Exclusive semileptonic decay of D and B mesons in the independent quark model

N. Barik
Department of Physics, Utkal University, Bhubaneswar-751004, India

P. C. Dash
Department of Physics, P. N. College, Khurda-752057, India
(Received 31 October 1994; revised manuscript received 31 May 1995)

We investigate the exclusive semileptonic decay modes $D^0 \to K, K^*$ and $\bar{B}^0 \to D, D^*$ in a field-theoretic framework based on the independent quark model with a scalar-vector-harmonic potential. Our predictions for the relevant form factors and their q^2 dependence are in reasonable agreement with the expectations of HQET and those of several other models. We predict that the decay width ratio and the polarization ratio for D^0 decays are $\Gamma(D^0 \to K^{*-})/\Gamma(D^0 \to K^-) = 0.68$ and $\Gamma_L(D^0 \to K^{*-})/\Gamma_T(D^0 \to K^{*-}) = 0.52$ and those for $\bar{B}^0$ decays are $\Gamma(\bar{B}^0 \to D^{*+})/\Gamma(\bar{B}^0 \to D^+) = 1.87$ and $\Gamma_L(\bar{B}^0 \to D^{*+})/\Gamma_T(\bar{B}^0 \to D^{*+}) = 0.77$, respectively.

PACS number(s): 13.20.Fc, 12.39.Ki, 12.39.Pn, 13.20.Jf

I. INTRODUCTION

The study of the semileptonic decay of hadrons has been of great interest to particle physics since it helps not only in probing the quark structure of hadrons but also in providing means to measure the Cabbibo-Kobayashi-Maskawa (CKM) parameters necessary to realize the CP-violating effects within the minimal standard model picture. In particular, the semileptonic decays of heavy flavored mesons such as D and B have received considerable attention in recent years due to the emergence of new theoretical ideas such as heavy-quark symmetries leading to many interesting model-independent predictions in this sector. Significant progress has also been made through the ongoing efforts to acquire relatively more precise experimental data for these semileptonic processes [1–6]. The theoretical analysis of such decays usually requires a detailed knowledge of the transition form factors with their explicit q^2 (four-momentum transfer squared) dependence. The form factors which are in fact the manifestations of QCD bound-state characters of the hadrons involved in the process are yet to be solved theoretically from the first principle. Although the heavy-quark effective theory (HQET) [7,8], which corresponds to QCD in the limit of $\Lambda_{\rm QCD}/m_Q \to 0$, can relate different form factors to a single one called the Isgur-Wise function, it is not possible to predict theoretically the q^2 dependence of this function except through an appeal to the nonperturbative technique of lattice QCD [9]. Therefore the weak decay form factors required to describe the semileptonic decays are usually obtained by various phenomenological bound-state models.

So far there have been many such models [10–18] giving wide ranging predictions on the exclusive semileptonic decays of heavy flavored mesons. In the nonrelativistic constituent quark model of Isgur, Scora, Grinstein, and Wise (ISGW) [10], all the weak decay form factors, computed with the overlap integral of the nonrelativistic meson wave functions [11], have the same exponential q^2 dependence, which is not entirely compatible with the predictions of the heavy-quark symmetry. Altomari and Wolfenstein (AW) [12], in a similar nonrelativistic approach, determine the form factors at $q^2 = q^2_{\rm max}$ and then extrapolate them down to $q^2 = 0$ postulating the q^2 dependence through monopole forms. However the calculation of one of the form factors, namely a_+, is considered by them to be less trustworthy because of the exclusion of the significant effects due to the quadratic and higher-order terms involving the daughter meson momentum. Gilman and Singleton (GS) [13] use a modified quark model based on an approach similar to [12] and suggest rescaling the form factors in order to fit the available data. In a relativistic calculation of Bauer, Stech, and Wirbel (BSW), the form factors having the q^2 dependence in the monopole ansatz with the normalization at $q^2 = 0$ are computed from the overlap integrals of light-cone wave functions [14,15]. As an extension of this work, Korner and Schuler (KS) [16] adopt a monopole or dipole ansatz for the q^2 dependence of the form factors. But such relativistic treatments are not totally free from objections. Unlike the quark potential models, the phenomenology in these cases is yet to be tuned. Second, the computation of the form factors normalized at $q^2 \to 0$, requires the knowledge of the infinite momentum frame wave functions near the end points where they are usually small or least understood. Therefore it appears that a completely consistent calculation of the weak decay form factors in the framework of constituent quark model has not been accomplished yet. This may be mainly due to the fact that in the calculation of the hadronic matrix element, the truly relativistic bound-state character of the relevant hadrons has not been adequately represented.

We therefore consider it worthwhile to investigate the semileptonic decay of heavy flavored mesons (D and B) in a relativistic independent quark model whose predictive power have been successfully demonstrated in the radiative [19,20], leptonic [21], weak leptonic [22], and weak radiative [23] decays as well as in the study of several other hadronic phenomena [24–26]. We intend here to generate the q^2 dependence of the relevant form factors directly from the model without taking resort to any kind of pole ansatz. The model parameters being fixed in its earlier applications, the result of the present investigation would be the model predictions assessing its further credibility in yet another testing ground of the semileptonic decays.

This paper is organized in the following manner. First of all, in Sec. II we provide a brief outline of the general formalism adopted here for the analysis of the exclusive semileptonic decay of heavy flavored mesons. In Sec. III, we describe the model conventions and realize the invariant transition matrix element as well as the relevant form factors with their appropriate q^2 dependence directly from the model. Section IV embodies our results and discussions and Sec. V provides the summary conclusions.

II. GENERAL FORMALISM AND KINEMATICS

We are interested in the exclusive semileptonic decay of heavy flavored pseudoscalar mesons $(D^0, \bar{B}^0)$ into pseudoscalar (K, D) and vector (K^*, D^*) mesons. Such a process as depicted in Fig. 1 is picturized through the decay of the heavy quark Q in the parent meson M into a less heavy or light quark q in the daughter meson m along with the virtual W boson which ultimately decays into a charged lepton and its neutrino, where the constituent antiquark $\bar{q}'$ remains as the spectator. The detail formalism with the kinematics describing these processes is a standard exercise and has been derived and reported elsewhere [12,13]. However, for the sake of completeness we repeat here a brief outline of the general formalism adopted here as per [13].

For the decay process $M \to me\nu$, the invariant transition matrix element is generally written as

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \mathcal{V}_{Qq} L^\mu H_\mu \ , \tag{1}$$

where G_F is the effective Fermi coupling constant and $\mathcal{V}_{Qq}$ is the CKM parameter. The leptonic and hadronic parts of the amplitude here are

$$\begin{aligned} L^\mu &= \bar{u}_\nu \gamma^\mu (1-\gamma_5) v_e \ , \\ H_\mu &= \langle m(k, S_m) | J^h_\mu(0) | M(P, S_M) \rangle \ , \end{aligned} \tag{2}$$

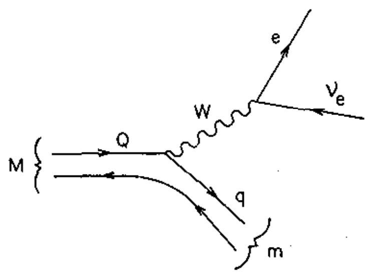

FIG. 1. The semileptonic decay of a heavy quark Q into a lighter quark q and a virtual W which becomes a lepton and neutrino.

where $J^h_\mu = V_\mu - A_\mu$. Here we take (M, m) to be the mass, (P, k) the four-momentum, and (S_M, S_m) the spin projection of the parent (M) and the daughter (m) meson, respectively. Taking (p, p') to be the four-momenta of the lepton pair (e^+, ν_e), the four-momentum transfer becomes $q = (P - k) = (p + p')$.

It is convenient to describe the kinematics by introducing the dimensionless variables $y = (q^2/M^2)$ and $x = (P \cdot p'/M^2)$. Neglecting the lepton mass, the kinematically allowed limits of y becomes

$$0 \le y \le \left(1 - \frac{m}{M}\right)^2 . \tag{3}$$

The coordinate system fixed here is such that the daughter meson momentum is along the negative z axis with the charged lepton momentum at an angle θ_e to the z axis [Fig. 2(a)] in the $e\nu$ frame. The y axis is oriented perpendicular to the plane containing the final momenta. In the $e\nu$ center-of-mass frame the kinematic quantities such as the energy momentum of the lepton pairs and the daughter meson are given, respectively, by

$$E_e = E_\nu = \frac{M}{2}\sqrt{y} \ , \tag{4}$$

$$E_m = \frac{M}{2\sqrt{y}} \left(1 + \frac{m^2}{M^2} - y\right) \ , \tag{5}$$

$$|\vec{k}| = K/\sqrt{y} \ , \tag{6}$$

with

$$K = \frac{M}{2} \left[\left(1 - \frac{m^2}{M^2} - y\right)^2 - 4\frac{m^2}{M^2} y \right]^{1/2} . \tag{7}$$

Such quantities in the parent meson rest frame are obtained as

$$\tilde{E}_e = Mx = \frac{K}{2}\cos\theta_e + \frac{M}{4}\left(1 - \frac{m^2}{M^2} + y\right) \ , \tag{8}$$

$$\tilde{E}_m = \frac{M}{2}\left(1 + \frac{m^2}{M^2} - y\right) \ , \tag{9}$$

$$|\tilde{k}| = K \ . \tag{10}$$

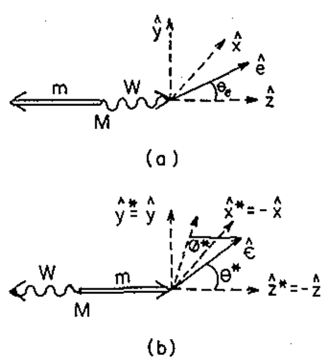

FIG. 2. Coordinate system for the semileptonic decay of a heavy meson: (a) the decaying virtual W and (b) the decaying final vector meson.

The kinematically allowed range for x depends on the value of y and can be determined from Eq. (8).

The hadronic matrix element in Eq. (2) is conventionally expressed in terms of the Lorentz-invariant form factors. For the semileptonic transition of the type $(0^- \to 0^-)$ where the pseudoscalar meson is in the final state, only the hadronic vector current contributes, which is expressed as

$$\langle m(k)|V_\mu(0)|M(P)\rangle = f_+(q^2)(P+k)_\mu + f_-(q^2)(P-k)_\mu \ . \tag{11}$$

On the other hand, for transitions of the type $(0^- \to 1^-)$ where a vector meson is in final state, the corresponding matrix elements are given by

$$\langle m(k,\epsilon^*)|V_\mu(0)|M(P)\rangle = ig(q^2)\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}(P+k)^\rho(P-k)^\sigma \ , \tag{12}$$

$$\langle m(k,\epsilon^*)|A_\mu(0)|M(P)\rangle = f(q^2)\epsilon^*_\mu + a_+(q^2)(\epsilon^*\cdot P)(P+k)_\mu + a_-(q^2)(\epsilon^*\cdot P)(P-k)_\mu \ , \tag{13}$$

where $\epsilon^* \equiv (\epsilon_0^*, \hat{\epsilon}^*)$ represents the vector meson polarization with $\epsilon^*\cdot k = 0$. Then in this case the spatial part of the hadronic current in the $(e\nu)$ frame can be written as

$$\vec{H} = 2i\sqrt{y}Mg(q^2)(\hat{\epsilon}^*\times\vec{k}) - f(q^2)\hat{\epsilon}^* - 2(\epsilon^*\cdot P)a_+(q^2)\vec{k} \ . \tag{14}$$

One can note here that the form factor $a_-(q^2)$ does not contribute to $\vec{H}$. For the pseudoscalar meson in the final state, the hadronic current matrix element in the $(e\nu)$ frame also receives no contribution from the form factor $f_-(q^2)$. It is quite convenient to express the invariant transition amplitude $\mathcal{M}$ given by Eq. (1) in the $(e\nu)$ center-of-mass frame. In that case the leptonic tensor $L^{\mu\nu} = L^\mu L^\nu$, which appears in $|\mathcal{M}|^2$, can have only the spatial component L^{ij} to be the nonvanishing ones in the limit of vanishing lepton mass and is given by

$$L^{ij} = 4M^2y[\delta^{ij} + \hat{n}^i\hat{n}^j - i\eta\epsilon^{ijl}\hat{n}^l] \ . \tag{15}$$

Here $\eta = -1(+1)$ for the final-state lepton pairs $e^+\nu_e(e^-\bar{\nu}_e)$ and $\hat{n}$ is the unit vector along the charged lepton direction in the $(e\nu)$ frame. In this frame the effective hadronic tensor in $|\mathcal{M}|^2$ would also turn out to be spacelike. It is therefore useful to expand $\vec{H}$ in terms of a helicity basis (effectively of the virtual W) in the form

$$\vec{H} = H_+\hat{e}_+ + H_-\hat{e}_- + +H_0\hat{e}_0 \ , \tag{16}$$

where

$$\hat{e}_\pm = \frac{1}{\sqrt{2}}(\mp\hat{x} - i\hat{y}), \quad \hat{e}_0 = \hat{z} \ . \tag{17}$$

The polarization vector $\hat{\epsilon}^*$ with polar and azimuthal angle (θ^*, ϕ^*) in the vector meson helicity frame [Fig. 2(b)], can be Lorentz transformed to the $(e\nu)$ frame to be expressed as

$$\hat{\epsilon}^* = \frac{1}{\sqrt{2}}\sin\theta^* e^{i\phi^*}\hat{e}_+ - \frac{1}{\sqrt{2}}\sin\theta^* e^{-i\phi^*}\hat{e}_- - \frac{E_m}{m}\cos\theta^*\hat{e}_0 \ . \tag{18}$$

The differential decay width for the exclusive decay process $(M \to me\nu)$ in the parent meson rest frame is given by

$$d\Gamma(M\to me\nu) = \frac{1}{2M}|\mathcal{M}|^2 d\Pi_3 \ , \tag{19}$$

where the three-body phase-space volume is represented by

$$d\Pi_3 = (2\pi)^4\delta^{(4)}(P-k-p-p')\Pi_f\frac{d^3k_f}{V^3 2E_f} \tag{20}$$

with the product symbol referring to the final particle momenta. The three-body phase-volume $d\Pi_3$ can be conveniently split into Lorentz-invariant pieces so as to take a particularly simple form

$$d\Pi_3 = \frac{M}{(4\pi)^5}K\,dy d\Omega_e d\tilde{\Omega}_m \ , \tag{21}$$

where $d\Omega_e$ is the solid angle of the electron in the $e\nu$ frame, $d\tilde{\Omega}_m$ is the solid angle of the daughter meson in the parent meson rest frame. This leads to the differential decay rate

$$\frac{d\Gamma}{dy d\Omega_e d\tilde{\Omega}_m} = \frac{1}{2}\frac{K}{(4\pi)^5}|\mathcal{M}|^2 \ , \tag{22}$$

where

$$|\mathcal{M}|^2 = \frac{G_F{}^2}{2}|\mathcal{V}_{Qq}|^2 L^{ij}H_iH_j^\dagger \ . \tag{23}$$

Now using the expansion of $\vec{H}$ in terms of the helicity basis as per Eqs. (14), (16), and (18), integrating over all angles and finally summing over the daughter meson polarization, one can obtain the differential decay rate in the general form as

$$\frac{d\Gamma}{dy} = \frac{G_F{}^2|\mathcal{V}_{Qq}|^2KM^2y}{96\pi^3}[|\bar{H}_+|^2 + |\bar{H}_-|^2 + |\bar{H}_0|^2] \ . \tag{24}$$

For the transition into vector meson final state, the contribution of $|\bar{H}_0|^2$ in Eq. (24) refers to longitudinal polarization mode where as that due to $[|\bar{H}_+|^2 + |\bar{H}_-|^2]$ refers to the transverse polarization mode. In this case the reduced helicity amplitudes $\bar{H}_\pm$ and $\bar{H}_0$ are obtainable in terms of the invariant form factors in the following manner:

$$\bar{H}_\pm = [f(q^2) \mp 2MKg(q^2)] \ , \tag{25}$$

$$\bar{H}_0 = \frac{M}{2m\sqrt{y}}\left[\left(1 - \frac{m^2}{M^2} - y\right)f(q^2) + 4K^2a_+(q^2)\right] \ . \tag{26}$$

However in case of the transition into a pseudoscalar meson final state; one can realize the appropriate expression for the decay rate by effectively identifying

$$\bar{H}_{\pm} = 0, \quad \bar{H}_0 = -2\frac{K}{\sqrt{y}} f_+(q^2) . \tag{27}$$

Hence in this case,

$$\frac{d\Gamma}{dy} = \frac{G_F{}^2|\mathcal{V}_{Qq}|^2 K^3 M^2}{24\pi^3} |f_+(q^2)|^2 . \tag{28}$$

Thus the study of the semileptonic decay essentially reduces to the study of the detail q^2 dependence of the invariant form factors such as $f_+(q^2)$, $f(q^2)$, $g(q^2)$, and $a_+(q^2)$, which we intend to extract from the explicit evaluation of the hadronic current matrix elements in an appropriate constituent quark model of relativistic independent quarks. We may point out here that the same exercise as has been described so far basing on the formalism of [13] can as well be performed on the basis of a different approach as per [12].

III. THE INDEPENDENT QUARK MODEL AND WEAK DECAY FORM FACTORS

The semileptonic decay of heavy flavored pseudoscalar mesons under investigation here physically occurs between the momentum eigenstates of the participating mesons. An exact field theoretic calculation should take into account the meson states represented by appropriate momentum wave packets reflecting their respective constituent quark-antiquark momentum distribution. The bound quark and antiquark inside the meson are in definite energy states having no definite momenta. Nevertheless, it is possible to find out a momentum probability amplitude for the constituent quark and antiquark inside the meson by suitable momentum-space projection of the corresponding bound quark or antiquark orbitals derivable in a suitable model, for which one may have to rely on certain simplifying assumptions. Defining suitably the meson states in the model, it can be possible to calculate the transition amplitude and hence the relevant hadronic matrix element corresponding to the diagram as shown in Fig. 1 describing the semileptonic decay processes. From the explicit calculation of the hadronic current matrix elements one can identify the invariant weak decay form factors with their appropriate q^2 dependence so as to be compared with the predictions based on heavy quark symmetry. The form factors can ultimately be utilized to compute the decay rates. In view of this we deem it essential to present briefly the outline and conventions of the constituent quark model adopted here.

A. The independent quark model

In the present model a meson, in general, is pictured as a color-singlet assembly of a quark and an antiquark independently confined by an effective flavor-independent potential [19–26]:

$$U(r) = \tfrac{1}{2}(1+\gamma^0)(ar^2 + V_0) . \tag{29}$$

This potential form is taken in the model as a phenomenological representation of the confining interaction which is expected to be generated by a nonperturbative multigluon mechanism. The quark-gluon interaction at the short distance originating from one-gluon-exchange and quark-pion interaction required in the nonstrange sector to preserve chiral symmetry are presumed to be residual interactions compared to the dominant confining interaction. Although these residual interactions treated perturbatively in the model are crucial in generating mass splittings [22,24,25,27], in the hadron spectroscopy, their role in the hadronic decay processes are considered less significant. Therefore, to a first approximation, it is believed that the zeroth-order quark dynamics inside the meson core, generated by the confining part of the interaction which is phenomenologically represented by $U(r)$ in the Eq. (29) can provide an adequate description for the semileptonic decay of D and B mesons. In this picture the independent quark Lagrangian density in zeroth order is given by

$$\mathcal{L}_q^0(x) = \bar{\psi}_q(x)\left[\frac{i}{2}\gamma^\mu \overleftrightarrow{\partial}_\mu - m_q - U(r)\right]\psi_q(x) . \tag{30}$$

The ensuring Dirac equation with $E_q' = E_q - V_0/2$, $m_q' = m_q + V_0/2$, $\lambda_q = (E_q' + m_q')$, and $r_{0q} = (a\lambda_q)^{-1/4}$, admits static solutions of positive and negative energy in zeroth order, which for the ground-state meson can be obtained in the form

$$\begin{aligned} \Phi_{q\lambda}^{(+)}(\vec{r}) &= \frac{1}{\sqrt{4\pi}}\begin{pmatrix} ig_q(r)/r \\ \vec{\sigma}\cdot\hat{r} f_q(r)/r \end{pmatrix}\chi_\lambda , \\ \Phi_{q\lambda}^{(-)}(\vec{r}) &= \frac{1}{\sqrt{4\pi}}\begin{pmatrix} \vec{\sigma}\cdot\hat{r} f_q(r)/r \\ -ig_q(r)/r \end{pmatrix}\tilde{\chi}_\lambda . \end{aligned} \tag{31}$$

The two-component spinors χ_λ and $\tilde{\chi}_\lambda$ stand for

$$\chi_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

and

$$\tilde{\chi}_\uparrow = \begin{pmatrix} 0 \\ -i \end{pmatrix}, \quad \tilde{\chi}_\downarrow = \begin{pmatrix} i \\ 0 \end{pmatrix} ,$$

respectively. The reduced radial parts in the upper and lower component solutions corresponding to the quark flavor q are

$$\begin{aligned} g_q(r) &= \mathcal{N}_q (r/r_{0q}) \exp(-r^2/2r_{0q}^2) , \\ f_q(r) &= -\left(\frac{\mathcal{N}_q}{\lambda_q r_{0q}}\right)(r/r_{0q})^2 \exp(-r^2/2r_{0q}^2) , \end{aligned} \tag{32}$$

where the normalization factor $\mathcal{N}_q$ is given by

$$\mathcal{N}_q^2 = 8\lambda_q/[\sqrt{\pi}\, r_{0q}(3E_q' + m_q')] . \tag{33}$$

The quark binding energy of zeroth order in the meson ground state is derivable from the bound-state condition

$$\sqrt{\lambda_q/a}(E'_q - m'_q) = 3 . \quad (34)$$

Thus knowing the quark-antiquark eigenmodes in the ground state of the mesons, it is possible to obtain their corresponding momentum distribution amplitude. Here we represent a meson state with momentum $\vec{P}$ and spin projection S_M as

$$|M(\vec{P}, S_M)\rangle = \frac{1}{\sqrt{N(\vec{P})}} \sum_{\lambda_1 \lambda_2 \in S_M} \zeta^M_{q_1 q_2}(\lambda_1, \lambda_2) \times \int d\vec{p}_1 d\vec{p}_2 \delta^{(3)}(\vec{p}_1 + \vec{p}_2 - \vec{P}) \mathcal{G}_M(\vec{p}_1, \vec{p}_2) \times b^\dagger_{q_1}(\vec{p}_1, \lambda_1) \tilde{b}^\dagger_{q_2}(\vec{p}_2, \lambda_2)|0\rangle , \quad (35)$$

where, $b^\dagger_{q_1}(\vec{p}_1, \lambda_1)$ and $\tilde{b}^\dagger_{q_2}(\vec{p}_2, \lambda_2)$ are, respectively, the quark and antiquark creation operators. $\zeta^M_{q_1 q_2}(\lambda_1, \lambda_2)$ stands for the appropriate SU(6)-spin-flavor coefficients for the meson $M(q_1, \bar{q}_2)$. $N(\vec{P})$ represents the overall normalization factor, which can be expressed in an integral form as

$$N(\vec{P}) = \int d\vec{p}_1 |\mathcal{G}_M(\vec{p}_1, \vec{P} - \vec{p}_1)|^2 . \quad (36)$$

This is obtainable from the meson-state normalization considered here in the form as

$$\langle M(\vec{P})|M(\vec{P}')\rangle = \delta^{(3)}(\vec{P} - \vec{P}') . \quad (37)$$

Finally, $\mathcal{G}_M(\vec{p}_1, \vec{p}_2)$ provides the effective momentum distribution amplitude for the quark and antiquark inside the meson. In an independent particle picture of the present model, $\mathcal{G}_M(\vec{p}_1, \vec{p}_2)$ can be expressed in terms of individual momentum distribution amplitudes $G_{q_1}(\vec{p}_1)$ and $\tilde{G}_{q_2}(\vec{p}_2)$ of the quark q_1 and antiquark $\bar{q}_2$, respectively. We follow here the ansatz as in [20–22] in a straightforward extension of the idea of Margolis and Mendel [28] so as to write

$$\mathcal{G}_M(\vec{p}_1, \vec{p}_2) = \sqrt{G_{q_1}(\vec{p}_1)\tilde{G}_{q_2}(\vec{p}_2)} . \quad (38)$$

Here $G_{q_1}(\vec{p}_1)$ can be obtained by a suitable momentum-space projection of the bound-quark orbital $\Phi^{(+)}_{q\lambda}(\vec{r})$ in Eq. (31) corresponding to the lowest eigenmode. If $G_{q_1}(\vec{p}_1; \lambda_1, \lambda'_1)$ is the amplitude of a bound quark in its eigenmode $\Phi^{(+)}_{q_1\lambda_1}(\vec{r})$ for being found in a state of definite momentum $\vec{p}_1$ and spin projection λ'_1, then

$$G_{q_1}(\vec{p}_1; \lambda_1, \lambda'_1) = \frac{u^\dagger_{q_1}(\vec{p}_1, \lambda'_1)}{\sqrt{2E_{p_1}}} \int d\vec{r} \Phi^{(+)}_{q_1\lambda_1}(\vec{r}) \exp(-i\vec{p}_1 \cdot \vec{r}) , \quad (39)$$

where $E_{p_1} = \sqrt{(\vec{p}\,^2_1 + m^2_{q_1})}$ and $u_{q_1}(\vec{p}_1, \lambda'_1)$ is the usual free Dirac spinor with the normalization

$$u^\dagger_q(\vec{p}, \lambda_1) u_q(\vec{p}, \lambda_2) = 2E_p \delta_{\lambda_1\lambda_2} = v^\dagger_q(\vec{p}, \lambda_1) v_q(\vec{p}, \lambda_2) ,$$
$$\bar{u}_q(\vec{p}, \lambda_1) u_q(\vec{p}, \lambda_2) = 2m_q \delta_{\lambda_1\lambda_2} = \bar{v}_q(\vec{p}, \lambda_1) v_q(\vec{p}, \lambda_2) , \quad (40)$$

and

$$\sum_\lambda u_q(\vec{p}, \lambda)\bar{u}_q(\vec{p}, \lambda) = (\not{p} + m_q) ,$$
$$\sum_\lambda v_q(\vec{p}, \lambda)\bar{v}_q(\vec{p}, \lambda) = (\not{p} - m_q) . \quad (41)$$

On further simplification with $\alpha_q = 1/2r^2_{0q}$, $G_{q_1}(\vec{p}_1, \lambda_1, \lambda'_1)$ reduces to the form

$$G_{q_1}(\vec{p}_1; \lambda_1, \lambda'_1) = G_{q_1}(\vec{p}_1)\delta_{\lambda_1\lambda'_1} , \quad (42)$$

where

$$G_{q_1}(\vec{p}_1) = \frac{i\pi \mathcal{N}_{q_1}}{2\alpha_{q_1}\lambda_{q_1}} \sqrt{(E_{p_1} + m_{q_1})/E_{p_1}}(E_{p_1} + E_{q_1}) \times \exp(-p^2_1/4\alpha_{q_1}) . \quad (43)$$

Thus $G_{q_1}(\vec{p}_1)$ essentially provides the momentum probability amplitude for a quark q_1 in its eigenmode $\Phi^{(+)}_{q_1\lambda_1}(\vec{r})$ to have a definite momentum $\vec{p}_1$ inside the meson. In a similar manner one can obtain the momentum probability amplitude $\tilde{G}_{q_2}(\vec{p}_2)$ for an antiquark in its eigenmode $\Phi^{(-)}_{q_2\lambda_2}(\vec{r})$ to realize that, for like flavors,

$$\tilde{G}_{\bar{q}_2}(\vec{p}_2) = G^*_{q_2}(\vec{p}_2) . \quad (44)$$

Such an ansatz for the effective momentum distribution amplitude $\mathcal{G}_M(\vec{p}_1, \vec{p}_2)$ has provided excellent and consistent descriptions for various hadronic phenomena [20–22].

B. Transition matrix for $M \to me\nu$

As discussed in Sec. II, the exclusive semileptonic processes are usually described by the invariant transition matrix expressed at the mesonic level in its familiar form as given in Eq. (1). Such a decay for a parent meson $M(Q\bar{q}')$ can basically be pictured as the weak transition of its constituent quark Q to a less heavy or light quark q belonging to the daughter meson $m(q\bar{q}')$; while the antiquark $\bar{q}'$, being common to both the participating mesons, remains as a mere spectator (Fig. 1). Then starting with such a basic weak transition at the constituent level, one can realize, on the basis of the model dynamics, the invariant transition matrix $\mathcal{M}$ at the mesonic level.

The S-matrix element corresponding to the diagram depicted in Fig. 1 which describes the semileptonic decay process $M \to me\nu$ in the parent meson rest frame can be written effectively as

$$S_{fi} = i\frac{G_F \mathcal{V}_{Qq}}{\sqrt{2}(2\pi)^4}\int d^4x_1 d^4x_2 d^4q \exp[(-iq(x_2-x_1)]$$
$$\times\langle e(\vec{p},\delta_1)\nu(\vec{p}\,',\delta_2)m(\vec{k},S_m)|:J_l^\mu(x_2)J_\mu^h(x_1):|M(\vec{P}=0,S_M)\rangle\ , \quad (45)$$

where, $q = (P-k) = (p+p')$, stands for the four-momentum transfer and with $\Gamma^\mu = \gamma^\mu(1-\gamma^5)$:

$$:J_l^\mu(x_2): \ = \bar{\psi}_e^{(-)}(x_2)\Gamma^\mu\psi_\nu^{(-)}(x_2)\ ,$$
$$:J_\mu^h(x_1): \ = \bar{\psi}_q^{(-)}(x_1)\Gamma_\mu\psi_Q^{(+)}(x_2)\ . \quad (46)$$

Using the lepton field expansion, the matrix element of the leptonic weak current $J_l^\mu(x_2)$ can be obtained in a straightforward manner, so that

$$\langle e(\vec{p},\delta_1)\nu(\vec{p}\,',\delta_2)|J_l^\mu(x_2)|0\rangle = \frac{1}{\sqrt{V^2 2E_e 2E_\nu}}e^{i(p+p')x_2}L^\mu \quad (47)$$

when

$$L^\mu = \bar{u}_e(\vec{p},\delta_1)\Gamma^\mu v_\nu(\vec{p}\,',\delta_2)\ . \quad (48)$$

Similarly, taking into account the appropriate momentum wave packets as given in Eq. (35) for the initial and final meson states and the usual quark field expansions in the hadronic weak current, we can obtain

$$\langle m(\vec{k},S_m)|J_\mu^h(x_1)|M(\vec{P}=0,S_M)\rangle = \int\frac{d\vec{p}_1\mathcal{G}_M(\vec{p}_1,-\vec{p}_1)\mathcal{G}_M(\vec{k}+\vec{p}_1,-\vec{p}_1)e^{ikx_1}}{\sqrt{V^2N_M(0)N_m(\vec{k})2E_{k+p_1}2E_{p_1}}}\langle S_m|J_\mu(0)|S_M\rangle\ , \quad (49)$$

where the symbolically represented spin-matrix element piece is

$$\langle S_m|J_\mu(0)|S_M\rangle = \sum_{\lambda_1,\lambda_2\in S_M;\lambda_1',\lambda_2'\in S_m}\zeta_{Q\bar{q}'}^M(\lambda_1,\lambda_2)\zeta_{q\bar{q}'}^m(\lambda_1',\lambda_2')\bar{u}_q(\vec{k}+\vec{p}_1,\lambda_1')\Gamma_\mu u_Q(\vec{p}_1,\lambda_1)\ . \quad (50)$$

E_{p_1} and $E_{p_1'}$ here stand for the energy of the nonspectator quark of the initial and the final meson with the three momenta $\vec{p}_1$ and $\vec{p}\,'_1$, respectively. Now using Eqs. (47)–(50) in Eq. (45), one expects to obtain the S-matrix element in the standard form with the energy-momentum conservation explicitly depicted by an appropriate four-momentum δ function at the mesonic level. But such a realization at the composite level starting from a picture at the constituent quark level has never been so straightforward. This is because of the fact that although three-momentum conservation is automatically guaranteed at the mesonic level through appropriate δ function, the same is not so transparent in case of energy conservation. The energy conservation at the mesonic level can however be realized by extracting out the energy δ function $\delta(E_{p_1}-E_{p_1+k}-E_e-E_\nu)$ from within the quark level integral in the form $\delta(M-\tilde{E}_m-E_e-E_\nu)$ with the ansatz that $(E_{p_1}+E_{p_2})$ and $(E_{p_1+k}+E_{p_2})$ in the δ-function argument can be equated in an integrated sense to the parent meson mass M and daughter meson energy $\tilde{E}_m$, respectively. However, there may be some mismatch in this respect since the constituent level dynamics considered here is in zeroth order only which alone cannot ensure the complete bound-state character with the total mass energy of the mesons. Hence it requires appropriate corrective measures which is *ad hoc* introduced here by multiplying the integrand in the quark level integration by the mismatch factor taken in the form

$$\left(\frac{M}{(E_{p_1}+E_{p_2})}\right)^{1/2}\left(\frac{\tilde{E}_m}{(E_{p_1+k}+E_{p_2})}\right)^{1/2}.$$

Finally we also ensure the appropriate phase space at the mesonic level with the covariant normalization of the meson states so as to realize the S-matrix element in its standard form as

$$S_{fi} = (2\pi)^4\delta^{(4)}(P-k-p-p')(i\mathcal{M})$$
$$\times\frac{1}{\sqrt{V^4 2E_e 2E_\nu 2\tilde{E}_m 2M}}\ , \quad (51)$$

where $P\equiv(M,0,0,0)$. The invariant transition matrix element is then obtained in its familiar form as

$$\mathcal{M} = \frac{G_F}{\sqrt{2}}\mathcal{V}_{Qq}L^\mu H_\mu \quad (52)$$

with the hadronic amplitude in its appropriate form as

$$H_\mu = \frac{\tilde{E}_m M}{\sqrt{N_M(0)N_m(\vec{k})}}\int\frac{d\vec{p}_1\mathcal{G}_M(\vec{p}_1,-\vec{p}_1)\mathcal{G}_m(\vec{k}+\vec{p}_1,-\vec{p}_1)}{\sqrt{E_{p_1+k}E_{p_1}(E_{p_1+k}+E_{p_2})(E_{p_1}+E_{p_2})}}\langle S_m|\Gamma_\mu|S_M\rangle\ . \quad (53)$$

Thus with the transition matrix element realized in the familiar form with explicit expression for the hadronic amplitude derived from the present model, the general formalism as described in Sec. II can now be followed to describe the semileptonic processes.

C. Weak decay form factors

It has been shown in Sec. II that in the $(e\nu)$ center-of-mass frame; the leptonic amplitude being spacelike, the correspondingly relevant spacelike components of the hadronic amplitudes are expressed in terms of the weak decay form factors. The transition form factors being Lorentz invariant can be calculated in any suitable frame. We choose here the parent meson rest frame for the sake of convenience to calculate the relevant form factors (f_+, f, g, and a_+). This essentially involves the calculation of the hadronic amplitude from Eq. (53) and then comparing the result with the corresponding quantity evaluated through the form factor expansion method using appropriate Eqs. (11)–(13).

For the transition $(0^- \to 0^-)$, the only contributing hadronic vector current yields the relevant spin-matrix elements

$$\langle S_m|V_0|S_M\rangle = \frac{[(E_{p_1+k}+m_q)(E_{p_1}+m_Q)+\vec{p}_1^{\,2}]}{\sqrt{(E_{p_1+k}+m_q)(E_{p_1}+m_Q)}} \tag{54}$$

and

$$\langle S_m|\vec{V}|S_M\rangle = \frac{(E_{p_1}+m_Q)\vec{k}}{\sqrt{(E_{p_1+k}+m_q)(E_{p_1}+m_Q)}} . \tag{55}$$

On the other hand, for the transition $(0^- \to 1^-)$ both the vector current V_μ and the axial-vector current A_μ contribute giving the spin-matrix elements as

$$\langle S_{m^*}|V_0|S_M\rangle = 0 ,$$
$$\langle S_{m^*}|\vec{V}|S_M\rangle = \frac{i(E_{p_1}+m_Q)(\hat{\epsilon}^*\times\vec{k})}{\sqrt{(E_{k+p_1}+m_q)(E_{p_1}+m_Q)}} \tag{56}$$

and

$$\langle S_{m^*}|A_0|S_M\rangle = \frac{(E_{p_1}+m_Q)(\hat{\epsilon}^*\cdot\vec{k})}{\sqrt{(E_{k+p_1}+m_q)(E_{p_1}+m_Q)}} ,$$
$$\langle S_{m^*}|\vec{A}|S_M\rangle = \frac{[(E_{k+p_1}+m_q)(E_{p_1}+m_Q)-\vec{p}_1^{\,2}/3]}{\sqrt{(E_{k+p_1}+m_q)(E_{p_1}+m_Q)}}\hat{\epsilon}^* . \tag{57}$$

Here we have identified the spin-matrix elements with terms which give nonvanishing contribution to the integral defining the hadronic amplitude.

Now substituting the expressions in Eqs. (54) and (55) into Eq. (53) and comparing the results with the corresponding expressions obtainable from the form factor expansion in Eq. (11) of Sec. II, we find

$$(M+\tilde{E}_m)f_+ + (M-\tilde{E}_m)f_- = \int d\vec{p}_1 C(p_1)[(E_{p_1+k}+m_q)(E_{p_1}+m_Q)+\vec{p}_1^{\,2}] , \tag{58}$$

$$f_+ - f_- = \int d\vec{p}_1 (E_{p_1}+m_Q)C(p_1) , \tag{59}$$

where

$$C(p_1) = \frac{M\tilde{E}_m}{\sqrt{N_M(0)N_m(\vec{k})}} \frac{\mathcal{G}_M(\vec{p}_1,-\vec{p}_1)\mathcal{G}_m(\vec{k}+\vec{p}_1,-\vec{p}_1)}{\sqrt{E_{p_1+k}E_{p_1}(E_{p_1+k}+m_q)(E_{p_1}+m_Q)(E_{p_1+k}+E_{p_2})(E_{p_1}+E_{p_2})}} . \tag{60}$$

From Eqs. (58) and (59), the form factor f_+ is found to be

$$f_+ = \frac{1}{2M}\int d\vec{p}_1 C(p_1)[(E_{p_1}+m_Q)(E_{p_1+k}+m_q+M-\tilde{E}_m)+\vec{p}_1^{\,2}] . \tag{61}$$

Then considering the $(0^- \to 1^-)$ transitions, we evaluate, with the help of Eqs. (53), (56), and (57), the spatial components of the hadronic amplitude in parent rest frame as

$$\langle m(\vec{k},\hat{\epsilon}^*)|\vec{V}-\vec{A}|M(\vec{P}=0)\rangle = -[i\mathcal{A}(\hat{\epsilon}^*\times\vec{k})-\mathcal{B}\hat{\epsilon}^*], \tag{62}$$

where

$$\mathcal{A} = \int d\vec{p}_1 C(p_1)(E_{p_1}+m_Q) , \tag{63}$$

$$\mathcal{B} = \int d\vec{p}_1 C(p_1)[(E_{p_1+k}+m_q)(E_{p_1}+m_Q)-\vec{p}_1^{\,2}/3] . \tag{64}$$

The same can however be expressed in terms of the invariant form factors using the expansions given in Eqs. (12) and (13) of Sec. II as

$$\langle m(\vec{k},\hat{\epsilon}^*)|\vec{V}-\vec{A}|M(\vec{P}=0)\rangle = i2Mg(\hat{\epsilon}^*\times\vec{k}) - f\hat{\epsilon}^* - (a_+-a_-)(\hat{\epsilon}^*\cdot P)\vec{k} . \tag{65}$$

Then from the term by term comparison of Eq. (62) with Eq. (65); we can identify the appropriate expressions standing for the invariant form factors g and f as

$$g = -\frac{\mathcal{A}}{2M} = -\frac{1}{2M}\int d\vec{p}_1 C(p_1)(E_{p_1}+m_Q) , \tag{66}$$

$$f = -\mathcal{B} = -\int d\vec{p}_1 C(p_1)[(E_{p_1+k} + m_q)(E_{p_1} + m_Q) - \vec{p}_1^{\,2}/3] , \tag{67}$$

when

$$a_+ = a_- . \tag{68}$$

Finally from the calculation of the matrix element of the timelike component of the axial-vector current in the parent meson rest frame corresponding to the longitudinal spin polarization of the final vector meson, i.e., $\langle m(\vec{k}, \hat{\epsilon}^{*(L)})|A_0|M(\vec{P}=0)\rangle$ and using Eq. (68), we find that

$$f\epsilon_0^{*(L)} + a_+ 2M^2\epsilon_0^{*(L)} = -\mathcal{A}|\vec{k}| . \tag{69}$$

Since we have taken the spin-quantization axis opposite to the boost direction, the longitudinal polarization vector $\epsilon_\mu^{*(L)}$ is boosted to get a timelike component $\epsilon_0^{*(L)} = -|\vec{k}|/m$ with $\epsilon_0^{*(T)} = 0$. Taking this into account and using Eq. (66), we find, from Eq. (69),

$$a_+ = -\frac{1}{2M^2}(f + 2Mmg) . \tag{70}$$

The invariant form factors (f_+, g, f, and a_+) so derived in the model through the respective expressions in Eqs. (61), (66), (67), and (70) are believed to embody the appropriate q^2 dependence. These form factors can also be written in the dimensionless forms as often cited in the literature to treat all of them including $f_+(q^2)$ on the same footing. They are defined as

$$\begin{aligned} F_1(q^2) &= f_+(q^2) , \\ V(q^2) &= (M+m)g(q^2) , \\ A_1(q^2) &= (M+m)^{-1}f(q^2) , \\ A_2(q^2) &= -(M+m)a_+(q^2) . \end{aligned} \tag{71}$$

With these form factors we can determine the helicity amplitudes through Eqs. (27), (25), and (26) for the pseudoscalar and vector meson final states, respectively. Then it is straightforward to calculate the decay widths as well as the polarization ratios for the specific cases of semileptonic decays under investigation using Eqs. (24)–(28) in Sec. II. The q^2 dependence of the form factors can also be compared with the predictions according to HQET and other models.

IV. RESULTS AND DISCUSSIONS

Having derived in the present model the expressions for the weak decay form factors which parametrize the hadronic matrix elements of weak currents between the two participating meson states; a detailed study of the semileptonic transition $M \to me\nu$ becomes quite straightforward. We consider here in particular the semileptonic decay of heavy-flavored mesons D^0 and $\bar{B}^0$ in their specific exclusive channels such as (i) $D^0 \to K^- e^+ \nu_e$, $D^0 \to K^{*-} e^+ \nu_e$ and (ii) $\bar{B}^0 \to D^+ e^- \bar{\nu}_e$, $\bar{B}^0 \to D^{*+} e^- \bar{\nu}_e$. Our approach here is not so much as to search afresh the appropriate values of the model parameters (such as a, V_0, m_Q, and m_q) to realize a reasonable fit for the experimentally available data; on the basis of which one can extract the relevant weak decay form factors with their q^2 dependence in the entire kinematic range. Instead, we prefer to take the values of the flavor-independent potential parameters (a, V_0) and the appropriate quark mass parameters (m_Q, m_q) as obtained for the present model in its earlier applications to several other hadronic phenomena in the mesonic and baryonic sectors [20–26]. This approach would then provide the detailed predictions of the model which can be compared with the outcomes of other similar models as well as with the expectations based on the heavy-quark symmetry. Accordingly, the potential parameters of the model are

$$(a, V_0) \equiv (0.017\,166 \text{ GeV}^3, -0.1375 \text{ GeV}) . \tag{72}$$

The quark mass m_q and the corresponding quark binding energy E_q along with relevant model quantities such as λ_q and r_{0q} used in the present calculation are summarized in Table I. Such a choice of these model parameters has successfully explained in the perturbative calculation the ground-state masses of the light mesons $(\rho, \pi; K^*, K)$ [24,25,27] and heavy mesons $(D^*, D; B^*, B)$ [22] in good agreement with their experimental values. Since the theoretical uncertainty due to the perturbative approach cannot be overlooked here, we would prefer to use in our calculation the observed meson masses for the participating mesons. Finally the CKM parameters relevant for the D and B decays under consideration are taken here as

$$(\mathcal{V}_{cs}, \mathcal{V}_{bc}) \equiv (0.975, 0.043) . \tag{73}$$

With these parameters we first of all calculate the relevant weak decay form factors $f_+(q^2)$, $g(q^2)$, $f(q^2)$,

TABLE I. The quark mass m_q and the corresponding quark binding energy E_q together with λ_q and r_{0q}.

Quark q	m_q (GeV)	E_q (GeV)	λ_q (GeV)	r_{0q} (GeV)$^{-1}$
u	0.078 75	0.471 25	0.550 00	3.208 06
d	0.078 75	0.471 25	0.550 00	3.208 06
s	0.315 75	0.591 00	0.906 75	2.831 14
c	1.492 76	1.579 51	2.072 27	2.086 74
b	4.776 59	4.766 33	9.542 92	1.571 85

and $a_+(q^2)$ from their respective model expressions in Eqs. (61), (66), (67), and (70) of Sec. III. These form factors are numerically evaluated by the familiar Gaussian quadrature technique for any given value of q^2 or $y = q^2/M^2$ in the entire kinematic range of $0 \leq y \leq (1 - m/M)^2$. Casting them in dimensionless forms as given in Eq. (71) we display their q^2 dependence as obtained in the present model for D and B decays separately in Figs. 3 and 4, respectively. According to HQET these form factors over the entire kinematic range of their variables are expected to satisfy the heavy-quark symmetry relations [8] such as

$$F_1(q^2) \simeq V(q^2) \simeq A_2(q^2) \simeq \tilde{A}_1(q^2) . \tag{74}$$

Here

$$\tilde{A}_1(q^2) = \left[1 - \frac{q^2}{(M+m)^2}\right]^{-1} A_1(q^2) .$$

Heavy-quark symmetry also leads to model-independent normalization at zero recoil giving the values of these form factors at $q^2 = q^2_{\rm max}$ as [8]

$$\begin{aligned} F_1(q^2_{\rm max}) &\simeq \frac{M+m}{2\sqrt{Mm}} , \\ V(q^2_{\rm max}) \simeq A_2(q^2_{\rm max}) &\simeq \frac{M+m^*}{2\sqrt{Mm^*}} , \\ A_1(q^2_{\rm max}) &\simeq \frac{2\sqrt{Mm^*}}{M+m^*} . \end{aligned} \tag{75}$$

These symmetry relations are in fact model-independent consequences of QCD in the limit of heavy-quark mass $m_Q \gg \Lambda_{\rm QCD}$; which can be used as the benchmarks to test the consistency of our model calculation. The form factors for $B \to D, D^*$ transitions, where both the parent as well as the daughter meson contain heavy quark with mass m_b and $m_c \gg \Lambda_{\rm QCD}$, are expected to reasonably obey the asymptotic QCD predictions. However the same cannot be true in case of $D \to K, K^*$ transitions since the underlying assumption requiring s quark to be very heavy is certainly not a good approximation. In Fig. 4, we observe that the q^2 dependence of the form factors in case of $B \to D, D^*$ transitions shows moderate deviations from the heavy-quark symmetry relations given by Eq. (74) which are also compatible with outcomes of various other models [8]. We also obtain the values of the corresponding form factors at $q^2 = q^2_{\rm max}$ in the following manner:

$$\begin{aligned} F_1(q^2_{\rm max}) &\simeq 1.20(1.13) , \\ V(q^2_{\rm max}) &\simeq 1.45(1.13) , \\ A_2(q^2_{\rm max}) &\simeq 1.39(1.13) , \\ A_1(q^2_{\rm max}) &\simeq 0.88(0.89) . \end{aligned} \tag{76}$$

These are in reasonable agreement with the model-independent normalization values (in the parentheses) at zero recoil following from the heavy-quark symmetry. Thus we find here that in B-meson semileptonic decays the HQET predictions are realized quite reasonably and precisely. We calculate the ratios A_2/A_1 and V/A_1 at $q^2 = q^2_{\rm max}$ and present in Table II a comparison of these quantities with those obtained in various other models [6]. But in the case of $D \to K, K^*$ transitions it is not surprising to find from Fig. 3 that the form factors $F_1(q^2)$, $V(q^2)$, $A_2(q^2)$, and $\tilde{A}_1(q^2)$ are not close to one another over the entire kinematic range. Finally we obtain the form factors at $q^2 = 0$ and compare them with the corresponding values obtained from various other models. Tables III and IV provide such comparison in case of $D^0 \to K^-, K^{*-}$ and $\bar{B}^0 \to D^+, D^{*+}$ transitions, respectively.

After displaying the q^2 dependence of the relevant form factors generated in the present model, we evaluate numerically the decay rates $\Gamma(M \to me\nu)$, the polarization ratio Γ_L/Γ_T and the ratio $R = \Gamma(0^- \to 1^-)/\Gamma(0^- \to 0^-)$ for $D^0 \to K^-, K^{*-}$ and $\bar{B}^0 \to D^+, D^{*+}$ transitions. These results are listed in Table V in comparison with the predictions of some other models and the available experimental data. We observe that the decay rates and the ratio R for the decays $D^0 \to K^-, K^{*-}$ are in good

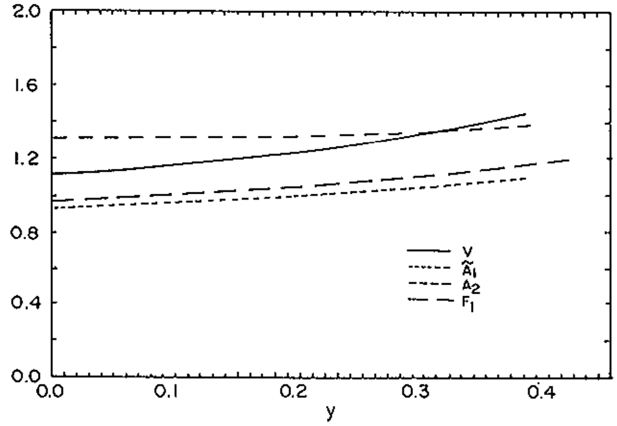

FIG. 4. Variation of the form factors relevant for the decays $\bar{B}^0 \to D^{*+}, D^+$ in the entire kinematic range of y.

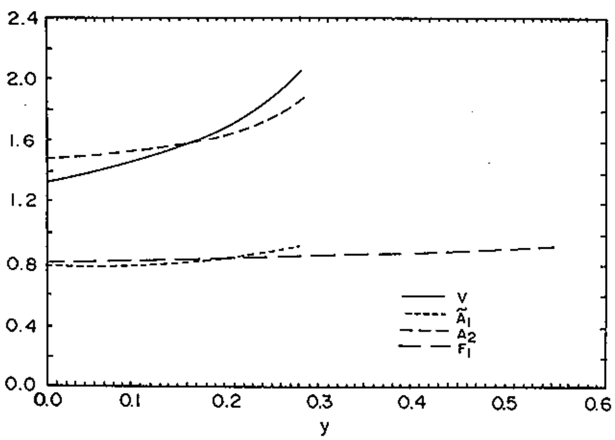

FIG. 3. Variation of the form factors relevant for the decays $D^0 \to K^{*-}, K^-$ in the entire kinematic range of y.

TABLE II. Measurement and predictions on the ratios of the form factors at $q^2 = q^2_{\rm max}$ for the decay $\bar{B}^0 \to D^* e\nu$. The two fits (a) and (b) of [29] correspond to different assumptions for the q^2 dependence of the form factors.

Ref.	A_2/A_1	V/A_1
CLEO II fit (a) [29]	1.02±0.24	1.07±0.57
CLEO II fit (b) [29]	0.79±0.28	1.32±0.62
(ISGW) [10]	1.14	1.27
(GS) [13]	1.16	1.38
(BSW) [15]	1.06	1.14
(KS) [16]	1.39	1.54
HQET based [7]	1.26	1.26
HQET based [8]	1.14	1.74
Present prediction	1.58	1.65

TABLE III. The predictions for the form factors at $q^2 = 0$ in the decay $D^0 \to K^-, K^{*-}$ along with those of other quark models and the experiment.

Form factor	(ISGW) [10]	(GS) [13]	(BSW) [15]	(WJ) [17]	Present prediction	Experiment [6][a]
$V(0)$	1.10	1.46	1.27	0.79	1.32	1.10±0.20
$A_1(0)$	0.80	0.74	0.88	0.59	0.77	0.56±0.04
$A_2(0)$	0.80	0.55	1.15	0.36	1.48	0.40±0.08
$F_1(0)$	0.80	0.70	0.75	0.70	0.80	0.75±0.03

[a]The value of the form factors quoted above is the average of the result of E691, E687, ad E653.

TABLE IV. The predictions for the form factors at $q^2 = 0$ in the decay $\bar{B}^0 \to D^+, D^{*+}$ along with those of other quark models and the experiment.

Form factor	(ISGW) [10]	(GS) [13]	(BSW) [15]	(WJ) [18]	Present prediction	Experiment [6]
$V(0)$		0.95	0.71	0.69	1.11	
$A_1(0)$		0.69	0.65	0.35	0.93	
$A_2(0)$		0.80	0.69	0.56	1.31	
$F_1(0)$			0.69	0.67	0.97	

TABLE V. Predictions on the decay width and the polarization ratio in the decays such as ($D^0 \to K^-, K^{*-}$) and ($\bar{B}^0 \to D^+, D^{*+}$).

	(ISGW) [10]	(GS) [13]	(BSW) [15]	(WJ) [17]/ [18]	Present prediction	Experiment [6]
$\Gamma(D^0 \to K^-)$ $\times(10^{10}\ {\rm s}^{-1})$	8.50	7.10	7.89	6.72	7.68	8.2±0.4
$\Gamma(D^0 \to K^{*-})$ $\times(10^{10}\ {\rm s}^{-1})$	9.13	9.50	9.03	4.28	5.20	4.6±0.4
$\frac{\Gamma_L(D^0 \to K^{*-})}{\Gamma_T(D^0 \to K^{*-})}$	1.09	1.21	0.90	1.44	0.52	1.23±0.13[a]
$\frac{\Gamma(D^0 \to K^{*-})}{\Gamma(D^0 \to K^-)}$	1.07	1.34	1.14	0.64	0.68	0.60±0.09±0.07 (CLEO II)
$\Gamma(\bar{B}^0 \to D^+)$ $\times(10^{10}\ {\rm s}^{-1})$	2.05	2.60	1.50	1.63	2.73	1.27±0.33
$\Gamma(\bar{B}^0 \to D^{*+})$ $\times(10^{10}\ {\rm s}^{-1})$	4.66	4.90	4.10	4.10	5.10	2.96±0.27
$\frac{\Gamma_L(\bar{B}^0 \to D^{*+})}{\Gamma_T(\bar{B}^0 \to D^{*+})}$	0.97	0.88	1.07	1.17	0.77	1.105±0.74±0.6 (CLEO)
$\frac{\Gamma(\bar{B}^0 \to D^{*+})}{\Gamma(\bar{B}^0 \to D^+)}$	2.27	1.88	2.67	2.46	1.87	$2.6^{+1.1+1.0}_{-0.8-0.8}$ (CLEO)

[a]Reference [3] gives the corresponding experimental value $(0.5^{+1.0+1.0}_{-0.1-0.2})$.

agreement with the experiment. Although our prediction for the polarization ratio $\Gamma_L(D^0 \to K^{*-})/\Gamma_T(D^0 \to K^{*-})$ is in agreement with the central value of the measurement of Mark III Collaboration [3], it is much below the more recent and precise experimental limit [6]. The present model therefore predicts that in the $D^0 \to K^{*-}e^+\nu$ semileptonic transition, the daughter meson K^{*-} is found to have its spin polarization predominantly transverse in nature, which is contrary to the current belief. The low polarization ratio here, in the present model, may be due to a relatively high value of $A_2(q^2)$ which contributes destructively towards the longitudinal decay mode. In fact, none of the quark model proposed so far except the one due to Jauss [17] has been entirely successful in describing all aspects of $D^0 \to K^-, K^{*-}$ transitions in perfect agreement with the available experimental data. In the case of $\bar{B}^0 \to D^+, D^{*+}$ transitions, we find that, although the decay rates $\Gamma(\bar{B}^0 \to D^+e^-\bar{\nu}_e)$ and $\Gamma(\bar{B}^0 \to D^{*+}e^-\bar{\nu}_e)$ obtained in the present model are higher in comparison with the available experimental data, the polarization ratio $\Gamma_L(\bar{B}^0 \to D^{*+})/\Gamma_T(\bar{B}^0 \to D^{*+})$ and the decay width ratio $\Gamma(\bar{B}^0 \to D^{*+})/\Gamma(\bar{B}^0 \to D^+)$ are found in reasonable agreement with the limits of the presently available imprecise data [6]. The ratio $R = \Gamma(\bar{B}^0 \to D^{*+})/\Gamma(\bar{B}^0 \to D^+) = 1.87$ is also not very different from the asymptotic QCD prediction around a value 2–3 following from the heavy-quark symmetry.

For a consistency check on the reliability of our calculation based on the general formalism [13] described in Sec. II, we have repeated the same calculation using a different but straightforward formalism as per [12] and have reproduced the above results. Thus we find that there exists some discrepancy between the present prediction and currently available experimental data. Nevertheless, we must remind ourselves that we have not made any attempt in the present calculation for readjusting the relevant model parameters in order to fit the experimental data. The experimental uncertainties in this sector are also too large at present to allow a stringent test.

V. CONCLUSIONS

We have studied the form factor dependence of the exclusive semileptonic decays of D and B mesons in the independent quark model. The possibility of disagreement of most of the quark models from the experiment in the charm meson decays is that the calculation of the form factors in those models is based on the assumptions which cannot be totally relied upon. The distinction of the present calculation is that the relevant form factors are here uniquely and unambiguously determined by the underlying relativistic quark dynamics without any specific end-point normalization to start with. The explicit q^2 dependence of the form factors is also derived from the model without assuming it to be monopole or dipole type.

Our prediction for the decay widths and the decay width ratio R relevant for the transitions $D^0 \to K^-e^+\nu$ and $D^0 \to K^{*-}e^+\nu$ are in very good agreement with the data. In the B meson decays we predict the decay widths ratio R and the polarization ratio to be comparable to the data at the present level of experimental uncertainty. The polarization of K^{*-}, though found out to be comparable to the measurement of the Mark Collaboration [3], is certainly below the current experimental limit. The precise prediction on the decay widths of B mesons and the polarization of K^{*-} in the D^0 decay would depend upon the close interplay between the future experiment and the developing phenomenology in these sectors. Thus within the working approximation adopted here, the present model provides a simple framework to explain reasonably the exclusive semileptonic decays of D and B mesons.

ACKNOWLEDGMENTS

One of us (P.C.D.) gratefully acknowledges the support of the Department of Education, Government of Orissa, India on providing study leave. We are thankful to Professor B. B. Deo for some useful discussions.

[1] E691 Collaboration, J. C. Anjos *et al.*, Phys. Rev. Lett. **62**, 722 (1989); **65**, 2630 (1990).
[2] Mark III Collaboration, J. Alder *et al.*, Phys. Rev. Lett. **62**, 1821 (1989).
[3] Mark III Collaboration, Z. Bai *et al.*, Phys. Rev. Lett. **66**, 1011 (1991).
[4] E653 Collaboration, K. Kodama *et al.*, Phys. Rev. Lett. **66**, 1819 (1991).
[5] ARGUS Collaboration, H. Albrecht *et al.*, Phys. Lett. B **255**, 634 (1991); **219**, 121 (1989).
[6] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994).
[7] N. Isgur and M. Wise, Phys. Lett. B **232**, 113 (1990); **237**, 527 (1990); M. B. Wise, in *Particle Physics—The Factory Era*, Proceedings of the Winter Institute, Lake Louise, Canada, 1991, edited by B. A. Campbell *et al.* (World Scientific, Singapore, 1991), and references therein.
[8] M. Neubert, Phys. Lett. B **264**, 455 (1991); Phys. Rep. **245**, 259 (1994), and references therein.
[9] M. Cristafulli, G. Martinelli, and C. T. Sachrajda, Phys. Lett. B **223**, 90 (1989); C. Bernard, A. El-Khandra, and A. Soni in *Lattice '88*, Proceedings of the International Symposium, Batavia, Illinois, edited by A. S. Kronfeld and P. B. Mackenzie [Nucl. Phys. B (Proc. Suppl.) **9**, 186 (1989)]; V. Lubiez, G. Martinelli, and C. T. Sachrajda, Nucl. Phys. **B356**, 301 (1991).
[10] B. Grinstein, N. Isgur, and M. B. Wise, Phys. Rev. Lett. **56**, 298 (1986); Caltech Report No. CALT-68-1311, 1986 (unpublished); N. Isgur, D. Scora, B. Grinstein, and M. B. Wise, Phys. Rev. D **39**, 799 (1989); N. Isgur and D. Scora, *ibid.* **40**, 1491 (1989).
[11] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985); R. Kokoski and N. Isgur, Phys. Rev. D **35**, 907 (1987).
[12] T. Altomari and L. Wolfenstein, Phys. Rev. Lett. **58**, 1583 (1987); Phys. Rev. D **37**, 681 (1988).
[13] J. G. Korner and G. A. Schuler, Mainz Report No. MZ-TH/88-14, 1988 (unpublished); Phys. Lett. B **226**, 185

(1989); H. Hagiwara, A. D. Matin, and M. F. Wade, *ibid.* **228**, 144 (1989); F. J. Gilman and R. L. Singleton, Jr., Phys. Rev. D **41**, 142 (1990).
[14] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C **29**, 637 (1985).
[15] M. Bauer and M. Wirbel, Z. Phys. C **42**, 671 (1989).
[16] J. G. Korner and G. A. Schuler, Z. Phys. C **38**, 511 (1988).
[17] Wolfgang Jaus, Z. Phys. C **54**, 611 (1992).
[18] Wolfgang Jaus, Phys. Rev. D **41**, 3394 (1990).
[19] N. Barik, P. C. Dash, and A. R. Panda, Phys. Rev. D **46**, 3856 (1992).
[20] N. Barik and P. C. Dash, Phys. Rev. D **49**, 299 (1994).
[21] N. Barik, P. C. Dash, and A. R. Panda, Phys. Rev. D **47**, 1001 (1993).
[22] N. Barik and P. C. Dash, Phys. Rev. D **47**, 2788 (1993).
[23] N. Barik and P. C. Dash, Mod. Phys. Lett. A **10**, 103 (1995).
[24] N. Barik, B. K. Dash, and P. C. Dash, Pramana J. Phys. **29**, 543 (1987).
[25] N. Barik and B. K. Dash, Phys. Rev. D **33**, 1925 (1986).
[26] N. Barik, B. K. Dash, and M. Das, Phys. Rev. D **32**, 1725 (1985); N. Barik and B. K. Dash, *ibid.* **34**, 2092 (1986); **34**, 2803 (1986).
[27] B. E. Palladino and P. Leal Ferriera, IFT Sao Paulo Report No. IFT/P-35/88 (unpublished).
[28] B. Margolis and R. R. Mendel, Phys. Rev. D **28**, 468 (1983).
[29] CLEO Collaboration, S. Sanghera *et al.*, Phys. Rev. D **47**, 791 (1993).