Remarks on monopoles in noncompact lattice QED

M. Göckeler,[1,2] R. Horsley,[2,*] P. E. L. Rakow,[2,3] and G. Schierholz[2,4]
[1] *Institut für Theoretische Physik, Rheinisch-Westfälische Technische Hochschule Aachen, Sommerfeldstraße, D-52056 Aachen, Germany*
[2] *Gruppe Theorie der Elementarteilchen, Höchstleistungsrechenzentrum HLRZ, c/o Forschungszentrum, Postfach 1913, D-52425 Jülich, Germany*
[3] *Institut für Theoretische Physik, Freie Universität Berlin, Arnimallee 14, D-14195 Berlin, Germany*
[4] *Deutsches Elektronen-Synchrotron DESY, Notkestraße 85, D-22603 Hamburg, Germany*
(Received 15 February 1994; revised manuscript received 3 January 1995)

We investigate the conjecture that monopoles in noncompact lattice QED condense, and that this phase transition is responsible for the breaking of chiral symmetry. The comparison of analytic and numerical results shows that we have a quantitative understanding of monopoles in both the quenched and dynamical cases. We see no evidence of monopole condensation.

PACS number(s): 11.15.Ha, 11.30.Rd, 12.20.Ds

I. INTRODUCTION

In a series of papers [1–5] we have investigated strongly coupled QED, both on the lattice in the noncompact formulation and in the continuum using Schwinger-Dyson equations. The strong coupling region is of interest because of the existence of a second order chiral phase transition. This implies a continuum limit, and the interesting question is whether or not it describes an interacting theory. Our calculations of the renormalized charge e_R and fermion mass m_R demonstrated that whenever m_R goes to zero in lattice units (i.e., the ultraviolet cutoff is removed) then e_R goes to zero. This suggests that the theory is noninteracting in the continuum limit in accordance with the general belief that nonasymptotically free theories are trivial. It is encouraging that the two approaches, namely, lattice and Schwinger-Dyson, agree with each other. Further support of this picture comes from other authors [6,7]. An interesting alternative approach to simulating lattice QED may be found in [8,9].

However, this picture has been queried by Hands, Kocić, Kogut, and collaborators [10–17], who investigated the behavior of magnetic monopoles near the phase transition. Using a new monopole "order parameter," the cluster susceptibility χ_c, introduced by Hands and Wensley [18], they conclude that monopoles condense in the chirally broken phase. The occurrence of this proposed second order monopole phase transition is important because such a transition would imply that the lattice model "has a critical point which should map onto a continuum model with real monopole excitations" [12,13]. This would cast doubt on conclusions about continuum physics drawn from lattice calculations, as monopoles are presumably absent in continuum QED. Furthermore, because dual superconductivity and charge confinement are to be expected whenever monopoles condense it is proposed that "unquenched lattice QED confines quarks in its strong coupling phase" [12,13]. This is in conflict with our picture [5] where we have found free electrons and massless photons in the broken phase.

In the strong coupling limit $\beta \to 0$, or the limit when we have a large number of flavors, the action is dominated by the fermion determinant, which is a compact object in the sense that it only depends on the compactified link variables e^{iA_μ}, A_μ being the gauge field. It was found that the compact U(1) Wilson action has a first order phase transition [19] which is driven by monopole condensation [19,20]. Hence it is conceivable that monopoles play a role in the noncompact case as well. Indeed, simulations with very large numbers of fermions [21] suggest that the phase transition becomes first order.

In this paper we shall investigate the relevance of monopoles for the phase transition. We solve the quenched case analytically [22] and look at the dynamical fermion case numerically. We find no evidence of monopole condensation.

II. LATTICE MONOPOLES

The action for noncompact lattice QED with dynamical staggered fermions can be found in [1]. The electromagnetic field is defined by $F_{\mu\nu}(x) = \Delta_\mu A_\nu(x) - \Delta_\nu A_\mu(x)$, where Δ_μ is the lattice forward derivative. To define monopoles [19,18] we decompose $F_{\mu\nu}$ into an integer valued string field $N_{\mu\nu}$ and a compact field $f_{\mu\nu}$ which lies in the range $(-\pi, \pi]$:

$$F_{\mu\nu} = 2\pi N_{\mu\nu} + f_{\mu\nu} \ . \qquad (2.1)$$

The Bianchi identity tells us that $F_{\mu\nu}$ summed over any closed surface always gives zero. This does not apply to the $N_{\mu\nu}$ and $f_{\mu\nu}$ fields separately. This allows the common definition of a conserved monopole current on the dual links:

*Present address: Institut für Physik, Humboldt-Universität zu Berlin, Invalidenstraße 110, D-10115 Berlin, Germany.

0556-2821/96/53(3)/1508(5)/$06.00

$$M_\mu(x) = \frac{1}{4\pi}\epsilon_{\mu\nu\rho\sigma}\Delta_\nu f_{\rho\sigma}(x+\hat{\mu})$$
$$= -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\Delta_\nu N_{\rho\sigma}(x+\hat{\mu}) \ . \qquad (2.2)$$

Each component of M_μ can take the values $0, \pm 1, \pm 2$.

Later on we shall be interested in the monopole susceptibility. This is defined by [23]

$$\chi_m = -\frac{4\pi^2}{12}\sum_x \langle x^2 M_\mu(x) M_\mu(0)\rangle \ . \qquad (2.3)$$

In the infinite volume limit further manipulations lead to the equivalent form

$$\chi_m = \tfrac{1}{6}\sum_x \langle f_{\mu\nu}(x) f_{\mu\nu}(0)\rangle = \frac{4\pi^2}{6}\sum_x \langle N_{\mu\nu}(x) N_{\mu\nu}(0)\rangle \ . \qquad (2.4)$$

We have used Eq. (2.4) for our measurements. If the monopoles condense we would expect χ_m to diverge. We shall also look at the monopole density ρ and string density σ:

$$\rho = \frac{1}{4V}\sum_{x,\mu}|M_\mu(x)| \quad \text{and} \quad \sigma = \frac{1}{4V}\sum_{x,\mu<\nu}|N_{\mu\nu}(x)| \ . \qquad (2.5)$$

At a phase transition ρ and σ would show nonanalytic behavior.

III. ANALYTIC RESULTS

In [15] it is claimed that even in quenched noncompact QED there is "an authentic second order phase transition" at which the magnetic monopoles condense. Because in the quenched case the action is quadratic and the partition function Gaussian, we can derive analytic formulas for most quantities [22]. First let us note that the free energy is an analytic function of β, a fact which is hard to reconcile with the existence of a phase transition. Of course, this does not exclude singularities in nonlocal functions of the gauge field such as the chiral condensate.

The simplest quantity related to monopole physics is the string density. The probability distribution for a single $F_{\mu\nu}$ field is a Gaussian: $\Psi(F) = \pi^{-1/2}\beta^{1/2}e^{-\beta F^2}$. The distribution is completely determined because we know the width $\langle F^2\rangle = 1/(2\beta)$. This gives

$$\sigma(\beta) = \tfrac{3}{2}\langle|N(F)|\rangle = \tfrac{3}{2}\sum_{n=0}^{\infty}\mathrm{erfc}[(2n+1)\pi\beta^{1/2}] \ . \qquad (3.1)$$

On a lattice with volume V the width of the Gaussian is reduced to $\langle F^2\rangle = (V-1)/(2V\beta)$. Thus σ on a finite lattice can be bound by replacing β by $\beta V/(V-1)$ in Eq. (3.1). One sees that finite size effects are negligible even on rather small lattices.

To find the monopole density ρ we need the probability distribution for the six F fields on the faces of a cube. The most general Gaussian form consistent with cubic symmetry and the Bianchi identity is

$$\Psi(F_1,\ldots,F_6) = \pi^{-5/2}6^{1/2}(a-b)^{3/2}(a+b)\beta^{5/2}$$
$$\times\delta(F_1+\cdots+F_6)$$
$$\times\exp\{-\beta a(F_1^2+\cdots+F_6^2)$$
$$-2\beta b(F_1F_6+F_2F_5+F_3F_4)\} \ . \qquad (3.2)$$

The outwardly directed "plaquettes" have been labeled so that F_n and F_{7-n} are on opposite faces. The parameters a and b are fixed by the known expectation values $\langle F_1^2\rangle = 1/(2\beta)$ and $\langle F_1F_2\rangle = -\gamma/(2\beta)$, where $\gamma = 0.215\,563\ldots$ on an infinite lattice. So a and b must take the values $a = (1+\gamma)/[12\gamma(1-2\gamma)]$ and $b = (1-5\gamma)/[12\gamma(1-2\gamma)]$. The monopole density $\rho(\beta)$ is

$$\rho(\beta) = \langle|M_\mu|\rangle = \int_{-\infty}^{\infty} dF_1\cdots dF_6\Psi(F_1,\ldots,F_6)$$
$$\times|N_1+\cdots+N_6| \ . \qquad (3.3)$$

The ratio $4\sigma/\rho$ is useful for giving a picture of the monopole distribution, because it is the average length of string joining a monopole-antimonopole pair. At large β this ratio is 1; there is always an antimonopole directly adjacent to every monopole. In the interesting region around $\beta = 0.24$, $4\sigma(\beta)/\rho(\beta)$ has only grown to ≈ 1.4, indicating that monopoles and antimonopoles are on a short leash (i.e., are tightly bound). In Fig. 1 these formulas are checked against the Monte Carlo data [18]. The agreement is excellent.

To check that ρ is analytic for $\beta > 0$ we expand $\rho(\beta)$ as a series of the form

$$\rho(\beta) = \beta^{5/2}\sum_{n=0}^{\infty} c_n(\beta_0-\beta)^n \qquad (3.4)$$

about an arbitrary point β_0. The coefficients c_n are given by the integrals

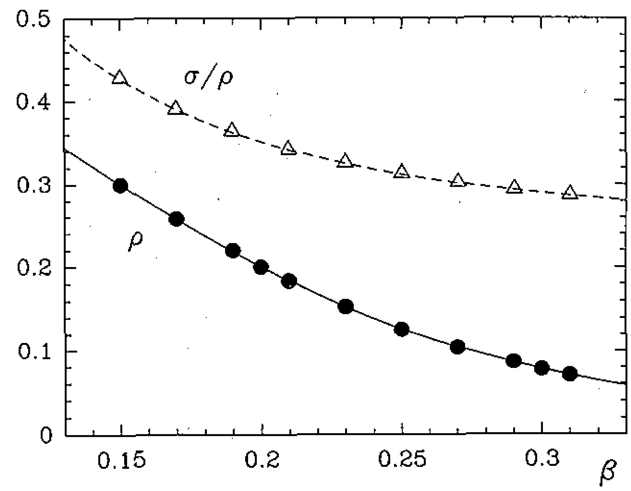

FIG. 1. The monopole density ρ and the σ to ρ ratio as a function of β for quenched QED. The symbols represent the data from Ref. [18] together with our data, while the curves show the analytic results on an infinite lattice.

$$c_n = \beta^{-5/2}\langle |M_\mu| [a(F_1^2 + \cdots + F_6^2) + 2b(F_1F_6 + F_2F_5 + F_3F_4)]^n \rangle / n! \quad (3.5)$$

evaluated at $\beta = \beta_0$. Using the fact that all the factors in the integrand are positive and that $|N_1 + \cdots + N_6| \le 2$ establishes the bounds

$$0 < c_n < \frac{1}{3}\frac{(2n+3)!}{n!(n+1)!}\frac{1}{4^n}\beta_0^{-5/2-n} . \quad (3.6)$$

The coefficients c_n must decrease at least as rapidly as this upper bound does, so we know that the series Eq. (3.4) is convergent with a radius of convergence of (at least) β_0. A convergent series expansion rules out the existence of any essential singularities in ρ. A similar proof holds for correlation functions involving a finite number of f's and N's.

To calculate the susceptibility χ_m from Eq. (2.4) we need to know the compact photon propagator $\langle f_i f_j \rangle$. This can be found from the general two-"plaquette" distribution $\Psi(F_i, F_j)$ which is a Gaussian with parameters fully determined by the noncompact photon propagator [24]. [The argument is analogous to that used to fix the parameters in (3.2).] At finite β there is no divergence: $\chi_m(\beta)$ is a correlation function involving f's (or N's) for which the proof of analyticity applies. At $\beta = 0$, χ_m has the value $\frac{2}{3}\pi^2$. After $\beta \approx 0.1$ the curve drops exponentially in β.

From the results presented in this section pure noncompact QED would appear to be a counterexample to any proposed link between the cluster susceptibility χ_c of [18] and monopole condensation [12,13]. χ_c diverges in pure noncompact QED [15], but we have seen that there is neither monopole condensation nor confinement.

IV. THE DYNAMICAL CASE

We shall now investigate what happens if dynamical fermions are included. We have measured monopole properties on gauge configurations saved from [5].

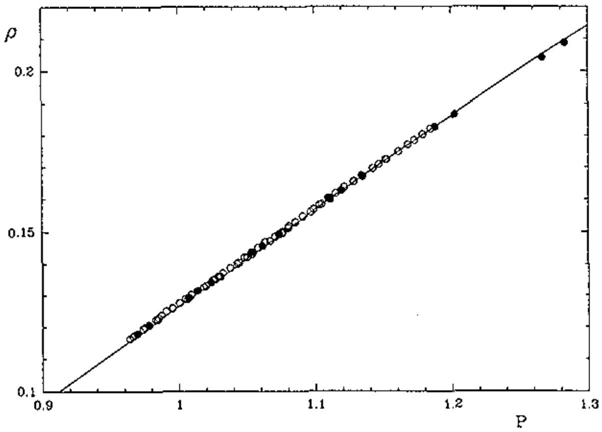

FIG. 2. The monopole density as a function of the plaquette energy. The data are for four flavors of dynamical staggered fermions. The solid symbols represent our data, while the open symbols represent data from Ref. [17]. The curve is the analytic result for the quenched case on an infinite lattice.

If, as seems likely from the last section, the monopole properties are determined by very short distance fluctuations of the electromagnetic fields, we could expect that these properties are determined by the plaquette energy values, because the plaquette energy is a good measure of the fluctuation strength. Therefore we have plotted the monopole density against $P \equiv (1/12)\sum_{\mu<\nu}\langle F_{\mu\nu}^2 \rangle$ in Fig. 2 using data from Refs. [5,17]. Our results are given in Table I. We also show the analytic curve calculated in the quenched case. We find surprisingly good agreement between the data and the analytic result, indicating that the inclusion of dynamical fermions does not change our previous conclusions. The data come from a wide range of bare masses, m=0.005–0.16, and plotting against P has brought them all onto a universal curve. In Fig. 3 we show the ratio σ/ρ. Plotting against P has again brought measurements at different masses onto the same curve. In Fig. 4 measurements of monopole susceptibility χ_m are plotted against P and compared with the analytic

TABLE I. The monopole density ρ, the string density σ, and the monopole susceptibility χ_m on a 12^4 lattice with four flavors of dynamical staggered fermions. Also given are the plaquette energy values $P = (1/12)\sum_{\mu<\nu}\langle F_{\mu\nu}^2 \rangle$.

β	m	P	ρ	σ	χ_m
0.17	0.04	1.2832(6)	0.2087(2)	0.0762 1(9)	2.729(107)
	0.02	1.2669(6)	0.2041(2)	0.0739 5(9)	2.816(89)
0.18	0.04	1.2022(6)	0.1865(3)	0.0653 3(13)	2.371(150)
	0.02	1.1881(6)	0.1826(3)	0.0635 5(12)	2.813(146)
0.19	0.04	1.1343(5)	0.1673(2)	0.0566 2(7)	2.320(82)
	0.02	1.1194(5)	0.1626(2)	0.0545 9(7)	2.198(88)
	0.01	1.1106(9)	0.1603(3)	0.0535 5(11)	2.050(122)
0.02	0.04	1.0739(3)	0.1493(2)	0.0490 1(7)	2.110(85)
	0.02	1.0617(4)	0.1457(2)	0.0474 5(6)	1.969(77)
	0.01	1.0548(6)	0.1437(2)	0.0466 6(10)	2.095(121)
0.21	0.04	1.0234(4)	0.1342(2)	0.0428 9(6)	1.890(63)
	0.02	1.0133(4)	0.1314(2)	0.0417 6(6)	1.806(64)
	0.01	1.0068(4)	0.1292(2)	0.0409 1(9)	1.899(89)
0.22	0.04	0.9779(3)	0.1205(2)	0.0376 6(6)	1.595(56)
	0.02	0.9692(3)	0.1179(2)	0.0367 0(5)	1.476(51)

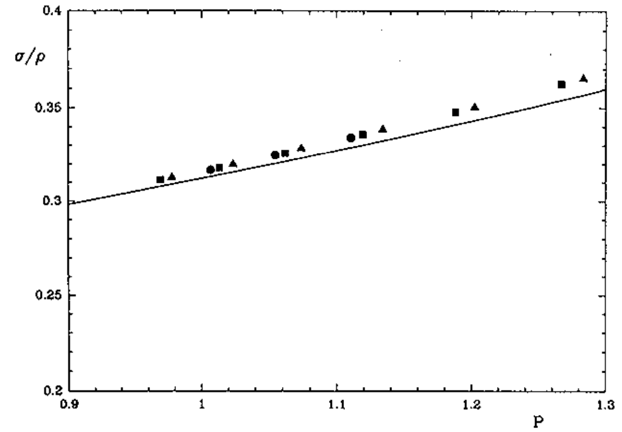

FIG. 3. The σ to ρ ratio as a function of the plaquette energy. The data are for four flavors of dynamical staggered fermions as given in Table I. The triangle is $m = 0.04$, the square is $m = 0.02$, and the circle is $m = 0.01$. The curve is the analytic result for the quenched case on an infinite lattice.

result. Within the errors we find agreement with the analytic quenched result, and do not see any divergence of χ_m. (The transition reported in [17] is at $P \approx 1.025$.)

V. DISCUSSION

In this work we have investigated monopoles in noncompact lattice QED. In the quenched case we have derived analytic formulas and can prove that there are no singularities in the quantities we have looked at. The same formulas (with no adjustable parameters) also describe the dynamical case when quantities are plotted against the plaquette energy, which measures the strength of the electromagnetic field. Thus we have arrived at a quantitative understanding of monopoles in both the quenched and dynamical cases.

The cluster susceptibility of Hands and Wensley [18] is

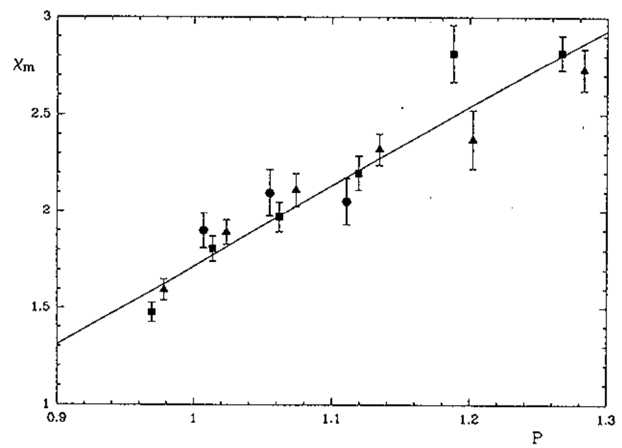

FIG. 4. The monopole susceptibility χ_m as a function of the plaquette energy. The data symbols are the same as in the previous figure. The curve is the analytic result for the quenched case on an infinite lattice.

$$\chi_c = \left\langle \frac{\left(\sum_{n=4}^{n_{\max}} g_n n^2\right) - n_{\max}^2}{\sum_{n=4}^{n_{\max}} g_n n} \right\rangle, \qquad (5.1)$$

where n is the number of dual sites in a cluster linked together by monopole world lines, g_n is the number of clusters of size n, and $n_{\max}$ is the size of the largest cluster. On an infinite lattice this susceptibility would diverge at the point where infinite clusters first appear, and has therefore long been used to find percolation thresholds [it is essentially the $S(p)$ of [26] or the χ^f of [27]]. It is, however, not clear what connection this susceptibility has to monopole condensation or charge confinement. Since χ_c is not a Green's function, a divergence of χ_c does not imply an infinite correlation length and so does not indicate a second order phase transition. Thus it can mislead if it is used to locate phase transitions.

The connection between monopole percolation and monopole condensation is not obvious, so we have tested the proposed link by applying conventional measures of monopole condensation [19,23]. These tests do not display any signal at the percolation threshold.

It is not surprising that when the monopole density becomes large ($\rho \gtrsim 0.15$) percolation takes place. However, percolation is not necessarily connected with condensation or with any other field-theoretic or thermodynamic property of the theory. Indeed, it is rather easy to find examples where the percolation threshold and the "authentic" phase transition are at different couplings. One example is the Ising model of higher dimension, where the percolation threshold lies at higher β than the phase transition [25]. The three-dimensional (3D) Ising model with a magnetic field h has percolation thresholds at finite h even at $\beta = 0$ where there is certainly no phase transition. Randomly distributed sites on a cubic lattice percolate when the concentration reaches $\approx 32\%$ [26]. This concentration is reached when $h \approx \pm 0.38$. Despite the occurrence of these percolation thresholds there are certainly no phase transitions at nonzero h in the Ising model. Therefore a cluster susceptibility can give "false positives" if used as an order parameter to detect phase transitions.

It is worth noticing that in dynamical noncompact QED χ_c diverges in places where there is no phase transition. The chiral phase transition takes place only at $m = 0$, while at finite m quantities such as $\langle \bar{\chi}\chi \rangle$ and P are smooth functions of β, as can be seen from the fact that all authors make successful fits to $\langle \bar{\chi}\chi \rangle$ with functions which are analytic for all nonzero m. However, χ_c diverges not only at $m = 0$ but for all m including $m = \infty$, (see Refs. [16,17]). (Even if the phase transition extends to small $m \sim 0.025$ as tentatively suggested in Ref. [9] the essential point is not changed, as the percolation threshold extends all the way to $m = \infty$.) That $\langle \bar{\chi}\chi \rangle$ is smooth at the percolation threshold where χ_c diverges suggests that these quantities are unrelated. Another example of a divergence in χ_c with no corresponding singularity in $\langle \bar{\chi}\chi \rangle$ is given by the quenched theory, in which the percolation threshold lies within the broken phase, and $\langle \bar{\chi}\chi \rangle$ shows no sign of a second singularity at the percolation threshold.

In conclusion, the papers [10–17] on monopoles in noncompact QED do not establish that monopoles are relevant in the continuum limit of the lattice theory or that confinement takes place at strong coupling, and so do not invalidate the picture of the chiral phase transition presented in Refs. [1–5]. In dynamical QED we have already checked [5] that the potential is Coulombic and the photon does not acquire a mass. This is also inconsistent with confinement at low β. In particular, we consider that the measurements of the strength of the Coulomb force and of the beta function in the neighborhood of the critical point [2,5] are relevant to the question of the triviality of QED.

ACKNOWLEDGMENTS

This work was supported in part by the Deutsche Forschungsgemeinschaft. Furthermore we would like to thank E. Seiler for drawing our attention to Ref. [25].

[1] M. Göckeler, R. Horsley, E. Laermann, P. Rakow, G. Schierholz, R. Sommer, and U.-J. Wiese, Nucl. Phys. **B334**, 527 (1990).
[2] M. Göckeler, R. Horsley, E. Laermann, P. Rakow, G. Schierholz, R. Sommer, and U.-J. Wiese, Phys. Lett. B **251**, 567 (1990); **256**, 562(E) (1991).
[3] P. E. L. Rakow, Nucl. Phys. **B356**, 27 (1991).
[4] G. Schierholz, in *Lattice '89*, Proceedings of the International Symposium, Capri, Italy, edited by R. Petronzio *et al.* [Nucl. Phys. B (Proc. Suppl.) **17**, 684 (1990)]; P. Rakow, in *1990 International Workshop on Strong Coupling Gauge Theories and Beyond*, edited by T. Muta and K. Yamawaki (World Scientific, Singapore, 1991), p. 249; G. Schierholz, *ibid.*, p. 279; R. Horsley *et al.*, in *Lattice '90*, Proceedings of the International Symposium, Tallahassee, Florida, edited by U. M. Heller, A. D. Kennedy, and S. Sanielevici [Nucl. Phys. B (Proc. Suppl.) **20**, 639 (1991)]; M. Göckeler, *ibid.*, p. 642; G. Schierholz, *ibid.*, p. 623.
[5] M. Göckeler, R. Horsley, P. Rakow, G. Schierholz, and R. Sommer, Nucl. Phys. **B371**, 713 (1992).
[6] S. P. Booth, R. D. Kenway, and B. J. Pendleton, Phys. Lett. B **228**, 115 (1989); S. W. de Souza and R. D. Kenway, *ibid.* **248**, 423 (1990); Nucl. Phys. **B354**, 39 (1991); A. Horowitz, Phys. Lett. B **244**, 306 (1990); Phys. Rev. D **43**, 43 (1991); in *Lattice '90* [4], p. 635.
[7] K.-I. Kondo and H. Nakatani, Nucl. Phys. **B351**, 236 (1991); D. Atkinson, in *1990 International Workshop on Strong Coupling Gauge Theories and Beyond* [4], p. 258.
[8] V. Azcoiti, G. Di Carlo, and A. F. Grillo, Mod. Phys. Lett. A **7**, 3561 (1992); in *Lattice '92*, Proceedings of the International Symposium, Amsterdam, The Netherlands, edited by J. Smit and P. van Baal [Nucl. Phys. B (Proc. Suppl.) **30**, 745 (1993)].
[9] V. Azcoiti, G. Di Carlo, and A. F. Grillo, Int. J. Mod. Phys. A **8**, 4235 (1993).
[10] S. J. Hands, J. B. Kogut, and A. Kocić, Nucl. Phys. **B357**, 467 (1991).
[11] S. Hands, in *Lattice '89* [4], p. 675.
[12] J. B. Kogut, in *1990 International Workshop on Strong Coupling Gauge Theories and Beyond* [4], p. 299.
[13] S. J. Hands, J. B. Kogut, R. Renken, A. Kocić, D. K. Sinclair, and K. C. Wang, Phys. Lett. B **261**, 294 (1991).
[14] K. C. Wang, in *Lattice '90* [4], p. 646.
[15] A. Kocić, J. B. Kogut, and S. J. Hands, Phys. Lett. B **289**, 400 (1992).
[16] S. J. Hands, R. L. Renken, A. Kocić, J. B. Kogut, D. K. Sinclair, and K. C. Wang, Nucl. Phys. **B413**, 503 (1994).
[17] A. Kocić, J. B. Kogut, and K. C. Wang, Nucl. Phys. **B398**, 405 (1993).
[18] S. Hands and R. Wensley, Phys. Rev. Lett. **63**, 2169 (1989).
[19] T. A. DeGrand and D. Toussaint, Phys. Rev. D **22**, 2478 (1980).
[20] J. B. Kogut and E. Dagotto, Phys. Rev. Lett. **59**, 617 (1987); H. C. Hege and A. Nakamura, in *Lattice '88*, Proceedings of the International Symposium, Batavia, Illinois, edited by A. S. Kronfeld and P. B. Mackenzie [Nucl. Phys. B (Proc. Suppl.) **9**, 235 (1989)].
[21] E. Dagotto, A. Kocić, and J. B. Kogut, Nucl. Phys. **B331**, 500 (1990); V. Azcoiti, G. Di Carlo, and A. F. Grillo, Phys. Lett. B **305**, 275 (1993).
[22] P. E. L. Rakow, in *Lattice '92* [8], p. 591.
[23] J. L. Cardy, Nucl. Phys. **B170**, 369 (1980).
[24] R. Horsley and U. Wolff, Phys. Lett. **105B**, 290 (1981).
[25] M. Aizenman, J. Bricmont, and J. L. Lebowitz, J. Stat. Phys. **49**, 859 (1987).
[26] J. W. Essam, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and M. S. Green (Academic, London, 1972), Vol. 2, p. 197, and references therein.
[27] G. Grimmett, *Percolation* (Springer-Verlag, Berlin, 1989).