

# Inclusive $S$ -wave charmonium productions in $B$ decays

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The inclusive  $S$ -wave charmonium production rates in  $B$  decays are considered using the Bodwin-Braaten-Lepage approach, including the relativistic corrections and the color-octet mechanism suggested as a possible solution to the  $\psi'$  puzzle at the Fermilab Tevatron. We first consider relativistic and radiative corrections to  $J/\psi \rightarrow e^+e^-$  and  $J/\psi \rightarrow$  light hadrons (LH's), in order to determine two nonperturbative parameters  $\langle J/\psi | O_1(^3S_1) | J/\psi \rangle$ ,  $\langle J/\psi | P_1(^3S_1) | J/\psi \rangle$  in the factorization formulas for these decays. Using these two matrix elements and including the color-octet  $c\bar{c}(^3S_1)$  state contribution, we get a moderate increase in the decay rates for  $B$  decays into  $J/\psi$  (or  $\psi'$ ) +  $X$ . Our results  $B(B \rightarrow J/\psi \text{ (or } \psi') + X) = 0.58 \text{ (0.23)\%}$  for  $M_b = 5.3 \text{ GeV}$  get closer to the recent CLEO data. As a by-product, we prefer a larger decay rate for  $\eta_c \rightarrow$  LH's compared to the present data.

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## I. INTRODUCTION

It has been commonly believed that inclusive production rates of a heavy quarkonium state in various high energy processes can be adequately described by perturbative QCD (PQCD) within the color-singlet model [1]. However, recent observations of inclusive  $J/\psi$  and  $\psi'$  productions at high  $p_T$  at the Fermilab Tevatron suggest that a new mechanism is called for beyond the color-singlet model [2]. For the  $J/\psi$  production, the lowest order subprocess comes from parton fusions, which are smaller than the data by more than an order of magnitude. If one includes the gluon fragmentations into  $J/\psi$  and into  $\chi_{cJ}(1P)$  states followed by  $\chi_{cJ}(1P) \rightarrow J/\psi + \gamma$  (which is the next-to-leading order in  $\alpha_s$ ), one gets higher theoretical estimates that still underestimate the experimental yield by a factor of 3–5. For the  $\psi'$  production, the situation is even worse. Even with the gluon fragmentation included, the theoretical production rate falls below the data by a factor of  $\sim 30$  or so.

In order to resolve this puzzle, basically two scenarios have been suggested up to now: (i) the existence of new charmonium states above the  $D\bar{D}$  threshold, which can decay into  $J/\psi$  and  $\psi'$  with appreciable branching ratios [3–5], and (ii) the importance of the gluon fragmentation into a pointlike color-octet  $S$ -wave  $c\bar{c}(^3S_1)$  state and its subsequent evolution into  $\psi'$  [6]. Both scenarios are quite intriguing in a sense that they call for new elements of physics within the standard model, *new spectroscopy* or *new production mechanism* for charmo-

onium states. It would be useful to explore and test these suggestions in places other than  $p\bar{p}$  colliders such as the Tevatron. In Ref. [7], one of us has explored the consequences of the first scenario, finding that these hypothetical  $\chi_{cJ}(2P)$  states should be observed at the level of (0.3–0.5) (in branching ratio) in the decay channels of  $B \rightarrow \chi_{c,J=1,2}(2P) + X$  (followed by  $\chi_{c,J=1,2} \rightarrow \psi' + \gamma$  with  $\approx 10\%$  in branching ratio, if the first scenario is to work).

In this paper, we explore the consequences of the second scenario in the inclusive decays of  $B$  mesons into a  $S$ -wave charmonium. It is well known that the lowest order results in the heavy quark velocity and the strong coupling constant for  $\Gamma(B \rightarrow J/\psi + X)_{\text{direct}}$  are smaller than the CLEO data by a factor of  $\sim 3$  [8]. The situation does not get better even if the next-to-leading order corrections in  $\alpha_s$  are included in the nonleptonic effective weak Hamiltonian for  $B$  decays [9]. In Ref. [8], only the color-singlet contribution has been included, since the color-octet contributions are higher order in  $v^2$ , hence suppressed relative to the singlet contributions. However, the Wilson coefficient for the color-singlet contribution is much suppressed compared to that for the color-octet contribution. Therefore, the color-octet contribution, being suppressed by  $v^4$ , may be numerically important, because of the larger Wilson coefficient. This is similar to the case of the gluon fragmentation into  $\chi_{cJ}(1P)$  states [10], for which the color-octet contribution is lower order in  $\alpha_s$  compared to the color-singlet contribution, whereas both of them are of the same order in  $v^2$ . Also, in the case of  $\psi'$  production through the gluon fragmentation into a color-octet  $c\bar{c}$  state, the octet contribution is higher by  $v^4$  compared to the color-singlet contribution, but this is compensated by the large short distance factor  $1/\alpha_s^2$  compared to the color-singlet contribution [6].

In Sec. II, the  $S$ -wave charmonium production rates in  $B$  decays are calculated in the framework of non-

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relativistic QCD (NRQCD) [11] including the relativistic corrections and a color-octet  $[c\bar{c}(^3S_1)]$  contribution which is of  $O(v^4)$  compared to the nonrelativistic limit. The results contain three nonperturbative parameters:  $\langle 0|O_1^H(^3S_1)|0\rangle$ ,  $\langle 0|P_1^H(^3S_1)|0\rangle$ , and  $\langle 0|O_8^H(^3S_1)|0\rangle$ . Among these three parameters appearing in the heavy quarkonium productions, the first two color-singlet matrix elements can be related to other parameters  $\langle H|O_1(^3S_1)|H\rangle$  and  $\langle H|P_1(^3S_1)|H\rangle$ , which enter in the heavy quarkonium decays in the vacuum saturation approximation [11] via

$$\langle 0|O_1^H|0\rangle \approx (2J+1) \langle H|O_1|H\rangle [1 + O(v^4)]. \quad (1)$$

In Sec. III A, the two parameters  $\langle H|O_1(^3S_1)|H\rangle$  and  $\langle H|P_1(^3S_1)|H\rangle$  (with  $H = J/\psi, \psi'$ ) are determined by analyzing the decays of  $J/\psi$  and  $\psi'$  into light hadrons

(LH's) and  $e^+e^-$ . Implications of this analysis on the decays of  $\eta_c$  into light hadrons and  $\gamma\gamma$  are discussed. Our result prefers a larger decay rate for  $\eta_c \rightarrow$  LH's. In Sec. III B, we present numerical estimates for  $S$ -wave charmonium production rates in  $B$  decays, using the factorization formulas obtained in Sec. II and  $\langle 0|O_1^H(^3S_1)|0\rangle$ ,  $\langle 0|P_1^H(^3S_1)|0\rangle$  obtained in Sec. III A. We also discuss the polarization of the  $J/\psi$  in  $B$  decays. In Sec. IV, the results are summarized, and possible improvements of the present work are speculated.

## II. HIGHER ORDER CORRECTIONS

The effective Hamiltonian for  $b \rightarrow c\bar{c}q$  (with  $q = d, s$ ) is written as [8]

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}V_{cq}^* \left[ \frac{2C_+ - C_-}{3} \bar{c}\gamma_\mu(1 - \gamma_5)c \bar{q}\gamma^\mu(1 - \gamma_5)b + (C_+ + C_-) \bar{c}\gamma_\mu(1 - \gamma_5)T^a c \bar{q}\gamma^\mu(1 - \gamma_5)T^a b \right], \quad (2)$$

where  $C_\pm$ 's are the Wilson coefficients at the scale  $\mu \approx M_b$ . We have neglected penguin operators, since their Wilson coefficients are small and thus they are irrelevant to our case. To leading order in  $\alpha_s(M_b)$  and to all orders in  $\alpha_s(M_b) \ln(M_W/M_b)$ , the above Wilson coefficients are

$$C_+(M_b) \approx 0.87, \quad C_-(M_b) \approx 1.34. \quad (3)$$

According to the factorization theorem for the  $S$ -wave charmonium productions in  $B$  decays, one has [8]

$$\Gamma(b \rightarrow J/\psi + X) = \frac{\langle 0|O_1^{J/\psi}(^3S_1)|0\rangle}{3M_c^2} \hat{\Gamma}_1(b \rightarrow (c\bar{c})_1(^3S_1) + X), \quad (4)$$

$$\Gamma(b \rightarrow \eta_c + X) = \frac{\langle 0|O_1^{\eta_c}(^1S_0)|0\rangle}{M_c^2} \hat{\Gamma}_1(b \rightarrow (c\bar{c})_1(^1S_0) + X), \quad (5)$$

in the nonrelativistic limit, where  $\hat{\Gamma}_1$  are rates for hard subprocesses of  $b$  quark decaying into a  $c\bar{c}$  pair with suitable angular momentum and vanishing relative momentum in the color singlet:

$$\hat{\Gamma}_1(b \rightarrow (c\bar{c})_1(^3S_1) + s, d) = (2C_+ - C_-)^2 \left(1 + \frac{8M_c^2}{M_b^2}\right) \hat{\Gamma}_0, \quad (6)$$

$$\hat{\Gamma}_1(b \rightarrow (c\bar{c})_1(^1S_0) + s, d) = (2C_+ - C_-)^2 \hat{\Gamma}_0, \quad (7)$$

with

$$\hat{\Gamma}_0 \equiv |V_{cb}|^2 \left(\frac{G_F^2}{144\pi}\right) M_b^3 M_c \left(1 - \frac{4M_c^2}{M_b^2}\right)^2. \quad (8)$$

The operator  $O_1^H(^{2S+1}S_J)$  is defined in terms of heavy quark field operators in NRQCD.<sup>1</sup> Its matrix element  $\langle 0|O_1^H(^{2S+1}S_J)|0\rangle$  contains the nonperturbative effects in the heavy quarkonium production processes, and is proportional to the probability that a  $c\bar{c}$  in a color-singlet  $S$ -wave state fragments into a color-singlet  $S$ -wave  $c\bar{c}$  bound state such as a physical  $J/\psi$ ,  $\eta_c$ , or  $\psi'$ . It is also related to the matrix element  $\langle H|O_1(^{2S+1}S_J)|H\rangle$  as in Eq. (1), and also to the nonrelativistic quarkonium wave function as follows:

$$\begin{aligned} \langle 0|O_1^{J/\psi}(^3S_1)|0\rangle &\approx 3 \langle J/\psi|O_1(^3S_1)|J/\psi\rangle \\ &\approx \left(\frac{9}{2\pi}\right) |R_\psi(0)|^2, \end{aligned} \quad (9)$$

<sup>1</sup>We follow the notation in Ref. [11], and will not give explicit forms for these dimension-6 operators in this paper.

in the nonrelativistic limit. Similar expressions hold for the case of nonperturbative matrix elements appearing in the  $\eta_c$  productions and its decays:

$$\langle 0|O_1^{\eta_c}(^1S_0)|0\rangle \approx \langle \eta_c|O_1(^1S_0)|\eta_c\rangle \approx \left(\frac{3}{2\pi}\right) |R_{\eta_c}(0)|^2. \quad (10)$$

Note that dependence on the radial quantum numbers  $n$  enters through the nonperturbative parameters  $\langle 0|O_1^H(^3S_1)|0\rangle$ .

Using the leptonic decay width of  $J/\psi$  and  $\psi'$ , one can determine

$$\langle J/\psi|O_1(^3S_1)|J/\psi\rangle \approx 2.4 \times 10^{-1} \text{ GeV}^3, \quad (11)$$

$$\langle \psi'|O_1(^3S_1)|\psi'\rangle \approx 9.7 \times 10^{-2} \text{ GeV}^3, \quad (12)$$

in the nonrelativistic limit with  $\alpha_s(M_c) = 0.27$ .<sup>2</sup> Also, to the lowest order in  $v^2$ , one has  $\langle \eta_c|O_1(^1S_0)|\eta_c\rangle = \langle J/\psi|O_1(^3S_1)|J/\psi\rangle$  because of heavy quark spin symmetry [11]. From these expressions with  $M_b \approx 5.3 \text{ GeV}$ , one can estimate the branching ratios for  $B$  decays into  $J/\psi + X$  and  $\psi' + X$ :

$$B(B \rightarrow J/\psi + X) = 0.23\% \quad (0.80 \pm 0.08)\%, \quad (13)$$

$$B(B \rightarrow \eta_c + X) = 0.14\% \quad (< 0.9\% \text{ (90\% C.L.)}), \quad (14)$$

$$B(B \rightarrow \psi' + X) = 0.08\% \quad (0.34 \pm 0.04 \pm 0.03)\%. \quad (15)$$

The recent data from CLEO [12] are shown in the parentheses, where the cascades from  $B \rightarrow \chi_{cJ}(1P) + X$  followed by  $\chi_{cJ} \rightarrow J/\psi + \gamma$  have been subtracted in the data shown. In view of these results, we may conclude that there are some important pieces missing in the calculations of decay rates for  $B \rightarrow (c\bar{c})_1(^3S_1) + X$  using the color-singlet model in the nonrelativistic limit.

In Ref. [8], it was noticed that the inclusion of the color-octet piece, which is often neglected in previous studies of the charmonium production in  $B$  decays, is mandatory in order to factorize the amplitude consistently without any infrared divergence in the case of  $B$  decays into the  $P$ -wave charmonium. This also leads to nonvanishing decay rates for  $B \rightarrow (h_c, \chi_{c0}, \chi_{c2}) + X$ , all of which vanish in the color-singlet model.

In view of this, we first estimate the color-octet contributions to  $B \rightarrow J/\psi + X$ , motivated by the suggestion that the color-octet mechanism might be the solution to the  $\psi'$  puzzle at the Tevatron. Although it is of higher order in  $v^2$  [ $\sim O(v^4)$ ], it can be important in the case of the inclusive  $B$  decays into  $J/\psi + X$ , since the Wilson coefficient of the color-singlet part is suppressed compared to that of the color-octet part by a factor of  $\sim \alpha_s$ . [In Eq. (2),  $(2C_+ - C_-) \approx 0.4$  and  $(C_+ + C_-) \approx 2.20$ .] Now, it is straightforward to calculate the contribution of  $(\bar{c}c)_8(^3S_1)$  to  $B \rightarrow J/\psi + X$ :

$$\Gamma(B \rightarrow (\bar{c}c)_8(^3S_1) + X \rightarrow J/\psi + X) = \frac{\langle 0|O_8^{J/\psi}(^3S_1)|0\rangle}{2M_c^2} (C_+ + C_-)^2 \left(1 + \frac{8M_c^2}{M_b^2}\right) \hat{\Gamma}_0, \quad (16)$$

$$\Gamma(B \rightarrow (\bar{c}c)_8(^1S_0) + X \rightarrow \eta_c + X) = \frac{3 \langle 0|O_8^{\eta_c}(^1S_0)|0\rangle}{2M_c^2} (C_+ + C_-)^2 \hat{\Gamma}_0. \quad (17)$$

[A similar expression holds for the  $B \rightarrow \psi'$  (or  $\eta'$ ) +  $X$  except that  $\langle 0|O_8^H(^3S_1)|0\rangle$  and  $M_c/M_b$  should change appropriately in order to account for the phase space effects.]

Here, a new nonperturbative parameter  $\langle 0|O_8(^3S_1)|0\rangle$  comes in, which creates a  $(\bar{c}c)$  pair in the color-octet state, projects into the subspace of states which contain  $J/\psi$  in the asymptotic future, and then annihilates the  $(\bar{c}c)$  pair at the creation point. The matrix element of this operator is proportional to the probability of the color-octet  $(\bar{c}c)_8(^3S_1)$  to fragment into the physical  $J/\psi$

state in the long distance scale. This type of a color-octet operator was first considered in the gluon fragmentation function into  $P$ -wave charmonia in Ref. [10], and then in the gluon fragmentation into  $\psi'$  to solve the  $\psi'$  puzzle at the Tevatron. Braaten and Fleming fixed  $\langle 0|O_8^{\psi'}(^3S_1)|0\rangle$  to be  $4.2 \times 10^{-3} \text{ GeV}^3$  (for  $M_c \approx 1.5 \text{ GeV}$ ), in order to fit the total cross section for the inclusive  $\psi'$  production cross section at the Tevatron, and found that this value of  $\langle 0|O_8^{\psi'}(^3S_1)|0\rangle$  yields the  $p_T$  spectrum for the  $\psi'$  production which nicely agrees with the measured shape. Then, Cho and Leibovich performed a complete analysis for the color-octet contribution to  $\Upsilon$  and  $\Psi$  productions at the Tevatron for both low and high  $p_T$  regions [13]. Their results are

$$\langle 0|O_8^{J/\psi}(^3S_1)|0\rangle = 1.2 \times 10^{-2} \text{ GeV}^3, \quad (18)$$

$$\langle 0|O_8^{\psi'}(^3S_1)|0\rangle = 7.3 \times 10^{-3} \text{ GeV}^3. \quad (19)$$

Taking the ratio between the color-octet and the color-singlet contributions, one gets

<sup>2</sup>The radiative corrections in  $\alpha_s$  have not been included here for consistency. To be consistent with the velocity counting rules in NRQCD in the Coulomb gauge for the heavy quarkonia [11], one has to include the relativistic corrections as well, since  $v \sim \alpha_s(Mv)$  in the heavy quarkonium system. If one includes the  $O(\alpha_s)$  radiative corrections to  $J/\psi \rightarrow l^+l^-$  without relativistic corrections, one gets a larger  $\langle 0|O_1^{J/\psi}(^3S_1)|0\rangle$  compared to the lowest order result, Eq. (11),  $\langle 0|O_1^{J/\psi}(^3S_1)|0\rangle \approx 4.14 \times 10^{-1} \text{ GeV}^3$ . Relativistic corrections give a further enhancement. See Eq. (48) below.

$$\frac{\Gamma(B \rightarrow (c\bar{c})_8(^3S_1) + X \rightarrow H + X)}{\Gamma(B \rightarrow (c\bar{c})_1(^3S_1) + X \rightarrow H + X)} = \frac{3\langle 0|O_8^H(^3S_1)|0\rangle}{2\langle 0|O_1^H(^3S_1)|0\rangle} \frac{(C_+ + C_-)^2}{(2C_+ - C_-)^2} \quad (20)$$

$$= 0.76 \quad (1.14) \text{ for } H = J/\psi \text{ } (\psi'). \quad (21)$$

Thus, we find that the color-octet  $(c\bar{c})_8(^3S_1)$  contributions to  $B \rightarrow J/\psi$  (or  $\psi'$ ) +  $X$  are about 76% (114%) of the color-singlet contributions in the nonrelativistic limit.

There is another subprocess in the lower order in  $v^2$  compared to the color-octet contribution considered above: the relativistic corrections to the color-singlet component which is an order of  $O(v^2)$  compared to (4),(5). Extending the Feynman rule in the presence of a heavy quarkonium [14], we can derive that the relativistic corrections to  $B \rightarrow J/\psi$  (or  $\eta_c$ ) +  $X$  can be written as the factorized form

$$\Gamma(b \rightarrow J/\psi + X) = -\frac{\langle 0|P_1^{J/\psi}(^3S_1)|0\rangle}{9M_c^4} \hat{\Gamma}_1(b \rightarrow (c\bar{c})_1(^3S_1) + X), \quad (22)$$

$$\Gamma(b \rightarrow \eta_c + X) = -\frac{\langle 0|P_1^{\eta_c}(^1S_0)|0\rangle}{M_c^4} \hat{\Gamma}_1(b \rightarrow (c\bar{c})_1(^1S_0) + X). \quad (23)$$

Here again, one can use the vacuum saturation approximation, Eq. (1),

$$\langle 0|P_1^H(^{2S+1}S_J)|0\rangle \approx (2J+1) \langle H|P_1(^{2S+1}S_J)|H\rangle. \quad (24)$$

The latter is related with the spin-weighted average of the  $S$ -wave heavy quarkonium wave functions in the following way:

$$\langle nS|P_1|nS\rangle = -\frac{3\text{Re}(\overline{R_{nS}^*} \nabla^2 R_{nS})}{2\pi} [1 + O(v^2)], \quad (25)$$

$\overline{R_{nS}}$  is the spin-weighted average of the  $S$ -wave wave functions [11]:

$$\overline{R_{nS}} \equiv \frac{1}{4} (3R_{\psi} + R_{\eta_c}). \quad (26)$$

In order to estimate the relativistic corrections, we need one more nonperturbative matrix element  $\langle 0|P_1^{nS}(^3S_1)|0\rangle$  or  $\text{Re}(\overline{R_{nS}^*} \nabla^2 R_{nS})$ . This is not available in the current literature now, and we will determine this parameter as well as  $\langle 0|O_1(^3S_1)|0\rangle$  in the following section. However, we note that the relativistic corrections make the decay rates for the  $S$ -wave charmonium productions in  $B$  meson decays decrease because  $\langle 0|P_1^{nS}(^3S_1)|0\rangle > 0$ .

### III. NUMERICAL ANALYSIS

#### A. Analysis of the $S$ -wave charmonium decays

In the previous section, we derived the  $S$ -wave charmonium production rates in  $B$  decays to  $O(v^2)$  for the color-singlet contributions, including one of the color-octet contributions at  $O(v^4)$ . The results depend on three nonperturbative parameters  $\langle 0|O_1(^3S_1)|0\rangle$ ,  $\langle 0|P_1(^3S_1)|0\rangle$ ,  $\langle 0|O_8(^3S_1)|0\rangle$ , or equivalently,  $|R_{\psi}(0)|^2$  and  $\text{Re}(\overline{R_S^*} \nabla^2 R_S)$  for the first two color-singlet matrix elements. Since the third parameter  $\langle 0|O_8(^3S_1)|0\rangle$  is fixed from the fit to the  $\psi'$  production at the Tevatron [6], we consider the other two parameters in this subsection. In order to determine these two parameters, one has to invoke the lattice calculations, some potential models. Or one can simply determine these parameters from the well-measured decay rates of  $J/\psi, \eta_c$  which depend on the same parameters. We choose the last method to fix two nonperturbative parameters  $\langle 0|O_1(^3S_1)|0\rangle \langle 0|P_1(^3S_1)|0\rangle$  in this work. For this purpose, we list the decay rates for  $\psi \rightarrow ggg + gg\gamma \rightarrow \text{LH}'s$  and  $\psi \rightarrow l^+l^-$  [11]:

$$\Gamma(\psi \rightarrow \text{LH}'s) = 2 \text{Im}f_1(^3S_1) \frac{\langle J/\psi|O_1(^3S_1)|J/\psi\rangle}{M_c^2} + 2 \text{Im}g_1(^3S_1) \frac{\langle J/\psi|P_1(^3S_1)|J/\psi\rangle}{M_c^4} + O(v^4\Gamma), \quad (27)$$

$$\Gamma(\psi \rightarrow e^+e^-) = 2 \text{Im}f_{ee}(^3S_1) \frac{\langle J/\psi|O_1(^3S_1)|J/\psi\rangle}{M_c^2} + 2 \text{Im}g_{ee}(^3S_1) \frac{\langle J/\psi|P_1(^3S_1)|J/\psi\rangle}{M_c^4} + O(v^4\Gamma). \quad (28)$$

We need to know the short distance coefficients  $\text{Im}f$ 's to  $O(\alpha_s)$ , and  $\text{Im}g$ 's to the leading order only in  $\alpha_s$ , because of the velocity counting rule in NRQCD [11]. From the results in the earlier literature [11,14], one can extract

$$\text{Im}f_1 \equiv \text{Im}f_{3g}(J/\psi \rightarrow ggg) + \text{Im}f_{1\gamma}(J/\psi \rightarrow gg\gamma), \quad (29)$$

$$\text{Im}g_1 \equiv \text{Im}g_{3g}(J/\psi \rightarrow ggg) + \text{Im}g_{1\gamma}(J/\psi \rightarrow gg\gamma),$$

where

$$\begin{aligned} \text{Im}f_{3g}(^3S_1) &= \frac{(\pi^2 - 9)(N_c^2 - 4)C_F}{54N_c} \alpha_s^3(M) \\ &\times \left[ 1 + [-9.46(2)C_F + 4.13(17)C_A - 1.161(2)n_f] \frac{\alpha_s}{\pi} \right], \end{aligned} \quad (30)$$

$$\text{Im}g_{3g}({}^3S_1) = -4 \times 4.33 \frac{(\pi^2 - 9)(N_c^2 - 4)C_F}{54N_c} \alpha_s^3(M), \quad (31)$$

$$\text{Im}f_{1\gamma}({}^3S_1) = \frac{2(\pi^2 - 9)C_F Q^2 \alpha}{3N_c} \alpha_s^2(M) \left[ 1 + (-9.46C_F + 2.75C_A - 0.774n_f) \frac{\alpha_s}{\pi} \right], \quad (32)$$

$$\text{Im}g_{1\gamma}({}^3S_1) = -4 \times 4.33 \frac{2(\pi^2 - 9)C_F Q^2 \alpha}{3N_c} \alpha_s^2(M), \quad (33)$$

$$\text{Im}f_{ee}({}^3S_1) = \frac{\pi Q^2 \alpha^2}{3} \left[ 1 - 4C_F \frac{\alpha_s}{\pi} \right], \quad (34)$$

$$\text{Im}g_{ee}({}^3S_1) = -\frac{4}{3} \frac{\pi Q^2 \alpha^2}{3}, \quad (35)$$

$$\text{Im}f_{qq}({}^3S_1) = \pi Q^2 (\Sigma_i Q_i^2) \alpha^2 \left[ 1 - \frac{13}{4} C_F \frac{\alpha_s}{\pi} \right], \quad (36)$$

$$\text{Im}g_{qq}({}^3S_1) = -\frac{4}{3} \pi Q^2 (\Sigma_i Q_i^2) \alpha^2. \quad (37)$$

In the above expressions,  $N_c = C_A = n_f = 3$  and  $C_F = 4/3$ , and  $Q$  is the electric charge of a heavy quark in the unit of the proton charge. The strong coupling constant  $\alpha_s(M)$  is defined in the modified minimal subtraction ( $\overline{\text{MS}}$ ) scheme for QCD with  $n_f$  light quarks, renormalized at the scale  $\mu = M$ . The last two are relevant to  $J/\psi \rightarrow \gamma^* \rightarrow$  hadrons.

Similar expressions for the  $\eta_c$  decays are

$$\Gamma(\eta_c \rightarrow \text{LH's}) = 2 \text{Im}f_1({}^1S_0) \frac{\langle \eta_c | O_1({}^1S_0) | \eta_c \rangle}{M_c^2} + 2 \text{Im}g_1({}^1S_0) \frac{\langle \eta_c | P_1({}^1S_0) | \eta_c \rangle}{M_c^4} + O(v^4\Gamma), \quad (38)$$

$$\Gamma(\eta_c \rightarrow \gamma\gamma) = 2 \text{Im}f_{\gamma\gamma}({}^1S_0) \frac{\langle \eta_c | O_1({}^1S_0) | \eta_c \rangle}{M_c^2} + 2 \text{Im}g_{\gamma\gamma}({}^1S_0) \frac{\langle \eta_c | P_1({}^1S_0) | \eta_c \rangle}{M_c^4} + O(v^4\Gamma), \quad (39)$$

where

$$\text{Im}f_1({}^1S_0) = \frac{\pi C_F}{2N_c} \alpha_s^2(M) \left[ 1 + \left\{ \left( \frac{\pi^2}{4} - 5 \right) C_F + \left( \frac{199}{18} - \frac{13\pi^2}{24} \right) C_A - \frac{8}{9} n_f \right\} \frac{\alpha_s}{\pi} \right], \quad (40)$$

$$\text{Im}g_1({}^1S_0) = -\frac{4}{3} \frac{\pi C_F}{2N_c} \alpha_s^2(M), \quad (41)$$

$$\text{Im}f_{\gamma\gamma}({}^1S_0) = \pi Q^4 \alpha^2 \left[ 1 + \left( \frac{\pi^2}{4} - 5 \right) C_F \frac{\alpha_s}{\pi} \right], \quad (42)$$

$$\text{Im}g_{\gamma\gamma}({}^1S_0) = -\frac{4}{3} \pi Q^4 \alpha^2. \quad (43)$$

The Particle Data Group (PDG) lists the measured data for these decays as [15]

$$\Gamma(\psi \rightarrow \text{LH's}) = (60.72 \pm 1.72) \text{ keV}, \quad (44)$$

$$\Gamma(\psi \rightarrow e^+e^-) = (5.26 \pm 0.37) \text{ keV}, \quad (45)$$

$$\Gamma(\eta_c \rightarrow \text{LH's}) = 10.3^{+3.8}_{-3.4} \text{ MeV}, \quad (46)$$

$$\Gamma(\eta_c \rightarrow \gamma\gamma) = 7.0^{+2.0}_{-1.7} \text{ keV}. \quad (47)$$

Since the data on  $\eta_c$  decays are not precise enough yet, we use the data on  $J/\psi$  decays only in order to determine two parameters  $\langle J/\psi | O_1({}^3S_1) | J/\psi \rangle$  and  $\langle J/\psi | P_1({}^3S_1) | J/\psi \rangle$ . Since we do not have enough inputs available at this level, we use  $\alpha_s(M)$  to be 0.25 – 0.28 instead of treating it as a free parameter. Although this choice is not fully systematic from the view point of perturbative QCD, our numerical results presented below should not strongly depend on the exact value of  $\alpha_s(M)$ . From  $\psi \rightarrow \text{LH's}$  and  $\psi \rightarrow e^+e^-$ , we determine

$$\langle J/\psi | O_1({}^3S_1) | J/\psi \rangle \approx 0.440 (0.490) \text{ GeV}^3, \quad (48)$$

$$\langle J/\psi | P_1({}^3S_1) | J/\psi \rangle \approx 0.025 (0.031) \text{ GeV}^5, \quad (49)$$

for  $\alpha_s = 0.25$  (0.28), respectively. For  $\psi'$ , we get

$$\langle \psi' | O_1({}^3S_1) | \psi' \rangle \approx 0.177 (0.198) \text{ GeV}^3, \quad (50)$$

$$\langle \psi' | P_1({}^3S_1) | \psi' \rangle \approx 0.008 (0.011) \text{ GeV}^5. \quad (51)$$

Note that the radiative corrections to the  $J/\psi$  and  $\psi'$  decays are fairly large with or without the  $\langle 0 | P_1({}^3S_1) | 0 \rangle$  term. Radiative corrections increase  $\langle J/\psi | O_1({}^3S_1) | J/\psi \rangle$  in Eq. (9) by  $\sim 80\%$ . The relativistic correction term  $\langle J/\psi | P_1({}^3S_1) | J/\psi \rangle$  is about  $\sim 9\%$  of Eq. (9), hence decreases the  $B \rightarrow J/\psi + X$  rate [Eq. (4)] by  $\sim 2\%$ .

Next, let us determine  $\langle \eta_c | O_1({}^1S_0) | \eta_c \rangle$  which is expected to be

$$\langle \eta_c | O_1({}^1S_0) | \eta_c \rangle = \langle J/\psi | O_1({}^3S_1) | J/\psi \rangle [1 + O(v^2)], \quad (52)$$

due to the heavy quark spin symmetry. Since  $\langle 0 | P_1({}^3S_1) | 0 \rangle$  is independent of the total spin  $S$ , one can determine  $\langle \eta_c | O_1({}^1S_0) | \eta_c \rangle$  from one of the decays, Eq. (38) or Eq. (39), and then predict the other and compare with the measured rate. Also, the relation (52) should be respected in order to be consistent with

NRQCD and the heavy quark spin symmetry. If we use (46) as an input, we get (with  $\alpha_s = 0.25$ )

$$\langle \eta_c | O_1(^1S_0) | \eta_c \rangle \approx (0.149_{-0.046}^{+0.053}) \text{ GeV}^3, \quad (53)$$

$$\Gamma(\eta_c \rightarrow \gamma\gamma) = (2.8_{-1.0}^{+1.1}) \text{ keV}, \quad (54)$$

the former of which severely violates the heavy quark spin symmetry, (52). For  $\alpha_s = 0.28$ , the numbers above change into  $(0.116_{-0.035}^{+0.039}) \text{ GeV}^3$  and  $(1.8_{-0.7}^{+0.8}) \text{ keV}$ . On the other hand, if we use (47) as an input with  $\alpha_s = 0.25$ , we get

$$\langle \eta_c | O_1(^1S_0) | \eta_c \rangle \approx (0.326_{-0.080}^{+0.092}) \text{ GeV}^3, \quad (55)$$

$$\Gamma(\eta_c \rightarrow \text{LH}'s) = (23_{-8}^{+7}) \text{ MeV}, \quad (56)$$

For  $\alpha_s = 0.28$ , the numbers become  $(0.341_{-0.083}^{+0.097}) \text{ GeV}^3$  and  $(32_{-8}^{+9}) \text{ MeV}$ , respectively. Now, the relation (48) is better obeyed, although the difference between  $\langle J/\psi | O_1(^3S_1) | J/\psi \rangle$  and  $\langle \eta_c | O_1(^1S_0) | \eta_c \rangle$  is not that small, and the predicted rate for  $\eta_c \rightarrow \text{LH}'s$  is quite large compared to the data, (46). One may conclude that the factorization formulation by Bodwin, Braaten, and Lepage (BBL) in terms of NRQCD predicts rather large value of  $\eta_c \rightarrow \text{LH}'s$ , compared to the current experimental values, considering the large uncertainties in the measurements. A better determination of  $\Gamma(\eta_c \rightarrow \text{LH}'s)$  would test the validity of the factorization approach to  $O(v^2)$ .

### B. Results for $B$ decays

Since all the relevant nonperturbative parameters are in hand now, we are ready to estimate the branching ratio for  $B \rightarrow J/\psi + X$  which includes  $O(v^2)$  corrections and one of the  $O(v^4)$  color-octet contributions. Adding up the change in  $\langle 0 | O_1^H(^3S_1) | 0 \rangle$  and the color-octet contributions, we get a moderate increase in the branching ratio by a factor of  $\approx (1.0 + 0.80 - 0.02 + 0.76) = 2.54$  of the lowest order prediction (13) for the  $B \rightarrow J/\psi + X$  case and  $(1.0 + 0.80 - 0.02 + 1.14) = 2.92$  for the  $\psi'$  case,

$$B(B \rightarrow J/\psi + X) = 0.58\%, \quad (57)$$

$$B(B \rightarrow \psi' + X) = 0.23\%, \quad (58)$$

compared to the data,  $(0.80 \pm 0.08)\%$  and  $(0.34 \pm 0.04 \pm 0.03)\%$ . We note that the agreements between theoretical estimates and the data are improved, after radiative corrections and the color-octet mechanism have been included. There is a residual uncertainty related to the  $b$  quark mass  $M_b$ . In this work, we have chosen  $M_b \approx 5.3 \text{ GeV}$ , and normalized the decay rate to that of the semileptonic  $B$  decay in order to reduce the uncertainty from the less known  $M_b$  [8]. If we use  $M_b = 4.5 \text{ GeV}$ , for example, all the decay rates should be multiplied by a factor of  $5.3/4.5 \approx 1.18$ . For  $B \rightarrow \eta_c + X$ , our prediction is tampered by the less known  $\langle 0 | O_1^{\eta_c}(^1S_0) | 0 \rangle$  as well as  $\langle 0 | O_8^{\eta_c}(^1S_0) | 0 \rangle$ . Still, we expect that the decay rate increases by a factor of  $\sim 2$  or more over the lowest order result, (14).

Let us finally consider the polarization of  $J/\psi$ 's produced in  $B$  decays. It is convenient to define the two parameters  $\zeta$  and  $\alpha$  as

$$\zeta = \frac{\Gamma_T(B \rightarrow J/\psi + X)}{\Gamma_{T+L}(B \rightarrow J/\psi + X)}, \quad (59)$$

$$\alpha = \frac{3\zeta - 2}{2 - \zeta}. \quad (60)$$

The quantity  $\alpha$  can be readily measured through the polar angle distribution of dileptons in  $J/\psi \rightarrow l^+l^-$  in the rest frame of  $J/\psi$ :

$$\frac{d\Gamma(J/\psi \rightarrow l^+l^-)}{d\cos\theta} \propto 1 + \alpha \cos^2\theta, \quad (61)$$

where  $\theta$  is the angle between the flight direction of a lepton in the rest frame of  $J/\psi$  and the flight direction of  $J/\psi$  in the rest frame of the initial  $B$  meson. For the unpolarized  $J/\psi$ , we would have  $\zeta = 2/3$  ( $\alpha = 0$ ) corresponding to the flat  $\cos\theta$  distribution of dileptons. Assuming the factorization for  $B \rightarrow J/\psi + X$  in the color-singlet component in the effective Hamiltonian (2), one finds that  $J/\psi$ 's produced in  $B$  decays are polarized with [16]

$$\alpha = -\frac{M_b^2 - M_\psi^2}{M_b^2 + M_\psi^2} \approx -0.49 \quad (-0.36) \quad (62)$$

or  $\zeta = 0.41$  (0.49) for  $M_b = 5.3$  (4.5) GeV, which nicely compares with the CLEO measurement [17],  $\alpha_{\text{expt}} = -0.44 \pm 0.16$ . One may wonder if the color-octet mechanism considered in this work can change the polarization of  $J/\psi$  substantially. However, the structure of the amplitude for  $b \rightarrow J/\psi + X$  due to the color-octet  $^3S_1$  state is the same as that due to the color-singlet mechanism (including both the lowest and the next-to-leading order terms in  $v^2$ ). Namely, the amplitude for  $b \rightarrow J/\psi + X$  is proportional to

$$\mathcal{M} \propto \epsilon_\mu \bar{s}\gamma^\mu (1 - \gamma_5)b. \quad (63)$$

Thus, the prediction for  $\alpha$  remains the same even in the presence of the  $(\bar{c}c)_8(^3S_1)$  color-octet contribution. This is in sharp contrast with the case of  $\psi'$  production through gluon fragmentation into the color-octet  $c\bar{c}$  state which in turn evolves into  $\psi'$ . In the case of gluon fragmentation [ $g \rightarrow (c\bar{c})(^3S_1)_8 \rightarrow J/\psi + X$ ], the initial gluon [with  $q^2 \approx (2M_c)^2$  and  $q^0 \gg 2M_c$ ] is almost on shell, being almost transverse up to  $q^2/q_0^2$ . Thus, the polarization of the color-octet  $c\bar{c}$  is also almost transverse. Because of the spin symmetry of the heavy quark system, the polarization of the daughter  $J/\psi$  is the same as the parental color-octet  $c\bar{c}$  state.

### IV. CONCLUSIONS

In this work, we considered the relativistic corrections and a color-octet contribution to  $S$ -wave charmonium production in  $B$  decays. Our results, (57) and (58), give a moderate increase to the previous analyses based on the color-singlet mechanism in the nonrelativistic limit. Compared to the previous analyses of the lowest order in  $v^2$  and  $\alpha_s$ , we get an  $\sim 80\%$  increase in  $\langle J/\psi | O_1(^3S_1) | J/\psi \rangle$  from the radiative corrections to  $J/\psi \rightarrow e^+e^-$  and  $J/\psi \rightarrow \text{LH}$ , an  $\sim 2\%$  decrease from the relativistic correction through the  $\langle J/\psi | P_1(^3S_1) | J/\psi \rangle$  term, and an  $\sim 76\%$  increase in the decay rate from the color-octet contribution  $\langle 0 | O_8^{J/\psi}(^3S_1) | 0 \rangle$ . Thus, the color-octet mechanism, which has been proposed as a

possible solution to the  $\psi'$  puzzle at the Tevatron, could give an enhancement of the  $B \rightarrow J/\psi + X$  decay rate by a moderate amount.

It should be kept in mind that what we considered in this work is only one of the color-octet operators that may contribute to  $B \rightarrow J/\psi + X$ . We have chosen only  $\langle 0|O_8^{J/\psi}(^3S_1)|0\rangle$ , since we know the numerical value of

$$\Gamma(B \rightarrow (c\bar{c})_8(^1S_0) + X \rightarrow J/\psi + X) = \frac{3\langle 0|O_8^{J/\psi}(^1S_0)|0\rangle}{2M_c^2} (C_+ + C_-)^2 \hat{\Gamma}_0, \quad (64)$$

$$\Gamma(B \rightarrow (c\bar{c})_8(^3S_1) + X \rightarrow \eta_c + X) = \frac{\langle 0|O_8^{\eta_c}(^3S_1)|0\rangle}{2M_c^2} (C_+ + C_-)^2 \left(1 + \frac{8M_c^2}{M_b^2}\right) \hat{\Gamma}_0, \quad (65)$$

and similar expressions for the contributions of  $\langle 0|O_8^H(^3P_J)|0\rangle$ :

$$\Gamma(B \rightarrow (c\bar{c})_8(^3P_1) + X \rightarrow J/\psi + X) = \frac{\langle 0|O_8^{J/\psi}(^3P_1)|0\rangle}{M_c^4} (C_+ + C_-)^2 \left(1 + \frac{8M_c^2}{M_b^2}\right) \hat{\Gamma}_0, \quad (66)$$

$$\Gamma(B \rightarrow (c\bar{c})_8(^3P_1) + X \rightarrow \eta_c + X) = \frac{\langle 0|O_8^{\eta_c}(^3P_1)|0\rangle}{M_c^4} (C_+ + C_-)^2 \left(1 + \frac{8M_c^2}{M_b^2}\right) \hat{\Gamma}_0. \quad (67)$$

In order to estimate the effects of these color-octet matrix elements  $\langle 0|O_8^{J/\psi}(^1S_0)|0\rangle$  and  $\langle 0|O_8^{J/\psi}(^3P_1)|0\rangle$ , we need to consider other processes as well, such as  $\gamma + p \rightarrow J/\psi + X$  and  $e^+e^- \rightarrow \gamma^* \rightarrow J/\psi + X$  [18]. For example, the height of the elastic peak for photoproduction of  $J/\psi$  depends on these color-octet matrix elements [6]. Complete analysis of color-octet contributions to the  $S$ -wave charmonia in  $\gamma p$  collisions is called for as well [19]. Once these new color-octet matrix elements are determined from other processes, our results in this work will provide an independent test of the hypothesis of color-octet mechanism as a possible solution to the  $\psi'$  anomaly at the Tevatron.

We have also analyzed the leptonic and the inclusive hadronic decays of  $J/\psi$  and  $\psi'$  to  $O(v^2)$  in the framework of the BBL factorization scheme, and did extract two nonperturbative parameters  $\langle H|O_1(^3S_1)|H\rangle$  and  $\langle H|P_1(^3S_1)|H\rangle$  with  $H = J/\psi$  and  $\psi'$ . These are important inputs in many other theoretical calculations of the  $S$ -wave charmonia production in various high energy

this matrix element from the work of Braaten and Fleming [6]. This matrix element is rather special in the sense that it is the only color-octet operator which is relevant to  $g \rightarrow (c\bar{c})_8(^3S_1) \rightarrow J/\psi + X$  in the leading order in  $v^2$  and  $\alpha_s$ . However, for the  $B$  decays, other color-octet operators can contribute as well: e.g.,

processes and their subsequent decays. As a by-product, we have found that the inclusive hadronic decay rate for  $\eta_c$  may be larger than the current PDG value by a factor of  $\sim 2$ , if the BBL factorization formulas to  $O(v^2)$  work with the charmonium system. The better measurements of  $\Gamma(\eta_c \rightarrow \text{LH})$  would test our predictions based on the factorization approach for the heavy quarkonium decays in the framework of NRQCD to  $O(v^2)$ .

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