Excited heavy mesons beyond leading order in the heavy quark expansion

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We examine the decays of excited heavy mesons, including the leading power corrections to the heavy quark limit. We find a new and natural explanation for the large deviation of the width of the $D_1(2420)$ from the heavy quark symmetry prediction. Our formalism leads to detailed predictions for the properties of the excited bottom mesons, some of which recently have been observed. Finally, we present a detailed analysis of the effect of power corrections and finite meson widths on the angular distributions which may be measured in heavy meson decays.

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I. INTRODUCTION

The excitation spectrum of charmed and bottom mesons has received considerable recent theoretical and experimental attention. The discovery of the B_1 and the B_2^* mesons [1], and the measurement of their masses and widths, complements the improving data being acquired on their charmed cousins, the D_1 and D_2^* [2]. At the same time, the theoretical understanding of the production and decay of these mesons has profited from the application of the heavy quark effective theory (HQET) and the ideas behind it, in particular the enlarged spin-flavor symmetry of QCD which obtains in the limit $m_c, m_b \to \infty$ [3].

We will begin this paper with a review of the experimental situation, and of the implications of heavy quark symmetry for excited heavy mesons. We will then investigate systematically the effect of the leading corrections to the heavy quark limit. We focus on corrections which violate the heavy spin symmetry, since it is predictions which follow from this symmetry which are tested by current data. We will find a new and natural explanation for the anomalously large width of the D_1. Our formalism leads to detailed predictions for the properties of the B_1 and B_2^*, as well as for their strange counterparts. We close by examining various angular distributions in the strong decays of heavy mesons, since with sufficiently precise data they will eventually provide detailed tests of our proposal.

II. HEAVY-LIGHT MESON SPECTRUM

The excitation spectrum of heavy mesons takes a particularly simple form in the limit $m \to \infty$, where $m = m_c$ or $m = m_b$. In this limit, the spin of the heavy quark decouples, and both the spin J of the meson and the angular momentum J_ℓ of the light degrees of freedom become

good quantum numbers. For each state in which the light degrees of freedom have spin and parity J_ℓ^P, there is a degenerate doublet of meson states with $J^P = J_\ell^P \pm \frac{1}{2}$. The mass splitting between these doublets arises only from effects of order Λ_{QCD}^2/m in the heavy quark expansion.

In this language, the ground state heavy mesons M and M^* have light degrees of freedom with $J_\ell^P = \frac{1}{2}^-$, and there are low-lying excited P-wave states with $J_\ell^P = \frac{1}{2}^+$ and $J_\ell^P = \frac{3}{2}^+$. The current experimental situation is summarized in Table I. Here we quote errors only for the excited D mesons [2,4]. The data on the excited B mesons are the result of a fit for which no errors are given [1], and the different charge states of the B^*, B_1, and B_2^* have not been resolved. States for which no masses are given in Table I have not yet been observed, although they are expected to exist.

Heavy quark symmetry imposes a number of constraints on the strong decays of these states. Since such decays are entirely transitions of the light degrees of freedom, the four possible decays of the two members of a doublet with given J_ℓ^P to the two members of another doublet with $J_\ell^{P\prime}$ are all essentially a single process. In the strict $m \to \infty$ limit, in which all such doublets are completely degenerate, this fact leads to the simple prediction that the two excited states should have exactly the same width. In fact, it is more accurate to use the actual masses of the states in calculating phase space effects, imposing the heavy quark symmetry only on the level of the matrix elements. Even though this approach is not technically consistent in the sense of the $1/m$ expansion, it allows us to incorporate certain $1/m$ effects which are numerically quite substantial.

For example, since the decay $D_2^* \to D\pi$ can take place only with the π in an orbital d wave, the same also must be true of the transitions $D_2^* \to D^*\pi$ and $D_1 \to D^*\pi$ (the matrix element for $D_1 \to D\pi$ vanishes). Incorporating phase space effects, the partial width for the transition $D_i \to D_f\pi$ is proportional to $(M_{D_f}/M_{D_i})|\mathbf{p}_\pi|^5$, times a mass-independent matrix element. This fact, plus some elementary spin counting, allows one to make the predictions [5]

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TABLE I. The lowest-lying charmed and bottom meson excitations. States with no masses listed have not yet been observed.

Spin			D system [2,4]			B system [1,4]		
J_ℓ^P	J^P	state	M (MeV)	Γ (MeV)	state	M (MeV)	Γ (MeV)
$\frac{1}{2}^-$	0^-	D^0	1865	$\tau = 0.42\,\mathrm{ps}$	B^0	5279	$\tau = 1.50\,\mathrm{ps}$
		$D^\pm$	1869	$\tau = 1.06\,\mathrm{ps}$	$B^\pm$	5279	$\tau = 1.54\,\mathrm{ps}$
		D_s	1969	$\tau = 0.47\,\mathrm{ps}$	B_s	5375	$\tau = 1.34\,\mathrm{ps}$
	1^-	D^{*0}	2007	< 2.1	B^*	5325	
		$D^{*\pm}$	2010	< 0.13			
		D_s^*	2110	< 4.5	B_s^*		
$\frac{1}{2}^+$	0^+	D_0^*			B_0^*		
	1^+	D_1'			B_1'		
$\frac{3}{2}^+$	1^+	D_1^0	2421 ± 3	20 ± 7	B_1	5725	20
		$D_1^\pm$	2425 ± 3	26 ± 9			
		D_{s1}	2535	< 2.3	B_{s1}	5874	1
	2^+	D_2^{*0}	2465 ± 4	28 ± 10	B_2^*	5737	25
		$D_2^{*\pm}$	2463 ± 4	27 ± 12			
		D_{s2}^*	2573 ± 2	16 ± 6	B_{s2}^*	5886	1

$$\frac{\Gamma(D_2^{*0} \to D^+\pi^-)}{\Gamma(D_2^{*0} \to D^{*+}\pi^-)} = 2.3\,, \quad \frac{\Gamma(D_1^0)}{\Gamma(D_2^{*0})} = 0.30\,. \qquad (2.1)$$

(We choose the modes for which the data are most accurate.) Experimentally, one finds [2,4][1]

$$\frac{\Gamma(D_2^{*0} \to D^+\pi^-)}{\Gamma(D_2^{*0} \to D^{*+}\pi^-)} = 2.2 \pm 0.9\,, \quad \frac{\Gamma(D_1^0)}{\Gamma(D_2^{*0})} = 0.71\,. \quad (2.2)$$

Clearly, one of these predictions works extremely well, while the other works not at all. Why might this be so?

One common explanation is that the D_1 has a small mixing with the D_1', which decays in an s wave rather than a d wave, and hence is expected to be considerably broader [5]. Such a mixing is allowed when spin symmetry violating $1/m$ effects are included, as the D_1 and D_1' both transform as $J^P = 1^+$ under the Lorentz group and differ only in their values of J_ℓ^P. Since even a small mixing is important if it is with a much broader state, this provides a simple explanation of why a $1/m$ correction of the natural size might lead to an anomalously large correction to the total width of the D_1.

Unfortunately, it is not one which is particularly favored by the present data. One may measure the angular distribution of the emitted pion and determine directly whether it is in an s wave or a d wave. The situation is complicated by the fact that the pion angular distribution depends not only the ratio of the s- and d-wave partial widths, but also on the relative phase η between the two matrix elements. Particularly in their data on

D_1^0 decay [2], CLEO finds that a large s-wave component is compatible only with a restricted region in $\cos\eta$. By no means is such a scenario ruled out, but it is less than generic.

In addition, there is no evidence for a significant s-wave component in the decay of the D_{s1}, which is related by flavor SU(3) symmetry to the D_1. One may use heavy quark symmetry to predict the d-wave width of the D_{s1} in terms of the width of the D_{s2}^*, analogous to the second relation of Eq. (2.1). The D_{s1} and D_{s2}^* decay via K emission to D and D^*. Given the measured $\Gamma(D_{s2}^*)$, one predicts

$$\Gamma(D_{s1}) = 0.3\,\mathrm{MeV}\,, \qquad (2.3)$$

far below the CLEO upper limit. Still, there is little room for a large additional s-wave component. To see this, one may use flavor SU(3) and the upper limit on $\Gamma(D_{s1})$ to predict an upper limit on $\Gamma(D_1)$ [6]. Since $|\mathbf{p}_\pi|$ in $D_1 \to D^*\pi$ is typically larger than $|\mathbf{p}_K|$ in $D_{s1} \to D^*K$, the upper limit on $\Gamma(D_1)$ will be much more stringent if the decay is assumed to be primarily s wave, as opposed to d wave. One then obtains independent upper limits on the s-wave and d-wave components of $\Gamma(D_1)$:

$$\Gamma_s(D_1) < 3\,\mathrm{MeV}\,, \qquad (2.4)$$
$$\Gamma_d(D_1) < 105\,\mathrm{MeV}\,.$$

The correlated limits are somewhat stronger (see Ref. [6]). However, we see immediately that under these assumptions it is impossible to accommodate a large s-wave component in D_1 decay.

Of course, these assumptions might not be very good. Flavor SU(3) could fail badly here. One test of SU(3) is to use the width of the D_2^* to predict the width of the D_{s2}^*, assuming d-wave decay and including the correct phase space for the K. One finds $\Gamma(D_{s2}^*) = (9 \pm 3)\,\mathrm{MeV}$,

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[1]We extract the ratio $\Gamma(D_1^0)/\Gamma(D_2^{*0})$ from the data in Table I. We do not assign an error, because the correct error depends crucially on the correlations between the measurements of the widths, which we do not know.

in reasonable agreement with experiment. [Perhaps it would be more correct to include the SU(3)-violating factor f_π^2/f_k^2 in this prediction; however, doing so makes the agreement with experiment worse.] However, it is possible that the D_{s1}-D'_{s1} mixing is very different from the D_1-D'_1 mixing, since the angle depends delicately on the interplay of a mixing matrix element and a mass splitting, both of which receive SU(3)-violating corrections. Still, we are not encouraged that this explanation for the anomalously large D_1 width is the correct one.

As an alternative, it has been suggested [7] that the D_1 width receives a large contribution from the emission of two pions which resonate through a ρ meson, with no analogous enhancement in D_2^* decay. Two-pion decays in which one of the pions resonates with a broad D'_1 or D_0^* have also been considered [8], but are not thought to contribute significantly to the total width. In the next section, we will introduce a simpler explanation for the D_1 width, which arises naturally at higher order in the heavy quark expansion.

III. CHIRAL LAGRANGIAN

The strong decays of excited mesons involve the emission of soft pions and kaons, and hence it is useful to analyze these interactions with the help of chiral perturbation theory. To be concrete, we will specify to the charm system; the generalization to bottom is at all points straightforward. The chiral Lagrangian appropriate to the analysis of ground state and excited heavy mesons has been derived elsewhere [8,9], here we will simply recall the basic points.

The heavy mesons are represented by matrix superfields which carry a representation not only of the Lorentz group but also of the SU(2) heavy quark spin symmetry. In addition, they transform as $\bar{3}$'s under flavor SU(3), since by convention our heavy mesons contain a single heavy quark (rather than an antiquark). For the ground state and lowest excited states discussed in the previous section, these superfields take the form [10]

$$H_a = \frac{(1+\slashed{v})}{2\sqrt{2}} \left[D_a^{*\mu}\gamma_\mu - D_a\gamma^5 \right] ,$$

$$S_a = \frac{(1+\slashed{v})}{2\sqrt{2}} \left[D_{1a}'^{\mu}\gamma_\mu\gamma^5 - D_{0a}^* \right] ,$$

$$T_a^\mu = \frac{(1+\slashed{v})}{2\sqrt{2}} \left[D_{2a}^{*\mu\nu}\gamma_\nu - D_{1a}^\nu\sqrt{\tfrac{3}{2}}\,\gamma^5(\delta_\nu^\mu - \tfrac{1}{3}\gamma_\nu\gamma^\mu + \tfrac{1}{3}\gamma_\nu v^\mu) \right] , \tag{3.1}$$

where v^μ is the four-velocity of the heavy meson, and a is the flavor index. These fields are normalized nonrelativistically. Under a heavy quark spin rotation S_Q, the superfields transform as $H \to S_Q H$, etc., while under a Lorentz transformation S_L they transform as $H \to S_L H S_L^\dagger$, etc.

The heavy mesons interact with the octet of pseudo Goldstone bosons, which are treated with the usual nonlinear formalism. The Lagrangian is written in terms of an exponentiated matrix of boson fields, $\xi = \exp(i\mathcal{M}/f_\pi)$, where

$$\mathcal{M} = \begin{pmatrix} \sqrt{\tfrac{1}{2}}\,\pi^0 + \sqrt{\tfrac{1}{6}}\,\eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\tfrac{1}{2}}\,\pi^0 + \sqrt{\tfrac{1}{6}}\,\eta & K^0 \\ K^- & \overline{K}^0 & -\sqrt{\tfrac{2}{3}}\,\eta \end{pmatrix} \tag{3.2}$$

and $f_\pi \approx 135\,\mathrm{MeV}$. Under $SU(3)_L \times SU(3)_R$, the field ξ transforms as $\xi \to L\xi U^\dagger = U\xi R^\dagger$, where U is a matrix which depends on $\mathcal{M}$. For generators within the diagonal subgroup, $U = L = R$.

The pseudo Goldstone bosons couple to the heavy superfields through the covariant derivative $D_{ab}^\mu = \delta_{ab}\partial^\mu + \frac{1}{2}(\xi^\dagger\partial^\mu\xi + \xi\partial^\mu\xi^\dagger)_{ab}$ and the axial vector field $A_{ab}^\mu = \frac{i}{2}(\xi^\dagger\partial^\mu\xi - \xi\partial^\mu\xi^\dagger)_{ab}$. Under $SU(3)_L \times SU(3)_R$, the fields transform as $A \to UAU^\dagger$, $H \to UH$, $D_\mu H \to UD_\mu H$, and so on. The kinetic part of the chiral Lagrangian is then given by

$$\mathcal{L}_{\mathrm{kin}} = \frac{1}{8}f_\pi^2 \mathrm{Tr}\left[\partial^\mu\Sigma\partial_\mu\Sigma^\dagger\right] - \mathrm{Tr}\left[\overline{H}iv\cdot DH\right]$$

$$+\mathrm{Tr}\left[\overline{S}(iv\cdot D - \Delta_S)S\right]$$

$$+\mathrm{Tr}\left[\overline{T}^\mu(iv\cdot D - \Delta_T)T_\mu\right] , \tag{3.3}$$

where $\Sigma = \xi^2$. The trace is taken with respect to spinor and flavor indices, which we suppress. The excitation energies Δ_S and Δ_T are defined with respect to the spin-averaged masses of the excited doublets. The heavy fields obey the equations of motion $iv\cdot DH = 0$, $iv\cdot DS = \Delta_S S$, and $iv\cdot DT^\mu = \Delta_T T^\mu$.

The interactions between the various fields are constrained by heavy spin symmetry and chiral symmetry,[2] as well as by Lorentz invariance (including parity and time reversal) and velocity reparametrization invariance [6,8,11]. We are interested in matrix elements between the ground state doublet and each of the excited states, for which we introduce the interaction terms

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[2]Since we are only interested in transitions involving a single light meson, flavor SU(3), under which the pions transform linearly, would be sufficient for our analysis. We employ the full formalism of nonlinear representations simply as a convenience.

$$\mathcal{L}_s = f \operatorname{Tr} \left[\overline{H} S\gamma^\mu\gamma^5 A_\mu \right] + \text{H.c.} ,$$

$$\mathcal{L}_d = \frac{h}{\Lambda_\chi} \operatorname{Tr} \left[\overline{H} T^\mu\gamma^\nu\gamma^5 (iD_\mu A_\nu + iD_\nu A_\mu) \right] + \text{H.c.} \quad (3.4)$$

These are the leading terms in the chiral expansion, which is an expansion in derivatives and fields over the chiral symmetry-breaking scale $\Lambda_\chi \sim 1\,\text{GeV}$. Note that $\mathcal{L}_d$, which mediates the d-wave decays of the D_1 and D_2^*, appears at higher order than $\mathcal{L}_s$, which mediates the s-wave decays of the D_0 and D_1'. Velocity reparametrization invariance plays a particularly important role in constraining $\mathcal{L}_d$ [6].

One may include bottom mesons by introducing new superfields H_b, S_b, and T_b^μ to represent the ground state and excited B mesons. These fields appear in analogues of the terms $\mathcal{L}_{\text{kin}}$, $\mathcal{L}_s$, and $\mathcal{L}_d$, with the *same* parameters Δ_S, Δ_T, f, and h as in the case of charm. This doubling of terms persists at higher orders, again with identical coefficients, except that explicit factors of $1/m$ should be taken to be $1/m_c$ or $1/m_b$ as appropriate.

The chiral Lagrangian, as developed so far, is sufficient to make the heavy spin symmetry predictions discussed in Sec. II. However, as we have seen, certain of these predictions work better than others; in particular, the ratio of partial widths $\Gamma(D_2^* \to D^*\pi)/\Gamma(D_2^* \to D\pi)$ is well predicted by the heavy spin symmetry, while the ratio of full widths $\Gamma(D_1)/\Gamma(D_2^*)$ is not. Evidently, corrections to the heavy quark limit can be quite large. Why is this so in some cases, but not in others? To gain insight into this question, it is necessary to go to subleading order in the $1/m$ expansion.

The most general extension of the chiral Lagrangian to order $1/m$ is extremely unwieldy. However, we are interested only in those $1/m$ corrections which break the heavy spin symmetry. We will find that when we neglect systematically spin symmetry-conserving terms, the resulting theory is sufficiently constrained to yield interesting information.

The $1/m$ corrections to the chiral Lagrangian arise from $1/m$ corrections to the heavy quark Lagrangian, which is derived from the full QCD Lagrangian in the $m \to \infty$ limit. This expansion takes the form [12]

$$\mathcal{L}_{\text{HQET}} = \bar{h}iv \cdot Dh + \frac{1}{2m}\bar{h}(iD)^2 h + \frac{1}{2m}\bar{h}\sigma^{\mu\nu}(\tfrac{1}{2}gG_{\mu\nu})h$$
$$+ \cdots , \quad (3.5)$$

where h is the HQET heavy quark field. The effect of the subleading terms $\bar{h}(iD)^2 h$ and $g\bar{h}\sigma^{\mu\nu}G_{\mu\nu}h$ on the chiral expansion may be treated in the same manner as other symmetry-breaking perturbations to the fundamental theory such as finite light quark masses. Namely, we introduce a "spurion" field which carries the same representation of the symmetry group as does the perturbation in the fundamental theory, and then include this spurion in the chiral Lagrangian in the most general symmetry-conserving way. When the spurion is set to the constant value which it has in QCD, the symmetry breaking is transmitted to the effective theory. In the case of finite light quark masses, for example, the symmetry-breaking term in QCD is $\bar{q}M_q q$, where

$M_q = \operatorname{diag}(m_u, m_d, m_s)$. Introducing a spurion M_q which transforms as $M_q \to L M_q R^\dagger$ under chiral SU(3), we then include terms in the ordinary chiral Lagrangian such as $\mu \operatorname{Tr}[M_q\Sigma + M_q\Sigma^\dagger]$.

In the present case, only the second of the two correction terms in $\mathcal{L}_{\text{HQET}}$ violates the heavy spin symmetry. We include its effect in the chiral Lagrangian by introducing a spurion $\Phi_s^{\mu\nu}$ which transforms as $\Phi_s^{\mu\nu} \to S_Q\Phi_s^{\mu\nu}S_Q^\dagger$ under a heavy quark spin rotation S_Q. This spurion is introduced in the most general manner consistent with heavy quark symmetry, and is then set to the constant $\Phi_s^{\mu\nu} = \frac{1}{2m}\sigma^{\mu\nu}$ to yield the leading spin symmetry-violating corrections to the chiral Lagrangian. We will restrict ourselves to terms in which $\Phi_s^{\mu\nu}$ appears exactly once.

The simplest spin symmetry-violating effect is to break the degeneracy of the heavy meson doublets. This occurs through the terms

$$\lambda_H \operatorname{Tr}\left[\overline{H}\Phi_s^{\mu\nu}H\sigma_{\mu\nu}\right]$$

$$-\lambda_S \operatorname{Tr}\left[\overline{S}\Phi_s^{\mu\nu}S\sigma_{\mu\nu}\right] - \lambda_T \operatorname{Tr}\left[\overline{T}^\alpha\Phi_s^{\mu\nu}T_\alpha\sigma_{\mu\nu}\right] . \quad (3.6)$$

The dimensionful coefficients are fixed once the masses of the mesons are known. For the ground state D and D^*, for example, we find

$$\lambda_H = \frac{1}{8}\left[M_{D^*}^2 - M_D^2\right] = (260\,\text{MeV})^2 . \quad (3.7)$$

This value is entirely consistent with what one would obtain, instead, with the B and B^* mesons. For the D_1 and D_2^*, we find

$$\lambda_T = \frac{3}{16}\left[M_{D_2^*}^2 - M_{D_1}^2\right] = (190\,\text{MeV})^2 . \quad (3.8)$$

Note that $\sqrt{\lambda_H}$ and $\sqrt{\lambda_T}$ are of order hundreds of MeV, the scale of the strong interactions.

We are interested in the spin symmetry-violating corrections to transitions in the class $T^\mu \to H\pi$, which will arise from terms analogous to $\mathcal{L}_d$ but with one occurrence of $\Phi_s^{\mu\nu}$. The spin symmetry, along with the symmetries which constrained $\mathcal{L}_d$, requires that any such term be of the generic form

$$\frac{1}{\Lambda_\chi}\operatorname{Tr}\left[\overline{H}\Phi_s^{\mu\nu}T^\alpha C_{\mu\nu\alpha\beta\kappa}\gamma^5\left(iD^\beta A^\kappa + iD^\kappa A^\beta\right)\right] + \text{H.c.} ,$$

$$(3.9)$$

where $C_{\mu\nu\alpha\beta\kappa}$ is an arbitrary product of Dirac matrices and may depend on the four-velocity v^λ. This would seem to allow for a lot of freedom, but it turns out that there is only a *single* term which respects both parity and time reversal invariance:

$$\mathcal{L}_{d1} = \frac{h_1}{2m\Lambda_\chi}\operatorname{Tr}\left[\overline{H}\sigma^{\mu\nu}T^\alpha\sigma_{\mu\nu}\gamma^\kappa\gamma^5\left(iD_\alpha A_\kappa + iD_\kappa A_\alpha\right)\right]$$
$$+ \text{H.c.} \quad (3.10)$$

We expect the new coefficient h_1, which has mass dimension 1, to be of order hundreds of MeV.

Finally, there may be additional correction terms

which come about by the application of velocity reparametrization invariance (VRI) [11] to the leading interaction term $\mathcal{L}_d$. This is a "symmetry"[3] which, as does a gauge symmetry, arises because of a redundancy in the variables appearing in the Lagrangian. The four-velocity v^λ which describes the heavy meson field is arbitrary up to terms of order $1/m$, and the Lagrangian must be constructed so as to be invariant under reparametrizations of the form $v^\lambda \to v^\lambda + \epsilon^\lambda/m$, where the components of ϵ^λ are of order Λ_{QCD}. The heavy meson fields also transform nontrivially under velocity reparametrization. VRI is a symmetry which constrains the new terms which may appear at higher order in the $1/m$ expansion in terms of those which are already there. The terms which VRI generates at order $1/m^{n+1}$ may be found by making the replacements [6,11]

$$v^\lambda \to v^\lambda + \frac{1}{m}iD^\lambda,$$

$$H \to H + \frac{1}{2m}[\gamma^\nu, iD_\nu H],$$

$$S \to S + \frac{1}{2m}\{\gamma^\nu, iD_\nu S\}, \qquad (3.11)$$

$$T^\mu \to T^\mu + \frac{1}{2m}[\gamma^\nu, iD_\nu T^\mu] - \frac{1}{m}v^\mu iD_\nu T^\nu$$

in the Lagrangian at order $1/m$. By the same token, all terms at order $1/m^{n+1}$ with derivatives acting on heavy meson fields must be consistent with such replacements at one order lower.

New interaction terms of the same order as $\mathcal{L}_{d1}$ will be generated when we make the replacements (3.11) in the leading term $\mathcal{L}_d$. However, massaging the new terms with integration by parts and application of the equations of motion, we find that they all may be written in the form $\text{Tr}\left[\overline{H}\,T^\mu f(\partial, A)\right]$, for some Dirac-matrix-valued function $f(\partial, A)$. Hence they do not break the heavy spin symmetry and we may ignore them for our analysis.

The other potentially important effect of $1/m$ corrections on the decay of the D_1 is a possible mixing between this state and the D_1'. The D_1' transforms the same as the D_1 under the Lorentz group, but differently under the heavy spin symmetry, and so spin symmetry-violating effects can mix the two states. Such a mixing can have a dramatic effect on the width of the D_1, which now may decay via s-wave pion emission.

The following parity- and time-reversal-invariant term will induce a nonzero matrix element for the mixing of the D_1 and D_1':

$$\mathcal{L}_{\text{mix}} = g_1 \text{Tr}\left[\overline{S}\Phi_s^{\mu\nu}T_\mu\sigma_{\nu\alpha}v^\alpha\right] + \text{H.c.} \qquad (3.12)$$

Unfortunately, the magnitude of the mixing matrix element does not by itself determine the mixing angle ψ. To find ψ, we also need the lowest order splitting Δ_T-Δ_S between the masses of the two states. Then the mixing angle is given by

$$\tan\psi = \frac{\sqrt{\delta^2 + \delta_g^2} - \delta}{\delta_g}, \qquad (3.13)$$

where $\delta = \frac{1}{2}(\Delta_T - \Delta_S)$ and $\delta_g = -\sqrt{\frac{2}{3}}\,g_1/m$. Since the D_1' has not yet been observed,[4] we do not know Δ_S; hence, it is more convenient to treat ψ itself as a free parameter, rather than write it as in Eq. (3.13).

IV. EXPERIMENTAL IMPLICATIONS

A. D_1 and D_2^* widths

The new interaction term $\mathcal{L}_{d1}$ affects the decays of D_1 and D_2^* in ways that do not necessarily respect the heavy spin symmetry predictions (2.1). It is straightforward to compute the single-pion partial widths of these excited states in terms of the coupling constants h and h_1. Lorentz invariance requires that the decays of the D_2^* still involve the emission of a d-wave pion, and we find

$$\Gamma(D_2^{*0} \to D\pi) = \frac{1}{10\pi}\frac{M_D}{M_{D_2^*}}\frac{4|\mathbf{p}_\pi|^5}{\Lambda_\chi^2 f_\pi^2}\left[h - \frac{h_1}{m_c}\right]^2,$$

$$\Gamma(D_2^{0*} \to D^*\pi) = \frac{3}{20\pi}\frac{M_{D^*}}{M_{D_2^*}}\frac{4|\mathbf{p}_\pi|^5}{\Lambda_\chi^2 f_\pi^2}\left[h - \frac{h_1}{m_c}\right]^2. \qquad (4.1)$$

Here and in Eq. (4.2) below we include emission of both charged and neutral pions, neglecting the small phase space differences between the two channels. Note that the heavy quark symmetry prediction for the ratio of the D_2^* partial widths is unaffected by the correction $\mathcal{L}_{d1}$. This is good, because we have seen that the lowest order prediction works quite well already. Truncating Eq. (4.1) at lowest order, inserting the experimental width of the D_2^*, and taking $\Lambda_\chi = 1\,\text{GeV}$, we obtain the estimate $h \approx 0.3$.

The effect of $\mathcal{L}_{d1}$ on D_1 decay is more complicated, because the new term can mediate both d-wave and s-wave decays. The decay width is given by

$$\Gamma(D_1 \to D^*\pi) = \frac{1}{4\pi}\frac{M_{D^*}}{M_{D_1}}\frac{4|\mathbf{p}_\pi|^5}{\Lambda_\chi^2 f_\pi^2}\left[\left(h + \frac{5h_1}{3m_c}\right)^2 + \frac{8h_1^2}{9m_c^2}\right]. \qquad (4.2)$$

The first term corresponds to a d-wave pion and the second to an s-wave pion. Note that here the s-wave width is

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[3]We use quotes because this is not a physical symmetry of nature; i.e., there is no associated conserved charge.

[4]It is possible that the $D_J(2440)^\pm$, reported by the TPC Collaboration [13] in the $D^{*0}\pi^\pm$ channel with a width of 40 MeV, is the $D_1'^\pm$. However, this observation has not been confirmed.

also suppressed by $|\mathbf{p}_\pi|^5$, and is in no sense intrinsically larger than the d-wave width. It is consistent to neglect it at this order, since we have not included $1/m^2$ interaction terms in the Lagrangian. The corrected ratio of the D_2^* width to the D_1 width is then given by

$$\frac{\Gamma(D_1^0)}{\Gamma(D_2^{*0})} = 0.30 \left[1 + \frac{16}{3} \frac{h_1}{m_c h} + O\left(\frac{1}{m_c^2} \right) \right] , \qquad (4.3)$$

where we assume that the widths are saturated by the one-pion decays. We see that in this prediction, the correction is enhanced by a large numerical prefactor. If we generalize Eq. (4.3) to include a mixture of d-wave and s-wave pion emission, we find

$$\frac{\Gamma(D_1)}{\Gamma(D_2^*)} = 0.30 \left[\cos^2 \psi \left(1 + \frac{16 h_1}{3 m_c h} \right) + \sin^2 \psi \frac{3 f^2}{4 (h/\Lambda_\chi)^2} \frac{E_\pi^2}{|\mathbf{p}_\pi|^4} \right] . \qquad (4.4)$$

With $\Lambda_\chi = 1\,\text{GeV}$, $h = 0.3$, and $m_c = 1.5\,\text{GeV}$, this reduces to

$$\frac{\Gamma(D_1)}{\Gamma(D_2^*)} = 0.30 \left[\left(1 + \frac{h_1}{85\,\text{MeV}} \right) \cos^2 \psi + 77 f^2 \sin^2 \psi \right] \overset{\text{expt}}{=} 0.71 . \qquad (4.5)$$

We may saturate the experimental width of the D_1 with d-wave decay by taking $h_1 = 115\,\text{MeV}$, which is quite reasonably small in view of the sizes of the similar corrections λ_H and λ_T. Alternatively, if $h_1 = 0$, then $(77 f^2 - 1) \sin^2 \psi = 1.37$, or $f \sin \psi \approx 0.13$ if f is of order one.

B. Predictions for excited B mesons

The heavy spin symmetry may also be combined with the heavy flavor symmetry to predict the masses and widths of the B mesons in terms of those in the charm sector. The splittings between excited doublets and the ground state should be independent of heavy quark mass, while spin symmetry-violating intradoublet splittings scale like $1/m$. If we define the spin-averaged masses and mass splittings

$$\overline{M}_B = \tfrac{3}{4} M_{B^*} + \tfrac{1}{4} M_B ,$$
$$\overline{M}_{B^{**}} = \tfrac{5}{8} M_{B_2^*} + \tfrac{3}{8} M_{B_1} , \qquad (4.6)$$
$$\Delta M_{B^{**}} = M_{B_2^*} - M_{B_1} ,$$

and analogously for charm, the heavy quark symmetries predict

$$\overline{M}_{B^{**}} - \overline{M}_B = \overline{M}_{D^{**}} - \overline{M}_D ,$$
$$\Delta M_{B^{**}} = \frac{m_c}{m_b} \Delta M_{D^{**}} . \qquad (4.7)$$

With $m_c/m_b = 1/3$, and averaging over the charged and neutral charmed mesons, these relations yield

$$M_{B_1} = 5780\,\text{MeV} ,$$
$$M_{B_2^*} = 5794\,\text{MeV} ,$$
$$M_{B_{s1}} = 5886\,\text{MeV} , \qquad (4.8)$$
$$M_{B_{s2}^*} = 5899\,\text{MeV} .$$

Compared with the data in Table I, we see that the measured masses are somewhat lower than expected, especially for the nonstrange mesons. The leading corrections to the predictions for $\overline{M}_{B^{**}}$ and $\overline{M}_{B^{**}}$ are of order

$$\delta \sim \Lambda_{\text{QCD}}^2 \left(\frac{1}{2 m_c} - \frac{1}{2 m_b} \right) \sim 40\,\text{MeV} , \qquad (4.9)$$

where we have estimated a QCD scale $\Lambda_{\text{QCD}} \sim 400\,\text{MeV}$, and so the accuracy with which the predictions (4.8) work is more or less what one would expect.

It is somewhat more delicate to make predictions for the widths of the excited B mesons, since these depend on the available phase space, hence on the values of the heavy meson masses. For the B_1 and B_{s1}, they also depend on what one assumes about $J_\ell^P = \tfrac{3}{2}^+$ and $\tfrac{1}{2}^+$ mixing in the bottom sector. Let us introduce a notation for the pion momenta which arise in these decays:

$$B_2^* \to B\pi: \qquad \mathbf{p}_{2B} ;$$

$$B_2^* \to B^*\pi: \qquad \mathbf{p}_{2B^*} ;$$

$$B_1 \to B^*\pi: \qquad \mathbf{p}_{1B^*} ;$$

and similarly for charm. Then, assuming the dominance of the one pion decay channel, the width of the B_2^* is related to that of the D_2^* via

$$\frac{\Gamma(B_2^*)}{\Gamma(D_2^*)} = \frac{M_{D_2^*}}{M_{B_2^*}} \left\{ \frac{0.4 |\mathbf{p}_{2B}|^5 M_B + 0.6 |\mathbf{p}_{2B^*}|^5 M_{B^*}}{0.4 |\mathbf{p}_{2D}|^5 M_D + 0.6 |\mathbf{p}_{2D^*}|^5 M_{D^*}} \right\} , \qquad (4.10)$$

from which we find the prediction

$$\Gamma(B_2^*) = (16 \pm 6)\,\text{MeV} , \qquad (4.11)$$

with the masses given in Table I. This width is somewhat low, but it is extremely sensitive to the mass of the B_2^* and grows rapidly with $M_{B_2^*}$. Perhaps here we have a hint that when the masses of the B_1 and B_2^* are better measured, they will be closer to the values predicted by heavy quark symmetry.

We may also generalize Eq. (4.4) to predict the ratio of the widths of the B_1 and the B_2^*, assuming once again that one-pion decays dominate. Leaving the dependence on the meson masses explicit, we find

$$\frac{\Gamma(B_1)}{\Gamma(B_2^*)} = \frac{M_{B_2^*}}{M_{B_1}} \left[\frac{|\mathbf{p}_{1B^*}|^5 M_{B^*}}{0.4|\mathbf{p}_{2B}|^5 M_B + 0.6|\mathbf{p}_{2B^*}|^5 M_{B^*}} \right] \left\{ \cos^2\psi_b \left(1 + \frac{16h_1}{3m_b h} \right) + \sin^2\psi_b \frac{3f^2}{4(h/\Lambda_\chi)^2} \frac{E_{1B^*}^2}{|\mathbf{p}_{1B^*}|^4} \right\}. \quad (4.12)$$

Since the mixing is generated by the spin symmetry-violating operator $\mathcal{L}_{\text{mix}}$ (3.12), it should scale inversely with the heavy quark mass. Hence for small mixing angles, we might expect $\psi_b \approx (m_c/m_b)\psi$. However, as we see in Eq. (3.13), ψ_b also depends delicately on the mass splitting between the B_1 and the B_1', and so we should put no particular trust in this estimate of ψ_b. Instead, we will make specific predictions of $\Gamma(B_1)$ only in the two limits of pure d-wave and pure s-wave decays of *both* the D_1 and the B_1, using the masses in Table I:

$$\Gamma(B_1)/\Gamma(B_2^*) = 0.9 \qquad \text{pure } d \text{ wave},$$
$$\Gamma(B_1)/\Gamma(B_2^*) = 1.4 \qquad \text{pure } s \text{ wave}. \qquad (4.13)$$

For the s-wave case, we take $\psi_b = \psi = \pi/2$ and choose f to give the correct D_1 width. While this extreme limit is not favored by the data on D_1 decay, it yields a useful upper bound on $\Gamma(B_1)$. We see that d-wave dominance is somewhat favored by the current data, from which one finds $\Gamma(B_1)/\Gamma(B_2^*) \approx 0.8$.

Finally, we make predictions for the one-kaon widths of the excited strange B mesons. Applying the analogue of Eq. (4.10), we find

$$\Gamma(B_{s2}^*) = (7 \pm 3)\,\text{MeV} \qquad (4.14)$$

and, from the analogue of Eq. (4.12),

$$\Gamma(B_{s1})/\Gamma(B_{s2}^*) = 0.4, \qquad (4.15)$$

where the B_{s1} is assumed to decay via the emission of a K in a d wave. These predictions will be tested as the data on the B_{s2}^* and B_{s1} improve.

C. Angular distributions

While the explanation of the width of the D_1 which we have presented is certainly consistent with the data, we

would like to be able to test it in somewhat more detail. We may do so by considering the angular distributions of the pions emitted in its decay. These distributions will depend on the chiral Lagrangian parameters h, h_1, and f, and on the mixing angle ψ. Finally, with our phase conventions the coefficient of $\mathcal{L}_s$ may be complex, and so we will take $f \to f \exp(i\eta)$.

In addition to the two constraints on the set of parameters $\{h, h_1, f, \eta, \psi\}$ from the experimental widths of the D_2^* and the D_1, we will now assume that the mixing angle ψ is small. Hence we will drop terms which are suppressed by $\sin\psi$, unless they are enhanced by a large phase space factor. Note that for values of f of order unity, the constraint (4.5) requires that ψ not be too large. An estimate of ψ based on a quark wave function model gives $\psi \approx 9°$ [14], which perhaps also supports the use of this approximation.

1. Two-pion distributions

In the decays $(D_1, D_2^*) \to D^*\pi_1 \to D\pi_1\pi_2$, the angle between the two pions contains information about the initial spin state. Let α be the angle between the momenta $\mathbf{p}_{\pi1}$ and $\mathbf{p}_{\pi2}$, as measured in the rest frame of the excited meson. Since the D_1 and D_2^* are separated by approximately 40 MeV and have intrinsic widths of 20–30 MeV, they overlap considerably. The distribution in $\cos\alpha$ is a function of where the pion's energy places it in relation to the two resonances. The formalism of Ref. [15] may be used to extend the results of Ref. [5] to the case where the finite widths Γ_{D_1} and $\Gamma_{D_2^*}$ are taken into account. We obtain

$$\frac{d\Gamma}{d\cos\alpha} \propto \left. \frac{d\Gamma}{d\cos\alpha} \right|_{D_2^*} + \left. \frac{d\Gamma}{d\cos\alpha} \right|_{D_1}, \qquad (4.16)$$

where

$$\left. \frac{d\Gamma}{d\cos\alpha} \right|_{D_2^*} = \frac{3\sin^2\alpha}{(E_\pi - \Delta_{21})^2 + \Gamma_{D_2^*}^2/4}, \qquad (4.17)$$

$$\left. \frac{d\Gamma}{d\cos\alpha} \right|_{D_1} = \frac{\mathcal{A}_{d1}^2(1 + 3\cos^2\alpha) - \mathcal{A}_s\mathcal{A}_{d1}2\sqrt{2}(1 - 3\cos^2\alpha)\cos\eta\sin\psi + 2\mathcal{A}_s^2\sin^2\psi}{(E_\pi - \Delta_{11})^2 + \Gamma_{D_1}^2/4}.$$

Here Δ_{21} and Δ_{11} are the resonant pion energies (averaged over charge states, $\Delta_{21} = 417\,\text{MeV}$ and $\Delta_{11} = 383\,\text{MeV}$), and

$$\mathcal{A}_s = \frac{\sqrt{3}}{2} \frac{f}{(h/\Lambda_\chi)} \frac{E_\pi}{E_\pi^2 - m_\pi^2},$$

$$\mathcal{A}_{d1} = 1 + \frac{8h_1}{3m_c h} \qquad (4.18)$$

give the relative strength of the s-wave and d-wave transitions. Since $\mathcal{A}_s$ contains a potentially large phase space enhancement, we keep $\mathcal{A}_s \sin\psi$ in our expressions. The

two terms in Eq. (4.16) are normalized correctly with respect to each other, but the overall normalization is arbitrary. Note that while the resonances overlap, they do not interfere in this distribution, a feature which follows directly from Lorentz invariance. In Fig. 1 we show the distribution in $\cos\alpha$ for $E_\pi = (380, 400, 420)\,\text{MeV}$, for several scenarios of D_1 decay: pure d wave ($\mathcal{A}_s = 0$ and $\psi = 0$), pure s wave ($\mathcal{A}_{d1} = 0$ and $\psi = \pi/2$), and a mixed case with $h_1 = 0$ and $\psi = 9°$, and $|f| = 0.85$. This latter case (actually, two cases, with $f = \pm 0.85$) corresponds to the situation in which there is no enhancement of the d-wave width, and the s-wave width is adjusted to give
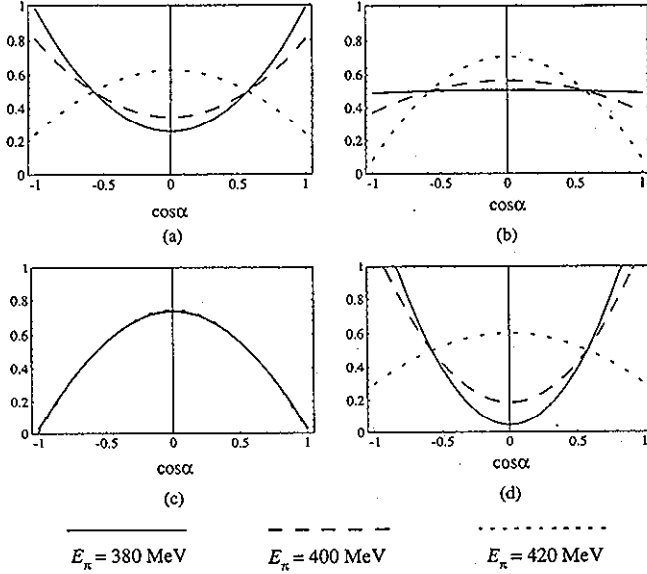
FIG. 1. The differential distribution in $\cos\alpha$, for the D_1 decaying (a) in a pure d wave; (b) in a pure s wave; (c) in a mixture with $h_1=0$, $\psi = 9°$, and $f = -0.85$; (d) in a mixture with $h_1=0$, $\psi = 9°$, and $f = 0.85$. The solid curve is for $E_\pi = 380$ MeV, the long dashed curve for $E_\pi = 400$ MeV, and the short dashed curve for $E_\pi = 420$ MeV. We have included the finite widths of the D_1 and D_2^*, and have set $\eta = 0$.

the correct total width of the D_1. We have also chosen a value of ψ taken from the quark model [14], and have set $\eta = 0$. We see that it should be possible to distinguish between these various scenarios.

2. One-pion distributions

We may also consider another distribution which is tied more closely to the fragmentation process by which the excited heavy meson is initially produced. In the decay $(D_1, D_2^*) \to D^*\pi$, let θ be the angle between the momentum of the pion and the fragmentation axis, as measured in the excited meson rest frame. In this frame, the fragmentation axis points back to the hard event in which the heavy quark was initially produced. The angular distribution in $\cos\theta$ depends not only on the quantities
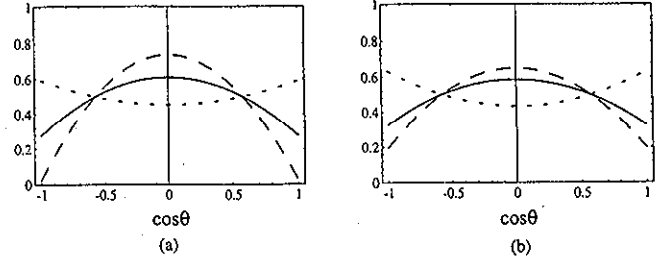

FIG. 2. The differential distribution in $\cos\theta$ (a) without and (b) with the finite widths of the D_1 and D_2^*. We show curves for $E_\pi = (380, 400, 420)$ MeV, identified as in Fig. 1, and have set $\psi = 9°$, $f = -0.85$, $h_1 = w_{3/2} = \eta = 0$.

$\{h, h_1, f, \eta, \psi\}$, but on the helicity distribution parameter $w_{3/2}$ [15]. This parameter describes the alignment with which the light degrees of freedom of $J_\ell = \frac{3}{2}$ are produced in the creation of the D_1 or D_2^*. The probabilities of the various helicity states along the fragmentation axis are given by

$$P(J_\ell^3 = \tfrac{3}{2}) = P(J_\ell^3 = -\tfrac{3}{2}) = \tfrac{1}{2}w_{3/2},$$
$$P(J_\ell^3 = \tfrac{1}{2}) = P(J_\ell^3 = -\tfrac{1}{2}) = \tfrac{1}{2}(1 - w_{3/2}). \qquad (4.19)$$

The parameter $w_{3/2}$ is a nonperturbative parameter of QCD, which is well defined only in the heavy quark limit. In Ref. [15], data from ARGUS [16] on the decay $D_2^* \to D\pi$ was used to set the 90% confidence level upper limit $w_{3/2} < 0.24$. Models based on perturbative QCD have yielded an estimate $w_{3/2} \approx 0.25$ [17,18].

The distribution in $\cos\theta$ is considerably more complicated when finite width effects are included, in part, because the interference between the D_1 and D_2^* resonances may not be neglected. (We note that when $w_{3/2}$ is extracted from the decay $D_2^* \to D\pi$, there are no interference effects, since the decay $D_1 \to D\pi$ is prohibited.) After a straightforward calculation, we find

$$\frac{d\Gamma}{d\cos\theta} \propto \left.\frac{d\Gamma}{d\cos\theta}\right|_{D_2^*} + \left.\frac{d\Gamma}{d\cos\theta}\right|_{D_1} + \left.\frac{d\Gamma}{d\cos\theta}\right|_{D_1-D_2^*}, \qquad (4.20)$$

where

$$\left.\frac{d\Gamma}{d\cos\theta}\right|_{D_2^*} = \frac{3(1 + \cos^2\theta) + 2w_{3/2}(1 - 3\cos^2\theta)}{(E_\pi - \Delta_{21})^2 + \Gamma_{D_2^*}^2/4}, \qquad (4.21)$$

$$\left.\frac{d\Gamma}{d\cos\theta}\right|_{D_1} = \frac{1}{(E_\pi - \Delta_{11})^2 + \Gamma_{D_1}^2/4}\{\mathcal{A}_{d1}^2\left[3(1 + \cos^2\theta) + 2w_{3/2}(1 - 3\cos^2\theta)\right]$$
$$+ \mathcal{A}_s\mathcal{A}_{d1}2\sqrt{2}\sin\psi\cos\eta(-1 + 2w_{3/2})(1 - 3\cos^2\theta) + 4\mathcal{A}_s^2\sin^2\psi\}$$

$$\left.\frac{d\Gamma}{d\cos\theta}\right|_{D_1-D_2^*} = \frac{(-1 + 2w_{3/2})(1 - 3\cos^2\theta)}{2[(E_\pi - \Delta_{21})^2 + \Gamma_{D_2^*}^2/4][(E_\pi - \Delta_{11})^2 + \Gamma_{D_1}^2/4]} \qquad (4.22)$$

$$\times\{(\mathcal{A}_{d1} - \mathcal{A}_s\sqrt{2}\sin\psi\cos\eta)[4(E_\pi - \Delta_{11})(E_\pi - \Delta_{21}) + \Gamma_{D_1}\Gamma_{D_2^*}]$$
$$- \mathcal{A}_s 2\sqrt{2}\sin\psi\sin\eta(E_\pi\Gamma_{D_1} - E_\pi\Gamma_{D_2^*} + \Delta_{11}\Gamma_{D_2^*} - \Delta_{21}\Gamma_{D_1})\}.$$
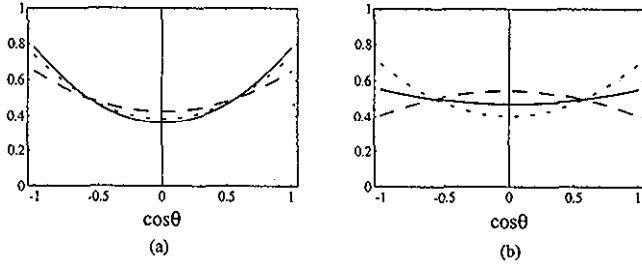
(a)   (b)

FIG. 3. The differential distribution in $\cos\theta$, for $w_{3/2} = 0$, in the case of (a) pure d-wave D_1 decay and (b) pure s-wave D_1 decay. We have set $f = -0.17$, $\eta = 0$, and $E_\pi = (380, 400, 420)\,\text{MeV}$.

(a)   (b)

FIG. 5. The differential distribution in $\cos\theta$, for $w_{3/2} = 0.75$, in the case of (a) pure d-wave D_1 decay and (b) pure s-wave D_1 decay. We have set $f = -0.17$, $\eta = 0$, and $E_\pi = (380, 400, 420)\,\text{MeV}$.

Once again, the three terms in Eq. (4.20) are normalized only with respect to each other.

To explore the importance of including the finite widths of the excited resonances, in Fig. 2 we compare the distribution in $\cos\theta$ with (a) $\Gamma(D_1) = \Gamma(D_2^*) = 0$ and (b) $\Gamma(D_1) = 22\,\text{MeV}$, $\Gamma(D_2^*) = 28\,\text{MeV}$, and scanning over $E_\pi = (380, 400, 420)\,\text{MeV}$. For the purpose of illustration, we set $w_{3/2} = 0$ and choose the parameters $h_1 = 0$, $\psi = 9°$, and $f = -0.85$ for the D_1 decay.

In Fig. 3, we set $w_{3/2} = 0$ and compare pure d-wave and pure s-wave D_1 decays, for $f = -0.17$ and $E_\pi = (380, 400, 420)\,\text{MeV}$. In Fig. 4 we do the same for $w_{3/2} = 0.25$, in Fig. 5 for $w_{3/2} = 0.75$, and in Fig. 6 for $w_{3/2} = 1.0$. (For $w_{3/2} = 0.5$, all the distributions are flat.) Together, Figs. 3–6 give some idea of the sensitivity of the distribution in $\cos\theta$ to the various parameters describing the decay. Note that the sensitivity to $w_{3/2}$ is considerably enhanced if the D_1 decays via d-wave emission rather than s-wave emission.

V. SUMMARY AND OUTLOOK

Our detailed analysis of the leading effects of heavy spin symmetry violation on the properties of excited charmed and bottom mesons has led to a number of inter-
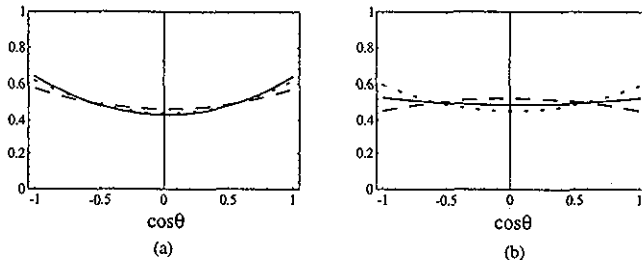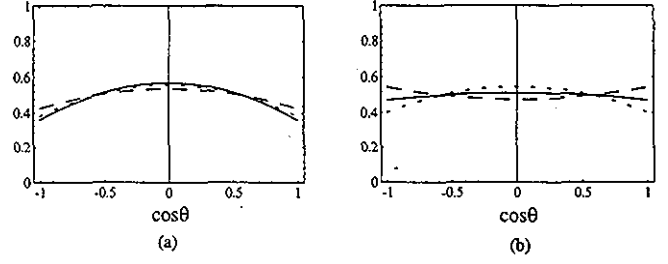
esting results. In particular, the width of the D_1, previously thought to be anomalously large, is seen actually to be of a natural size. Our predictions for the properties of excited bottom mesons agree well with the minimal data which exist so far, and will be tested soon in more detail. We presented detailed angular distributions for strong decays of excited charmed mesons, which will eventually provide a more stringent test of the predictions of our formalism.

Excited heavy mesons are important both for their own sake and for the insight they give into the heavy quark expansion. Since the most accurate determinations of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{cb} involve the theoretical application of HQET, it is crucial to understand how well the $m_c, m_b \to \infty$ limit approximates the real world. It is a matter of more than academic interest whether the large width of the D_1 can be explained naturally *within* the heavy quark expansion, because the answer to this question affects our willingness to trust that the charm quark may be treated as heavy in other contexts. Similarly, it is worthwhile to do one's best to extract parameters such as $w_{3/2}$, f, and ψ from pion angular distributions in strong decays. Doing so, we learn not only about excited heavy mesons themselves, but about whether one can indeed explain their properties consistently in the context of the heavy quark expansion.
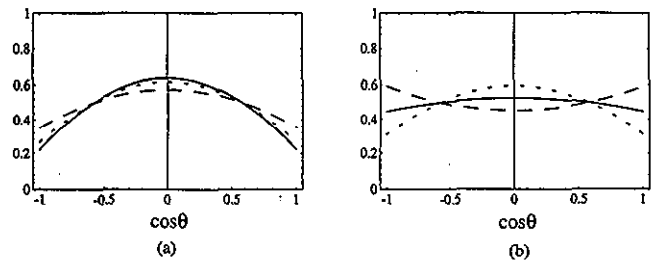

(a)   (b)

FIG. 4. The differential distribution in $\cos\theta$, for $w_{3/2} = 0.25$, in the case of (a) pure d-wave D_1 decay and (b) pure s-wave D_1 decay. We have set $f = -0.17$, $\eta = 0$, and $E_\pi = (380, 400, 420)\,\text{MeV}$.

(a)   (b)

FIG. 6. The differential distribution in $\cos\theta$, for $w_{3/2} = 1.0$, in the case of (a) pure d-wave D_1 decay and (b) pure s-wave D_1 decay. We have set $f = -0.17$, $\eta = 0$, and $E_\pi = (380, 400, 420)\,\text{MeV}$.

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