [7]. When the quark magnetic moments are taken to be the same in all isomultiplets, the baryon magnetic moments are connected by sum rules. These sum rules are clearly violated by the experimental data, indicating that the quark magnetic moments are not the same in different isomultiplets. Lattice calculations [8] also indicate that they are different in different baryons. Thus, the quark magnetic moments will be denoted by μ_f^B, where f is the quark flavor and B is the baryon isomultiplet.

The spin structure variables in the magnetic moments are not *a priori* the same as in deep inelastic scattering experiments and axial-vector form factors. However, in many models [9] they are proportional to these spin polarizations. Since the equations for the magnetic moments are homogeneous in the quark magnetic moments and the spin polarizations, the quark magnetic moments can be deduced from experimental data by normalizing the spin polarizations with the value of the weak axial-vector form factor $g_A^{np}=\Delta u-\Delta d=1.2573$ [10]. The spin polarizations we deduce will then be the relevant ones for the spin as measured in deep inelastic scattering experiments. Later we will use data of the quark magnetic moments from the octet baryons, and these are normalized in this way.

In the decuplet there are 4 different isomultiplets and thus there are in principle 12 quark magnetic moments. We intro-

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duce the flavor-breaking parameters T and U describing the flavor breaking among the quarks, which is assumed to be the same for all isomultiplets. Thus $\mu_u^B = T\mu_d^B$ and $\mu_s^B = U\mu_d^B$ independently of B. In what follows we will assume that T and U are the same for both the octet and decuplet baryons.[1] This assumption can in principle be tested when more data of the decuplet magnetic moments are available. There are therefore 6 parameters for the 12 quark magnetic moments.

In each baryon there are also spin structure variables. In the octet there are three different ones, Δu, Δd, and Δs, defined as the spin polarization of the u, d, and s quarks in the proton in the spin up state. However, in the decuplet it turns out that there are only two different spin polarization parameters, denoted Δu and Δs. These are both defined for the Δ^{++} resonance in the $J_z = +3/2$ state and are normalized for convenience of notation with a factor of 3, so that $3\Delta u$, $3\Delta s$, and $3\Delta s$ are the quark spin polarizations in Δ^{++} of the u, d, and s quarks, respectively. The reason for there being only two different spin polarization parameters in the decuplet is that the spin structure there is much simpler than in the octet due to the flavor functions being fully symmetric, which does not allow mixed symmetry for the spins.

In the NQM the spin polarization parameters have for the decuplet the values $\Delta u = 1$ and $\Delta s = 0$. In a more sophisticated model, due to virtual quark antiquark pairs, gluonic corrections, spontaneously broken chiral symmetry, emission and capture of Goldstone bosons, etc., they might have more general values.

SU(3) relations are used to construct the spin structure of the states. The flavor breaking is assumed to occur in the quark magnetic moment operators by the effective masses and possible vertex corrections. The magnetic moments of the decuplet particles can then be written

$$\mu(\Delta^{++}) = \mu_d^{\Delta}[3T\Delta u + (3+3U)\Delta s], \tag{3a}$$

$$\mu(\Delta^{+}) = \mu_d^{\Delta}[(2T+1)\Delta u + (T+2+3U)\Delta s], \tag{3b}$$

$$\mu(\Delta^{0}) = \mu_d^{\Delta}[(T+2)\Delta u + (2T+1+3U)\Delta s], \tag{3c}$$

$$\mu(\Delta^{-}) = \mu_d^{\Delta}[3\Delta u + (3T+3U)\Delta s], \tag{3d}$$

$$\mu(\Sigma^{*+}) = \mu_d^{\Sigma^*}[(2T+U)\Delta u + (T+3+2U)\Delta s], \tag{3e}$$

$$\mu(\Sigma^{*0}) = \mu_d^{\Sigma^*}[(T+1+U)\Delta u + (2T+2+2U)\Delta s], \tag{3f}$$

$$\mu(\Sigma^{*-}) = \mu_d^{\Sigma^*}[(2+U)\Delta u + (3T+1+2U)\Delta s], \tag{3g}$$

$$\mu(\Xi^{*0}) = \mu_d^{\Xi^*}[(2U+T)\Delta u + (2T+3+U)\Delta s], \tag{3h}$$

$$\mu(\Xi^{*-}) = \mu_d^{\Xi^*}[(2U+1)\Delta u + (3T+2+U)\Delta s], \tag{3i}$$

$$\mu(\Omega^{-}) = \mu_d^{\Omega}[3U\Delta u + (3+3T)\Delta s]. \tag{3j}$$

Combining Eqs. (3b), (3c), (3e), (3g), (3h), and (3i) we get the system of equations

$$[3(T+1) - A'(T-1)]\Delta u + [3(T+1+2U) + A'(T-1)]\Delta s = 0, \tag{4a}$$

$$[T+1+U - B'(T-1)]\Delta u + [2(T+1+U) + B'(T-1)]\Delta s = 0, \tag{4b}$$

$$[1+T+4U - C'(T-1)]\Delta u + [5T+5+2U + C'(T-1)]\Delta s = 0, \tag{4c}$$

where

$$A' = \frac{\mu(\Delta^{+}) + \mu(\Delta^{0})}{\mu(\Delta^{+}) - \mu(\Delta^{0})}, \tag{5}$$

$$B' = \frac{\mu(\Sigma^{*+}) + \mu(\Sigma^{*-})}{\mu(\Sigma^{*+}) - \mu(\Sigma^{*-})}, \tag{6}$$

$$C' = \frac{\mu(\Xi^{*0}) + \mu(\Xi^{*-})}{\mu(\Xi^{*0}) - \mu(\Xi^{*-})}. \tag{7}$$

The set of equations (4) constitutes an overdetermined system. We therefore get two different equations from the conditions that the secular determinant of either (4a)+(4b) or (4b)+(4c) should vanish. Both of these contain the root $U = -1 - T$, which we discard. The other roots are

$$U = \frac{1}{2}[1 + T + (A' - 2B')(1-T)], \tag{8}$$

$$U = \frac{1}{2}[1 + T + (2B' - C')(1-T)]. \tag{9}$$

A similar analysis for the octet baryons gives the relation [6]

$$U = \frac{1}{2}[1 + T + D(1-T)], \tag{10}$$

where $D = \sqrt{AB - AC + BC} = (0.78 \pm 0.02)\mu_N$, and

$$A = \frac{\mu(p) + \mu(n)}{\mu(p) - \mu(n)}, \tag{11}$$

$$B = \frac{\mu(\Sigma^{+}) + \mu(\Sigma^{-})}{\mu(\Sigma^{+}) - \mu(\Sigma^{-})}, \tag{12}$$

$$C = \frac{\mu(\Xi^{0}) + \mu(\Xi^{-})}{\mu(\Xi^{0}) - \mu(\Xi^{-})}. \tag{13}$$

Our assumption that the symmetry breaking is the same in the decuplet as in the octet predicts the sum rules

$$A' - 2B' = 2B' - C' = (0.78 \pm 0.02)\mu_N. \tag{14}$$

[1] In the NQM the values of T and U are $T = \mu_u/\mu_d = -1.91$, $U = \mu_s/\mu_d = 0.63$ when the three quark magnetic moments are allowed to be free parameters.

In the present case there are two more baryons than in the nucleon-hyperon case and one spin polarization less. It might therefore seem possible to get the spin structure directly from the magnetic moment data. However, as we mentioned above, the equations are homogeneous in the quark moments and the spin polarizations. We therefore still lack a normalization for the spin polarizations. Hence, the set of equations above can only determine the relative magnitude of the spin polarizations and quark magnetic moments. Also the magnetic moment of Ω^- cannot be given in the model, since the value of μ_d^Ω is a free parameter. We therefore have nine data with seven parameters, constituting an overconstrained system, giving the possibility of a nontrivial test of the model.

For example using only the Δ resonances we can solve for the spin polarization in terms of μ_d^Δ. The relevant formulas are

$$3\Delta u=\frac{1}{\mu_d^\Delta(T-1)(T+U+1)}[(2T+1+3U)\mu(\Delta^+) -(T+2+3U)\mu(\Delta^0)], \tag{15}$$

$$3\Delta s=\frac{1}{\mu_d^\Delta(T-1)(T+U+1)}[-(T+2)\mu(\Delta^+) +(2T+1)\mu(\Delta^0)]. \tag{16}$$

The total spin polarization $\Delta\Sigma(3/2)$ is given by the sum of the coefficients of the quark magnetic moments, i.e., in our model of μ_d^B, $T\mu_d^B$, and $U\mu_d^B$. This gives for all 3/2 baryons $\Delta\Sigma(3/2)=3(\Delta u+2\Delta s)$. Thus

$$\Delta\Sigma(3/2)=\frac{3}{\mu_d^\Delta(T-1)(T+U+1)}[(U-1)\mu(\Delta^+) +(T-U)\mu(\Delta^0)]. \tag{17}$$

However, we can also express the spin sum in terms of two other of these moments as

$$\Delta\Sigma(3/2)=\frac{3}{\mu_d^\Delta(T-1)(T+U+1)}[(T+U-2)\mu(\Delta^{++}) +(2T-U-1)\mu(\Delta^-)]. \tag{18}$$

We will not pursue this line further here, since there is at present not enough data to test these relations.

III. PREDICTING DECUPLET MAGNETIC MOMENTS AND SPIN POLARIZATIONS FROM THE OCTET DATA

Lacking most of the data to carry out an analysis, we will from now on instead work to predict the magnetic moments of the 3/2 resonances using data from the nucleon-hyperon system. We will then also be able to obtain the quark spin polarizations of the spin 3/2 resonances.

As we mentioned above, the flavor symmetry breaking is assumed to be the same for the decuplet as for the octet. This assumption can in the future be tested in the way indicated above, by determining the constants T and U from the magnetic moments in the decuplet.

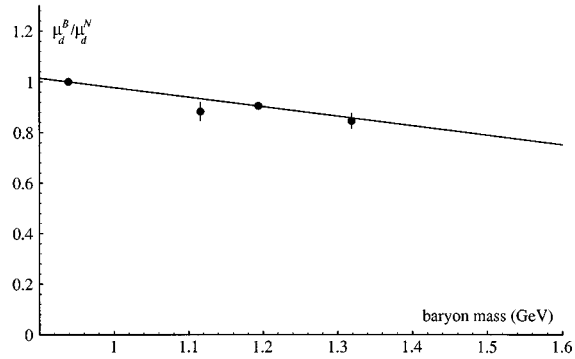

FIG. 1. μ_d^B/μ_d^N as a function of the mass of B. The straight line is a least squares fit to the four data points. If the line is extrapolated to Ω^-, all μ_d^B's for the decuplet can be found.

From the measured magnetic moments of the octet baryons we can calculate the μ_d^B's, once the spin polarizations are known. Since the value of $\Delta\Sigma$ for the octet (the nucleons) is still not too well determined, and does not give too precise a determination of T, we will take two typical cases, one consistent with the most recent data, and for the other we take $T=-2$, corresponding to no isospin symmetry breaking. New evaluations of $\Delta\Sigma$ for the nucleons give different although overlapping values. A recent determination by the Spin Muon Collaboration (SMC) [11] gives $\Delta\Sigma=0.20\pm0.11$, whereas Ellis and Karliner [12] have analyzed data to get $\Delta\Sigma=0.31\pm0.07$.

The values we will use are $\Delta\Sigma=0.27$, corresponding to $T=-1.80$ and $\Delta\Sigma=0.08$ for $T=-2$ (no isospin symmetry breaking). The formulas used to extract the μ_d^B's are [6]

$$\mu_d^N=\frac{\mu(p)-\mu(n)}{(T-1)g_A^{np}}, \tag{19}$$

$$\mu_d^\Lambda=\frac{6\mu(\Lambda)}{(T-1)g_A^{np}}\,\frac{B-C}{2AB-2AC+2CD-CB-BD}, \tag{20}$$

$$\mu_d^\Sigma=\frac{\mu(\Sigma^+)-\mu(\Sigma^-)}{(T-1)g_A^{np}}\,\frac{B-C}{D-C}, \tag{21}$$

$$\mu_d^\Xi=\frac{\mu(\Xi^0)-\mu(\Xi^-)}{(T-1)g_A^{np}}\,\frac{B-C}{D-B}, \tag{22}$$

where $g_A^{np}=1.2573$ and A, B, and C are given by (11)–(13).

In Fig. 1 the ratios μ_d^B/μ_d^N are plotted as a function of the mean mass of B. We see that they can be well fitted by a linear function, and we therefore extrapolate this function out to Ω^- and use the ensuing data set for the μ_d^B's, where the intermediate values are interpolated from this graph. These values are given in Table I. We will analyze this linear relation among the μ_d^B's in Sec. IV.

With the parameters of Table I we can now determine the quark spin polarizations in the decuplet from the measured magnetic moments of Δ^{++} and Ω^- and use this to predict the other magnetic moments.

The result is

TABLE I. The parameters used in the model. The decuplet quark magnetic moments are interpolated from a linear fit to the octet ones. The error of $0.05\mu_N$ for all decuplet moments is an estimate of the theoretical uncertainty. All moments are given in μ_N.

	Case $T=-2$	Case $T=-1.80$
$\mu_u/\mu_d \equiv T$	-2	-1.80
$\mu_s/\mu_d \equiv U$	0.67 ± 0.03	0.70 ± 0.03
μ_d^N	-1.25 ± 0.01	-1.34 ± 0.01
μ_d^Λ	-1.10 ± 0.05	-1.18 ± 0.05
μ_d^Σ	-1.13 ± 0.01	-1.21 ± 0.01
μ_d^Ξ	-1.06 ± 0.04	-1.13 ± 0.04
μ_d^Δ	-1.11 ± 0.05	-1.19 ± 0.05
$\mu_d^{\Sigma^*}$	-1.04 ± 0.05	-1.11 ± 0.05
$\mu_d^{\Xi^*}$	-0.97 ± 0.05	-1.04 ± 0.05
μ_d^Ω	-0.90 ± 0.05	-0.97 ± 0.05

$$3\Delta u = \frac{\mu_d^\Omega(1+T)\mu(\Delta^{++}) - \mu_d^\Delta(1+U)\mu(\Omega^-)}{\mu_d^\Delta\mu_d^\Omega[T(T+1)-U(U+1)]}, \quad (23)$$

$$3\Delta s = \frac{-\mu_d^\Omega U\mu(\Delta^{++}) + \mu_d^\Delta T\mu(\Omega^-)}{\mu_d^\Delta\mu_d^\Omega[T(T+1)-U(U+1)]}. \quad (24)$$

Inserting the experimental values given in Table II gives

$$3\Delta u = 0.4\pm1.1, \quad (25)$$

$$3\Delta s = -2.0\pm0.8, \quad (26)$$

$$\Delta\Sigma(3/2) = 3(\Delta u + 2\Delta s) = -3.6\pm2.6, \quad (27)$$

for $T=-2$, and

$$3\Delta u = -1.9\pm2.8, \quad (28)$$

$$3\Delta s = -4.3\pm2.5, \quad (29)$$

$$\Delta\Sigma(3/2) = 3(\Delta u + 2\Delta s) = -10.5\pm7.9, \quad (30)$$

for $T=-1.80$.

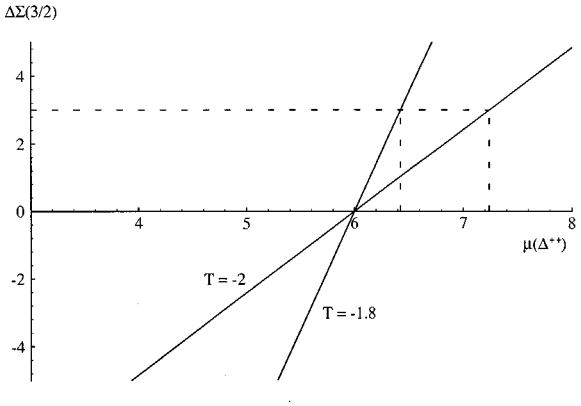

FIG. 2. $\Delta\Sigma(3/2)$ as a function of $\mu(\Delta^{++})$ for $T=-2$ and $T=-1.8$. We see that $\Delta\Sigma(3/2)=0$ for $\mu(\Delta^{++})=6.0\mu_N$ and that $\Delta\Sigma(3/2)=3$ for $\mu(\Delta^{++})=7.2\mu_N$ when $T=-2$, and for $\mu(\Delta^{++})=6.4\mu_N$ when $T=-1.8$. $\Delta\Sigma(3/2)$ depends very strongly on T, and therefore no precise prediction can be made.

The result indicates that the decuplet spin polarization with the present data taken at face value is large and negative. However, the error on the magnetic moment of Δ^{++} is rather large and the value might according to Ref. [10] be anywhere between $3.7\mu_N$ and $7.5\mu_N$. We have therefore instead plotted the value of $\Delta\Sigma(3/2)$ as a function of $\mu(\Delta^{++})$ in Fig. 2. From this we see that $\Delta\Sigma(3/2)$ passes zero at around $6.0\mu_N$ independently of the value of T. Only at $\mu(\Delta^{++})\approx7.2\mu_N$ does $\Delta\Sigma(3/2)$ reach the NQM value of 3 when $T=-2$. For $T=-1.8$, $\Delta\Sigma(3/2)$ reaches the value 3 at $\mu(\Delta^{++})\approx6.4\mu_N$.

We clearly need a better measurement of $\mu(\Delta^{++})$ to truly fix the value of $\Delta\Sigma(3/2)$, but in the face of existing data we feel confident to say that $\Delta\Sigma(3/2)$ is much smaller than the NQM value of 3, in analogy to the situation for the nucleon spin.

For completeness we have included a prediction of the magnetic moments of the decuplet, assuming that $\mu(\Delta^{++})\approx6.0\mu_N$, corresponding to $\Delta\Sigma(3/2)=0$, in Table II. This value also gives $3\Delta u=1.9$ and $3\Delta s=-0.9$, when $T=-2$, which looks more realistic.

Using (23) and (24) the magnetic moments of the decuplet particles can be predicted. The values are given in

TABLE II. Our prediction of the decuplet baryon magnetic moments, compared to the nonrelativistic quark model. Most of the errors in the predictions come from the large error in $\mu(\Delta^{++})$. In the last column we have used $\mu(\Delta^{++})=6.0\mu_N$ as an input corresponding to $\Delta\Sigma(3/2)=0$. The magnetic moments are given in μ_N.

	Experiment	NQM	Our predictions	
$\mu(\Delta^{++})$	4.5 ± 1.0 [15]	5.56	4.5 ± 1.0(input)	6.0(input)
$\mu(\Delta^{+})$	-	2.73	1.9 ± 0.6	2.8
$\mu(\Delta^{0})$	-	-0.09	-0.7 ± 0.3	-0.3
$\mu(\Delta^{-})$	-	-2.92	-3.3 ± 0.3	-3.5
$\mu(\Sigma^{*+})$	-	3.09	2.0 ± 0.6	3.0
$\mu(\Sigma^{*0})$	-	0.27	-0.4 ± 0.3	0.0
$\mu(\Sigma^{*-})$	-	-2.56	-2.9 ± 0.2	-3.0
$\mu(\Xi^{*0})$	-	0.63	-0.1 ± 0.3	0.3
$\mu(\Xi^{*-})$	-	-2.20	-2.4 ± 0.2	-2.5
$\mu(\Omega^{-})$	-2.024 ± 0.056 [1]	-1.84	-2.024 ± 0.056 (input)	-2.024 ± 0.056 (input)

Table II. We first remark that these values are independent of T. To see this let us introduce the combinations

$$\frac{\mu(\Delta^{++})}{3\mu_d^{\Delta}} = T\Delta u + (1+U)\Delta s \tag{31}$$

and

$$\frac{\mu(\Omega^{-})}{3\mu_d^{\Omega}} = U\Delta u + (1+T)\Delta s. \tag{32}$$

In terms of these two numbers we can construct all other magnetic moments as linear combinations of them. We have

$$\mu(\Delta^{+}) = \frac{3D+1}{3(D+1)}\mu(\Delta^{++}) + \frac{2}{3(D+1)}\frac{\mu_d^{\Delta}}{\mu_d^{\Omega}}\mu(\Omega^{-}), \tag{33a}$$

$$\mu(\Delta^{0}) = \frac{3D-1}{3(D+1)}\mu(\Delta^{++}) + \frac{4}{3(D+1)}\frac{\mu_d^{\Delta}}{\mu_d^{\Omega}}\mu(\Omega^{-}), \tag{33b}$$

$$\mu(\Delta^{-}) = \frac{D-1}{D+1}\mu(\Delta^{++}) + \frac{2}{D+1}\frac{\mu_d^{\Delta}}{\mu_d^{\Omega}}\mu(\Omega^{-}), \tag{33c}$$

$$\mu(\Sigma^{*+}) = \frac{2\mu_d^{\Sigma^*}}{3\mu_d^{\Delta}}\mu(\Delta^{++}) + \frac{\mu_d^{\Sigma^*}}{3\mu_d^{\Omega}}\mu(\Omega^{-}), \tag{33d}$$

$$\mu(\Sigma^{*0}) = \frac{2D}{3(D+1)}\frac{\mu_d^{\Sigma^*}}{\mu_d^{\Delta}}\mu(\Delta^{++}) + \frac{D+3}{3(D+1)}\frac{\mu_d^{\Sigma^*}}{\mu_d^{\Omega}}\mu(\Omega^{-}), \tag{33e}$$

$$\mu(\Sigma^{*-}) = \frac{2D-2}{3(D+1)}\frac{\mu_d^{\Sigma^*}}{\mu_d^{\Delta}}\mu(\Delta^{++}) + \frac{D+5}{3(D+1)}\frac{\mu_d^{\Sigma^*}}{\mu_d^{\Omega}}\mu(\Omega^{-}), \tag{33f}$$

$$\mu(\Xi^{*0}) = \frac{\mu_d^{\Xi^*}}{3\mu_d^{\Delta}}\mu(\Delta^{++}) + \frac{2\mu_d^{\Xi^*}}{3\mu_d^{\Omega}}\mu(\Omega^{-}), \tag{33g}$$

$$\mu(\Xi^{*-}) = \frac{D-1}{3(D+1)}\frac{\mu_d^{\Xi^*}}{\mu_d^{\Delta}}\mu(\Delta^{++}) + \frac{2D+4}{3(D+1)}\frac{\mu_d^{\Xi^*}}{\mu_d^{\Omega}}\mu(\Omega^{-}), \tag{33h}$$

where $D=\sqrt{AB-AC+BC}$. We have here used the relation (10) between U and T. Since the ratios between the μ_d^B's are independent of T, the baryon magnetic moments also are.

The ratios between the μ_d^B's within the decuplet found from the linear fit resemble very much those found in lattice gauge theory calculations [2].

The magnetic moments differ notably from those of the NQM, mainly because of the low experimental value of $\mu(\Delta^{++})$. But we also note that the recent precise measurement of $\mu(\Omega^{-})=(-2.024\pm 0.056)\mu_N$ [1] is off from the NQM value $3\mu_s=-1.84\mu_N$ by several standard deviations.

IV. MASS DEPENDENCE OF THE μ_d's

Here we will study the linear relation among the μ_d^B's that we found when analyzing the octet data in the model.

We first remark that the quark magnetic moments obtained with this model are really only effective moments, as was already mentioned in the Introduction.

In the quark model the magnetic moment of a quark is of the form

$$\mu_f = \frac{e_f}{2m_f}, \tag{34}$$

e_f being the quark charge. The dependence on B could then be due to a variation of the effective quark mass m_f on B. A model of this kind has been considered by Chao [13].

Another interpretation is suggested by a further study of Eq. (1). Let us introduce the ratio

$$\alpha(B) = \frac{\mu_d^B}{\mu_d^N}, \tag{35}$$

which as a function of the mean mass of B is plotted in Fig. 1. With this function we can now rewrite Eq. (1) as

$$\mu(B^i) = \sum_{f,f'} \mu_f \alpha(B) M(B^i)_{ff'} \Delta f', \tag{36}$$

where $\mu_f=\mu_f^N$, $f,f'=u,d,s$. Since the $\alpha(B)$'s are independent of f, we can take them outside the sum to get

$$\mu(B^i) = \alpha(B)\sum_{f,f'} \mu_f M(B^i)_{ff'} \Delta f'. \tag{37}$$

This equation suggests that the magnetic moments of heavier states for some reason decrease with increasing mass relative to the values expected from the nucleon data. This might take place through collective effects that increase with mass, e.g., the moment of inertia, and/or gluonic collective effects, etc. One example of this could be Skyrmion collective effects, like in the chiral bag model [4].

Finally a third way of interpreting the result is also possible.

In our analysis the Δf's are kept constant and the same in the whole octet. We then get B-dependent μ_d's in the form

$$\mu_d^B = \alpha(B)\mu_d^N. \tag{38}$$

However, we also note that by a slight reinterpretation of our model we could associate the $\alpha(B)$ factor instead with the Δf's. In this case the quark magnetic moments would be fixed throughout and the spin polarizations would be varying linearly with the mass of B in the form

$$\Delta f^{B^i} = \sum_{f'} M(B^i)_{ff'} \alpha(B) \Delta f' \tag{39}$$

for each flavor f.

Since the baryon masses are well accounted for by mass formulas with the same quark masses in both the octet and the decuplet, Eq. (34) suggests that the quark magnetic moments might also be constant in the two supermultiplets. This last interpretation is therefore not unreasonable. Our empirically found mass dependence of the μ_d^B's would then be induced by us keeping the Δf's fixed in the analysis.

Clearly, as long as it comes to analyzing only the baryon magnetic moments, these different interpretations of Eq. (36) are equivalent, and all relations between magnetic moments will be the same in the three interpretations.

In the last interpretation, however, the quark spin polarizations would decrease with increasing isomultiplet mass. Thus $\Delta\Sigma=0.27$ for nucleons will decrease to ≈ 0.23 for the Ξ particles. Also the spin polarizations as well as $\Delta\Sigma(3/2)$ should be renormalized from the value presented here by a factor $\alpha(B)$ for the $B^i(3/2)$ resonances, since we have normalized the spin polarization relative to the nucleons. This means in particular that $\Delta\Sigma(3/2)$ for the Δ resonances should instead have the value $\Delta\Sigma(3/2)=-3.2\pm 2.6$ for $T=-2$. These questions could be illuminated if we could measure the spin polarizations in the spin 3/2 resonances.

In all interpretations we have assumed that $\alpha(B)$ is valid for both the octet and decuplet. This implies that the magnetic moments of the baryons are predicted to be vanishingly small as $\alpha(B)$ goes to zero as a function of the mass m_B of B. This happens at $m_B\approx 3.6$ GeV if the linear relation is still valid in this region. At any rate it is clearly highly interesting to try to measure magnetic moments of very massive baryon states and to study them with lattice calculations.

Further studies of the last interpretation will be presented elsewhere [14].

In any case we can test the linear relation among the quark magnetic moment ratios by the value of μ_d^Ω obtained from a fit to, say, $\mu(\Omega^-)$ once the values of Δu and Δs are given from any other two magnetic moments measured in the future.

V. SUMMARY AND CONCLUSIONS

We have studied the magnetic moments of the spin 3/2 resonances in a model allowing a general parametrization of flavor symmetry breaking and arbitrary quark spin polarizations. As we saw in an earlier analysis of the octet magnetic moments, the flavor breaking is small when the quark spin of the nucleon is small. At face value of the measured magnetic moments, the isospin symmetric case $T=-2$ suggests a negative quark spin content in the 3/2 resonances of the order of -3.6 ± 2.6 as compared to the NQM value of 3. However, the experimental data are still not good enough to give a precise determination.

Using the two measured magnetic moments we predict the magnetic moments of the other 3/2 resonances independently of the symmetry breaking. They are quite different from the quark model ones [as are already the measured values of $\mu(\Delta^{++})$ and $\mu(\Omega^-)$], and could, when measured, be used as a probe to study the consistency of the model. In fact, already three experimental data are enough to test most of the assumptions made in our model.

We also find that the quark magnetic moment ratios in our analysis decrease linearly with mass of the host particle and discuss various interpretations of this effect. It would be very interesting to try to measure magnetic moments of very massive baryon states and to study them with lattice calculations, to see how far this linear dependence holds.

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