Time dependence of coherent $P^0\bar{P}^0$ decays and CP violation at asymmetric B factories

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A generic formalism is presented for the time-dependent or time-integrated decays of any coherent $P^0\bar{P}^0$ system ($P^0 = K^0, D^0, B^0_d$, or B^0_s). To meet various possible measurements at asymmetric B factories, we reanalyze some typical signals of CP violation in the coherent $B^0_d\bar{B}^0_d$ transitions. The advantage of proper time cuts is illustrated for measuring mixing parameters and CP violation. We show that the direct and indirect CP asymmetries are distinguishable in neutral B decays to CP eigenstates. We also remark on the non-negligible effects of final-state interactions on CP violation in some B_d decays to non-CP eigenstates. The possibility to detect the CP-forbidden processes at the $\Upsilon(4S)$ resonance is explored in some detail.

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I. INTRODUCTION

It is known in particle physics that mixing between a neutral meson P^0 and its CP-conjugate state $\bar{P}^0$ provides a mechanism whereby interference in the decay amplitudes can occur, leading to the possibility of CP violation [1]. To date, K^0-$\bar{K}^0$ mixing and B^0_d-$\bar{B}^0_d$ mixing have been measured, and the CP-violating signal induced by K^0-$\bar{K}^0$ mixing has been unambiguously established [2]. Compared with B^0_d-$\bar{B}^0_d$ mixing, B^0_s-$\bar{B}^0_s$ (D^0-$\bar{D}^0$) mixing is expected to be quite large (very small) in the context of the standard electroweak model [3]. Today the B^0_d-$\bar{B}^0_d$ system is playing an important role in studying flavor mixing and CP violation beyond the neutral kaon system. The B^0_s-$\bar{B}^0_s$ and D^0-$\bar{D}^0$ systems are more interesting in practice for probing new physics that is out of reach of the standard model predictions.

A feasible way to study CP violation arising from P^0-$\bar{P}^0$ mixing is to measure the coherent decays of $P^0\bar{P}^0$ pairs produced at appropriate resonances. For instance,

$$\phi \to K^0\bar{K}^0 , \qquad \psi'' \to D^0\bar{D}^0 ,$$
$$\Upsilon(4S) \to B^0_d\bar{B}^0_d , \qquad \Upsilon(5S) \to B^0_s\bar{B}^0_s .$$

At present, efforts are underway to develop asymmetric B factories at KEK and SLAC laboratories [4], while asymmetric ϕ factory options are also under consideration [5]. The main purpose of these machines is to probe CP violation (and to test other discrete symmetries or conservation laws) by measuring the time-dependent transitions. By now some phenomenological analyses of coherent $K^0\bar{K}^0$ and $B^0_d\bar{B}^0_d$ decays have been made in the literature [6–8]. These works have outlined the main features of CP violation in the K^0-$\bar{K}^0$ or B^0_d-$\bar{B}^0_d$ systems, although many of their formulas and results rely on the characteristics of the system itself or some model-dependent approximations. A generic formalism, which can describe the common properties of coherent $P^0\bar{P}^0$ decays, is still lacking. In addition, little attention has been paid to the advantage of proper time cuts for measuring the mixing parameters and CP asymmetries in coherent weak decays.

In this paper we shall present a generic and concise formalism for the time-dependent or time-integrated decays of any coherent $P^0\bar{P}^0$ system. This formalism should be very useful for phenomenological applications, because it is independent of the contexts of the standard model or its various nonstandard extensions. To meet various possible measurements at the forthcoming B factories, we shall carry out a reanalysis of the typical signals of CP violation expected to appear in B^0_d-$\bar{B}^0_d$ mixing, in B_d decays to CP eigenstates or non-CP eigenstates, and in CP-forbidden transitions at the $\Upsilon(4S)$ resonance. Our analysis differs from the previous ones in the following four aspects: (1) we illustrate the advantage of proper time cuts for measurements of the mixing parameters and CP-violating asymmetries; (2) we highlight the distinguishable effect of direct CP violation on CP asymmetries in neutral B decays to CP eigenstates; (3) we remark on the effects of final-state interactions on CP violation in B_d decays to non-CP eigenstates; and (4) we explore in some detail the possibility to detect the CP-forbidden decays at B factories. This work concentrates only on analytical studies. A more comprehensive discussion about the present topic, together with numerical predictions, will be given elsewhere.

II. FORMALISM OF COHERENT $P^0\bar{P}^0$ DECAYS

The time-dependent wave function for a $P^0_{\rm phys}\bar{P}^0_{\rm phys}$ pair at rest can be written as

$$\frac{1}{\sqrt{2}}[|P^0_{\rm phys}(\mathbf{K},t)\rangle \otimes |\bar{P}^0_{\rm phys}(-\mathbf{K},t)\rangle + C|P^0_{\rm phys}(-\mathbf{K},t)\rangle \otimes |\bar{P}^0_{\rm phys}(\mathbf{K},t)\rangle] , \quad (1)$$

where $\mathbf{K}$ is the three-momentum vector of the P meson, and $C = -$ or $+$ is the charge-conjugation parity of the $P^0_{\rm phys}\bar{P}^0_{\rm phys}$ pair. The proper time evolution of an initially ($t = 0$) pure P^0 or $\bar{P}^0$ is given by

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$$
|P^0_{\rm phys}(t)\rangle = g_+(t)|P^0\rangle + (q/p)g_-(t)|\bar{P}^0\rangle \, ,
$$

$$
|\bar{P}^0_{\rm phys}(t)\rangle = (p/q)g_-(t)|P^0\rangle + g_+(t)|\bar{P}^0\rangle \, , \tag{2}
$$

where the mixing parameters p and q connect the flavor eigenstates $|\overset{(-)}{P}{}^0\rangle$ to the mass eigenstates $|P_{1,2}\rangle$ through $|P_1\rangle = p|P^0\rangle + q|\bar{P}^0\rangle$ and $|P_2\rangle = p|P^0\rangle - q|\bar{P}^0\rangle$, and

$$
g_\pm(t) = \frac{1}{2}\exp\left[-\left(im+\frac{\Gamma}{2}\right)t\right]\left\{\exp\left[+\left(i\Delta m - \frac{\Delta\Gamma}{2}\right)\frac{t}{2}\right] \pm \exp\left[-\left(i\Delta m - \frac{\Delta\Gamma}{2}\right)\frac{t}{2}\right]\right\} . \tag{3}
$$

Here we have defined $m = (m_1+m_2)/2$, $\Delta m = (m_2 - m_1)$, $\Gamma = (\Gamma_1+\Gamma_2)/2$, and $\Delta\Gamma = (\Gamma_1 - \Gamma_2)$, where $\Gamma_{1,2}$ and $m_{1,2}$ are the widths and masses of $P_{1,2}$.

Now we consider the case that one of the two P mesons (with the momentum $\mathbf{K}$) decays to a final state f_1 at proper time t_1 and the other (with $-\mathbf{K}$) to f_2 at t_2. f_1 and f_2 may be either hadronic or semileptonic states. After a lengthy calculation, the joint decay rate for having such an event is given as

$$
\begin{aligned}
R(f_1,t_1;f_2,t_2)_C \propto |A_{f_1}|^2|A_{f_2}|^2 \exp(-\Gamma t_+)\Bigg[&\frac{1}{2}|\xi_C+\zeta_C|^2\exp\left(-\frac{\Delta\Gamma}{2}t_C\right) + \frac{1}{2}|\xi_C-\zeta_C|^2\exp\left(+\frac{\Delta\Gamma}{2}t_C\right) \\
&- \left(|\xi_C|^2 - |\zeta_C|^2\right)\cos(\Delta m t_C) + 2{\rm Im}\left(\xi_C^*\zeta_C\right)\sin(\Delta m t_C)\Bigg] ,
\end{aligned} \tag{4}
$$

where

$$
A_{f_i} = \langle f_i|H|P^0\rangle \, , \quad \bar{A}_{f_i} = \langle f_i|H|\bar{P}^0\rangle \, , \quad \rho_{f_i} = \frac{\bar{A}_{f_i}}{A_{f_i}} \, , \quad (i=1,2) \, ; \tag{5a}
$$

and

$$
t_C = t_2 + C t_1 \, , \quad \xi_C = (p/q) + C(q/p)\rho_{f_1}\rho_{f_2} \, , \quad \zeta_C = \rho_{f_2} + C\rho_{f_1} \, . \tag{5b}
$$

The time-independent decay rate is obtainable from Eq. (4) by integrating $R(f_1,t_1;f_2,t_2)_C$ over t_1 and t_2:

$$
\begin{aligned}
R(f_1,f_2)_C \propto |A_{f_1}|^2|A_{f_2}|^2 \Bigg[&\frac{|\xi_C+\zeta_C|^2}{2(1+y)(1+Cy)} + \frac{|\xi_C-\zeta_C|^2}{2(1-y)(1-Cy)} \\
&-\frac{1-Cx^2}{(1+x^2)^2}\left(|\xi_C|^2-|\zeta_C|^2\right) + \frac{2(1+C)x}{(1+x^2)^2}{\rm Im}\left(\xi_C^*\zeta_C\right)\Bigg] ,
\end{aligned} \tag{6}
$$

where $x = \Delta m/\Gamma$ and $y = \Delta\Gamma/(2\Gamma)$ are two measurables of the P^0-$\bar{P}^0$ system. Note that Eqs. (4) and (6) are useful at both symmetric and asymmetric flavor factories.

An asymmetric e^+e^- collider running at the threshold of production of $(P^0_{\rm phys}\bar{P}^0_{\rm phys})_C$ pairs will offer the possibility to measure the decay-time difference $t_- = (t_2 - t_1)$ between $P^0_{\rm phys} \to f_1$ and $\bar{P}^0_{\rm phys} \to f_2$. Usually it is difficult to measure the $t_+ = (t_2+t_1)$ distribution in either linacs or storage rings, unless the bunch lengths are much shorter than the decay lengths [4,5,9]. Hence it is more practical to study the t_- distribution of the joint decay rates. Here and hereafter we use t to denote t_- for simplicity. Integrating $R(f_1,t_1;f_2,t_2)_C$ over t_+, we obtain the decay rates (for $C=\pm$) as

$$
\begin{aligned}
R(f_1,f_2;t)_- \propto |A_{f_1}|^2|A_{f_2}|^2 e^{-\Gamma|t|}\Bigg[&\frac{1}{2}|\xi_- + \zeta_-|^2 e^{-y\Gamma t} + \frac{1}{2}|\xi_- - \zeta_-|^2 e^{+y\Gamma t} \\
&- \left(|\xi_-|^2 - |\zeta_-|^2\right)\cos(x\Gamma t) + 2{\rm Im}\left(\xi_-^*\zeta_-\right)\sin(x\Gamma t)\Bigg]
\end{aligned} \tag{7a}
$$

and

$$
\begin{aligned}
R(f_1,f_2;t)_+ \propto |A_{f_1}|^2|A_{f_2}|^2 e^{-\Gamma|t|}\Bigg[&\frac{|\xi_+ + \zeta_+|^2}{2(1+y)}e^{-y\Gamma|t|} + \frac{|\xi_+ - \zeta_+|^2}{2(1-y)}e^{+y\Gamma|t|} \\
&-\frac{|\xi_+|^2 - |\zeta_+|^2}{\sqrt{1+x^2}}\cos\left(x\Gamma|t| + \phi_x\right) + \frac{2{\rm Im}\left(\xi_+^*\zeta_+\right)}{\sqrt{1+x^2}}\sin(x\Gamma|t|+\phi_x)\Bigg] ,
\end{aligned} \tag{7b}
$$

where $\phi_x = \arctan x$ signifies a phase shift. One can check that integrating $R(f_1,f_2;t)_C$ over t, where $t \in (-\infty,+\infty)$, will lead to the time-independent decay rates $R(f_1,f_2)_C$ given in Eq. (6). Equation (7) provides us two basic formulas for investigating coherent $B^0\bar{B}^0$ (or $K^0\bar{K}^0$) decays at asymmetric B (or ϕ) factories.

Another possibility is to measure the time-integrated decay rates of $(P^0_{\rm phys}\bar{P}^0_{\rm phys})_C$ with a proper time cut, which can sometimes increase the sizes of CP asymmetries [10]. In practice, appropriate time cuts can also suppress background and improve statistic accuracy of signals. If the decay events in the time region $t \in [+t_0,+\infty]$ or $t \in (-\infty,-t_0]$ are used, where $t_0 \geq 0$, the respective decay rates can be defined by

$$\hat{R}(f_1, f_2; +t_0)_C \equiv \int_{+t_0}^{+\infty} R(f_1, f_2; t)_C dt \, ,$$
$$\hat{R}(f_1, f_2; -t_0)_C \equiv \int_{-\infty}^{-t_0} R(f_1, f_2; t)_C dt \, . \tag{8}$$

By use of Eq. (7), we obtain

$$\hat{R}(f_1, f_2; \pm t_0)_- \propto |A_{f_1}|^2 |A_{f_2}|^2 e^{-\Gamma t_0} \left[\frac{|\xi_- \pm \zeta_-|^2}{4(1+y)} e^{-y\Gamma t_0} + \frac{|\xi_- \mp \zeta_-|^2}{4(1-y)} e^{+y\Gamma t_0} - \frac{|\xi_-|^2 - |\zeta_-|^2}{2\sqrt{1+x^2}} \cos(x\Gamma t_0 + \phi_x) \pm \frac{\mathrm{Im}\,(\xi_-^* \zeta_-)}{\sqrt{1+x^2}} \sin(x\Gamma t_0 + \phi_x) \right] \tag{9a}$$

and

$$\hat{R}(f_1, f_2; \pm t_0)_+ \propto |A_{f_1}|^2 |A_{f_2}|^2 e^{-\Gamma t_0} \left[\frac{|\xi_+ + \zeta_+|^2}{4(1+y)^2} e^{-y\Gamma t_0} + \frac{|\xi_+ - \zeta_+|^2}{4(1-y)^2} e^{+y\Gamma t_0} - \frac{|\xi_+|^2 - |\zeta_+|^2}{2(1+x^2)} \cos(x\Gamma t_0 + 2\phi_x) + \frac{\mathrm{Im}\,(\xi_+^* \zeta_+)}{1+x^2} \sin(x\Gamma t_0 + 2\phi_x) \right] . \tag{9b}$$

It is easy to check that

$$\hat{R}(f_1, f_2; +0)_C + \hat{R}(f_1, f_2; -0)_C \;=\; R(f_1, f_2)_C \, . \tag{10}$$

One can observe that in $\hat{R}(f_1, f_2; \pm t_0)_C$ different terms are sensitive to the time cut t_0 in different ways. Thus it is possible to enhance a CP-violating term (and suppress the others) via a suitable cut t_0.

The formulas given above are applicable to all coherent decays of the K^0-$\bar{K}^0$, D^0-$\bar{D}^0$, B^0_d-$\bar{B}^0_d$, and B^0_s-$\bar{B}^0_s$ systems. Denoting the decay amplitudes of $P_n \to f_i$ by $A^{(n)}_{f_i}$ $(n, i = 1, 2)$ and the ratio of $A^{(2)}_{f_i}$ to $A^{(1)}_{f_i}$ by η_{f_i}, one can also express the joint decay rates in terms of $A^{(n)}_{f_i}$ and η_{f_i} through the transformations

$$A_{f_i} \;=\; \frac{1}{2p} \left[A^{(1)}_{f_i} + A^{(2)}_{f_i} \right] \, ,$$
$$\bar{A}_{f_i} \;=\; \frac{1}{2q} \left[A^{(1)}_{f_i} - A^{(2)}_{f_i} \right] \, , \tag{11}$$
$$\rho_{f_i} \;=\; \frac{p}{q} \frac{1 - \eta_{f_i}}{1 + \eta_{f_i}} \, .$$

Such notations are usually favored in the K^0-$\bar{K}^0$ system [6]. In the following we shall apply the above formalism to the coherent $(B^0_{d,\mathrm{phys}} \bar{B}^0_{d,\mathrm{phys}})_C$ decays and CP violation at the $\Upsilon(4S)$ resonance, a basis of the forthcoming B factories [4,9].

III. SIGNALS OF CP VIOLATION AT B FACTORIES

The unique experimental advantages of studying b-quark physics at the $\Upsilon(4S)$ resonance are well known. For symmetric e^+e^- collisions the produced B mesons are almost at rest and their mean decay length is only about 20 μm, a distance which is insufficient for identifying the decay vertices or measuring the decay time difference [9,10]. If the colliding e^+ and e^- beams have different energies, the product of collisions will move with a significant relativistic boost factor in the laboratory[1] (along the direction of the more energetic beam). This can cause the two B mesons far apart in space, such that the distance between their decay vertices becomes measurable. It is then possible to study the time distribution of the joint decay rates and CP asymmetries.

A. CP violation in B^0_d-$\bar{B}^0_d$ mixing

We first consider the joint decays $(B^0_{d,\mathrm{phys}} \bar{B}^0_{d,\mathrm{phys}})_C \to (l^\pm X^\mp_a)(l^\pm X^\mp_b)$, which lead to dilepton events in the final states. Keeping the $\Delta Q = \Delta B$ rule and CPT symmetry, we have

$$|\langle l^+ X^-_i | H | B^0_d \rangle| = |\langle l^- X^+_i | H | \bar{B}^0_d \rangle| \;\equiv\; |A_{li}| \, ;$$
$$|\langle l^- X^+_i | H | B^0_d \rangle| = |\langle l^+ X^-_i | H | \bar{B}^0_d \rangle| \;=\; 0 \, , \tag{12}$$

where $i = a$ or b. Subsequently we use $N^{\pm\pm}_C(t)$ and $N^{+-}_C(t)$ to denote the time-dependent like-sign and opposite-sign dilepton numbers, respectvely. Similarly, let $\hat{N}^{\pm\pm}_C(\pm t_0)$ and $\hat{N}^{+-}_C(\pm t_0)$ denote the time-integrated dilepton events with the time cut t_0. With the help of Eq. (7), we obtain

[1]For a moving $\Upsilon(4S)$ system, the momentum of the B mesons in the $\Upsilon(4S)$ rest frame can be ignored. This safe approximation has been discussed in Refs. [9,10].

$$
\begin{aligned}
N_-^{++}(t) &\propto |p/q|^2|A_{la}|^2|A_{lb}|^2 e^{-\Gamma|t|}\left[\cosh(y\Gamma t)-\cos(x\Gamma t)\right] ,\\
N_-^{--}(t) &\propto |q/p|^2|A_{la}|^2|A_{lb}|^2 e^{-\Gamma|t|}\left[\cosh(y\Gamma t)-\cos(x\Gamma t)\right] ,\\
N_-^{+-}(t) &\propto 2|A_{la}|^2|A_{lb}|^2 e^{-\Gamma|t|}\left[\cosh(y\Gamma t)+\cos(x\Gamma t)\right] ;
\end{aligned}
\tag{13a}
$$

and

$$
\begin{aligned}
N_+^{++}(t) &\propto |p/q|^2|A_{la}|^2|A_{lb}|^2 e^{-\Gamma|t|}\left[\frac{\cosh(y\Gamma|t|)+y\sinh(y\Gamma|t|)}{1-y^2}-\frac{\cos(x\Gamma|t|+\phi_x)}{\sqrt{1+x^2}}\right] ,\\
N_+^{--}(t) &\propto |q/p|^2|A_{la}|^2|A_{lb}|^2 e^{-\Gamma|t|}\left[\frac{\cosh(y\Gamma|t|)+y\sinh(y\Gamma|t|)}{1-y^2}-\frac{\cos(x\Gamma|t|+\phi_x)}{\sqrt{1+x^2}}\right] ,\\
N_+^{+-}(t) &\propto 2|A_{la}|^2|A_{lb}|^2 e^{-\Gamma|t|}\left[\frac{\cosh(y\Gamma|t|)+y\sinh(y\Gamma|t|)}{1-y^2}+\frac{\cos(x\Gamma|t|+\phi_x)}{\sqrt{1+x^2}}\right] .
\end{aligned}
\tag{13b}
$$

If y is not very small in comparison with x, its size and sign should (in principle) be determinable from the above equations.[2]

Since both $N_C^{\pm\pm}(t)$ and $N_C^{+-}(t)$ are even functions of proper time t, one finds

$$
\hat{N}_C^{\pm\pm}(+t_0) \;=\; \hat{N}_C^{\pm\pm}(-t_0) \;\equiv\; \frac{1}{2}\hat{N}_C^{\pm\pm}(t_0) , \qquad \hat{N}_C^{+-}(+t_0) \;=\; \hat{N}_C^{+-}(-t_0) \;\equiv\; \frac{1}{2}\hat{N}_C^{+-}(t_0) . \tag{14}
$$

The time-integrated observables of CP violation and B_d^0-$\bar{B}_d^0$ mixing can be defined as

$$
\mathcal{A}_C^{+-}(t_0) \;\equiv\; \frac{\hat{N}_C^{++}(t_0)-\hat{N}_C^{--}(t_0)}{\hat{N}_C^{++}(t_0)+\hat{N}_C^{--}(t_0)} , \qquad \mathcal{S}_C^{+-}(t_0) \;\equiv\; \frac{\hat{N}_C^{++}(t_0)+\hat{N}_C^{--}(t_0)}{\hat{N}_C^{+-}(t_0)} . \tag{15}
$$

Using Eqs. (9) and (14), we obtain

$$
\mathcal{A}_-^{+-}(t_0) \;=\; \mathcal{A}_+^{+-}(t_0) \;=\; \frac{|p/q|^2-|q/p|^2}{|p/q|^2+|q/p|^2} , \tag{16}
$$

which is independent of the time cut t_0. The nonvanishing $\mathcal{A}_C^{+-}(t_0)$ implies CP violation in B_d^0-$\bar{B}_d^0$ mixing. In addition, we find

$$
\begin{aligned}
\mathcal{S}_-^{+-}(t_0) &= \frac{|p/q|^2+|q/p|^2}{2}\,\frac{\cosh(y\Gamma t_0)+y\sinh(y\Gamma t_0)-z\cos(x\Gamma t_0+\phi_x)}{\cosh(y\Gamma t_0)+y\sinh(y\Gamma t_0)+z\cos(x\Gamma t_0+\phi_x)} ,\\
\mathcal{S}_+^{+-}(t_0) &= \frac{|p/q|^2+|q/p|^2}{2}\,\frac{(1+y^2)\cosh(y\Gamma t_0)+2y\sinh(y\Gamma t_0)-z^2\cos(x\Gamma t_0+2\phi_x)}{(1+y^2)\cosh(y\Gamma t_0)+2y\sinh(y\Gamma t_0)+z^2\cos(x\Gamma t_0+2\phi_x)} ,
\end{aligned}
\tag{17}
$$

where $z=(1-y^2)/\sqrt{1+x^2}$.

In the context of the standard model, $|q/p|\approx 1$ and $y\approx 0$ are two good approximations [3]. Thus Eq. (17) is simplified as

$$
\begin{aligned}
\mathcal{S}_-^{+-}(t_0) &\approx \frac{\sqrt{1+x^2}-\cos(x\Gamma t_0+\phi_x)}{\sqrt{1+x^2}+\cos(x\Gamma t_0+\phi_x)} \xrightarrow{t_0=0} \frac{x^2}{2+x^2} ,\\
\mathcal{S}_+^{+-}(t_0) &\approx \frac{(1+x^2)-\cos(x\Gamma t_0+2\phi_x)}{(1+x^2)+\cos(x\Gamma t_0+2\phi_x)} \xrightarrow{t_0=0} \frac{3x^2+x^4}{2+x^2+x^4} .
\end{aligned}
\tag{18}
$$

We show the evolution of $\mathcal{S}_C^{+-}(t_0)$ with t_0 in Fig. 1, where the experimental input is $x\approx 0.7$ [2] (this leads to $\phi_x\approx 35°$). One observes that an appropriate time cut can significantly increase the ratio of the same-sign dilepton events to the opposite-sign ones. Practically time cuts should be a useful way to enhance the signals of B_d^0-$\bar{B}_d^0$ mixing, only if the cost of the total number of events is not too large.

B. CP asymmetries in B_d decays to CP eigenstates

Neutral B decays to CP eigenstates are favored in both theory and experiments to study quark mixing and CP violation. At the $\Upsilon(4S)$ resonance, the produced $B^0_{d,\mathrm{phys}}$ and $\bar{B}^0_{d,\mathrm{phys}}$ mesons exist in a coherent state until one of them decays. Thus one can use the semileptonic decay of one B_d meson to tag the flavor of the other meson decaying to a flavor-nonspecific hadron state. Let us consider the joint transitions $(B^0_{d,\mathrm{phys}}\bar{B}^0_{d,\mathrm{phys}})_C \to (l^{\mp}X^{\pm})f$, where f denotes a hadronic CP eigenstate such as $\psi K_S, D^+D^-$, or $\pi^+\pi^-$. To a good degree of accuracy in the standard model, we have $|q/p|\approx 1$ and $y\approx 0$. With the help of Eq. (7), the time-dependent decay rates

[2]A more detailed discussion has been given by Dass and Sarma in Ref. [7], where only the case of $C=-$ was taken into account.

are given as

$$R(l^{\mp},f;t)_{-} \propto |A_l|^2|A_f|^2 e^{-\Gamma|t|}\left[\frac{1+|\lambda_f|^2}{2} \pm \frac{1-|\lambda_f|^2}{2}\cos(x\Gamma t) \mp \text{Im}\lambda_f \sin(x\Gamma t)\right] \tag{19a}$$

and

$$R(l^{\mp},f;t)_{+} \propto |A_l|^2|A_f|^2 e^{-\Gamma|t|}\left[\frac{1+|\lambda_f|^2}{2} \pm \frac{1}{\sqrt{1+x^2}}\frac{1-|\lambda_f|^2}{2}\cos(x\Gamma|t|+\phi_x) \mp \frac{1}{\sqrt{1+x^2}}\text{Im}\lambda_f \sin(x\Gamma|t|+\phi_x)\right] , \tag{19b}$$

where $\lambda_f = (q/p)\rho_f$. The time-dependent CP asymmetries, defined by

$$\mathcal{A}_C(t) \equiv \frac{R(l^-,f;t)_C - R(l^+,f;t)_C}{R(l^-,f;t)_C + R(l^+,f;t)_C} , \tag{20}$$

can be explicitly expressed as

$$\mathcal{A}_-(t) = \mathcal{U}_f \cos(x\Gamma t) + \mathcal{V}_f \sin(x\Gamma t) ,$$

$$\mathcal{A}_+(t) = \frac{1}{\sqrt{1+x^2}}\left[\mathcal{U}_f \cos(x\Gamma|t|+\phi_x) + \mathcal{V}_f \sin(x\Gamma|t|+\phi_x)\right] , \tag{21}$$

where

$$\mathcal{U}_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2} , \qquad \mathcal{V}_f = \frac{-2\text{Im}\lambda_f}{1+|\lambda_f|^2} . \tag{22}$$

We find that $\mathcal{A}_C(t)$ contains both the direct CP asymmetry in the decay amplitude ($|\lambda_f| \neq 1$ or $\mathcal{U}_f \neq 0$) and the indirect one from interference of mixing and decay ($\text{Im}\lambda_f \neq 0$ or $\mathcal{V}_f \neq 0$). Measuring the time distribution of $\mathcal{A}_\pm(t)$ can distinguish between these two sources of CP violation [8].

There are two ways to combine the time-integrated decay events (with the time cuts $\pm t_0$), leading to two types of CP asymmetries:

$$\mathcal{A}_C^{(1)}(t_0) \equiv \frac{\left[\hat{R}(l^-,f;+t_0)+\hat{R}(l^-,f;-t_0)\right] - \left[\hat{R}(l^+,f;+t_0)+\hat{R}(l^+,f;-t_0)\right]}{\left[\hat{R}(l^-,f;+t_0)+\hat{R}(l^-,f;-t_0)\right] + \left[\hat{R}(l^+,f;+t_0)+\hat{R}(l^+,f;-t_0)\right]} ,$$

$$\mathcal{A}_C^{(2)}(t_0) \equiv \frac{\left[\hat{R}(l^-,f;+t_0)+\hat{R}(l^+,f;-t_0)\right] - \left[\hat{R}(l^+,f;+t_0)+\hat{R}(l^-,f;-t_0)\right]}{\left[\hat{R}(l^-,f;+t_0)+\hat{R}(l^+,f;-t_0)\right] + \left[\hat{R}(l^+,f;+t_0)+\hat{R}(l^-,f;-t_0)\right]} . \tag{23}$$

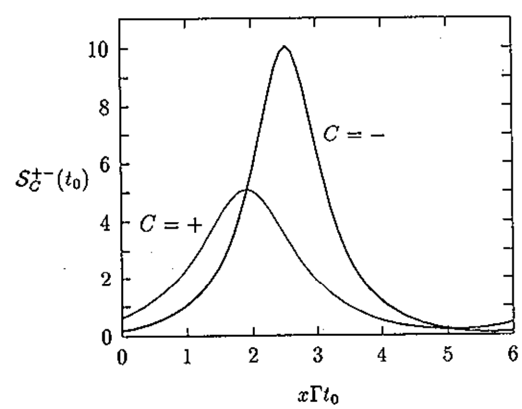

FIG. 1. Ratios of the same-sign dilepton events to the opposite-sign ones as functions of the time cut t_0 at the $\Upsilon(4S)$ resonance.

With the help of Eq. (9), we obtain

$$\mathcal{A}_-^{(1)}(t_0) = \frac{\cos(x\Gamma t_0+\phi_x)}{\sqrt{1+x^2}}\mathcal{U}_f ,$$

$$\mathcal{A}_-^{(2)}(t_0) = \frac{\sin(x\Gamma t_0+\phi_x)}{\sqrt{1+x^2}}\mathcal{V}_f ; \tag{24a}$$

and

$$\mathcal{A}_+^{(1)}(t_0) = \frac{\cos(x\Gamma t_0+2\phi_x)}{1+x^2}\mathcal{U}_f + \frac{\sin(x\Gamma t_0+2\phi_x)}{1+x^2}\mathcal{V}_f ,$$

$$\mathcal{A}_+^{(2)}(t_0) = 0 . \tag{24b}$$

Clearly the asymmetries $\mathcal{A}_-^{(1)}(t_0)$ and $\mathcal{A}_-^{(2)}(t_0)$ signify direct and indirect CP violation, respectively. They can be separated from each other on the $\Upsilon(4S)$ resonance.

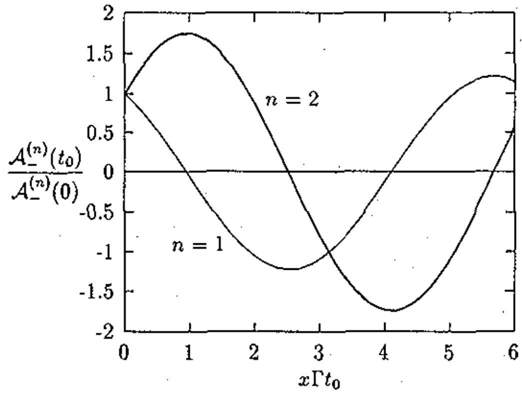

FIG. 2. Ratios of the CP asymmetries $\mathcal{A}_-^{(n)}(t_0)$ to $\mathcal{A}_-^{(n)}(0)$ as functions of the time cut t_0 on the $\Upsilon(4S)$ resonance.

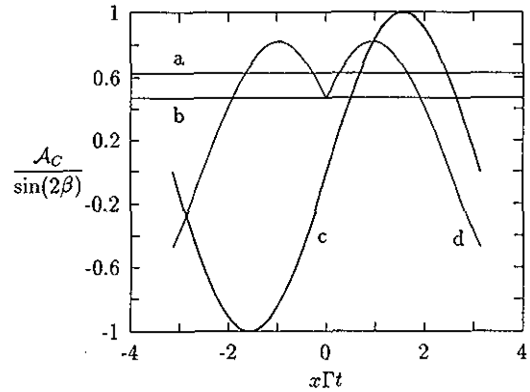

FIG. 3. CP asymmetries (a) $\mathcal{A}_+^{(1)}(0)$, (b) $\mathcal{A}_-^{(2)}(0)$, (c) $\mathcal{A}_-(t)$, and (d) $\mathcal{A}_+(t)$ in $B_d \to \psi K_S$ at the $\Upsilon(4S)$ resonance. Here $x \approx 0.7$ (or $\phi_x \approx 35^\circ$) has been used.

In Fig. 2 we show the ratios $\mathcal{A}_-^{(n)}(t_0)/\mathcal{A}_-^{(n)}(0)$ ($n=1$ and 2) as functions of t_0. A proper time cut can certainly increase the CP asymmetries (at some cost of decay events). In practice, it can also suppress background and improve statistic accuracy of signals. Note that a suitable cut of decay time is (in principle) able to isolate the direct or indirect CP asymmetry in $\mathcal{A}_+^{(1)}(t_0)$. For example,

$$\mathcal{A}_+^{(1)}\left(\frac{\pi}{x\Gamma} - \frac{2\phi_x}{x\Gamma}\right) = -\frac{1}{1+x^2}\mathcal{U}_f\,,$$
$$\mathcal{A}_+^{(1)}\left(\frac{\pi}{2x\Gamma} - \frac{2\phi_x}{x\Gamma}\right) = \frac{1}{1+x^2}\mathcal{V}_f\,. \tag{25}$$

At symmetric B factories, it is possible to measure a large CP asymmetry $\mathcal{A}_+^{(1)}(0)$.

For illustration, we take a look at the CP asymmetries in $B_d \to \psi K_S$ and ϕK_S. The former is dominated by the tree-level quark diagrams, thus $|\lambda_{\psi K_S}| = 1$ or $\mathcal{U}_{\psi K_S} = 0$ is a good approximation. Using the notation $q/p \approx e^{-2i\beta}$, where β corresponds to an inner angle of the Kobayashi-Maskawa (KM) unitarity triangle [2], we have $\mathcal{V}_{\psi K_S} = -\mathrm{Im}\lambda_{\psi K_S} = \sin(2\beta)$. In Fig. 3 the nonzero CP asymmetries $\mathcal{A}_-(t)$, $\mathcal{A}_+(t)$, $\mathcal{A}_-^{(2)}(0)$, and $\mathcal{A}_+^{(1)}(0)$ for $B_d \to \psi K_S$ are explicitly illustrated. It is clear that the weak phase β can be well determined from one of the four types of CP asymmetries. In comparison with $B_d \to \psi K_S$, the decay modes B_d^0 vs $\bar{B}_d^0 \to \phi K_S$ occur only through the strong and electroweak penguin diagrams. Hence the effect of direct CP violation is non-negligible in the latter processes. Practically, a nonvanishing $\mathcal{U}_{\phi K_S}$ should be isolated first from the observation of CP violation in $B_u^\pm \to \phi K^\pm$, where the flavors of the initial mesons are automatically tagged by the decays themselves. Such an asymmetry can in principle be confirmed by measuring $\mathcal{A}_-(t)$ or $\mathcal{A}^{(1)}(t_0)$ in $B_d \to \phi K_S$. The determination of both $\mathcal{U}_{\phi K_S}$ and $\mathcal{V}_{\phi K_S}$ in experiments will lead to a better understanding of the penguin diagrams and the mechanism of CP violation in theory.

C. CP asymmetries in B_d decays to non-CP eigenstates

In this subsection we consider the case that both B_d^0 and $\bar{B}_d^0$ mesons decay to common non-CP eigenstates, but their amplitudes are governed by different KM factors. Most of such decays occur through the quark transitions $\overset{(-)}{b} \to u\bar{c}\overset{(-)}{q}$ and $c\bar{u}\overset{(-)}{q}$ ($q = d$ or s), and a typical example is B_d^0 vs $\bar{B}_d^0 \to D^\pm\pi^\mp$ (see Fig. 4). In this case, there is not measurable direct CP violation between a decay mode $B_{d,\mathrm{phys}}^0 \to f$ and its CP-conjugate counterpart $\bar{B}_{d,\mathrm{phys}}^0 \to \bar{f}$, because of $|\langle\bar{f}|H|\bar{B}_d^0\rangle| = |\langle f|H|B_d^0\rangle|$. For a joint transition at the $\Upsilon(4S)$ resonance, we obtain the time-dependent decay rates as

$$R(l^-,f;t)_- \propto |A_l|^2|A_f|^2 e^{-\Gamma|t|}\left[\frac{1+|\lambda_f|^2}{2} + \frac{1-|\lambda_f|^2}{2}\cos(x\Gamma t) - \mathrm{Im}\lambda_f\sin(x\Gamma t)\right]\,,$$
$$R(l^+,\bar{f};t)_- \propto |A_l|^2|A_f|^2 e^{-\Gamma|t|}\left[\frac{1+|\bar{\lambda}_{\bar{f}}|^2}{2} + \frac{1-|\bar{\lambda}_{\bar{f}}|^2}{2}\cos(x\Gamma t) - \mathrm{Im}\bar{\lambda}_{\bar{f}}\sin(x\Gamma t)\right]\,; \tag{26}$$

and

$$R(l^-,f;t)_+ \propto |A_l|^2|A_f|^2 e^{-\Gamma|t|}\left[\frac{1+|\lambda_f|^2}{2} + \frac{1}{\sqrt{1+x^2}}\frac{1-|\lambda_f|^2}{2}\cos(x\Gamma|t|+\phi_x) - \frac{1}{\sqrt{1+x^2}}\mathrm{Im}\lambda_f\sin(x\Gamma|t|+\phi_x)\right]\,,$$
$$R(l^+,\bar{f};t)_+ \propto |A_l|^2|A_f|^2 e^{-\Gamma|t|}\left[\frac{1+|\bar{\lambda}_{\bar{f}}|^2}{2} + \frac{1}{\sqrt{1+x^2}}\frac{1-|\bar{\lambda}_{\bar{f}}|^2}{2}\cos(x\Gamma|t|+\phi_x) - \frac{1}{\sqrt{1+x^2}}\mathrm{Im}\bar{\lambda}_{\bar{f}}\sin(x\Gamma|t|+\phi_x)\right]\,, \tag{27}$$

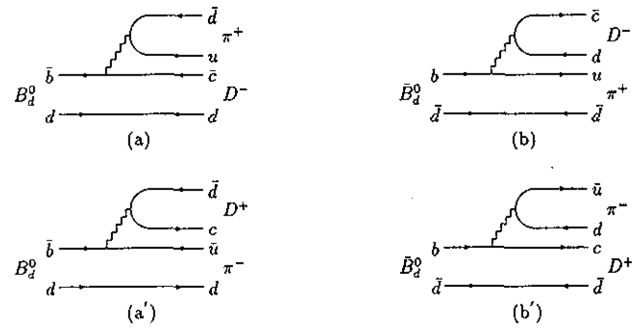

FIG. 4. Dominant quark diagrams for B^0_d vs $\bar{B}^0_d \to D^-\pi^+$ and $D^+\pi^-$.

where $\bar{\lambda}_{\bar{f}} = (p/q)\bar{\rho}_{\bar{f}}$ with the definition $\bar{\rho}_{\bar{f}} = \langle \bar{f}|H|B^0_d\rangle / \langle \bar{f}|H|\bar{B}^0_d\rangle$. In obtaining Eqs. (26) and (27), $|q/p| \approx 1$ and $y \approx 0$ have been used. Consequently we have $|\bar{\lambda}_{\bar{f}}| = |\lambda_f|$. The time-dependent CP asymmetries

$$\mathcal{A}_C(t) \equiv \frac{R(l^-, f; t)_C - R(l^+, \bar{f}; t)_C}{R(l^-, f; t)_C + R(l^+, \bar{f}; t)_C} \tag{28}$$

are then expressible as

$$\mathcal{A}_-(t) = \frac{-\mathrm{Im}\left(\lambda_f - \bar{\lambda}_{\bar{f}}\right)\sin(x\Gamma t)}{1 + |\lambda_f|^2 + F(\lambda_f, \bar{\lambda}_{\bar{f}}, x\Gamma t)} \tag{29a}$$

and

$$\mathcal{A}_+(t) = \frac{-\mathrm{Im}\left(\lambda_f - \bar{\lambda}_{\bar{f}}\right)\sin(x\Gamma t + \phi_x)}{\sqrt{1+x^2}\,(1 + |\lambda_f|^2) + F(\lambda_f, \bar{\lambda}_{\bar{f}}, x\Gamma |t| + \phi_x)}\,, \tag{29b}$$

where F is a function defined by

$$F(z_1, z_2, z_3) \equiv \left(1 - |z_1|^2\right)\cos z_3 - \mathrm{Im}(z_1 + z_2)\sin z_3\,. \tag{30}$$

Clearly, CP violation is induced by the interplay of decay and B^0_d-$\bar{B}^0_d$ mixing in this type of transitions.

In a similar way, one can obtain the time-integrated CP asymmetries

$$\mathcal{A}_C \equiv \frac{R(l^-, f)_C - R(l^+, \bar{f})_C}{R(l^-, f)_C + R(l^+, \bar{f})_C} \tag{31}$$

with the help of Eq. (6). We find

$$\mathcal{A}_- = 0 \tag{32a}$$

and

$$\mathcal{A}_+ = \frac{-2x\mathrm{Im}\left(\lambda_f - \bar{\lambda}_{\bar{f}}\right)}{2 + x^2 + x^4 + x^2(3+x^2)|\lambda_f|^2 - 2x\mathrm{Im}\left(\lambda_f + \bar{\lambda}_{\bar{f}}\right)}\,. \tag{32b}$$

In some previous studies $\bar{\lambda}_{\bar{f}} = \lambda^*_f$ was used in order to simplify final-state interactions and to make numerical estimates possible. Some isospin analyses have shown that this assumption is not valid for many decay modes such as $B_d \to D^{(*)}\pi$ and $D^{(*)}K$, since each final state contains at least two different isospin configurations (see, e.g., Refs. [11,12]).

By use of both the isospin analysis and the current data, the author [12] has found that the effects of final-state interactions on CP violation are significant in $B_d \to D\pi$ and $D^*\pi$, but negligible in $B_d \to D\rho$ to an acceptable degree of accuracy. For the latter processes, the interference terms $\mathrm{Im}\left(\lambda_{D^+\rho^-} - \bar{\lambda}_{D^-\rho^+}\right)$ and $\mathrm{Im}\left(\lambda_{D^0\rho^0} - \bar{\lambda}_{\bar{D}^0\rho^0}\right)$ turn out to be proportional to $\sin(2\beta + \gamma)$, where β and γ are two inner angles of the well-known KM unitarity triangle [2]. Thus there are also possibilities to extract the KM phase parameter from the CP asymmetries in some neutral B decays to non-CP eigenstates, if a detailed isospin study of them can be realized with the help of reliable experimental data.

D. CP-forbidden transitions

We finally consider the CP-forbidden decay modes

$$(B^0_{d,\text{phys}}\bar{B}^0_{d,\text{phys}})_{\mp} \to (f_a f_b)_{\pm}\,, \tag{33}$$

where $f_{a,b}$ denote the CP eigenstates with the same or opposite CP parities. It should be emphasized that for such decays CP-violating signals can be established by measuring the joint decay rates other than the decay rate asymmetries [3]. In practice, this implies that neither flavor tagging nor time-dependent measurements are necessary.

On the $\Upsilon(4S)$ resonance, the typical CP-forbidden channels include $(B^0_{d,\text{phys}}\bar{B}^0_{d,\text{phys}})_- \to (ff)_+$ with $f = \psi K_S$, ψK_L, D^+D^-, and $\pi^+\pi^-$. Taking $f = X_{\bar{c}c}K_S$ for example, where $X_{\bar{c}c}$ denote all the possible charmonium states that can form the odd CP eigenstates with K_S (see Fig. 5), we have the safe approximation $\rho_{X_{\bar{c}c}K_S} \approx -1$. In addition, $y \approx 0$ and $q/p \approx e^{-2i\beta}$ are another two good approximations. With the help of Eqs. (6) and (7a), one obtains the branching fractions

$$B(X_{\bar{c}c}K_S, X_{\bar{c}c}K_S; t)_- \propto B^2_{X_{\bar{c}c}K_S}\sin^2(2\beta)e^{-\Gamma|t|}[1 - \cos(x\Gamma t)]\,, \tag{34}$$

$$B(X_{\bar{c}c}K_S, X_{\bar{c}c}K_S)_- \propto B^2_{X_{\bar{c}c}K_S}\sin^2(2\beta)\frac{x^2}{1+x^2}\,,$$

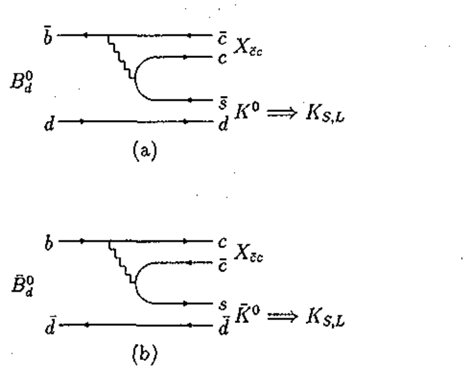

FIG. 5. Dominant quark diagrams for B^0_d vs $\bar{B}^0_d$ decays to $X_{\bar{c}c}K_{S,L}$. Here $X_{\bar{c}c}$ denote the possible charmonium states that can form the odd (even) CP eigenstates with K_S (K_L).

where $B_{X_{\bar{c}c}K_S}$ denotes the branching ratio of $B_d^0 \to X_{\bar{c}c}K_S$. Clearly the above joint decay rates are forbidden by CP symmetry ($\beta = 0$ or $\pm\pi$). In practice, a sum over the available final states $X_{\bar{c}c}K_S$ can statistically increase the number of decay events:

$$B_- \equiv \sum_{X_{\bar{c}c}} B(X_{\bar{c}c}K_S, X_{\bar{c}c}K_S)_- \propto \frac{x^2}{1+x^2}\sin^2(2\beta)\sum_{X_{\bar{c}c}}\left(B^2_{X_{\bar{c}c}K_S}\right) . \quad (35)$$

Similarly, one can derive the branching fractions for $f = X_{\bar{c}c}K_L$.

Just above the $\Upsilon(4S)$ resonance, an interesting type of CP-forbidden channel should be $(B^0_{d,\text{phys}}\bar{B}^0_{d,\text{phys}})_+ \to [(X_{\bar{c}c}K_S)(X_{\bar{c}c}K_L)]_-$. Neglecting CP violation in the kaon system, we have $\rho_{X_{\bar{c}c}K_L} \approx -\rho_{X_{\bar{c}c}K_S} \approx 1$ and $B_{X_{\bar{c}c}K_S} \approx B_{X_{\bar{c}c}K_L}$ to a good degree of accuracy. From Eqs. (6) and (7b), it is straightforward to obtain

$$B(X_{\bar{c}c}K_S, X_{\bar{c}c}K_L; t)_+ \propto B^2_{X_{\bar{c}c}K_S}\sin^2(2\beta)e^{-\Gamma|t|}\left[1 - \frac{\cos(x\Gamma|t| + \phi_x)}{\sqrt{1+x^2}}\right] , \quad (36)$$

$$B(X_{\bar{c}c}K_S, X_{\bar{c}c}K_L)_+ \propto B^2_{X_{\bar{c}c}K_S}\sin^2(2\beta)\frac{3x^2+x^4}{1+2x^2+x^4} .$$

Summing over the possible states $(X_{\bar{c}c}K_S)(X_{\bar{c}c}K_L)$, we find

$$B_+ \equiv \sum_{X_{\bar{c}c}} B(X_{\bar{c}c}K_S, X_{\bar{c}c}K_L)_+ \propto \frac{3x^2+x^4}{1+2x^2+x^4}\sin^2(2\beta)\sum_{X_{\bar{c}c}}\left(B^2_{X_{\bar{c}c}K_S}\right) . \quad (37)$$

In Fig. 6 we show the relative sizes of the effective branching fractions B_- and B_+ in the region $0.17 \leq \sin(2\beta) \leq 0.99$, limited by the current data [13]. For our purpose, the suitable $X_{\bar{c}c}$ states include ψ, ψ', ψ'', η_c, η'_c, etc.[3] Since such channels occur through the same tree-level quark diagram (see Fig. 5), their branching ratios $B_{X_{\bar{c}c}K_S}$ (or $B_{X_{\bar{c}c}K_L}$) are expected to be of the same order. Thus a combination of the available decay modes is able to enhance the event number of a single mode by several times. In a similar way, one can study the CP-forbidden transitions $(B^0_{d,\text{phys}}\bar{B}^0_{d,\text{phys}})_- \to (X_{\bar{c}c}K_S\pi^0)(X_{\bar{c}c}K_S\pi^0)$, $(X_{\bar{c}c}K_L\pi^0)(X_{\bar{c}c}K_L\pi^0)$, and $(B^0_{d,\text{phys}}\bar{B}^0_{d,\text{phys}})_+ \to (X_{\bar{c}c}K_S\pi^0)(X_{\bar{c}c}K_L\pi^0)$. For a more detailed discussion about CP violation in the semi-inclusive decays B_d^0 vs $\bar{B}_d^0 \to (\bar{c}c)K_S$ or $(\bar{c}c)K_L$, we refer the reader to Ref. [14].

[3]Note that $\psi' \to \psi\pi\pi$, $\psi'' \to D\bar{D}$, and $\eta'_c \to \eta_c\pi\pi$.

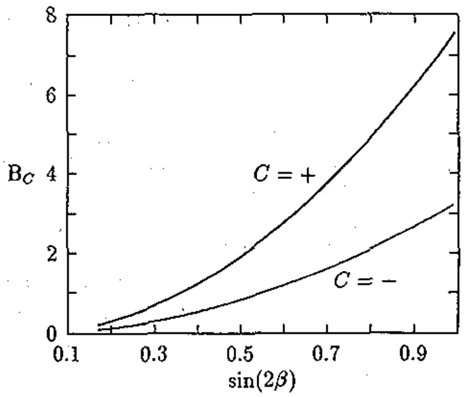

FIG. 6. Relative sizes of the effective branching fractions B_C (in arbitrary units) as functions of the CP-violating angle β at the $\Upsilon(4S)$ resonance.

IV. SUMMARY

In keeping with the experimental efforts to study flavor mixing and CP violation, we have presented a generic formalism for the time-dependent and time-integrated decays of all possible P^0-$\bar{P}^0$ systems. This formalism is useful for various phenomenological applications at the forthcoming flavor factories, where a large amount of coherent $P^0\bar{P}^0$ events will be produced and accumulated. In our calculations, the $\Delta Q = \Delta P$ rule and CPT symmetry have been assumed. Relaxing these two constraints one can obtain the more general formulas, which should be useful for searching for CPT violation or $\Delta Q = \Delta P$ breaking in the K^0-$\bar{K}^0$ [6] and B^0-$\bar{B}^0$ systems [15,16].

To meet various possible measurements of CP violation at asymmetric B factories, we have carried out a reanalysis of four types of CP-violating signals expected to appear in the coherent $B_d^0\bar{B}_d^0$ transitions at the $\Upsilon(4S)$ resonance. Although some comprehensive works have been done on this topic, our present one differs from them in several aspects. We illustrate the advantage of proper time cuts for measuring the B_d^0-$\bar{B}_d^0$ mixing parameters and CP asymmetries. It is shown that direct and indirect CP-violating effects are distinguishable in neutral B decays to CP eigenstates. The effect of final-state interactions on CP asymmetries in many B_d decays to non-CP eigenstates may be significant. In addition, we explore the possibility to measure some CP-forbidden processes as a direct test of CP symmetry breaking at B factories.

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