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COMMENTS

Comments are short papers which criticize or correct papers of other authors previously published in the **Physical Review**. *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Comment on "θ-term renormalization in (2+1)-dimensional CP^{N-1} model with a θ term"

I. N. Kondrashuk*
Bogolubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna (Moscow Region), Russia

A. V. Kotikov†
Laboratoire de Physique Theorique ENSLAPP, LAPP, B.P. 110, F-74941, Annecy-le-Vieux Cedex, France
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It is found that in a recent paper by Park the first coefficient of the nonzero β function for the Chern-Simons term in the $1/N$ expansion of the CP^{N-1} model is untrue numerically. The correct result is given. The main conclusions of Park's paper are not changed.

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In a recent paper [1], Park investigated the $1/N$ expansion in the (2+1)-dimensional CP^{N-1} model with a Chern-Simons (or θ) term and showed that the θ term does acquire infinite radiative corrections in the first order of $1/N$. We repeated these calculations and found complete agreement with these conclusions but a different value of the β function of the θ charge:

$$\beta(\theta) = \frac{320}{9\pi^2}\,\frac{\theta^3}{(1+\theta^2)^2}\,\frac{1}{N}.$$

The disagreement between our and Park's results is in the calculation of diagrams (5a)–(5e) from [1]. *First,* the diagrams (5d) and (5e) can be represented, respectively, as (5b) and (5c) but with the reverse orientation of arrows in one of the two circles. Since every circle has only two $\overline{n}A_\mu n$ vertices containing momentum and one $\overline{n}\alpha n$ vertex, the contribution of diagram (5b) is independent of the orientation of the arrows and will not change if arrows are reversed in one of the two circles. Hence the contributions from (5b) and (5d), for example, do not cancel each other as was proposed in [1] but are summed. The infinite parts of the four diagrams (5b) – (5e) coincide and equal

$$-\mu^{-2\epsilon}\frac{1}{18\pi^2}\,\frac{\theta}{1+\theta^2}\,\epsilon^{\mu\rho\nu}p^\rho\frac{1}{\epsilon}\frac{1}{N}.$$

Second, our calculation of the contribution of diagram (5a) yields a result which is twice as small as Park's. We assume that the reason for this may be a wrong double count of the orientation of the arrow (a change of orientation of the arrow does not result in a new diagram). And *in the third place,* our last note is that, for the correspondence between the Lagrangian and Feynman rules, the coefficient of the θ term in the Lagrangian must be twice as large as the one written by the author. Comparing this Lagrangian with the one in the author's previous paper [2], we confirm our assumption.

We calculated the singular parts of the contributions of diagrams (5a), (5b), and (5d) from [1] in the following way. The leading (at large p^2, where p is the external momentum) contribution of every diagram, which leads to the renormalization of θ, has the form $A\epsilon^{\mu\rho\nu}p^\rho/(p^2)^{l\epsilon}$, where l is the loop number and A is the required coefficient. After differentiation with respect to p^σ, the singular part of every diagram[1] does not depend on the momentum p and may be found by Vladimirov's method [3], where the external momentum is put equal to zero (in principle, there is the necessity of introducing also some masses to preserve the solution from infrared singularities but this is not the case).

The whole sum of the infinite parts of all diagrams (5a)–(5e) is

$$-\mu^{-2\epsilon}\frac{5}{9\pi^2}\,\frac{\theta}{1+\theta^2}\,\epsilon^{\mu\rho\nu}p^\rho\frac{1}{\epsilon}\frac{1}{N}.$$

*Electronic address: IKOND@THSUN1.JINR.DUBNA.SU
†On leave of absence from Particle Physics Laboratory, JINR, Dubna, Russia. Electronic address: KOTIKOV@LAPPHP0.IN2P3.FR; KOTIKOV@SUNSE.JINR.DUBNA.SU

[1]More exactly kR' of the diagram, but in our case it coincides with the singular part because there are only $1/\epsilon$ terms.

To cancel this infinity we must add to the Lagrangian the corresponding counterterm, which results in the following expression for the bare charge:

$$\theta_0 = \mu^{-2\epsilon}\left(\theta + \frac{80}{9\pi^2}\,\frac{\theta}{1+\theta^2}\,\frac{1}{\epsilon}\frac{1}{N}\right).$$

From here we can derive the β function that is written above.

To conclude, note that the main result of [1] about the occurrence of infinite renormalization of the θ term in the case of the $1/N$ expansion does not lose its importance. The function $\beta(\theta)$ is nonzero and all the main conclusions of the paper [1] are not the subject of a critical review in our comment.

Note only that the results of [1] are in contradiction with the usual weak-coupling expansion where the nonrenormalization theorem was established (see [4]). Technically, the appearance of the nonzero β function in the $1/N$ expansion is quite clear. There is $1/N$ resummation of the photon propagator. Another (half-integer) power of p^2 is obtained in the ultraviolet range and ultraviolet singularities start to appear already in the leading order of the $1/N$ expansion. Will higher order contributions lead to the permanent saturation of this effect? It is an open question.

Perhaps the calculation of the next $(1/N^2)$ correction might help to illuminate this process. However, usually the calculation of higher order contributions in the framework of the $1/N$ expansion is not a very simple problem.

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