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Galactic halos as boson stars

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We investigate a boson star with a self-interacting scalar field as a model of galactic halos. The model has slightly increasing rotation curves and allows wider ranges of the mass (m) and coupling (λ) of the halo dark matter particle than the noninteracting model previously suggested. The two quantities are related by $\lambda^{\frac{1}{2}}(m_P/m)^2 \gtrsim 10^{50}$.

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It is well known that the flatness of the galactic rotation curves indicates the presence of dark matter around galactic halos. However, the properties of dark matter are still mysterious. For example, why doesn't the dark matter in halos fall towards the center of the galaxy and form black holes? The answer to the above question may be a good criterion for good halo model.

There is a thermal distribution model [1] where the density profile $\rho \sim r^{-2}$ and a spherical infall model [2] where $\rho \sim r^{-2.25}$. Recently Sin [3,4] suggested a new model of the halos composed of pseudo Nambu-Goldstone bosons (PNGB's). According to the model, the condensation of ultralight PNGB's whose Compton wavelength $\lambda_{\rm Comp} = \frac{\hbar}{mc}$ is about $R_{\rm halo}$ is responsible for the halo formation. The cosmological role of the ultra light PNGB's was studied in the late time phase transition model [5] to reconcile the smoothness in the background radiation with the large scale structure.

Before Sin's work, an astronomical object which consists of PNGB dark matter was suggested by some authors [6]. In their model the force against gravitational collapse comes from the momentum uncertainty of the quantum-mechanical uncertainty principle. Since the typical length scale R in this model is the Compton wavelength $\lambda_{\rm Comp} \sim \frac{1}{m}$ of the particle, the typical mass scale of the object is $M \sim \frac{R}{G} \sim \frac{m_P^2}{m}$.

Similarly, in Sin's model galactic halos are the objects of the self-gravitating Bose liquid whose collapse prevented by the uncertainty principle.

The typical halo has a radius $R_{\rm halo} \sim 100$ kpc$\sim 10^{24}$ cm and a mass $M_{\rm halo} \sim 10^{12} M_\odot \sim 10^{45}$ g, so one find the mass m of the PNGB whose de Broglie wavelength $\sim R_{\rm halo}$ is about 10^{-26} eV. Note that the de Broglie length $\sim \frac{c}{v}\lambda_{\rm Comp}$ is more adequate for our purpose.

The self-gravitating condensed states are described by the nonlinear Schrödinger equation

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + GmM_0\int_0^{r'} dr'\frac{1}{r'^2} \times \int_0^r dr'' 4\pi r''^2|\psi|^2\psi(r), \qquad (1)$$

which was known as the equation of the Newtonian limit of the boson star fields [7]. The normalization constant M_0 is chosen to give the total mass of the halo $M = M_0\int dr 4\pi r^2|\psi|^2$ as in Ref. [3]. The rotation velocity at radius r is given by

$$V(r) = \sqrt{\frac{GM(r)}{r}}, \qquad (2)$$

where $M(r)$ is the mass within r.

Integrating Eq. (1) numerically and using Eq. (2) Sin found slightly increasing rotation curves and a density profile $\rho \sim r^{-1.6}$.

What happens if there are repulsive self-interactions between the dark matter particles? To answer this and the stability question it is more desirable to study the relativistic field equations than the Schrödinger equation.

The cold gravitational equilibrium configurations of a massive scalar field were found by solving the Klein-Gordon equations with gravity decades ago [8]. We find that these configurations, called boson stars [9], are adequate for the relativistic extension of Sin's model.

Consider a self-interacting complex scalar field and gravity whose action is given by

$$S = \int\sqrt{-g}d^4x\left[\frac{-R}{16\pi G} - \frac{g^{\mu\nu}}{2}\phi^*_{;\mu}\phi_{;\nu} - \frac{m^2}{2}|\phi|^2 - \frac{\lambda}{4}|\phi|^4\right]. \qquad (3)$$

Since halos seem to be spherical, we choose the Schwarzschild metric

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\Omega \qquad (4)$$

and assume spherically symmetric field solutions

$$\phi(r,t) = (4\pi G)^{-\frac{1}{2}}\sigma(r)e^{-i\omega t}. \qquad (5)$$

From the action, dimensionless time-independent Einstein and scalar wave equations appear as in Ref. [10]:

$$\frac{A'}{A^2x}+\frac{1}{x^2}\left[1-\frac{1}{A}\right]=\left[\frac{\Omega^2}{B}+1\right]\sigma^2+\frac{\Lambda}{2}\sigma^4+\frac{\sigma'^2}{A}, \quad (6)$$

$$\frac{B'}{ABx}-\frac{1}{x^2}\left[1-\frac{1}{A}\right]=\left[\frac{\Omega^2}{B}-1\right]\sigma^2-\frac{\Lambda}{2}\sigma^4+\frac{\sigma'^2}{A}, \quad (7)$$

$$\sigma''+\left[\frac{2}{x}+\frac{B'}{2B}-\frac{A'}{2A}\right]\sigma'+A\left[\left(\frac{\Omega^2}{B}-1\right)\sigma-\Lambda\sigma^3\right]=0, \quad (8)$$

where $x = mr$, $\Omega = \frac{\omega}{m}$, $A \equiv [1 - 2\frac{M(x)}{x}]^{-1}$, and $\Lambda = \frac{\lambda m_P^2}{4\pi m^2}$. One may take $M(x)$ for the dimensionless mass of the boson star for large x.

Numerical solutions of the above equations have been studied by many authors [11–13]. The required boundary conditions are $M(0) = 0$, $\sigma'(0) = 0$, and $B(\infty) = 1$ and the free parameters are $\sigma(0)$ and Ω.

For the case $\Lambda = 0$ [12] it was found that there is a maximum mass $M_{\max} = 0.633\frac{m_P^2}{m}$ for the zero node solution. We will focus on the nonzero node solutions, because the rotation curve of the zero node solution falls too fast to explain the flatness of the rotation curves of many galaxies(see Ref. [4] for more arguments). This raises the stability problem of higher node solutions, which will be discussed later.

Maximum masses for higher node solutions are proportional to the node number n and about the same order as for the zero node case for small n. This with M_{halo} gives us $m \lesssim 10^{-22}$ eV.

Another constraint comes from the maximum center density stable against small radial perturbation [13], $\rho_c = 2.1 \times 10^{98} m^2$ g/cm$^3 > 10^{-24}$ g/cm^3, which is equivalent to $m \gtrsim 10^{-28}$ eV for the zero node solutions. So for the zero node solutions 10^{-28} eV $\lesssim m \lesssim 10^{-22}$ eV.

For the case $\Lambda \neq 0$, a new scale appears because of the repulsive force preventing the halo from gravitational collapse. In this case the typical length scale is $R \sim \Lambda^{\frac{1}{2}}/m$, thus the typical mass scale is $\frac{R}{G} \sim \Lambda^{\frac{1}{2}} m_P^2/m$; which is also of the order of the maximum mass as in the $\Lambda = 0$ case.

A numerical study [10] shows that $M_{\max} = 0.22\Lambda^{\frac{1}{2}}\frac{m_P^2}{m}$ for zero node solutions. From the fact that $M_{\max} > M_{\text{halo}}$ we find

$$\lambda^{\frac{1}{2}}\left(\frac{m_P}{m}\right)^2 \gtrsim 10^{50}. \quad (9)$$

This is a relation between the mass and coupling of the halo dark matter particle. For the perturbative case ($\lambda \lesssim 1$) the above relation implies $m \lesssim 10^3$ eV.

To treat particles as a classical field, we require that the interparticle distance should be smaller than their Compton wavelength. This gives $m \lesssim 10^{-2}$ eV.

Note that $\Lambda = \lambda m_P^2/4\pi m^2$ is very large even for very small λ due to the smallness of m relative to m_P; hence, the self-interaction effect is non-negligible.

Are there any realistic particle physics model satisfying the above relation? Unfortunately, the usual cosine potential $V(\phi) = \mu^4[1-\cos(\phi/f)]$ for PNGB's is inappropriate for our study, because the sign of the quartic coupling constant is negative in the Taylor expansion about the potential minima and ϕ is real. A real scalar field such as an axion may form an oscillating soliton star [14] rather than a boson star.

Instead, we consider the potential

$$V(\phi) = \mu^4\left[1+\left(\frac{\phi}{f}\right)^2\right]^2. \quad (10)$$

Inserting the mass and quartic coupling from the above potential into the relation in Eq. (9), we get $0.1(\frac{m_P}{\mu})^2 \gtrsim 10^{50}$ and equivalently $\mu \lesssim 10^2$ eV.

We also solve the equations numerically and find the dimensionless rotation velocity which is given by $V_{\text{rot}} = \sqrt{M(x)/x} = [\frac{1}{2}(1 - A^{-1})]^{\frac{1}{2}}$. The results are shown in Fig. 1 and Fig. 2.

Figure 1 shows rotation velocity curves for the cases $\Lambda = 0$ and $\Lambda = 300$. The parameters are $B(0) = 0.641$, $\sigma(0) = 0.1$, and $B(0) = 0.781$, $\sigma(0) = 0.01$, respectively. Figure 2 shows σ and a rotation velocity curve of the eight-node solution ($n = 9$).

It is interesting that the line connecting the minimum points of the rotation velocity is almost straight. For large n and $\Lambda \gg 1$ the mass profile is $\rho \sim r^{-1.7}$. The rotation curves are slightly increasing regardless of the self-interactions.

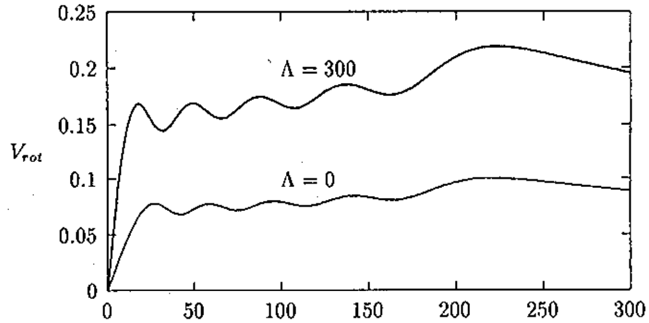

FIG. 1. Rotation velocities as a function of rescaled x for the parameters $\Lambda = 0$ (thin line) and $\Lambda = 300$ (thick line). Ω is 0.9. The real values of the x end are 80 and 220, respectively.

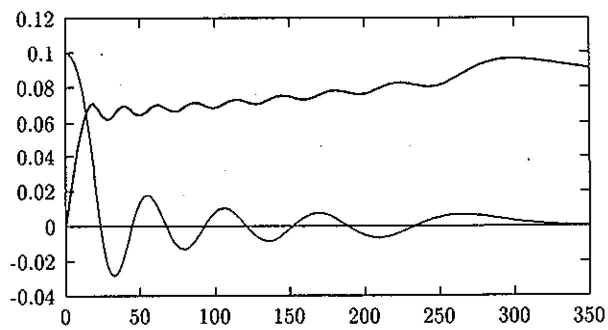

FIG. 2. Rotation velocity and 10σ as a function of position x for the $n = 9$ solution. The parameters are $\Lambda = 300$, $\Omega = 0.9$, $B(0) = 0.780$, and $\sigma(0) = 0.01$.

Including the visible matter may change the slope of the curves and explain the variety of the observed galaxy rotation curves as shown in Ref. [4].

We will now study the Newtonian limit of our model. The strength of the gravity of the halo, $GM_{\rm halo}/R_{\rm halo}$, is comparable to that of Earth. Therefore we can use the Newtonian limit $\Omega = \frac{\omega}{m} = \sqrt{1+(\frac{k}{m})^2} \to 1$, which is comparable to the Newtonian gravity approximation $2M(x)/x \ll 1$. Collecting terms to $O(\xi^2 = (\frac{k}{m})^2)$ one finds that for $\Lambda = 0$ the equations of motion are [15]

$$\nabla^2\sigma = \gamma\sigma, \tag{11}$$

$$\nabla^2\gamma = 2\sigma^2, \tag{12}$$

where $\gamma \equiv 1 - \frac{\Omega^2}{B}$.

Integrating Eq. (12) and inserting the result into Eq. (11) we find

$$\frac{1}{2}\nabla^2\sigma = \left(E + \int_0^x dx' \frac{1}{x'^2}\int_0^{x'} dx'' x''^2\sigma^2\right)\sigma, \tag{13}$$

which is a dimensionless version of Eq. (1). Here E is an integration constant. Therefore one may treat the Bose liquid model as a boson star model.

It is useful to study the scaling properties of Eq. (11) and Eq. (12) for analyzing numerical solutions. Rescaling the total number of charges $N = Q \propto \int \sigma^2 x^2 dx$ l times increases the mass l times. Equations (11) and (12) are invariant under this rescaling when

$$x \to l^{-1}x, \quad \sigma \to l^2\sigma, \quad \gamma \to l^2\gamma, \tag{14}$$

which is consistent with the model in Ref. [3]. So one may say that for the noninteracting case the heavier halos are smaller in size.

It is difficult to find the scaling properties for the case $\Lambda \neq 0$.

For the case $\Lambda \gg 1$ further rescaling $\sigma_* = \sigma\Lambda^{1/2}$, $x_* = x\Lambda^{-1/2}$, and $M_* = M\Lambda^{-1/2}$ neglecting terms to $O(\Lambda^{-1})$ yields the wave equations

$$\sigma_*^2 = (\Omega^2/B - 1) = -\gamma, \tag{15}$$

$$M_*' = \frac{1}{4}x_*^2(3\Omega^2/B + 1)(\Omega^2/B - 1), \tag{16}$$

$$\frac{B'}{ABx_*} - \frac{1}{x_*^2}(1 - A^{-1}) = \frac{1}{2}(\Omega^2/B - 1)^2, \tag{17}$$

which are also shown in Ref. [10]. Following the same procedure as in the $\Lambda = 0$ case, we get the Newtonian limits of Eq. (16) and Eq. (17),

$$\nabla^2\gamma = 2\sigma_*^2 = -2\gamma, \tag{18}$$

whose solutions are

$$\gamma = -\gamma_0 \frac{\sin(\sqrt{2}x_*)}{\sqrt{2}x_*} \tag{19}$$

and

$$\sigma_* = \sqrt{\frac{\gamma_0 \sin(\sqrt{2}x_*)}{\sqrt{2}x_*}}, \tag{20}$$

where $\gamma_0 = |\gamma(0)|$.

The above approximation is invalid when x_* is large and $n > 1$.

As expected, the typical length scale is $m^{-1}\Lambda^{1/2}$. These solutions do not show a simple scaling property; however, numerical study indicates that, when the central density is less than the critical value corresponding to $M_{\max}$, a heavier halo has a smaller radius for both the $\Lambda = 0$ and $\Lambda \neq 0$ cases [11].

Note the facts that the above arguments are valid when the node number is fixed and both the mass and radius of the boson star are increasing functions of the node number.

Now, let us discuss the stability of higher node solutions. There are studies [12,13] indicating that nonzero node solutions with $\Lambda = 0$ are unstable against fission and small radial perturbation. Since higher node solutions are unstable, they must be long lived to explain the age of galaxies.

One possible decay mechanism for the nonzero node solutions to the zero node solution is gravitational radiation [18]. The power of the gravitational radiation P is about $G(d^3I/dt^3)^2$, where the quadrapole moment $I \sim M_{\rm halo}R_{\rm halo}^2$. The available time scale is $T \sim (R_{\rm halo}^3/GM_{\rm halo})^{1/2}$, which is given by the virial theorem. The parameter $\alpha \equiv GM_{\rm halo}/R_{\rm halo}$ indicates how much halo is relativistic.

So we can find a crude estimate of the power $P \sim G^4(M_{\rm halo}/R_{\rm halo})^5 \sim \alpha^5/G \sim \alpha^5 \times 10^{59}$ ergs/s. Since $\alpha \sim 10^{-7}$ for the halo, $P \sim 10^{24}$ erg/s. The potential energy of halo, $GM_{\rm halo}^2/R_{\rm halo}$, is about 10^{58} erg, and therefore the time scale of decay by the gravitational radiation is much longer than the age of galaxies $\sim 10^{10}$ yr$\sim 10^{17}$ s.

For the $\Lambda \neq 0$ case, there is work indicating that higher node solutions are stable against the perturbation with fixed particle number [19]. However, there seems to be no work on the stability against a more general perturbations. Since it is still unclear that the higher node solution with $\Lambda \neq 0$ is stable, we must again estimate the lifetime of the halo against gravitational radiation. From Eq. (15) we get $\Lambda\sigma^4 = -\gamma\sigma^2$, which indicates that in the Newtonian limit the energy of the repulsive force is comparable to the gravitaional potential energy. So the energy distribution in the halo is not so different from the $\Lambda = 0$ case. Therefore we argue that the same procedure for calculating the gravitational radiation is applicable to the $\Lambda \neq 0$ case and the halo is long lived against the gravitational radiation.

Another cooling mechanism, the evaporation and collapse procedure, is also inefficient [3].

In conclusion, we find that self-interactions between the particles, even if weak, may play an important role in the boson star model of halos. Our work can easily be extended to the boson-fermion star [16] and the Q star [17].

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