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Electron-positron pair production in the Aharonov-Bohm potential

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In the framework of QED we evaluate the cross section for electron-positron pair production by a single photon in the presence of the external Aharonov-Bohm potential in first order of perturbation theory. We analyze energy, angular, and polarization distributions at different energy regimes: near the threshold and at high photon energies.
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I. INTRODUCTION

In the previous paper [1] we investigated the bremsstrahlung process for relativistic electrons scattered by the external Aharonov-Bohm (AB) potential (magnetic string). This process is supposed to be the most significant one among those accompanying the AB scattering [2]. The AB effect, the influence of magnetic fluxes on quantum systems, can be adequately interpreted by means of *phase factors* [3] $\exp\left(ie\oint A_\mu dx_\mu\right)$ which produce phase shifts in wave functions of charged particles. A number of remarkable experiments was made to observe the resulting interference pattern of an electron beam scattered by a thin solenoid. For a comprehensive review see [4,5]. In solid state physics the manifestation of the AB effect brought new unexpected results [6,7].

In addition to the bremsstrahlung process there exist other important quantum effects in the presence of the external AB field. We consider here in the framework of QED the production of an electron-positron pair by a single photon in first order. This process, as other analogous quantum processes, is possible only in the presence of external fields which provide the necessary momentum transfer. It happens, for example, in the Coulomb field [8] or a uniform magnetic field [9–11]. In these cases there are external local forces which influence the motion of the created charged particles. In the AB case, however, the pair creation seems to be somehow mysterious since it happens due to a global, topological reason. In fact the AB field provides the violation of the momentum conservation law. Therefore the mechanism that permits pair production bears some resemblance with processes near cosmic strings [12,13].

The theoretical study of the AB scattering for the Dirac electron [14,15] raised a mathematical problem related to the correct description of the behavior of electron wave functions on the magnetic string. We do not discuss this problem here but refer to [14,15,1]. The issue of spin changes slightly the interpretation of the AB effect. The interaction between spin and magnetic field leads to wave functions of Dirac particles which do not vanish on the magnetic string and thus, in a way, a local element is added to the nonlocality of the AB effect.

The paper is organized as follows. In Sec. II we consider briefly the Dirac equation in the presence of the AB potential and work out the electron and positron wave functions characterized by quantum numbers of a complete set of commuting operators. The exact scattering wave function for electrons and positrons are expressed in terms of partial waves. In Sec. III the matrix element for the pair production by a single photon is calculated and the effective differential cross section is evaluated. We analyze the behavior of the differential and total cross section at different energy regimes and discuss their particular features for the Dirac electron in Secs. IV and V. We use units such that $\hbar = c = 1$ and take $e < 0$ for the electron charge.

II. THE ELECTRON AND POSITRON SOLUTIONS TO THE DIRAC EQUATION IN THE AHARONOV-BOHM POTENTIAL

The Dirac equation in an external magnetic field reads

$$i\partial_t\psi = H\psi, \quad H = \alpha_i(p_i - eA_i) + \beta M\ , \tag{1}$$

where e is the electron charge. For matrices α and β we use

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2}$$

In cylindrical coordinates (ρ, φ, z) the kinetic momenta are given by

$$\pi_\rho = p_\rho = -i\partial_\rho, \quad \pi_\varphi = p_\varphi - eA_\varphi = -\frac{i}{\rho}\partial_\varphi - eA_\varphi,$$
$$p_3 = -i\partial_z\ , \tag{3}$$

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where

$$\sigma_\rho = \sigma_1 \cos\varphi + \sigma_2 \sin\varphi,$$
$$\sigma_\varphi = -\sigma_1 \sin\varphi + \sigma_2 \cos\varphi, \tag{4}$$

and σ_i are the Pauli matrices.

The vector potential for the *pure AB case* (magnetic string) has a nonzero angular component [2]

$$eA_\varphi = \frac{e\Phi}{2\pi\rho} = -\frac{\Phi}{\Phi_0\rho} = \frac{\phi}{\rho}, \tag{5}$$

where Φ is the magnetic flux and $\Phi_0 = 2\pi\hbar c/|e|$ is the magnetic flux quantum. It corresponds to a magnetic field with support on the z axis:

$$B_z = \frac{2\phi}{e\rho}\delta(\rho) \tag{6}$$

which points to the positive (negative) z direction for $\phi < 0$ ($\phi > 0$). Note that it is the fractional part δ of the magnetic flux $\phi = N + \delta$, $0 < \delta < 1$ which produces all physical effects. Its integral part N appears as a phase factor $\exp(iN\varphi)$ in solutions of the Dirac equation.

The exact solution of the Dirac equation for the scattering problem in the external AB field can be written in an integral form as it was done for the Schrödinger equation in the original paper by Aharonov and Bohm [2]. For our problem, however, cylindrical modes are more convenient.

For the Dirac equation in the AB field the complete set of commuting operators is

$$\hat{H}\,, \quad \hat{p_3} := -i\partial_z\,, \quad \hat{J}_3 := -i\partial_\varphi + \tfrac{1}{2}\Sigma_3\,,$$

$$\hat{S}_3 := \beta\Sigma_3 + \gamma\,\frac{p_3}{M}\,, \tag{7}$$

where $\gamma := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. The corresponding eigenvalue equations are given by

$$\hat{H}\psi = E\psi\,, \tag{8}$$
$$\hat{p_3}\psi = p_3\psi\,, \tag{9}$$
$$\hat{J}_3\psi = j_3\psi\,, \tag{10}$$
$$\hat{S}_3\psi = s\psi\,, \tag{11}$$

where $E = \sqrt{p_\perp^2 + p_3^2 + M^2}$ is the energy, p_3 and j_3 are the z components of linear and total angular momentum, respectively; $p_\perp$ denotes the radial momentum. The eigenvalue of $\hat{J}_3$ is half-integer and we rewrite it by introducing l, $j_3 =: l + N + 1/2$. Here l is an integer number and N is fixed as above. Note that $l + N$ denotes an integral part of the eigenvalue of $\hat{J}_3$ in contrast to the usual convention. The corresponding separation of a factor $\exp(iN\varphi)$ in the solutions of the Dirac equation will turn out to be convenient in the following calculations. We introduced in Eq. (7) the operator $\hat{S}_3$ and not the helicity operator $\hat{S}_t = \Sigma_i(p_i - eA_i)/p$ which is often used. Both of these operators commute in the relativistic case with the operators $\hat{H}$, $\hat{p}_3$, and $\hat{J}_3$, when a magnetic field of a fixed direction is present. This can be seen, for example, in [19]. We prefer to use $\hat{S}_3$ because in the nonrelativistic limit, which will be treated below, it describes the spin projection along the direction of the magnetic field. Its eigenvalue is given by $s = \pm\sqrt{1 + p_3^2/M^2}$. Solving these eigenvalue equations leads to a radial solution of Bessel type.

As independent radial solutions we choose Bessel functions of the first kind of positive and negative orders. Then the normalization condition for the partial modes with quantum numbers $j = (p_\perp, p_3, l, s)$,

$$\int d^3x\,\psi^\dagger(j,x)\psi(j',x)$$

$$= \delta_{j,j'} = \delta_{s,s'}\delta_{l,l'}\delta(p_3 - p_{3'})\frac{\delta(p_\perp - p'_\perp)}{\sqrt{p_\perp p'_\perp}}, \tag{12}$$

fixes the solutions (for electron states with $E > 0$) of these equations for values of l outside of the interval $-1 < l - \delta < 0$ thus removing Bessel functions with negative order which are not square integrable. One finds, for $l \neq 0$,

$$\psi_e(j,x) = \frac{1}{2\pi}\frac{1}{\sqrt{2E_p}}\,e^{-iE_pt + ip_3z}\,e^{iN\varphi}\exp\left(i\frac{\pi}{2}|l|\right)\begin{pmatrix} u \\ v \end{pmatrix}, \tag{13}$$

where

$$u = \frac{1}{\sqrt{2s}}\begin{pmatrix} \sqrt{E_p + sM}\sqrt{s+1}\,J_{\nu_1}(p_\perp\rho)\,e^{il\varphi} \\ i\epsilon_3\epsilon_l\sqrt{E_p - sM}\sqrt{s-1}\,J_{\nu_2}(p_\perp\rho)\,e^{i(l+1)\varphi} \end{pmatrix}, \tag{14}$$

$$v = \frac{1}{\sqrt{2s}}\begin{pmatrix} \epsilon_3\sqrt{E_p + sM}\sqrt{s-1}\,J_{\nu_1}(p_\perp\rho)\,e^{il\varphi} \\ i\epsilon_l\sqrt{E_p - sM}\sqrt{s+1}\,J_{\nu_2}(p_\perp\rho)\,e^{i(l+1)\varphi} \end{pmatrix}, \tag{15}$$

and

$$p_\perp := \sqrt{p^2 - p_3^2} = \sqrt{E_p^2 - M^2 - p_3^2},$$

$$s = \pm\sqrt{1 + \frac{p_3^2}{M^2}}, \quad \epsilon_3 := \mathrm{sgn}(sp_3), \tag{16}$$

$$\nu_1 := \begin{cases} l - \delta \\ -l + \delta \end{cases}, \quad \nu_2 := \begin{cases} l + 1 - \delta \\ -l - 1 + \delta \end{cases},$$

$$\epsilon_l := \begin{cases} 1 & \text{if } l \geq 0, \\ -1 & \text{if } l < 0. \end{cases} \tag{17}$$

Since the normalization condition method does not apply for $l = 0$ this case needs a separate discussion, which has been done in [17]. Fortunately it turns out that the corresponding solution is of the same form as for $l \neq 0$. Therefore the expressions above are valid not only for $l \neq 0$ but also for $l = 0$ so that it is allowed to include this case in (17). The critical mode $l = 0$ contains the irregular but square integrable Bessel functions of orders $-\delta$ and $-1+\delta$. Their inevitable appearance in solutions of the Dirac equation is an obvious consequence of the interaction between spin and magnetic field. This problem is a part of a general problem of the self-adjoint extension for the Hamilton operator in the presence of a singular potential (*pure AB case*), and it was discussed in the previous paper [1] (see also [14,16]). In [17] we presented an alternative method of treating the self-adjointness problem.

The complete set of solutions of the Dirac equations includes the negative energy electron states. Instead of them we introduce positron states ψ_p with $E > 0$ which can be obtained from electron states of negative energy by the charge conjugation operation

$$\psi \to \psi_c = C\bar{\psi}_{\mathrm{transp}}\,, \quad C = \alpha_2 \tag{18}$$

and replacing e by $-e$. ψ_c obeys the free Dirac equation as well as ψ does but with opposite sign of the electric charge and has quantum numbers $E, -p_3, -j_3, s$. One needs to replace $p_3 \to -p_3$, $j_3 \to -j_3$ ($l \to -l-1$) in the electron state of negative energy to obtain positron state with quantum numbers E, p_3, j_3, s.

The electron-positron field operator reads

$$\psi(x,t) = \int d\mu_j [\psi_e(j,x)a_j + \psi_p^c(j,x)b_j^\dagger], \tag{19}$$

with a_j and b_j being the annihilation operators for the electron and positron with given quantum numbers. It contains positive frequency functions ψ_e (electron states) and negative frequency functions ψ_p^c (positron states):

$$\psi_p^c(j,x) = \frac{1}{2\pi}\frac{1}{\sqrt{2E_p}}\, e^{iEt-ip_3z}\, e^{iN\varphi} \exp\left(i\frac{\pi}{2}|l|\right)\begin{pmatrix} y \\ w \end{pmatrix}, \tag{20}$$

where

$$y = \frac{1}{\sqrt{2s}}\begin{pmatrix} i\epsilon_l\sqrt{E_p - sM}\sqrt{s+1}\, J_{\nu_2'}(p_\perp\rho)\, e^{-i(l+1)\varphi} \\ \epsilon_3\sqrt{E_p + sM}\sqrt{s-1}\, J_{\nu_1'}(p_\perp\rho)\, e^{-il\varphi} \end{pmatrix}, \tag{21}$$

$$w = -\frac{1}{\sqrt{2s}}\begin{pmatrix} i\epsilon_3\epsilon_l\sqrt{E_p - sM}\sqrt{s-1}\, J_{\nu_2'}(p_\perp\rho)\, e^{-i(l+1)\varphi} \\ \sqrt{E_p + sM}\sqrt{s+1}\, J_{\nu_1'}(p_\perp\rho)\, e^{-il\varphi} \end{pmatrix} \tag{22}$$

with

$$\nu_1' := \begin{cases} l+\delta \\ -l-\delta \end{cases}, \quad \nu_2' := \begin{cases} l+1+\delta & \text{if } l \geq 0\,, \\ -l-1-\delta & \text{if } l < 0\,. \end{cases} \tag{23}$$

The expressions (13)–(17) and (20)–(23) present the partial electron and positron wave functions in terms of cylindrical modes. These states do not describe outgoing particles with definite linear momenta at infinity. In order to calculate the cross section of the pair production process we need the *electron and positron scattering wave functions.* In external fields there exist two independent exact solutions of the Dirac equation which behave at large distances like a plane wave (propagating in the direction $\vec{p}$ given by $p_x = p_\perp \cos\varphi_p$, $p_y = p_\perp \sin\varphi_p$, p_z) plus an outgoing or ingoing cylindrical waves, correspondingly. For outgoing particles we need to take wave functions which contain ingoing cylindrical waves. In this case the interaction of the created electron and positron with the external magnetic field will be described correctly [8].

The corresponding scattering wave functions can be obtained by superpositions of the cylindrical modes:

$$\Psi_e(J,x) := \sum_l c_l^{(e)}\, \psi_e(j_p,x) \tag{24}$$

and

$$\Psi_p^c(J,t) := \sum_n c_n^{(p)}\, \psi_p^c(j_q,x) \tag{25}$$

with the coefficients

$$c_l^{(e)} := e^{-il\varphi_p}\, e^{-i\frac{\pi}{2}\epsilon_l\delta}\,, \quad c_n^{(p)} := e^{i(n+1)(\varphi_q+\pi)}\, e^{i\frac{\pi}{2}\epsilon_n\delta}\,, \tag{26}$$

where J is a collective index for the linear momentum at infinity and s.

In the terms of the wave functions (24) and (25) the electron-positron field operator reads

$$\psi(x,t) = \int d\mu_J [\Psi_e(J,x)a_J + \Psi_p^c(J,x)b_J^\dagger], \tag{27}$$

with a_J and b_J being the annihilation operators for the electron and positron with quantum numbers of the scattering states.

The external AB field has no influence on the *photon wave function.* In cylindrical coordinates it reads

$$A_\mu^\lambda(\vec{k},x) = \frac{e_\mu^{(\lambda)}}{\sqrt{2\omega_k}} e^{-i\omega_k t + ik_3 z} e^{ik_\perp\rho\cos(\varphi-\varphi_k)}\,, \tag{28}$$

where the polarization vectors

$$e^{(\sigma)} := (0,\, -\sin\varphi_k,\, \cos\varphi_k,\, 0)\,,$$
$$e^{(\pi)} := \frac{1}{\omega_k}(0,\, -k_3\cos\varphi_k,\, -k_3\sin\varphi_k,\, k_\perp)\,, \tag{29}$$

correspond to two linear transversal polarization states. In the coordinate frame with $k_3 = 0$ in which we will perform all calculations, the polarization vector $e^{(\pi)}$ is directed along the z axis and $e^{(\sigma)}$ is orthogonal to the magnetic string.

III. MATRIX ELEMENTS AND DIFFERENTIAL CROSS SECTIONS FOR PAIR PRODUCTION BY A SINGLE PHOTON $\gamma \to e^- + e^+$

The differential cross section of the pair production process describes the distribution of the created particles with respect to their quantum numbers. For these one may take the angular momenta which correspond to cylindrical modes. But usually final states are related to plane wave states, and in our case to the scattering states (24) and (25). The cylindrical modes have a vanishing radial flux and therefore do not describe ingoing or outgoing particles. They are, however, convenient for calculating matrix elements, and we use these matrix elements as starting point for calculation of the differential cross section which refers to scattering states. As far as the total cross section is concerned one can use any final states.

A. Matrix elements for cylindrical modes

The matrix element for pair production of an electron with quantum numbers $j_p = (p_\perp, p_3, l, s)$ and a positron with quantum numbers $j_q = (q_\perp, q_3, n, r)$ by a single photon with quantum numbers $(\vec{k},\ \lambda)$ for physical states $\lambda = \sigma,\ \pi$ has the usual form

$$\widetilde{M}(j_p, j_q; \vec{k}, \lambda) = -i\langle j_q, j_p|S^{(1)}|\vec{k}, \lambda\rangle = -e\int d^4x\, \bar{\psi}_e(j_p, x)\ A^{\lambda}_{\mu}(\vec{k}, x)\gamma_\mu\ \psi^c_p(j_q, x)\ , \tag{30}$$

whereby gamma matrices are written in terms of Pauli matrices as

$$\gamma_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}, \tag{31}$$

so that

$$e^{(\lambda)}_\mu \gamma_\mu = \begin{pmatrix} 0 & \alpha_\lambda \\ -\alpha_\lambda & 0 \end{pmatrix},\ \alpha_\sigma = \begin{pmatrix} 0 & -ie^{-i\varphi_k} \\ ie^{i\varphi_k} & 0 \end{pmatrix},$$

$$\alpha_\pi = \begin{pmatrix} k_\perp & -k_3 e^{-i\varphi_k} \\ -k_3 e^{i\varphi_k} & -k_\perp \end{pmatrix}\frac{1}{\omega_k}. \tag{32}$$

Using expressions (13), (20), and (28) we can rewrite the matrix element (30) in the form

$$\widetilde{M}_\lambda(j_p, j_q) = -e\frac{1}{2\sqrt{2\omega_k E_q E_p}}\exp\left(-i\frac{\pi}{2}(|l| - |n|)\right)\delta(E_p + E_q - \omega_k)\delta(p_3 + q_3 - k_3)\ m_\lambda \tag{33}$$

with

$$m_\lambda := \int \rho\, d\rho\, d\varphi\, e^{ik_\perp \rho \cos(\varphi - \varphi_k)}\ \left[u^\dagger(p)\alpha_\lambda w(q) + v^\dagger(p)\alpha_\lambda y(q)\right]$$

$$= e^{-i(l+n+1)\varphi_k}\int \rho\, d\rho\, d\varphi\, e^{ik_\perp \rho \cos(\varphi - \varphi_k)} K_\lambda(\rho, \varphi). \tag{34}$$

The Dirac equation (1) in the external AB field is invariant under boost transformations along the string direction. This means that it is sufficient to treat the case of normal incidence of the photon on the magnetic string, and therefore we may perform all calculations in the coordinate system in which $k_3 = 0$. No information will be lost but calculations become simpler in this case.

For the polarization state $\lambda = \sigma$ we have

$$K_\sigma(\rho, \varphi) := iR_\sigma\left[\sqrt{E_p + sM}\sqrt{E_q + rM}\ J_{\nu_1}(p_\perp \rho)J_{\nu_1'}(q_\perp \rho)\ e^{-i(l+n)(\varphi - \varphi_k)} + \epsilon_l \epsilon_n \sqrt{E_p - sM}\sqrt{E_q - rM}\ J_{\nu_2}(p_\perp \rho)J_{\nu_2'}(q_\perp \rho)\ e^{-i(l+n+2)(\varphi - \varphi_k)}\right], \tag{35}$$

with

$$R_\sigma := \frac{1}{2\sqrt{sr}}\left[\sqrt{s+1}\sqrt{r+1} - \epsilon_3(p)\epsilon_3(q)\sqrt{s-1}\sqrt{r-1}\,\right] \tag{36}$$

and, for the polarization state $\lambda = \pi$,

$$K_\pi(\rho, \varphi) := iR_\pi\left[\epsilon_n\sqrt{E_p + sM}\sqrt{E_q - rM}\ J_{\nu_1}(p_\perp \rho)J_{\nu_2'}(q_\perp \rho) - \epsilon_l\sqrt{E_p - sM}\sqrt{E_q + rM}\ J_{\nu_2}(p_\perp \rho)J_{\nu_1'}(q_\perp \rho)\right] e^{-i(l+n+1)(\varphi - \varphi_k)} \tag{37}$$

with

$$R_\pi := \frac{1}{2\sqrt{sr}}\left[\epsilon_3(p)\sqrt{s-1}\sqrt{r+1} - \epsilon_3(q)\sqrt{s+1}\sqrt{r-1}\,\right]. \tag{38}$$

Integrating over φ we obtain

$$m_\sigma = 2\pi i e^{-i(l+n+1)\varphi_k} e^{i\frac{\pi}{2}(l+n)} R_\sigma \int \rho\, d\rho \left[\sqrt{E_p+sM}\sqrt{E_q+rM}\; J_{\nu_1}(p_\perp\rho) J_{\nu_1'}(q_\perp\rho) J_{l+n}(k_\perp\rho) - \epsilon_l\epsilon_n\sqrt{E_p-sM}\sqrt{E_q-rM}\; J_{\nu_2}(p_\perp\rho) J_{\nu_2'}(q_\perp\rho) J_{l+n+2}(k_\perp\rho)\right] \tag{39}$$

and

$$m_\pi = 2\pi i e^{-i(l+n+1)\varphi_k} e^{i\frac{\pi}{2}(l+n+1)} R_\pi \int \rho\, d\rho \left[\epsilon_n\sqrt{E_p+sM}\sqrt{E_q-rM}\; J_{\nu_1}(p_\perp\rho) J_{\nu_2'}(q_\perp\rho) J_{l+n+1}(k_\perp\rho) - \epsilon_l\sqrt{E_p-sM}\sqrt{E_q+rM}\; J_{\nu_2}(p_\perp\rho) J_{\nu_1'}(q_\perp\rho) J_{l+n+1}(k_\perp\rho)\right]. \tag{40}$$

It follows from the energy conservation law, $\omega_k = E_p + E_q$, that the photon's radial momentum obeys the inequality $k_\perp > p_\perp + q_\perp$. The excess of radial momentum, $k_\perp - (p_\perp + q_\perp)$, is transmitted to the flux tube. For this case, using formulas [6.578(3) and 6.522(14)] of [18], one can see that the integrals over ρ vanish unless $(l+\frac{1}{2})(n+\frac{1}{2}) < 0$. This inequality is satisfied at $l \geq 0, n < 0$ and $n \geq 0, l < 0$, and the nonvanishing integrals are of the type

$$J(\alpha,\beta) := \int_0^\infty \rho\, d\rho\, J_\alpha(p_\perp\rho\sin A\cos B) J_\beta(q_\perp\rho\cos A\sin B) J_{\beta-\alpha}(k_\perp\rho) = \frac{2\sin\pi\alpha}{\pi k_\perp^2\cos(A+B)\cos(A-B)}\left(\frac{\sin A}{\cos B}\right)^\alpha\left(\frac{\sin B}{\cos A}\right)^\beta \tag{41}$$

with $p_\perp = k_\perp\sin A\cos B,\ q_\perp = k_\perp\sin B\cos A$.

Denoting

$$a := \frac{\sin A}{\cos B} = \frac{p_\perp}{E_p + \sqrt{q_3^2+M^2}}, \quad b := \frac{\sin B}{\cos A} = \frac{q_\perp}{E_q + \sqrt{q_3^2+M^2}}\,, \tag{42}$$

we have in terms of the integral (41) for the matrix elements (39)

$$\begin{aligned} m_\sigma = {} & 2\pi i e^{-i(l+n+1)\varphi_k} e^{i\frac{\pi}{2}(l+n)} R_\sigma \Big\{\Theta(l\geq 0)\Theta(n<0)(-1)^{l+n}\Big[\sqrt{E_p+sM}\sqrt{E_q+rM}\; J(l-\delta,-n-\delta) \\ & + \sqrt{E_p-sM}\sqrt{E_q-rM}\; J(l+1-\delta,-n-1-\delta)\Big] \\ & + \Theta(l<0)\Theta(n\geq 0)\Big[\sqrt{E_p+sM}\sqrt{E_q+rM}\; J(-l+\delta,n+\delta) \\ & + \sqrt{E_p-sM}\sqrt{E_q-rM}\; J(-l-1+\delta,n+1+\delta)\Big]\Big\} \\ = {} & -\frac{4iR_\sigma\, e^{-i(l+n+1)\varphi_k + i\frac{\pi}{2}|l-n|}\sin\pi\delta}{\sqrt{k_\perp^4 - 2k_\perp^2(p_\perp^2+q_\perp^2) + (p_\perp^2-q_\perp^2)^2}}\, a^{|l|}\, b^{|n|} \\ & \times\Big[\Theta(l\geq 0)\Theta(n<0)(ab)^{-\delta}\left(\sqrt{E_p+sM}\sqrt{E_q+rM} - \frac{a}{b}\sqrt{E_p-sM}\sqrt{E_q-rM}\right) \\ & - \Theta(l<0)\Theta(n\geq 0)(ab)^{\delta}\left(\sqrt{E_p+sM}\sqrt{E_q+rM} - \frac{b}{a}\sqrt{E_p-sM}\sqrt{E_q-rM}\right)\Big] \end{aligned} \tag{43}$$

and, for the matrix element (40),

$$m_\pi = -2\pi i e^{-i(l+n+1)\varphi_k} e^{i\frac{\pi}{2}(l+n+1)} R_\pi \Big\{ \Theta(l\ge 0)\Theta(n<0)(-1)^{l+n+1}\Big[\sqrt{E_p+sM}\sqrt{E_q-rM}\; J(l-\delta,-n-1-\delta)$$
$$+ \sqrt{E_p-sM}\sqrt{E_q+rM}\; J(l+1-\delta,-n-\delta)\Big]$$
$$-\Theta(l<0)\Theta(n\ge 0)\Big[\sqrt{E_p+sM}\sqrt{E_q-rM}\; J(-l+\delta,n+1+\delta)$$
$$+ \sqrt{E_p-sM}\sqrt{E_q+rM}\; J(-l-1+\delta,n+\delta)\Big]\Big\}$$
$$= \frac{4R_\pi\, e^{-i(l+n+1)\varphi_k+i\frac{\pi}{2}|l-n|}\sin\pi\delta}{\sqrt{k_\perp^4-2k_\perp^2(p_\perp^2+q_\perp^2)+(p_\perp^2-q_\perp^2)^2}}\, a^{|l|}\, b^{|n|}$$
$$\times\Big[\Theta(l\ge 0)\Theta(n<0)(ab)^{-\delta}\Big(\frac{1}{b}\sqrt{E_p+sM}\sqrt{E_q-rM}-a\sqrt{E_p-sM}\sqrt{E_q+rM}\Big)$$
$$-\Theta(l<0)\Theta(n\ge 0)(ab)^{\delta}\Big(b\sqrt{E_p+sM}\sqrt{E_q-rM}-\frac{1}{a}\sqrt{E_p-sM}\sqrt{E_q+rM}\Big)\Big]. \qquad (44)$$

The partial wave analysis of the pair production process caused by a photon which passes the AB string shows a rather unexpected feature: The process turns out to be forbidden unless the quantum numbers l and n of the outgoing electron and positron have opposite signs. This in turn implies that (the expectation values of) their kinetic angular momentum projections, $[\vec{r}\times(\vec{p}-e\vec{A})]_3 = -i\partial_\varphi-\phi$ have opposite signs. (For a detailed discussion see [1].) In the framework of a semiclassical picture this means that created charged particles need to pass the magnetic string in opposite directions. Apparently this is necessary for the ingoing photon to give the excess of its radial momentum, $k_\perp-(p_\perp+q_\perp)$, to the string and to create a real electron-positron pair from the vacuum.

We draw attention to another characteristic trait of the pair creation process. It takes place although the incoming photon is not influenced by the magnetic string *directly*. The process happens since the created charged particles interact with the AB potential. The magnetic string distorts the states of the virtual electron-positron pairs in the vacuum. The incoming photon interacts with these virtual pairs, and it can transform them into real pairs if the conditions for momentum transfer (and energy conservation) are satisfied.

B. Differential cross section for scattering states

It is now easy to calculate the matrix element for the electron-positron pair production by a photon with respect to the electron (24) and positron (25) scattering states. For an incoming photon with momentum $\vec{k}$ and polarization λ which creates electron and positron with momenta $\vec{p}$, $\vec{q}$ and spins s, r, correspondingly, the matrix element of the process reads

$$M_\lambda := -i\langle(\vec{q},r),(\vec{p},s)|S^{(1)}|(\vec{k},\lambda)\rangle$$
$$= \sum_{l,n} c_l^{(e)*} c_n^{(p)}\, \widetilde{M}_\lambda(j_p,j_q)$$
$$= \frac{e\sqrt{2}\sin\pi\delta}{\sqrt{\omega_k E_q E_p}}\,\frac{\delta(E_p+E_q-\omega_k)\,\delta(p_3+q_3)}{\sqrt{k_\perp^4-2k_\perp^2(p_\perp^2+q_\perp^2)+(p_\perp^2-q_\perp^2)^2}}$$
$$\times R_\lambda\,\Sigma_\lambda\,, \qquad (45)$$

where the coefficients $c_l^{(e)*}$ and $c_n^{(p)}$ are given by Eq. (26) and we denote

$$i\Sigma_\sigma := \sum_{l,n} e^{il\varphi_{pk}+i(n+1)\varphi_{qk}}\, a^{|l|}\, b^{|n|}\Big[(ab)^{-\delta}\Theta(l\ge 0)\Theta(n<0)\Big(\sqrt{E_p+sM}\sqrt{E_q+rM}-\frac{a}{b}\sqrt{E_p-sM}\sqrt{E_q-rM}\Big)$$
$$-(ab)^{\delta}\Theta(l<0)\Theta(n\ge 0)\Big(\sqrt{E_p+sM}\sqrt{E_q+rM}-\frac{b}{a}\sqrt{E_p-sM}\sqrt{E_q-rM}\Big)\Big], \qquad (46)$$

$$\Sigma_\pi := \sum_{l,n} e^{il\varphi_{pk}+i(n+1)\varphi_{qk}}\, a^{|l|}\, b^{|n|}\Big[(ab)^{-\delta}[\Theta(l\ge 0)\Theta(n<0)\Big(\frac{1}{b}\sqrt{E_p+sM}\sqrt{E_q-rM}-a\sqrt{E_p-sM}\sqrt{E_q+rM}\Big)$$
$$-(ab)^{\delta}\Theta(l<0)\Theta(n\ge 0)\Big(b\sqrt{E_p+sM}\sqrt{E_q-rM}-\frac{1}{a}\sqrt{E_p-sM}\sqrt{E_q+rM}\Big)\Big] \qquad (47)$$

with $\varphi_{pk} := \varphi_p-\varphi_k$, $\varphi_{qk} := \varphi_q-\varphi_k$.

Performing the sums over l, n we obtain, for the polarization state $\lambda=\sigma$,

$$i\Sigma_\sigma = (ab)^{-\delta}\left(b\sqrt{E_p+sM}\sqrt{E_q+rM} - a\sqrt{E_p-sM}\sqrt{E_q-rM}\right)\Sigma$$
$$-(ab)^{\delta}\left(a\sqrt{E_p+sM}\sqrt{E_q+rM} - b\sqrt{E_p-sM}\sqrt{E_q-rM}\right)\, e^{-i\varphi_{pq}}\,\Sigma^* \tag{48}$$

and, for the polarization state $\lambda = \pi$,

$$\Sigma_\pi = (ab)^{-\delta}\left(\sqrt{E_p+sM}\sqrt{E_q-rM} - ab\sqrt{E_p-sM}\sqrt{E_q+rM}\right)\Sigma$$
$$-(ab)^{\delta}\left(ab\sqrt{E_p+sM}\sqrt{E_q-rM} - \sqrt{E_p-sM}\sqrt{E_q+rM}\right)\, e^{-i\varphi_{pq}}\,\Sigma^* \tag{49}$$

with

$$\Sigma := \frac{1}{1-a\,e^{i\varphi_{pk}}}\cdot\frac{1}{1-b\,e^{-i\varphi_{qk}}}. \tag{50}$$

Equation (45) together with (46)–(50) contain results for the pair production matrix elements with respect to scattering states.

Based on the matrix element (45) we evaluate the differential probability of electron-positron pair production by a single photon per unit length of the magnetic string and unit time

$$dW_\lambda = W_\lambda\, p_\perp dp_\perp d\varphi_p\, q_\perp dq_\perp d\varphi_q dp_3 dq_3\ , \tag{51}$$

where

$$W_\lambda := \frac{|M_\lambda|^2}{TL}\,\frac{e^2\,\sin^2\pi\delta}{8\pi^4\omega_k E_q E_p}\,\frac{\delta(E_p+E_q-\omega_k)\,\delta(p_3+q_3)}{k_\perp^4 - 2k_\perp^2(p_\perp^2+q_\perp^2)+(p_\perp^2-q_\perp^2)^2}\,R_\lambda^2\,|\Sigma_\lambda|^2. \tag{52}$$

Calculating

$$R_\sigma^2 = \frac{1+s_3r_3}{2} + \frac{q_3^2}{q_3^2+M^2}\frac{1-s_3r_3}{2},\quad R_\pi^2 = \frac{1-s_3r_3}{2} + \frac{q_3^2}{q_3^2+M^2}\frac{1+s_3r_3}{2}\ , \tag{53}$$

where $s_3 = \operatorname{sgn} s$, $r_3 = \operatorname{sgn} r$ and sorting terms with respect to the flux parameter δ,

$$|\Sigma_\lambda|^2 = (ab)^{-2\delta}P_\lambda^{(-)} + (ab)^{2\delta}P_\lambda^{(+)} + P_\lambda^{(0)}\ , \tag{54}$$

we find

$$\begin{aligned}
P_\sigma^{(-)} &= [b^2(E_p+sM)(E_q+rM) + a^2(E_p-sM)(E_q-rM) - 2abp_\perp q_\perp]\,|\Sigma|^2,\\
P_\sigma^{(+)} &= [a^2(E_p+sM)(E_q+rM) + b^2(E_p-sM)(E_q-rM) - 2abp_\perp q_\perp]\,|\Sigma|^2,\\
P_\sigma^{(0)} &= [-2ab(E_pE_q+srM^2) + (a^2+b^2)p_\perp q_\perp]\,|\Sigma|^2\,F(\varphi),\\
P_\pi^{(-)} &= [(E_p+sM)(E_q-rM) + a^2b^2(E_p-sM)(E_q+rM) - 2abp_\perp q_\perp]\,|\Sigma|^2,\\
P_\pi^{(+)} &= [a^2b^2(E_p+sM)(E_q-rM) + (E_p-sM)(E_q+rM) - 2abp_\perp q_\perp]\,|\Sigma|^2,\\
P_\pi^{(0)} &= [-2ab(E_pE_q-srM^2) + (1+a^2b^2)p_\perp q_\perp]\,|\Sigma|^2\,F(\varphi)\ ,
\end{aligned} \tag{55}$$

where

$$|\Sigma|^2 = \frac{1}{(1+a^2-2a\cos\varphi_{pk})(1+b^2-2b\cos\varphi_{qk})}, \tag{56}$$

$$|\Sigma|^2F(\varphi) := e^{i\varphi_{pq}}\Sigma^2 + e^{-i\varphi_{pq}}\Sigma^{*2}\ , \tag{57}$$

with

$$F(\varphi) := \frac{[2a-(1+a^2)\cos\varphi_{pk}][2b-(1+b^2)\cos\varphi_{qk}] - (1-a^2)(1-b^2)\sin\varphi_{pk}\sin\varphi_{qk}}{(1+a^2-2a\cos\varphi_{pk})(1+b^2-2b\cos\varphi_{qk})}. \tag{58}$$

The differential probability of the pair production process with respect to the variables $\vec{p}$, $\vec{q}$ is given by Eqs. (51)–(58).

The complete information about energy, angular, and polarization distributions of created electrons and positrons is contained in the effective differential cross section:

$$\frac{d\sigma_\lambda}{dE_q d\varphi_q d\varphi_p dq_3} = \frac{e^2 \sin^2 \pi\delta}{32\pi^4} \frac{R_\lambda^2 |\Sigma_\lambda|^2}{\omega_k^3(q_3^2 + M^2)} . \tag{59}$$

These distributions can be observed in experiments. Instead of $p_\perp$ and $q_\perp$ we made use of more convenient variables – the positron (electron) energy E_q (E_p) and the z component of the positron (electron) momentum q_3 ($-p_3$). They are, obviously, related by the equalities $E_p + E_q = \omega_k$, $p_3 + q_3 = 0$.

We rewrite the cross section (59) in a more detailed form

$$\frac{d\sigma_\lambda}{dE_q d\varphi_q d\varphi_p dq_3} = \frac{e^2 \sin^2 \pi\delta}{128\pi^4} \frac{c^{-\delta} S_\lambda^{(-)} + c^\delta S_\lambda^{(+)} + S_\lambda^{(0)}}{\omega_k^3(q_3^2 + M^2)(E_p - p_\perp \cos\varphi_{pk})(E_q - q_\perp \cos\varphi_{qk})} \tag{60}$$

with

$$c := (ab)^2 = \frac{(E_p - \sqrt{q_3^2 + M^2})(E_q - \sqrt{q_3^2 + M^2})}{(E_p + \sqrt{q_3^2 + M^2})(E_q + \sqrt{q_3^2 + M^2})} \tag{61}$$

and $p_\perp = \sqrt{E_p^2 - q_3^2 - M^2}$, $q_\perp = \sqrt{E_q^2 - q_3^2 - M^2}$.

Then we have, for the polarization state σ,

$$S_\sigma^{(\mp)} = (1 + s_3 r_3)(q_3^2 + M^2) \left[E_p^2 + E_q^2 - 2(q_3^2 + M^2) \mp s_3(q_3^2 + M^2)(E_p^2 - E_q^2)\right] + (1 - s_3 r_3) q_3^2 (E_p - E_q)^2 (1 \pm s_3), \tag{62}$$

$$S_\sigma^{(0)} = -(1 + s_3 r_3) 2 p_\perp q_\perp (q_3^2 + M^2) F(\varphi) , \tag{63}$$

and, for the polarization state π,

$$S_\pi^{(\mp)} = (1 - s_3 r_3)(q_3^2 + M^2)\omega_k^2 (1 \pm s_3) + (1 + s_3 r_3) q_3^2 \left[E_p^2 + E_q^2 - 2(q_3^2 + M^2) \mp s_3(E_p^2 - E_q^2)\right] , \tag{64}$$

$$S_\pi^0 = (1 + s_3 r_3) 2 p_\perp q_\perp q_3^2 F(\varphi) ; \tag{65}$$

with $s_3 := \operatorname{sgn} s$ and $r_3 := \operatorname{sgn} r$.

Equation (60) together with (61)–(65) and (58) gives the *final expression* for the effective differential cross section for the pair production process. We will discuss it at different energies of the incoming photon.

IV. THE CROSS SECTION FOR PAIR PRODUCTION AT DIFFERENT PHOTON ENERGIES

In this section we will analyze the angular and polarization distributions of the created electrons and positrons and find the total pair production cross section at different energies of the incoming photon.

A. Angular and polarization distributions

The angular distributions for created electrons and positrons are of a fairly complicated nature. They simplify considerably at low and high photon energies.

At *low photon energy*, just above the pair production threshold $2M$, $\omega - 2M \ll 2M$, we have

$$p_\perp \sim q_\perp \sim q_3 \ll M, \quad c \ll 1 ,$$

and

$$S_\sigma^{(\mp,0)} \sim S_\pi^{(0)} \ll S_\pi^{(\mp)} \approx 4M^4(1 - s_3 r_3)(1 \pm s_3).$$

It means that electron-positron pair of low energies are created mainly from π-polarized photons, and the differential cross section for pair production above the threshold reads in this case

$$\frac{d\sigma_\pi}{dE_q d\varphi_q d\varphi_p dq_3} \approx \frac{e^2 \sin^2 \pi\delta}{256\pi^4} (1 - s_3 r_3) \frac{c^{-\delta}(1 + s_3) + c^\delta (1 - s_3)}{M^3}. \tag{66}$$

The angular distributions for electrons and positrons of low energies are uniform in the plane perpendicular to the magnetic string but their dependence on the polar angle ϑ is rather intricate.

The polarizations of the electron and the positron depend strongly on the photon polarization state. Note that we are in the nonrelativistic limit and that s_3 (r_3) now agrees with the spin projection of the electron (positron). Created particles have spin projections of opposite signs, and electrons with positive spin projections (antiparallel to the magnetic string) are produced by π-polarized photons predominantly since $c \ll 1$. Their fraction increases with the flux parameter δ. In this case the interaction of the magnetic moments with the string magnetic field is attractive for both the electron and the positron, and their wave functions are localized near the string. The σ-polarized photon, on the other hand possesses a polarization vector perpendicular to the magnetic string and creates particles with spin projections s_3 and r_3 of equal signs which implies that their magnetic moments have opposite directions. Because of the interaction of the magnetic moment with the magnetic field of the string therefore only one of the created particles, either the electron or the positron, is attracted to the string, which leads to an enhancement of the wave function near the string. This means that one of the particles is located near the string while the other one is located at a certain distance and, in a heuristic picture, it is unlikely that the σ-polarized photon creates an electron-positron pair.

With increasing photon energies the dependence on the photon polarization disappears since also larger orbital momenta contribute to the cross section. This can be seen from Eqs. (43) and (44).

At *high photon energies*, $\omega \gg M$, the angular distributions are very simple. In this case the electron and positron are emitted predominantly in the forward direction, within a narrow cone surrounding the direction of motion of the photon. Because of the presence the factors $E_p - p_\perp \cos\varphi_{pk}$, $E_q - q_\perp \cos\varphi_{qk}$, and $q_3^2 + M^2 \sim E_q^2 \cos^2\vartheta$ in the denominator of (60) their angular distributions have sharp maxima in this direction ($\varphi_{pk} \sim \varphi_{qk} \sim 0$, $\vartheta \sim \pi/2$) and the effective angular aperture of the cone is given in order of magnitude by M/ω_k.

B. The total cross section

Let us now analyze the energy behavior of the total cross section. Integration of the differential cross section (60) over the azimuthal angles φ_p, φ_q leads to an additional factor $4\pi^2/(q_3^2 + M^2)$ and removes the term with $S_\lambda^{(0)}$ from the cross section:

$$\frac{d\sigma_\lambda}{dE_q dq_3} = \frac{e^2 \sin^2\pi\delta}{32\pi^2} \frac{c^{-\delta} S_\lambda^{(-)} + c^\delta S_\lambda^{(+)}}{\omega_k^3 (q_3^2 + M^2)^2}. \quad (67)$$

Performing the sums over polarizations of created electron and positron we obtain

$$\frac{d\sigma_\lambda}{dE_q dq_3} = \frac{e^2 \sin^2\pi\delta}{8\pi^2} \frac{c^{-\delta} + c^\delta}{\omega_k^3 (q_3^2 + M^2)^2} C_\lambda \quad (68)$$

with

$$C_\sigma := (q_3^2 + M^2)[E_p^2 + E_q^2 - 2(q_3^2 + M^2)] + q_3^2 (E_p - E_q)^2,$$
$$C_\pi := (q_3^2 + M^2)\omega_k^2 + q_3^2 [E_p^2 + E_q^2 - 2(q_3^2 + M^2)]. \quad (69)$$

Since the variables $p_\perp$ and $q_\perp$ are both positive the variable q_3 ranges from $-q_3^{\max}$ to $q_3^{\max}$ where $q_3^{\max} = \min\left(\sqrt{E_q^2 - M^2}, \sqrt{E_p^2 - M^2}\right)$. Introducing new variables

$$\varepsilon = |E_p - E_q|, \quad x = \sqrt{q_3^2 + M^2}, \quad (70)$$

we obtain the general expression for the total cross section for pair production by a photon of the energy ω_k and polarization λ:

$$\sigma_\lambda = \frac{e^2 \sin^2\pi\delta}{4\pi^2} \frac{1}{\omega_k^3} \int_0^{\omega_k - 2M} d\varepsilon \int_M^{\frac{\omega_k - \varepsilon}{2}} dx \frac{c^{-\delta} + c^\delta}{x^3 \sqrt{x^2 - M^2}} C_\lambda(\varepsilon, x)$$
$$= \frac{e^2 \sin^2\pi\delta}{4\pi^2} \frac{1}{\omega_k^3} \int_M^{\frac{\omega_k}{2}} dx \frac{1}{x^3\sqrt{x^2 - M^2}} \int_0^{\omega_k - 2x} d\varepsilon \, (c^{-\delta} + c^\delta) \, C_\lambda(\varepsilon, x) \, , \quad (71)$$

where

$$c = (ab)^2 = \frac{(\omega_k - 2x)^2 - \varepsilon^2}{(\omega_k + 2x)^2 - \varepsilon^2}, \quad (72)$$

$$C_\sigma(\varepsilon, x) = \tfrac{1}{2} x^2 (\omega_k^2 + \varepsilon^2 - 4x^2) + (x^2 - M^2)\varepsilon^2, \quad (73)$$

$$C_\pi(\varepsilon, x) = x^2 \omega_k^2 + \tfrac{1}{2}(x^2 - M^2)(\omega_k^2 + \varepsilon^2 - 4x^2). \quad (74)$$

The remaining integrals over ε and x cannot be found analytically for arbitrary values of the flux parameter δ. Even for the symmetric case $\delta = \frac{1}{2}$ we have a rather complicated integral

$$\sigma_\lambda = \frac{e^2}{2\pi^2} \frac{1}{\omega_k^3} \int_M^{\frac{\omega_k}{2}} dx \frac{1}{x^3\sqrt{x^2 - M^2}} \times \int_0^{\omega_k - 2x} d\varepsilon \frac{(\omega_k^2 - \varepsilon^2 + 4x^2)\, C_\lambda(\varepsilon, x)}{\sqrt{(\omega_k^2 - \varepsilon^2)^2 - 8x^2(\omega_k^2 + x^2) + 16x^4}}. \quad (75)$$

But the general expression (71) for the total cross section of pair production simplifies considerably at low and high photon energies to which we turn now.

At *low energies*, near the pair creation threshold, $\omega_k - 2M \ll M$ we have $C_\sigma \ll C_\pi \approx 4M^2$. Therefore we will consider only the polarization state π. It is then

$$c \approx \frac{(\omega_k - 2x)^2 - \varepsilon^2}{16M^2} \ll 1.$$

Dropping in (71) the term $c^\delta \ll c^{-\delta}$ and introducing new dimensionless variables t and y by

$$\varepsilon = (\omega_k - 2x)t, \quad x = M + \frac{\omega_k - 2M}{2} y,$$

we obtain

$$\begin{aligned}\sigma_\pi &\approx \frac{e^2 \sin^2 \pi\delta}{8\pi^2 M^2} \int_M^{\frac{\omega_k}{2}} dx \int_0^{\omega_k - 2x} d\varepsilon \frac{c^{-\delta}}{\sqrt{2M(x-M)}} \\ &= \frac{e^2 \sin^2 \pi\delta}{4\sqrt{2}\pi^2 M} \left(\frac{\omega_k - 2M}{2M}\right)^{\frac{3}{2}-2\delta} \\ &\quad \times B(1-\delta, 1-\delta)\, B(\tfrac{1}{2}, 2-2\delta) \\ &= \frac{r_0 \sin^2 \pi\delta}{\sqrt{2}\pi} \left(\frac{\omega_k - 2M}{2M}\right)^{\frac{3}{2}-2\delta} \\ &\quad \times B(1-\delta, 1-\delta)\, B(\tfrac{1}{2}, 2-2\delta)\ , \end{aligned} \quad (76)$$

where $r_0 = \frac{e^2}{4\pi M}$ is the classical electron radius and the constant $B(\mu, \nu)$ is the Euler's integral of the first kind. After integration over a small energy interval Δ above the threshold, $2M \geq \omega_k \leq 2M(1+\Delta)$ the integral cross section for pair production reads

$$\begin{aligned} I := \int_{2M}^{2M(1+\Delta)} \sigma_\pi(\omega_k)\, d\omega_k &\approx \frac{e^2 \sin^2 \pi\delta}{2\sqrt{2}\pi} \frac{\Delta^{\frac{5}{2}-2\delta}}{\frac{5}{2} - 2\delta} \\ &\quad \times B(1-\delta, 1-\delta)\, B(\tfrac{1}{2}, 2-2\delta). \end{aligned} \quad (77)$$

This quantity determines the output of electron-positron pairs produced by a photon per unit length of the magnetic string per unit time within the given energy interval. At $\delta = \frac{1}{2}$ we find

$$I = \frac{e^2\sqrt{2}}{3\pi} \Delta^{\frac{3}{2}}. \quad (78)$$

At *high photon energies*, $\omega_k \gg M$, the parameter c behaves like $c \sim 1$. In this case we have

$$\begin{aligned} \sigma_\lambda \approx \frac{e^2 \sin^2 \pi\delta}{2\pi^2} \frac{1}{\omega_k^3} \int_M^{\frac{\omega_k}{2}} dx \frac{1}{x^3\sqrt{x^2 - M^2}} \\ \times \int_0^{\omega_k - 2x} d\varepsilon\, C_\lambda(\varepsilon, x). \end{aligned} \quad (79)$$

Calculating the integral over ε we find that the main contribution to the asymptotic behavior of the cross section at $\omega_k \gg M$ arises from values of $x \sim M$. Performing integration over x we obtain

$$\sigma_\lambda \approx \frac{e^2 \sin^2 \pi\delta}{4\pi M} a_\lambda = r_0 \sin^2 \pi\delta\, a_\lambda \quad (80)$$

with $a_\sigma = \frac{2}{3}$ and $a_\pi = 1$.

At high photon energies the total cross section of the pair production tends asymptotically to constant values for both photon polarizations. This energy dependence is compatible with unitarity. (One might have expected that the singular *pure AB potential* leads to an increasing cross section and causes the violation of the perturbative theory at high energies, in a similar way as in the case of the idealized, infinitely thin cosmic string [12].)

However, we do not consider the high energy AB pair production as a realistic subject for experimental investigation. For a realization one needs to take much care about coherence of the high energy photon beam. We are going to estimate the possibility of the experimental observation of the AB pair production effect in a subsequent paper.

V. CONCLUSION

We have analyzed the electron-positron pair production by a single photon under the influence of a magnetic string in first-order perturbation theory, which, as other quantum processes connected with the AB effect, leads to rather unexpected results. Photons do not interact directly with magnetic fields, and the process which was considered here happens due to the interaction of the created charged particles in the final states with the AB potential.

In addition to the AB interaction, resulting from the nonintegrable phase factors, which all quantum particles suffer, spin particles interact with the magnetic field via their magnetic moments. This strongly influences their behavior near the flux tube. In the idealized case of an infinitely thin magnetic string their wave functions do not vanish on the string and the nonlocality of the AB effect is modified by a local interaction. This interaction leads to a specific behavior of the cross section.

We evaluated the differential cross section for the pair production, which contains complete information about energy, angular, and polarization distributions of the created particles, as well as the total cross section and analyzed them for different energy regimes. For low photon energies, just above the pair production threshold, electrons and positrons are produced predominantly by the π-polarized photons with polarization vectors directed along the magnetic string. This result may be the most interesting one with regard to possible experimental observations of the AB pair production process.

Of course, the observation of the AB effect, which is done by means of electron interference and electron holography [5], is not a simple task, and the experiments with photons which interact with the magnetic string and create electron-positron pairs requires a careful discussion. For these photons of rather high energies there exist additional effects which can obscure the AB pair production. In particular, this is the pair production by the photon in collision with material of the tube carrying the magnetic flux.

We draw attention to a remarkable feature which is characteristic for quantum processes in the presence of the AB string both for spinless and for spin particles.

The pair production process happens if the created electrons and positrons have angular momentum projection of opposite signs. In a sense the virtual charged particles need to circle the AB string to transform to real ones. In this case the photon can transmit a part of its perpendicular momentum to the string. We analyzed in detail how the total cross section of the process depends on the photon polarization. This analysis may be very important because it enables us to distinguish the pure effect from interfering effects accompanying the AB pair production process.

Finally we point out the analogy to the pair production process in the presence of a cosmic string [12]. In this case an additional term appears in the Dirac equation which results from the spin connection. It corresponds to the vector potential term in the AB case.

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