Phase structure of renormalizable four-fermion models in spacetimes of constant curvature

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A number of 2D and 3D four-fermion models which are renormalizable, in the $1/N$ expansion, in a maximally symmetric constant curvature space are investigated. To this purpose, a powerful method for the exact study of spinor heat kernels and propagators on maximally symmetric spaces is reviewed. The renormalized effective potential is found for any value of the curvature and its asymptotic expansion is given explicitly, both for small and for strong curvature. The influence of gravity on the dynamical symmetry-breaking pattern of some U(2) flavorlike and discrete symmetries is described in detail. The phase diagram in S^2 is constructed and it is shown that, for any value of the coupling constant, a curvature exists above which chiral symmetry is restored. For the case of H^2, chiral symmetry is always broken. In three dimensions, in the case of positive curvature, S^3, it is seen that curvature can induce a second-order phase transition. For H^3 the configuration given by the auxiliary fields equated to zero is not a solution of the gap equation. The physical relevance of the results is discussed.

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I. INTRODUCTION

Four-fermion models [1, 2], usually considered in the $1/N$ expansion, are interesting due to the fact that they provide an opportunity to carry out an explicit, analytical study of composite bound states and dynamical chiral symmetry breaking. At the same time, these theories, and especially their renormalizable two-dimensional (2D) [2] and 3D [3] variants, exhibit specific properties which are similar to the basic behaviors of some realistic models of particle physics. Moreover, this class of theories can be used for the description of the standard model (SM) itself or of some particle physics phenomena in the SM (see [4–7]). For example, the dynamical symmetry-breaking pattern of Nambu–Jona-Lasinio (NJL) models for the electroweak interaction, with the top quark as an order parameter, has been discussed in [4, 5].

Having in mind the applications of four-fermion models to the early universe and, in particular, the chiral symmetry phase transitions that take place under the action of the external gravitational field, there has been recently some activity in the study of 2D [8], 3D [9], and 4D [10–14] four-fermion models in curved spacetime (for a general introduction to quantum field theory in a curved spacetime, see [15]). The block-spin renormalization group (RG) approach and the similarities of the model with the Higgs-Yukawa one have been considered in [13] and [14], respectively.

The effective potential of composite fermions in curved spacetime has been calculated in different dimensions [8–11]. Dynamical chiral symmetry-breaking fermionic mass generation and curvature-induced phase transitions have been investigated in full detail. However, in most of these cases only linear curvature terms of the effective potential have been taken into account [9–12]. But it turns out in practice that it is often necessary to consider precisely the strong curvature effects to dynamical symmetry breaking. In fact we will see that going beyond the linear-curvature approximation can lead to qualitatively different results.

In this paper we will investigate some 2D and 3D four-fermion models which are renormalizable, in the $1/N$ expansion, in a maximally symmetric constant-curvature space (either of positive or of negative curvature). The renormalized effective potential will be found for any value of the curvature and the possibility of dynamical symmetry breaking in a curved spacetime will be carefully explored. Furthermore, the phase structure of the theory will be described in detail.

The paper is organized as follows. In the next section we calculate the effective potential of composite fermions in the Gross-Neveu model, in the spaces S^2 and H^2. The phase diagram in S^2 is constructed and it is shown that for any value of the coupling constant there exists a curvature above which chiral symmetry is restored. For the case of H^2, we show that chiral symmetry is always broken. The asymptotic expansions of the effective potential are given explicitly, both for small and for strong curvature. The three-dimensional case is studied in Sec. III. We consider two different four-fermion models: one

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which exhibits a continuous U(2) symmetry and another where we concentrate on two discrete symmetries which happen never to be simultaneously broken (see [16]). In Sec. IV we study explicitly the dynamical P and Z_2 symmetry-breaking pattern in H^3 and S^3. Finally, Sec. V is devoted to conclusions and some technical points of the procedure are summarized in two appendixes.

II. GROSS-NEVEU MODEL IN A SPACE OF CONSTANT CURVATURE

A. Case of the 2D de Sitter space S^2

In this subsection we will undertake the discussion of the Gross-Neveu model [2] in de Sitter space. This model, although rather simple in its conception, displays a quite rich structure, similar to that of realistic four-dimensional theories, as renormalizability, asymptotic freedom [2,17], and dynamical chiral symmetry breaking. Some discussions of chiral symmetry restoration in the Gross-Neveu model for different external conditions (such as an electromagnetic field, nonzero temperature or a change of the fermionic number density) have appeared in the past [18–20] (the influence of kink-antikink configurations on the phase transitions is described in [21,22].

The study of the Gross-Neveu model in an external gravitational field has been performed in Ref. [8] using the Schwinger method [23] (for other analyses of two-dimensional models in curved space, see [24]). Unfortunately, the generalization of the Schwinger procedure to curved spacetime is not free from ambiguities and this is why the result of Ref. [8] includes some mistakes.

In the calculation of the effective potential in the Gross-Neveu model on de Sitter space we will use a rigorous mathematical treatment of the fermionic propagator in (constant curvature) spacetime, which has been developed in Ref. [25]. We shall start from the action

$$S = \int d^2x \sqrt{-g}\left[\bar{\psi} i\gamma^\mu(x)\nabla_\mu\psi + \frac{\lambda}{2N}(\bar{\psi}\psi)^2\right], \quad (1)$$

where N is the number of fermions, λ the coupling constant, $\gamma^\mu(x) = \gamma^a e_a^\mu(x)$, with γ^a the ordinary Dirac matrix in flat space, and ∇_μ is the covariant derivative. By introducing the auxiliary field σ, it is convenient to rewrite (1) as

$$S = \int d^2x \sqrt{-g}\left[\bar{\psi} i\gamma^\mu(x)\nabla_\mu\psi - \frac{N}{2\lambda}\sigma^2 - \sigma\bar{\psi}\psi\right], \quad (2)$$

with $\sigma = -\frac{\lambda}{N}\bar{\psi}\psi$. Furthermore, in order to apply the results of Ref. [25], it is convenient to use Euclidean notation. Then (2) is written as

$$S = \int d^2x \sqrt{g}\left[\bar{\psi}\gamma^\mu(x)\nabla_\mu\psi + \frac{N}{2\lambda}\sigma^2 + \sigma\bar{\psi}\psi\right]. \quad (3)$$

Assuming that we work in de Sitter space and using the standard $1/N$ expansion, we get the effective potential in terms of the σ field as [8, 10, 11, 9]

$$V(\sigma) = \frac{\sigma^2}{2\lambda} + \mathrm{Tr}\int_0^\sigma D(x,x,s)\,ds, \quad (4)$$

where the propagator D is defined by

$$(\hat{\nabla} + s)D(x,y,s) = -\delta_2(x,y). \quad (5)$$

The curvature of both S^d and H^d can be written in the form

$$R = \frac{d(d-1)k}{a^2}, \quad (6)$$

with $k = 1$ for S^d and $k = -1$ for H^d; a stands for the radius of the manifold.

We consider first $k = 1$. Following [25] we begin the calculation by obtaining the "squareing" Green's function

$$\left(\hat{\nabla}^2 - s^2\right) G(x,y) = -\delta_2(x,y). \quad (7)$$

Then $D = \left(\hat{\nabla} - s\right) G(x,y)$. We resort to the *ansatz* [25]

$$G(x,y) = u(x,y)g(p), \quad (8)$$

where $u(x,y)$ yields a unit matrix in spinor indices when $y \to x$, and p is the distance between x and y along the geodesic that goes through these two points. Introducing the notation

$$\theta = \frac{p}{a}, \quad g(\theta) = \cos\frac{\theta}{2}\, h(\theta), \quad Z = \cos^2\frac{\theta}{2} \quad (9)$$

leads to the following equation for h:

$$\left[Z(1-Z)\frac{d^2}{dZ^2} + (2-3Z)\frac{d}{dZ} - 1 - s^2a^2\right] h(Z) = 0. \quad (10)$$

One can show [25] that the solution of (7) is given by a linear combination of the hypergeometric functions [26]. As boundary conditions for (7) it is convenient to choose the following. First, one selects the only singularity that appears for $\theta \to 0$. Second, one demands [25] that the singular part of this limit have the same form as in flat space,

$$g(\theta) \sim -\frac{1}{2\pi}\ln\theta, \quad \theta \to 0. \quad (11)$$

Using properties of the hypergeometric functions and the boundary conditions (11), the function D for coinciding arguments is found to be

$$D(x,x,s) = -s\lim_{\theta\to 0} g(\theta) = \frac{s}{4\pi}\left[\psi(1+isa) + \psi(1-isa) + 2\gamma + \ln\frac{\theta^2}{4}\right], \quad (12)$$

where (11) has been used explicitly. Here γ is the Euler constant, ψ the digamma function, and the two arguments of D are supposed to be separated by a small geodesic distance $p = \theta a$. Differentiating (4) with respect to σ and using (12), one gets

$$V'(\sigma) = \frac{\sigma}{\lambda} + \mathrm{Tr}\ D(x,x,\sigma) = \frac{\sigma}{\lambda}\left\{1 + \frac{\lambda}{2\pi}\left[2\gamma + \psi\left(1 + i|\sigma|\sqrt{\frac{2}{R}}\right) + \psi\left(1 - i|\sigma|\sqrt{\frac{2}{R}}\right) + \ln\frac{p^2 R}{8}\right]\right\}, \tag{13}$$

where R is the curvature.

As a renormalization condition we choose [8]

$$V''(\sigma)|_{R=0,\sigma=\mu} = \frac{1}{\lambda}. \tag{14}$$

By selecting counterterms of the form

$$\delta V = -\frac{1}{2\pi}\sigma^2 \ln\frac{\mu p\, e^{\gamma+1}}{2}, \tag{15}$$

adding them to (13), and using the asymptotics of $\psi(x)$, we find the following value for the derivative of the renormalized effective potential:

$$V'(\sigma) = \frac{\sigma}{\lambda}\left\{1 + \frac{\lambda}{2\pi}\left[\psi\left(1 + i|\sigma|\sqrt{\frac{2}{R}}\right) + \psi\left(1 - i|\sigma|\sqrt{\frac{2}{R}}\right) - 2 - \ln\frac{2\mu^2}{R}\right]\right\}. \tag{16}$$

As we can see, setting aside the different notation employed for σ, the terms involving $\psi(x)$ differ from the corresponding terms reported in [8].

Starting now from expression (16), different physical questions can be studied. In particular, the possibility to construct a corresponding phase diagram appears. To this end, let us calculate the second derivative

$$V''(\sigma) = \frac{1}{\lambda} + \frac{1}{2\pi}\left[\psi\left(1 + i|\sigma|\sqrt{\frac{2}{R}}\right) + \psi\left(1 - i|\sigma|\sqrt{\frac{2}{R}}\right) - 2 - \ln\frac{2\mu^2}{R}\right]$$
$$+\frac{i|\sigma|}{2\pi}\sqrt{\frac{2}{R}}\left[\zeta\left(2, 1 + i|\sigma|\sqrt{\frac{2}{R}}\right) - \zeta\left(2, 1 - i|\sigma|\sqrt{\frac{2}{R}}\right)\right]. \tag{17}$$

We can now study the behavior of the renormalized V near $\sigma = 0$. We have always

$$V'(0) = 0, \quad V''(0) = \frac{1}{\lambda}\left[1 - \frac{\lambda}{2\pi}\left(2\gamma + 2 + \ln\frac{2\mu^2}{R}\right)\right]. \tag{18}$$

With the notation

$$R_0 = 2\mu^2 e^{2(\gamma+1)}, \qquad \lambda_0 = 2\pi, \tag{19}$$

we obtain that the point $\sigma = 0$ is a minimum for $R > R_0 \exp(-\lambda_0/\lambda)$ and a maximum for $R < R_0 \exp(-\lambda_0/\lambda)$. That is, for any value of λ, there exists a value of the curvature above which chiral symmetry is restored. The connection of the critical curvature with the coupling constant is

$$R_{\rm cr} = R_0 e^{-\lambda_0/\lambda}. \tag{20}$$

A different expression for $R_{\rm cr}$ will be given below, in which the independence of $R_{\rm cr}$ from the renormalization scale μ is made apparent. For $R < R_{\rm cr}$ the chiral symmetry is broken and a dynamical fermion mass is generated.

As a next step one can investigate different limits of the expression (16) for the renormalized potential. For $R \to 0$, one obtains, from (16),

$$V'(\sigma) \sim \frac{\sigma}{\lambda}\left[1 + \frac{\lambda}{2\pi}\left(\ln\frac{\sigma^2}{\mu^2} - 2 + \frac{R}{12\sigma^2}\right)\right], \tag{21}$$

$$V(\sigma) \sim \frac{\sigma^2}{2\lambda}\left[1 + \frac{\lambda}{2\pi}\left(\ln\frac{\sigma^2}{\mu^2} - 3\right)\right] + \frac{R}{48\pi}\ln\frac{\sigma^2}{R}. \tag{22}$$

Notice that in (22) the constant of integration has been chosen having in mind the finiteness of V as $R \to 0$ [8]. At the same time, at $R \to \infty$ one gets

$$V'(\sigma) \sim \frac{\sigma}{\lambda}\left[1 + \frac{\lambda}{2\pi}\left(-2\gamma - 2 + 4\zeta(3)\frac{\sigma^2}{R} - \ln\frac{2\mu^2}{R}\right)\right], \tag{23}$$

$$V(\sigma) \sim \frac{\sigma^2}{2\lambda}\left[1 + \frac{\lambda}{2\pi}\left(-2\gamma - 2 + 2\zeta(3)\frac{\sigma^2}{R} - \ln\frac{2\mu^2}{R}\right)\right]. \tag{24}$$

The last expressions show the behavior of the effective potential at strong curvature. Analyzing (21)–(24) one can see that for small R chiral symmetry is broken, as happens in flat spacetime. However, in the limit of strong curvature chiral symmetry is restored. Thus, the study of the asymptotics is a further check of our general analysis

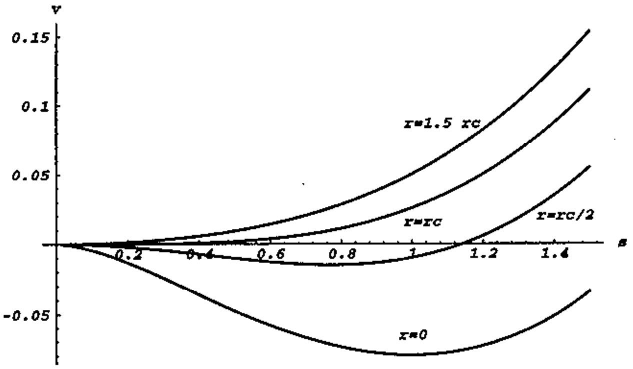

FIG. 1. In this figure, $v \equiv Va^2$, $s \equiv \sigma a$, and $r \equiv \frac{R}{M^2}$. One sees that for $r > rc \equiv \frac{R_{cr}}{M^2}$ the symmetry is restored in S^2.

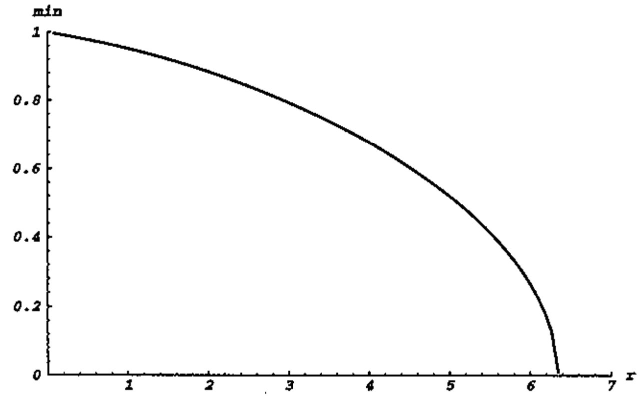

FIG. 2. In this figure, $r \equiv \frac{R}{M^2}$ and min $= a\sigma_{\rm min}$, where $\sigma_{\rm min}$ is the value of σ at which V attains its minimum. The continuous character of the transition which takes place in S^2 is clearly exhibited.

(phase diagrams).

If one defines M by

$$V''(\sigma)\,|_{R=0,\ \sigma=M} = 0,$$

then one can express the derivative of the effective potential in terms of this parameter as

$$V'(\sigma) = \frac{\sigma}{2\pi}\left[\psi\left(1 + i\sqrt{\frac{2\sigma^2}{R}}\right) + \psi\left(1 - i\sqrt{\frac{2\sigma^2}{R}}\right) + \ln\frac{R}{2M^2}\right].$$

We may now rephrase the criterion of symmetry restoration in terms of M by saying that the symmetry is restored when $R > R_{\rm cr} = 2M^2 \exp(2\gamma)$. The shape of the effective potential for different values of the quotient $\frac{R}{M^2}$ is shown in Fig. 1. The character of the transition is continuous, as illustrated by Fig. 2.

B. Case of the hyperbolic space H^2

In the hyperbolic space H^2 (negative curvature) the analysis can be carried out in a very similar way. After introducing the notation

$$\theta = \frac{p}{a}, \quad g(\theta) = \cosh\frac{\theta}{2}\, h(\theta), \quad Z = \cosh^2\frac{\theta}{2}, \tag{25}$$

Eq. (7) acquires the form [25]

$$\left[Z(1-Z)\frac{d^2}{dZ^2} + (2-3Z)\frac{d}{dZ} - 1 + s^2a^2\right] h(Z) = 0. \tag{26}$$

In H^2 we need to use other boundary conditions: the most rapid decrease with p going to infinity, and the same as in S^2 with p going to 0; as a result, using a similar procedure as in Sec. II A, we get

$$D(x,x,s) = \frac{s}{4\pi}\left[2\gamma + \psi(1+sa) + \psi(sa) + \ln\frac{\theta^2}{4}\right]. \tag{27}$$

The nonrenormalized expression for $V'(\sigma)$ is

$$V'(\sigma) = \frac{\sigma}{\lambda}\left\{1 + \frac{\lambda}{2\pi}\left[2\gamma + \psi\left(1 + |\sigma|\sqrt{\frac{2}{|R|}}\right) + \psi\left(|\sigma|\sqrt{\frac{2}{|R|}}\right) + \ln\frac{p^2|R|}{8}\right]\right\}. \tag{28}$$

Making the same renormalization as in Sec. II A, we obtain the renormalized effective potential

$$V'(\sigma) = \frac{\sigma}{\lambda}\left\{1 + \frac{\lambda}{2\pi}\left[2\psi\left(1 + |\sigma|\sqrt{\frac{2}{|R|}}\right) - 2 - \ln\frac{2\mu^2}{|R|}\right]\right\} - \sqrt{\frac{|R|}{8\pi^2}}, \tag{29}$$

$$V''(\sigma) = \frac{1}{\lambda}\left\{1 + \frac{\lambda}{2\pi}\left[2\psi\left(1 + |\sigma|\sqrt{\frac{2}{|R|}}\right) - 2 - \ln\frac{2\mu^2}{|R|}\right]\right\} + \frac{|\sigma|}{\pi}\sqrt{\frac{2}{|R|}}\,\zeta\left(2, 1 + |\sigma|\sqrt{\frac{2}{|R|}}\right). \tag{30}$$

A careful study of $V'(0)$ shows that due to the presence of the last term in (29), $\sigma = 0$ is never stationary for any value of λ and finite R. Owing to the fact that $V'(0) < 0$, chiral symmetry is always broken in H^2.

In the small curvature limit ($|R| \to 0$),

$$V(\sigma) = \frac{\sigma^2}{2\lambda}\left[1 + \frac{\lambda}{2\pi}\left(\ln\frac{\sigma^2}{\mu^2} - 3\right)\right] - \frac{|R|}{48\pi}\ln\frac{\sigma^2}{|R|}, \tag{31}$$

which coincides with (22), taking into account the change of sign for the curvature. In the strong curvature limit ($|R| \to \infty$),

$$V(\sigma) = \frac{\sigma^2}{2\lambda}\left[1 - \frac{\lambda}{2\pi}\left(2\gamma + 2 + \ln\frac{2\mu^2}{|R|} + \frac{\sqrt{2|R|}}{|\sigma|}\right)\right]. \tag{32}$$

The analysis of Eqs. (31) and (32) shows that the general conclusion about the chiral symmetry breaking at any finite R in H^2 is correct. In a similar way one can study the influence of curvature in the massive Gross-Neveu model (for a recent discussion of such model at nonzero temperature, see [26]).

It is interesting to note that in [29] it has been suggested to use a constant negative curvature space as a convenient infrared regulator in quantum field theory. The authors of [29] have argued that in a space of negative curvature the metric properties are effectively infinite dimensional, which gives the possibility to distinguish between different types of transitions. The two-dimensional XY model is disordered at any finite temperature due to vortice effects [29]. In our present work we use a slightly different representation of the propagators, taken from [25], and we do not study the questions addressed in [29]. However, our result that chiral symmetry is always broken in $\mathcal{H}^2$ can be seen, in some sense, to yield support to the findings of Callan and Wilczek on the role of a negative curvature space as an infrared regulator. It would be of some interest to try to translate the results on chiral symmetry breaking in the Gross-Neveu model on $\mathcal{H}^2$ to the language of [29] in more detail.

III. DYNAMICAL U(2) FLAVOR SYMMETRY BREAKING IN H^3 AND S^3

This section is devoted to the description of a three-dimensional four-fermion model which has a continuous flavorlike symmetry and how its breaking is affected by a gravitational background.

We consider the following model on a Riemannian manifold:

$$\mathcal{L}_E = \bar\psi \not{D}\psi - \frac{\lambda_B}{2N}\left[(\bar\psi\psi)^2 + (\bar\psi i\tau^1\psi)^2 + (\bar\psi i\gamma^5\psi)^2\right], \tag{33}$$

with

$$\gamma^\mu = \begin{pmatrix}\sigma^\mu & 0\\ 0 & -\sigma^\mu\end{pmatrix}, \quad \mu = 1,2,3, \quad \gamma^5 = i\begin{pmatrix}0 & \mathbb{1}_2\\ -\mathbb{1}_2 & 0\end{pmatrix} = -\tau^1\gamma^1\gamma^2\gamma^3, \quad \tau^1 = \begin{pmatrix}0 & \mathbb{1}_2\\ \mathbb{1}_2 & 0\end{pmatrix}, \tag{34}$$

and so we take a reducible, four-dimensional Dirac algebra.

The transformation of the bilinear terms $\bar\psi\psi$, $\bar\psi i\tau^1\psi$, and $\bar\psi i\gamma^5\psi$ under

$$\delta\psi = -iT_\alpha\theta^\alpha\psi, \qquad \delta\bar\psi = i\bar\psi\gamma^3 T_\alpha\gamma^3\theta^\alpha, \tag{35}$$

with $T_0 = 1$, $T_1 = \gamma^5$, $T_2 = \tau^1$, and $T_3 = i\tau^1\gamma^5$, is given by

$$\begin{pmatrix}\delta(\bar\psi\psi)\\ \delta(\bar\psi i\tau^1\psi)\\ \delta(\bar\psi i\gamma^5\psi)\end{pmatrix} = 2\begin{pmatrix}0 & -\theta^1 & -\theta^2\\ \theta^1 & 0 & -\theta^3\\ \theta^2 & \theta^3 & 0\end{pmatrix}\begin{pmatrix}\bar\psi\psi\\ \bar\psi i\tau^1\psi\\ \bar\psi i\gamma^5\psi\end{pmatrix}. \tag{36}$$

It is convenient to express this theory in terms of auxiliary fields: namely,

$$\mathcal{L}_E = \bar\psi\left(\not{D} + \sigma + i\tau^1\rho + i\gamma^5\pi\right)\psi + \frac{N}{2\lambda_B}(\sigma^2 + \rho^2 + \pi^2), \tag{37}$$

which yields

$$\Gamma_{N\to\infty}[\sigma,\rho,\pi] = \int dx\,\sqrt{g}\,\frac{N}{2\lambda_B}(\sigma^2 + \rho^2 + \pi^2) - N\ln\det\left(\not{D} + \sigma + i\tau^1\rho + i\gamma^5\pi\right). \tag{38}$$

We consider the constant configurations of σ, ρ, π. The rotational symmetries impose that the effective potential only depends on $\sigma^2 + \rho^2 + \pi^2$, which allows us to set $\rho = \pi = 0$. The regularized expression of the effective potential turns out to be

$$\int dx\,\sqrt{g}\,V[\sigma] = \int dx\,\sqrt{g}\,\frac{N}{2\lambda_B}\sigma^2 + \frac{N}{2}\mathrm{Tr}\int_{1/\Lambda^2}^{\infty}\frac{dt}{t}\,e^{-t\sigma^2}e^{\nabla^2 t}. \tag{39}$$

The reader may verify that the use of point-splitting regularization leads to the same results that we get below by regulating the lower limit in the proper time integral. On the right hand side we observe the appearance of the coincidence limit of the heat kernel. Resorting to the results of Appendix B, the outcome is (for H^3)

$$V[\sigma] = \frac{N}{2\lambda_B}\sigma^2 + \frac{N}{2\pi^{3/2}}\int_{1/\Lambda^2}^{\infty}\frac{dt}{t^{5/2}}\left(1+\frac{1}{2t}\right)e^{-t\sigma^2}e^{\nabla^2 t}. \tag{40}$$

The dependence on the radius of the manifold is not shown, but it can very easily be recovered from dimensional analysis. We obtain

$$\frac{1}{N}V[\sigma] = \frac{\sigma^2}{2\lambda_B} + \frac{|\sigma|}{(4\pi)^{3/2}a^2}\left[\Gamma\left(-\frac{1}{2},\frac{\sigma^2}{\Lambda^2}\right) + 2a^2\sigma^2\Gamma\left(-\frac{3}{2},\frac{\sigma^2}{\Lambda^2}\right)\right]. \tag{41}$$

Imposing the renormalization condition

$$\frac{1}{N}\frac{d^2}{d\sigma^2}V[\sigma=0, R=-6/a^2=0] = \frac{1}{\lambda}, \tag{42}$$

one gets

$$\frac{1}{\lambda_B} = \frac{1}{\lambda} + \frac{\Lambda}{\pi^{3/2}}, \tag{43}$$

and, in the limit $\Lambda \to \infty$,

$$\frac{1}{N}V[\sigma] = \frac{\sigma^2}{2\lambda} + \frac{|\sigma|}{\pi}\left(\frac{\sigma^2}{3} - \frac{1}{4a^2}\right). \tag{44}$$

From this one can easily see that the symmetry is always broken in H^3. In fact we can also notice the curious feature that the origin is not a solution of the gap equation. This appears to liken the situation of four-fermion models in three dimensions under the influence of a magnetic field (see [27]).

As for S^3, we have (see Appendix B)

$$V(\sigma) = \frac{\sigma^2}{2\lambda_B} - \frac{1}{2\,\mathrm{Vol}}\mathrm{Tr}\int_0^{\sigma^2} d\, s\frac{1}{s - \nabla\!\!\!/^2}. \tag{45}$$

From this formula one derives the regularized effective potential using the result from [24] that on S^N the solution of

$$(\nabla\!\!\!/^2 - m^2)G(y) = -\delta_N(y)$$

is given by $G(y) = U(y)g_N(\sigma)$, where $U(y)$ is a parallel transport matrix that is the identity at the coincidence limit, and

$$g_N(\theta) = \frac{\Gamma\left(\frac{N}{2}+im\right)\Gamma\left(\frac{N}{2}-im\right)}{(4\pi)^{N/2}\Gamma\left(\frac{N}{2}+1\right)}\cos\left(\frac{\theta}{2}\right)F\left(\frac{N}{2}+im,\ \frac{N}{2}-im,\ \frac{N}{2}+1,\ \cos^2\left(\frac{\theta}{2}\right)\right),$$

where $\theta = \frac{\rho}{a}$ (ρ is the geodesic distance, and a is the radius of the manifold). With this in mind and using the known properties of the hypergeometric functions one may arrive at the expression (in which we only keep terms divergent in ρ and terms independent from it)

$$V'(\sigma) = \frac{\sigma}{\lambda_B} - \frac{\sigma}{4\pi}\frac{\mathrm{Tr}\ \mathbb{1}_4}{\rho} + \frac{\mathrm{Tr}\ \mathbb{1}_4}{4\pi}\left(\frac{1}{4a^2}+\sigma^2\right)\tanh(\pi\sigma a). \tag{46}$$

Using the renormalization given by

$$\frac{1}{N}\frac{d^2}{d\sigma^2}V[\sigma=0, R=6/a^2=0] = \frac{1}{\lambda},$$

we get

$$V'(\sigma) = \frac{\sigma}{\lambda} + \frac{1}{\pi}\left(\frac{1}{4a^2}+\sigma^2\right)\tanh(\pi\sigma a). \tag{47}$$

This appears to be in agreement with the correspondent effective potential in the Gross-Neveu model [in the simplest version without continuous U(2) symmetry and discrete symmetries] on de Sitter space using, different from our approach, dimensional regularization (see [19]). We can now compare this result with the one that was found in Ref. [9], which was a study of three-dimensional theories in the small curvature limit. One can easily check that expression (46) is indeed compatible with those in [9]. What is most surprising in the weak curvature limit is that, in view of the results, one might be tempted to conclude that the origin is not a solution to the gap equation (as happens in H^3) and, furthermore, that there may be no solution at all. However, looking at the exact result (46) or (47), this is seen to be an artifact of the approximation. The effective potential given in expression (47) may give rise to a second-order phase transition, as we illustrate in Fig. 3.

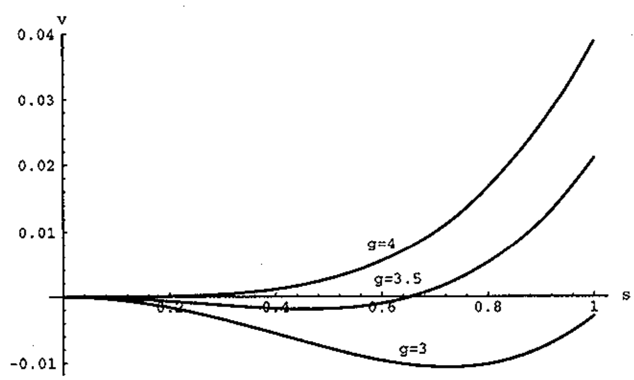

FIG. 3. Here, $v \equiv Va^3$, $s \equiv \sigma a$, and $g \equiv \frac{-\lambda}{a}$. It is clearly seen that there may be a second-order phase transition in S^3 and that it takes place when $g = 4$.

IV. DYNAMICAL P AND Z_2 SYMMETRY BREAKING IN H^3 AND S^3

In this section we analyze a model which displays two discrete symmetries. First of all, we present the model and later we describe its symmetries in some detail. To finish, we will describe the influence of gravity on the breaking of the symmetries. Using the representation for the γ^μ (which has no γ_5),

$$\gamma^\mu = \begin{pmatrix} \sigma^\mu & 0 \\ 0 & \sigma^\mu \end{pmatrix}, \tag{48}$$

it is also possible in this case to define a parity operation which admits the presence of a mass term: i.e.,

$$\mathcal{L}_E = \bar{\psi}\, \partial\!\!\!/ \psi + m\bar{\psi}\tau^3\psi, \tag{49}$$

with

$$P[\psi(x_1,x_2,x_3)] = \gamma^2\tau^1\psi(x_1,-x_2,x_3),$$
$$P[\bar{\psi}(x_1,x_2,x_3)] = -\bar{\psi}(x_1,-x_2,x_3)\gamma^1\tau^1]. \tag{50}$$

However, $\bar{\psi}\psi$ transforms as a pseudoscalar. There is another discrete symmetry of the kinetic term, given by

$$Z_2[\psi] = \tau^1\psi, \qquad Z_2[\bar{\psi}] = \bar{\psi}\tau^1. \tag{51}$$

Under this operation, the roles of the mass terms are reversed, e.g.,

$$Z_2[\bar{\psi}\tau^3\psi] = -\bar{\psi}\tau^3\psi, \qquad Z_2[\bar{\psi}\psi] = \bar{\psi}\psi. \tag{52}$$

With this in mind, it is immediate that

$$\mathcal{L}_E = \bar{\psi}\, D\!\!\!\!/\, \psi - \frac{\lambda_B}{2N}(\bar{\psi}\psi)^2 - \frac{\kappa_B}{2N}(\bar{\psi}\tau^3\psi)^2 \tag{53}$$

is invariant under both P and Z_2. But, as $\bar{\psi}\tau^3\psi$ is *not* invariant under Z_2, it can be taken as an order parameter for the Z_2 symmetry breaking. Likewise, $\langle\bar{\psi}\psi\rangle$ is an order parameter for the P symmetry breaking.

In terms of auxiliary fields,

$$\mathcal{L}_E = \bar{\psi}(D\!\!\!\!/ + \phi + \chi\tau^3)\psi + \frac{N}{2\lambda_B}\phi^2 + \frac{N}{2\kappa_B}\chi^2. \tag{54}$$

Proceeding along the same lines as before, we will impose equivalent renormalization conditions

$$\frac{d^2}{d\phi^2}V[\phi=0,R=0] = \frac{1}{\lambda}, \quad \frac{d^2}{d\chi^2}V[\chi=0,R=0] = \frac{1}{\kappa}. \tag{55}$$

In the case of hyperbolic space H^3 we get that the effective potential in the large-N limit is given by

$$\frac{1}{N}V[\phi,\chi] = \frac{\sigma_+^2+\sigma_-^2}{8}\left(\frac{1}{\lambda}+\frac{1}{\kappa}\right) + \frac{\sigma_+\sigma_-}{4}\left(\frac{1}{\lambda}-\frac{1}{\kappa}\right) + \frac{1}{2\pi}\left[|\sigma_+|\left(\frac{\sigma_+^2}{3}-\frac{1}{4a^2}\right) + |\sigma_-|\left(\frac{\sigma_-^2}{3}-\frac{1}{4a^2}\right)\right], \tag{56}$$

with $\sigma_\pm \equiv \phi \pm \chi$. We obtain the two cases

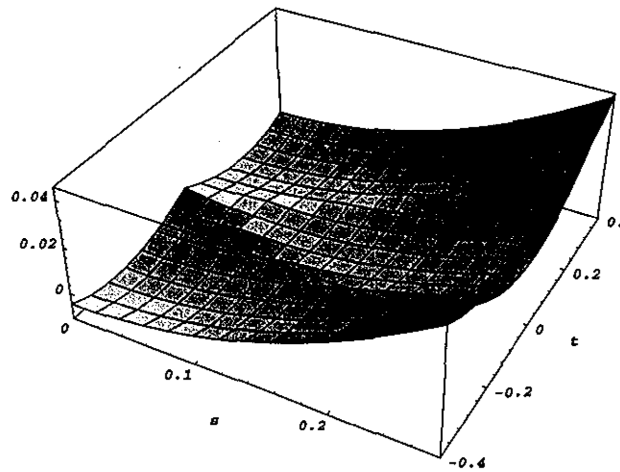

FIG. 4. This figure is a plot of Va^3 in H^3 in terms of $s \equiv \phi a$ and $t \equiv \chi a$. We have taken $\frac{a}{\lambda} = 1$ and $\frac{a}{\kappa} = 0.25$, and so $\frac{1}{\lambda} - \frac{1}{\kappa} > 0$ and Z_2 is broken.

$$\frac{1}{\lambda} - \frac{1}{\kappa} > 0 \quad \bar{\sigma}_+ = -\bar{\sigma}_-, \quad (\bar{\phi} = 0, \bar{\chi} \neq 0), \tag{57}$$

for the Z_2 symmetry breaking, and

$$\frac{1}{\lambda} - \frac{1}{\kappa} < 0 \quad \bar{\sigma}_+ = \bar{\sigma}_-, \quad (\bar{\phi} \neq 0, \bar{\chi} = 0), \tag{58}$$

for the P symmetry breaking, respectively. This is illustrated in Figs. 4 and 5, where we only consider positive values of ϕ, as the symmetry of the model allows us to reproduce the result for the region of negative ϕ immediately. It is also worth noting that either P or Z_2 is broken, but that it is impossible to have *both* symmetries broken. Figure 4 exemplifies the first situation: One sees that the minimum lies at $\phi = 0$, $\chi \neq 0$, and Z_2 is broken. In Fig. 5 we are in the second situation: The minimum lies at $\phi \neq 0$, $\chi = 0$, and P is broken.

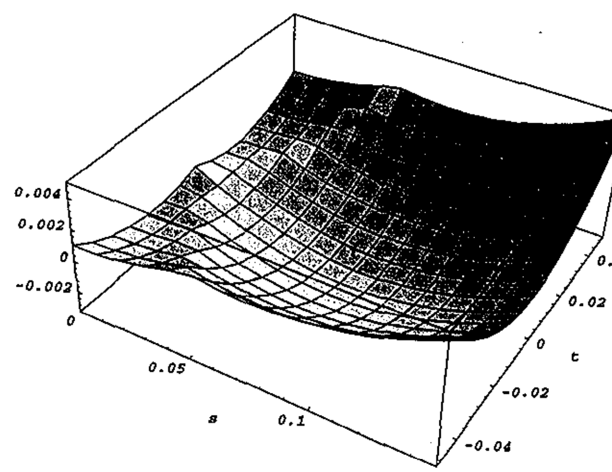

FIG. 5. This is also a plot of Va^3 in H^3, with s and t defined as in Fig. 4. Here we have $\frac{1}{\lambda} - \frac{1}{\kappa} < 0$ and P is broken.

As for the S^3 case, the reader may verify, along the same lines as above, that one encounters again two different situations: Either both P and Z_2 are unbroken or just one of them is broken. This is obtained without difficulty by repeating the same analysis, and we feel that further details are not necessary.

V. CONCLUSIONS

In this paper we have reviewed a powerful method for the exact study of spinor heat kernels and propagators on maximally symmetric spaces. We have used it with success in a number of different four-fermion models. The renormalized effective potential has been found, in each case, for any value of the curvature, and its asymptotic expansion has been given explicitly, both for small and for strong curvature.

We have described in great detail the influence of gravity on the dynamical symmetry-breaking pattern of some U(2) flavorlike and discrete symmetries. In particular, we have seen explicitly that the effect of a negative curvature is similar to that of the presence of a magnetic field.

For the two-dimensional Gross-Neveu model on S^2, where the chiral symmetry is a discrete one, we show the possibility of chiral symmetry breaking and of fermion mass generation. Note that the curvature of a two-dimensional de Sitter space acts here as some external parameter (such as temperature) which induces the chiral symmetry phase transition. In this sense and owing to the fact that we treat curved spacetime exactly, the de Sitter space cannot be considered to be some fluctuation over flat spacetime.

In the case of positive curvature S^3, we have checked that the scenario given in the framework of the small curvature expansion (e.g., fluctuations over flat space) changes dramatically when gravity is treated exactly. In particular we have found that the character of the phase transitions induced by gravity is continuous and that the origin is always a solution of the gap equation. A point to be duly remarked on is the fact that the techniques applied here to two- and three-dimensional models work equally well in four- and higher-dimensional ones. Of course, these models are not renormalizable in the standard way (see, however, Ref. [30] where mean-field renormalization of four-fermion models has been discussed), but one can still apply the above method in order to obtain the cutoff-dependent effective potential, which could certainly be useful for cosmological applications.

We shall now argue on the relevance of our results. We have investigated the phase structure of four-fermion models in 2D and 3D constant curvature space. We have used the mean-field approximation (actually, the leading approximation in the $\frac{1}{N}$ expansion), by taking into account only constant field configurations. In this approach, we have showed that the models can be treated analytically, and we have encountered the possibility of curvature-induced phase transitions (this fact was independently shown in a recent report [19]. A sensible question now is, what is the physical relevance of our results?

It is well known that spontaneous symmetry breaking in finite volume spaces is strictly speaking impossible. The fact that we have found the possibility of curvature-induced phase transitions between broken and unbroken phases of the theory (or symmetry breaking at some curvatures) indicates that the mean-field approximation is not good any longer to describe the physics near the critical curvature and above. The situation is analogous to that occurring in the Gross-Neveu model at nonzero temperature, where taking into account nonstatic (space-dependent) configurations $\sigma(x)$, called kink-antikink pairs, changes qualitatively the results on phase transitions at nonzero temperature [18, 31]. In this reference it is shown that at nonzero temperature there is a kink-antikink gas whose density is exponentially suppressed for large N; thus, the system appears divided into domains where the value of the order parameter is that given by mean-field theory. Analogously, one can expect that inclusion of space-dependent configurations in the models under consideration here may qualitatively change the results, and may always present the theory in the unbroken phase at any curvature for spaces of finite volume. Unfortunately, it is not clear to us at present how to perform this calculation technically, due to the fact that the theory is formulated in curved spacetime and thus the corresponding perturbation theory is considerably more difficult than in the case of flat space [18, 31], where such space-dependent configurations could certainly be taken into account. Hence, this topic must be left for further research.

Moreover, we would like to stress that a mean-field theory approach is still meaningful. First, the results of our study are in fact physically relevant for those curvatures where there is no symmetry breaking. Second, in all cases we discuss the small curvature case [Eqs. (22) and (31)] or the large curvature case [Eqs. (24) and (32)]. These cases correspond to large classes of spacetimes, not necessarily of finite volume. In this sense, the results obtained in such situation are of physical relevance and correctly describe the corresponding phase structure. Third, by analogy with the arguments in Ref. [31] we expect that space-dependent configurations do not show up in finite orders of the $\frac{1}{N}$ expansion. In other words, the results on the symmetry-breaking pattern (or phase transitions) in a finite segment of the whole system are physically relevant because the probability to find space-dependent configurations here is extremely small.

Finally, let us note that an interesting topic is also to study a formulation overlapping technique between the Schwinger-Dyson equations in a constant-curvature space and the effective potential approach. We expect to be able to address some of these questions in the near future.

ACKNOWLEDGMENTS

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APPENDIX A: GENERALITIES ABOUT SPINORS IN THREE DIMENSIONS

Some general facts about spinors in three dimensions that may be interesting to recall for the benefit of the reader are the following. The Dirac algebra $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$ has two irreducible representations: namely, (1) $\gamma^0 = \sigma^3$, $\gamma^1 = i\sigma^1$, $\gamma^2 = i\sigma^2$, and (2) $\gamma'_\mu = -\gamma_\mu$. On the other hand, there is no "γ^5" matrix of order 2 which anticommutes with all of the γ^μ's. A mass term of the form $m\bar{\psi}\psi$ in the Lagrangian explicitly violates parity, defined by

$$P : \psi(x^0, x^1, x^2) \longrightarrow \sigma^1 \psi(x^0, -x^1, x^2). \tag{A1}$$

However, if one uses a reducible representation of the Dirac matrices, such as

$$\gamma^0 = \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} i\sigma^1 & 0 \\ 0 & -i\sigma^1 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} i\sigma^2 & 0 \\ 0 & -i\sigma^2 \end{pmatrix}, \tag{A2}$$

then the mass term $m\bar{\psi}\psi$ does preserve parity, as defined by

$$P : \psi(x^0, x^1, x^2) \longrightarrow \tau^1 \gamma^1 \psi(x^0, -x^1, x^2), \tag{A3}$$

being

$$\tau^1 = \begin{pmatrix} 0 & 1\!\!1_2 \\ 1\!\!1_2 & 0 \end{pmatrix}, \quad \tau^2 = \begin{pmatrix} 0 & -i1\!\!1_2 \\ i1\!\!1_2 & 0 \end{pmatrix}, \quad \tau^3 = \begin{pmatrix} 1\!\!1_2 & 0 \\ 0 & -1\!\!1_2 \end{pmatrix}. \tag{A4}$$

The kinetic term is invariant under the U(2) transformations with generators $T_0 = 1\!\!1_4$, $T_1 = \gamma^5$, $T_2 = \tau^1$, and $T_3 = i\tau^1\gamma^5$, where

$$\gamma^5 = \tau^1\gamma^0\gamma^1\gamma^2 = i \begin{pmatrix} 0 & 1\!\!1_2 \\ -1\!\!1_2 & 0 \end{pmatrix}. \tag{A5}$$

The mass term breaks this symmetry down to U(1) $\times$ U(1) with the generators T_0, T_3.

APPENDIX B: SPINOR HEAT KERNEL IN MAXIMALLY SYMMETRIC SPACES (THE INTERTWINING METHOD)

In this appendix we review a powerful method to derive closed expressions for the spinor heat-kernel and propagators in maximally symmetric spaces (for more details see [24]).

The spinor heat kernel satisfies $\left(-\frac{\partial}{\partial t} + \nabla\!\!\!/^2\right) K(y, y_0; t) = 0$ and the initial condition $\lim_{t\to 0} K(y, y_0; t) = 1\, \delta_N(y, y_0)$. Substituting the ansatz $K(y,t) = U(y) f(\sigma, t)$ in the heat-kernel equation, one gets

$$-U\frac{\partial f}{\partial t} + U\nabla^a\nabla_a f + 2n^a(\nabla_a U)\frac{\partial f}{\partial \sigma} + (\nabla^a\nabla_a U) f - \frac{R}{4} U f = 0, \tag{B1}$$

where $n_a = \nabla_a \sigma$. Here the term linear in $\partial f/\partial\sigma$ and $\nabla_a U$ cancels out, provided that U satisfies the parallel transport equation

$$n^a \nabla_a U = 0, \qquad U(y_0) = 1. \tag{B2}$$

The Laplacian acting on f can be replaced by its radial part given by

$$\nabla^a\nabla_a f = \Box_N f = (\partial_\sigma^2 + (N-1)B\partial_\sigma) f, \qquad B = \begin{cases} \frac{1}{a}\cot\left(\frac{\sigma}{a}\right), & S^N, \\ \frac{1}{a}\coth\left(\frac{\sigma}{a}\right), & H^N, \end{cases} \tag{B3}$$

and the Laplacian acting on U is

$$\nabla^a\nabla_a U = -\frac{1}{4}A^2(N-1)U, \qquad A = \begin{cases} -\frac{1}{a}\tan\left(\frac{\sigma}{2a}\right), & S^N, \\ \frac{1}{a}\tanh\left(\frac{\sigma}{2a}\right), & H^N. \end{cases} \tag{B4}$$

The equation for the scalar f becomes

$$(-\partial_t + L_N) f = 0, \qquad L_N = \Box_N - \frac{R}{4} - \frac{1}{4}(N-1)A^2. \tag{B5}$$

The idea is now to relate the solutions of this equation for different N's. To this end one looks for an operator D such that $L_N D = D L_{N-2}$. A simple ansatz leads to

$$D = \begin{cases} \frac{1}{2\pi} \cos\frac{\theta}{2} \frac{\partial}{\partial(\cos\theta)} \left(\cos\frac{\theta}{2}\right)^{-1}, & \theta = \frac{\sigma}{a},\ S^N, \\ -\frac{1}{2\pi} \cosh\frac{x}{2} \frac{\partial}{\partial(\cosh x)} \left(\cosh\frac{x}{2}\right)^{-1}, & x = i\frac{\sigma}{a}, H^N. \end{cases} \tag{B6}$$

The odd-dimensional case is elementary, taking into account that $L_N D^{(N-1)/2} = D^{(N-1)/2} L_1$ and the known solution of $(-\partial_t + L_1)K_1 = 0$. This yields

$$\begin{aligned} K_N(y,t) &= U(y) \cosh\frac{x}{2} \left(-\frac{1}{2\pi} \frac{\partial}{\partial(\cosh x)}\right)^{(N-1)/2} \left(\cosh\frac{x}{2}\right)^{-1} \frac{e^{-x^2/(4t)}}{\sqrt{4\pi t}}, \quad H^N, \\ K_N(y,t) &= U(y) \sum_{n=-\infty}^{\infty} (-1)^n f_{dp}(\theta + 2\pi n, t), \\ & \quad f_{dp}(\theta + 2\pi n, t) = \cosh\frac{\theta}{2} \left(\frac{1}{2\pi} \frac{\partial}{\partial(\cosh\theta)}\right)^{(N-1)/2} \left(\cosh\frac{\theta}{2}\right)^{-1} \frac{e^{-\theta^2/(4t)}}{\sqrt{4\pi t}}, \quad S^N. \end{aligned} \tag{B7}$$

In the particular case $N = 3$ these expressions read, in the coincidence limit,

$$\lim_{y\to y_0} K(y,t) = \mathbb{1}_d \frac{1}{(4\pi t)^{3/2}} \left[1 + \frac{t}{2a^2}\right], \quad H^3,$$

$$\lim_{y\to y_0} K(y,t) = \mathbb{1}_d \frac{1}{(4\pi t)^{3/2}} \left[1 - \frac{t}{2a^2} + \sum_{n=1}^{\infty} (-1)^n \exp\left(-\frac{\pi^2 a^2 n^2}{t}\right) \left(2 - t - 4\pi^2 n^2 \frac{a^2}{t}\right)\right], \quad S^3,$$

where d is the dimension of the Dirac algebra, in our case $d = 4$.

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