New lower bound on $SU(4)_C$ gauge boson mass from CERN LEP measurements and $K_L \to \mu e$

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Using the most recent experimental limit on $K_L \to \mu\bar{e}$, the inputs from the CERN LEP measurements, and the top-quark mass, we obtain a new lower bound on the heavy $SU(4)_C$ gauge boson mass due to Pati and Salam to be $M_C > 906^{+241}_{-195}$ TeV.

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One of the most interesting extensions of the standard model is through the $SU(4)_C$ model of Pati and Salam [1] where leptons have been suggested as the fourth color. Apart from answering the question as to why leptons are different from quarks, the parity (P) and CP violations in weak interactions [2] and small neutrino masses [3] can be attributed to arise from spontaneous symmetry breakings if $SU(4)_C$ forms a part of a left-right symmetric gauge group $SU(2)_L \times SU(2)_R \times SU(4)_C \times P$ ($\equiv G_{224P}$, $g_{2L} = g_{2R}$) which might undergo spontaneous symmetry breaking to the standard model through $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ with or without the left-right discrete symmetry ($\equiv P$, $g_{2L} = g_{2R}$). A number of grand unified theories such as SO(10), SU(16), $SU(8)_L \times SU(8)_R$, SO(18), E_6, and $SU(4)^4$ contains G_{224P} as its subgroup. While G_{224P} might exist as a good symmetry at high mass scales, the left-right asymmetric gauge group G_{224} [3] with $g_{2L} \neq g_{2R}$ or $SU(2)_L \times U(1)_{I_{3R}} \times SU(4)_C$ might survive as a good symmetry at much lower mass scales. Whether or not these gauge symmetries emerge from grand unified theories (GUT's), their spontaneous breaking to the standard gauge group at a scale $M_C \gg M_Z$ is always accompanied by heavy $SU(4)_C$ gauge bosons leading to interesting muon-number-violating processes such as $K_L \to \mu\bar{e}$. Thus experimental limits on rare decay processes can set bounds on the $SU(4)_C$-breaking scale, signifying observable new physics beyond the standard model.

Several attempts were made earlier to obtain bounds on M_C. While Pati and Salam [1] obtained a rough estimate $M_C > 3 \times 10^4$ GeV from the ratio $\Gamma(K_L \to \mu\bar{e})/\Gamma(K_L \to \mu^+\mu^-)$, Dimopoulos, Raby, and Kane (DRK) [4] derive

$$(M_C/g_{4C}) > 3.1 \times 10^5 \text{ GeV} \tag{1}$$

using the experimental ratio $\Gamma(K_L \to \mu e)/\Gamma(K^+ \to \mu^+\nu)$. In (1), g_{4C} is the $SU(4)_C$ gauge coupling. Using renormalization group equations (RGE's), QCD corrections, and the same experimental ratio as DRK, Deshpande and Johnson [5] have obtained the improved bound

$$M_C > 3.5 \times 10^5 \text{ GeV} . \tag{2}$$

The purpose of this analysis is to obtain a more refined bound on M_C using the measurements from the CERN e^+e^- collider LEP, the available data on the top quark mass and including the strong and electroweak interaction effects on quark mass evolutions from $\mu = M_C$ to $\mu = M_Z$, whereas only strong interaction effects were included in this range in Ref. [5] using the old data. As in Ref. [5], we include the impact of $SU(3)_C$ on the evolution of the down quark masses from $\mu = M_Z$ to $\mu = 1$ GeV, but using more accurate data on $\alpha_S(M_Z)$ from CERN LEP and its extrapolations. For our estimation we also use the most recent experimental limit on $K_L \to \mu^{\mp}e^{\pm}$, which gives [6,7]

$$B(K_L \to \mu\bar{e}) \equiv \frac{\Gamma(K_L \to \mu^{\mp}e^{\pm})}{\Gamma(K_L \to \mu^+\nu_\mu)} < 10^{-10.28} . \tag{3}$$

We consider the symmetry-breaking chain

$$\begin{array}{ll} SU(2)_L \times SU(2)_R \times SU(4)_C & \\ \text{or} & \xrightarrow{M_C} SU(2)_L \times U(1)_Y \times SU(3)_C \\ SU(2)_L \times U(1)_{I_{3R}} \times SU(4)_C & \qquad\qquad \downarrow^{M_Z} \\ & \qquad\qquad U(1)_{\text{em}} \times SU(3)_C , \end{array}$$

where the particle content below M_C is taken to be minimal as in the standard model with three fermion generations and the standard Higgs scalar doublet. Using the effects of renormalization on the d- and s-quark masses on the theoretical expression [5], the branching ratio in (3) at $\mu = \mu_0 = 1$ GeV can be written as

$$B(K_L \to \mu\bar{e}) = \frac{4\pi^2\alpha_S^2(M_C)m_K^4 R}{G_F^2 \sin^2\theta_C m_\mu^2 (m_S + m_d)^2 M_C^4} , \tag{4}$$

where the factor R in the right-hand side (RHS) of (4) accounts for the evolution of the quark masses from $\mu = M_C$ to $\mu = \mu_0 = 1$ GeV:

$$R = \left\{ R_S \left[\frac{1}{\alpha_S(M_C)}\right]^{24/21} \right\} \left\{ R_{2L} \left[\frac{1}{\alpha_{2L}(M_C)}\right]^{27/19} \right\} \times \left\{ R_{1Y} \left[\frac{1}{\alpha_{1Y}(M_C)}\right]^{2/41} \right\} , \tag{5}$$

$$R_S = [\alpha_S(m_t)]^{24/21} \left[\frac{\alpha_S(M_Z)}{\alpha_S(m_t)}\right]^{24/23} \left[\frac{\alpha_S(m_b)}{\alpha_S(M_Z)}\right]^{24/23} \times \left[\frac{\alpha_S(m_C)}{\alpha_S(m_b)}\right]^{24/25} \left[\frac{\alpha_S(\mu_0)}{\alpha_S(m_C)}\right]^{24/27} , \tag{6}$$

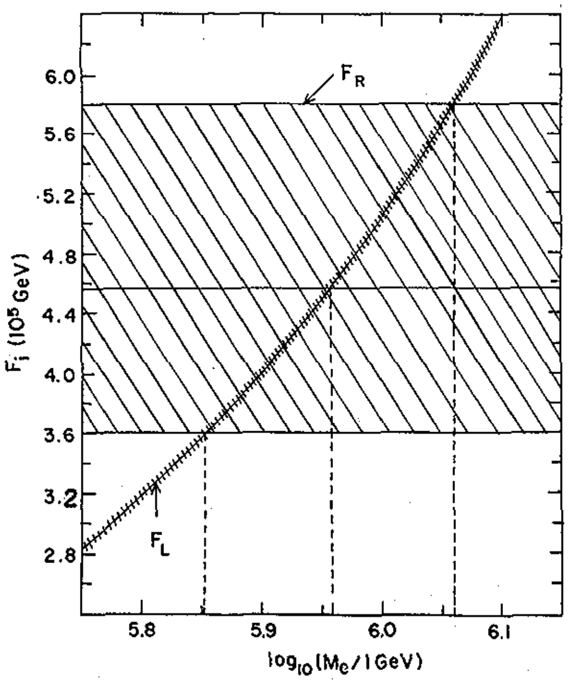

FIG. 1. Graphical representation of the method of obtaining a numerical solution on the lower bound on M_C. The horizontal shaded region (F_R) is the RHS of the inequality (11), whereas the curve F_L represents the LHS. The lower bound is confined within the dashed vertical lines. Here $\log_{10}$ stands for logarithm based on powers of ten.

$$R_{2L} = [\alpha_{2L}(m_t)]^{27/19}\left[\frac{\alpha_{2L}(M_Z)}{\alpha_{2L}(m_t)}\right]^{27/23} , \quad (7)$$

$$R_{1Y} = [\alpha_{1Y}(m_t)]^{2/41}\left[\frac{\alpha_{1Y}(M_Z)}{\alpha_{1Y}(m_t)}\right]^{6/103} . \quad (8)$$

The first, second, and third factors inside the three curly brackets on the RHS of (5) represent evolutions of the d- (or s-) quark mass via $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$, respectively. For the sake of simplicity, we have ignored a very small contribution to the quark mass evolution due to $U(1)_{em}$. In all earlier attempts [1,4,5], the electroweak effects on the d-quark mass evolution were ignored due to want of accurate values on the relevant gauge couplings. Also the top quark mass was not known. Now that the precision measurements at CERN LEP have led to the determination of the standard model gauge couplings and the top quark mass is given by the data from the Collider Detector at Fermilab (CDF) Collaboration [7–11],

$$\alpha_{1Y}^{-1}(M_Z) = 58.9 \pm 0.21 , \quad \alpha_{2L}^{-1}(M_Z) = 29.72 \pm 0.15 ,$$
$$\alpha_{3C}^{-1}(M_Z) = 8.33 \pm 0.60 , \quad m_t = 174 \pm 15 \text{ GeV} , \quad (9)$$

the strong and electroweak effects on mass evolutions are known more accurately. We use the following well-known values of the masses for neutral K meson [7], muon, current quark masses m_d and m_s [7,12], Fermi coupling constant, and Cabibbo-Kobayashi-Maskawa mixing angle [7]:

$$m_K = 0.493 \text{ GeV} , \quad m_\mu = 0.105 \text{ GeV} ,$$
$$m_s = 0.150 \text{ GeV} , \quad m_d = 0.010 \text{ GeV} ,$$
$$G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2} , \quad \sin\theta_C = 0.22 . \quad (10)$$

Using the renormalization of gauge couplings at the one-loop level and the values at M_Z from (9), we obtain

$$M_C\left[9.118 \pm 0.6 + \frac{7}{2\pi}\ln\frac{M_C}{m_t}\right]^{3/14}\left[30.114 \pm 0.15 + \frac{19}{12\pi}\ln\frac{M_C}{m_t}\right]^{-27/76}\left[58.54 \pm 0.21 - \frac{41}{20\pi}\ln\frac{M_C}{m_t}\right]^{-1/82} ,$$
$$> \left[\frac{4\pi^2 m_K^4 R_S R_{2L} R_{1Y}}{G_F^2 \sin^2\theta_C m_\mu^2 (m_s + m_d)^2} \times 10^{10.28}\right]^{1/4} . \quad (11)$$

The inequality in (11) is distinctly different from that derived in Ref. [5]. For the sake of convenience, we denote the LHS (RHS) of inequality (11) as the function F_L (F_R). Using input values from (9) and (10) gives

$$F_R = 456.9^{+123.3}_{-96.2} \text{ TeV} . \quad (12)$$

In order to obtain the bound, we plot the LHS graphically as a function of M_C with $m_t = 174 \pm 15$ GeV as shown by the solid curve in Fig. 1 where the horizontal shaded region represents F_R given by (12). The numerical solution for the allowed range of values on the lower bound for M_C is clearly demonstrated in Fig. 1 by the dashed vertical lines, leading to the improved estimates

$$M_C > 906^{+241}_{-195} \text{ TeV} . \quad (13)$$

As compared to the earlier bound $M_C > 350$ TeV [5], the lower limit has increased at least by a factor of 2 and our analysis reveals that the rare kaon decay experiments [6,7] have already probed into the $SU(4)_C$ gauge boson mass of 700 TeV. As stated earlier, the reasons for obtaining the improved bound in the present analysis are due to (i) the inclusion of the effects of $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ on the quark mass evolutions, (ii) more accurate information on the standard model gauge couplings based upon the precision CERN LEP measurements, (iii) information on the top quark mass, and (iv) the improved experimental limit on the branching ratio for $K_L \to \mu\bar{e}$. For the sake of simplicity, we have neglected two-loop effects and excluded the effects of $U(1)_{em}$ on the evolution of the down quark mass from M_Z to μ_0, which is negligible.

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