PHYSICAL REVIEW D VOLUME 53, NUMBER 3 1 FEBRUARY 1996

Semileptonic, radiative, and pionic decays of B, B^* and D, D^* mesons

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(Received 14 September 1994; revised manuscript received 25 April 1995)

A relativistic constituent quark model (RQM) is used to calculate the form factors for the semileptonic decays $B, D \to \pi(\rho)l\nu$, $B \to D(D^*)l\nu$, $D \to K(K^*)l\nu$, and the coupling constants for the radiative decays $B^* \to B\gamma$, $D^* \to D\gamma$. The quark model is combined with a soft pion theorem to derive the $B^*B\pi$ and $D^*D\pi$ coupling constants, which are used to calculate the rate for $D^* \to D\pi$. The parameters of the model are fixed by exploiting the duality of the vector meson dominance (VMD) picture and the picture of constituent quarks. This approach, which requires only that the predictions of the VMD model and the RQM are consistent, enables a parameter free determination of heavy quark properties, and leads to a q^2 dependence of form factors which is different from the usual pole approximation. The predicted rates for D and D^* mesons agree with the data without exception. This positive result supports the conclusion that properties of heavy mesons can be analyzed consistently in this framework.

PACS number(s): 13.20.Fc, 12.39.Ki, 12.40.Vv, 13.20.He

I. INTRODUCTION

The analysis of semileptonic decays of heavy mesons is of great practical interest since the respective Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, which determine the strength of the underlying quark decays, can be extracted and the quark structure of mesons can be probed by measurements of the decay rates. The rate Γ for the decay of a heavy meson is usually represented in terms of a reduced decay rate Γ_r:

$$\Gamma = |V_{if}|^2\Gamma_r \ , \tag{1.1}$$

where V_{if} is the CKM matrix element for a $q_i \to q_f$ transition, and Γ_r needs to be calculated. Γ_r can be expressed in terms of form factors which describe the relationship between the quark picture and the hadronic picture of the decay process. Usually a phenomenological model is used to calculate the form factors for the various decays and, with increasing precision, the experimental data can be used to check and refine the input to the model.

The CKM matrix elements that govern charm decays are rather well known, since the constraint of unitarity gives tight bounds for their magnitude. From [1] we quote

$$|V_{cd}| = 0.220 \pm 0.002, \quad |V_{cs}| = 0.9743 \pm 0.0007 \ .$$

Therefore, the calculated form factors can be compared directly with the data and models checked.

The situation with respect to the CKM matrix elements that govern b decays is different, since it is difficult to obtain a quantitative estimate of their magnitude based on the CKM matrix elements that are associated with the first two quark families. Semileptonic B decays are the only source of information about the CKM matrix elements V_{cb} and V_{ub}. Their determination from an analysis of these decays necessarily requires reliable estimates of the decay form factors. The value of V_{cb} extracted from the measured rates for the decays of $B \to Dl\nu$ and $B \to D^*l\nu$ is rather insensitive to the model used to calculate form factors, at least at the present level of accuracy of the data. The value of V_{ub} can be determined from the measured rates for the decays $B \to \pi e\nu$ and $B \to \rho e\nu$. The transition $B \to \pi$ is of particular interest, since the form factor needs to be known for the large range $0 \le q^2 \le (M_B - M_\pi)^2$, where q is the momentum transfer. The interpretation of this reaction depends very sensitively on the particular assumptions that form the basis of the model used to analyze the decay. For instance, theoretical predictions for $\Gamma_r(B^0 \to \pi^+ e\nu)$ range from 2.1×10^{12} s^{-1} [2] to 54.3×10^{12} s^{-1} [3].

Let us consider in more detail transitions like $B \to \pi l\nu$ and $D \to \pi l\nu$. For vanishing lepton masses, the rate for each decay is determined by one respective form factor $f_+(q^2)$. Certain quark models [4,5] and lattice calculations [6] evaluate the form factor for $q^2 = 0$ and postulate, motivated by vector dominance ideas, that a pole form holds for all values of q^2:

$$f_+(q^2) \simeq \frac{f_+(0)}{1 - q^2/\Lambda_1^2} \ , \tag{1.2}$$

where the value of Λ_1 is usually assumed not to be very different from M_V, the mass of the lowest resonance $V = B^*, D^*$. A very detailed investigation of these decay modes by means of QCD sum rules in [7] seemingly confirms these assumptions, since it concludes that $f_+(q^2)$ is well approximated by the pole behavior (1.2) with pole masses in the expected range. However, we shall argue below that this extension of the pole form (1.2) to high values of q^2 does not seem to be reliable.

An alternative determination of the form factor for a different range of values of q^2 is possible in the approach that combines the heavy quark effective theory (HQET) and chiral perturbation theory. It has been applied to the semileptonic decays of B and D mesons [8], with the result that the B^* and D^* poles are the dominant components of the form factor $f_+(q^2)$ for $B \to \pi$ and $D \to \pi$ transitions close to the maximum possible value $q^2 = q^2_{\rm max}$. For example, for high values of q^2 the form factor for the decay $B^0 \to \pi^+ e\nu$ is given by the vector meson dominance (VMD) formula

$$f_+(q^2) \simeq \frac{\sqrt{2} g_{B^* B\pi^+} f_{B^*}}{M^2_{B^*} - q^2} \ . \tag{1.3}$$

The factor $\sqrt{2}$ appearing in Eq. (1.3) is due to our definition of the vector decay constant f_{B^*}:

$$\langle 0|\bar{b}\gamma_\mu u|B^*;P'\rangle = i\varepsilon_\mu \sqrt{2} f_{B^*} \ . \tag{1.4}$$

The $B^*B\pi$ coupling constant $g_{B^*B\pi^+}$ is defined by

$$\langle B;P''|j_{\pi^0}(0)|B^*;P'\rangle = 2(\varepsilon P'') g_{B^*B\pi^0} \ , \tag{1.5}$$

$$g_{B^*B\pi^0} = g_{B^*B\pi^+}/\sqrt{2} \ ,$$

where ε_μ is the polarization vector of the B^* and j_π is the source of the pion field.

Analogous equations hold for $D \to \pi$ transitions. In the limit where the heavy-quark mass m_Q goes to infinity there are flavor-independent relations between coupling constants:

$$g = g_{D^*D\pi^+} \sqrt{2} f_\pi / M_{D^*} = g_{B^*B\pi^+} \sqrt{2} f_\pi / M_{B^*} \ , \tag{1.6}$$

where $f_\pi = 92.4 \pm 0.2$ MeV, and

$$\sqrt{M_D} f_D = f_{D^*}/\sqrt{M_{D^*}} = \sqrt{M_B} f_B = f_{B^*}/\sqrt{M_{B^*}} \ . \tag{1.7}$$

These relations are valid at leading order in a perturbation expansion in powers of $1/m_Q$ [9].

We shall show that a naive extrapolation of the vector meson dominance formula (1.3) to the point $q^2 = 0$ is not consistent with the pole formula (1.2), for in general the relevant parameters are different: $f_+(0) \neq \sqrt{2} g_{B^*B\pi^+} f_{B^*}/M^2_{B^*}$ and $\Lambda_1 \neq M_{B^*}$. In order to illustrate the quantitative consequences of these inequalities, it is instructive to compare the results of two representative calculations. The method of [4] is based on the pole form (1.2), extended to all values of q^2, and gives the result $\Gamma_r(B^0 \to \pi^+) = 7.4 \times 10^{12}\ {\rm s}^{-1}$. The method of [3] is based on the VMD formula (1.3), extended to all values of q^2. The HQET equations (1.6) and (1.7) are used to estimate the parameters for the $B \to \pi$ transitions from the experimental data for $D \to \pi$. The reduced rate is predicted to be $\Gamma_r(B^0 \to \pi^+) = 54.3 \times 10^{12}\ {\rm s}^{-1}$. Thus it is obvious that the two methods to not match.

A general ansatz which interpolates between the forms (1.2) and (1.3) is given by

$$f_+(q^2) = \frac{\sqrt{2} g_{B^*B\pi^+} f_{B^*}}{(M^2_{B^*} - q^2)[1 + (1 - q^2/M^2_{B^*}) G(q^2)]} \ . \tag{1.8}$$

The representation (1.8) is motivated also by the classic work of Gounaris and Sakurai [10], in which the VMD model is refined by taking into account the width of the vector meson, i.e., analytic properties and unitarity are included globally in a generalized Breit-Wigner form for the vector meson propagator. We shall use Eq. (1.8) as a convenient analytic expression to extrapolate the VMD form factor (1.3) outside its known range of validity to low values of q^2. The damping of the monopole form (1.3) is contained in the unknown function $G(q^2)$ which accounts for the structure of the background. For $B \to \pi$ transitions the background would consist of higher B^* resonances plus the $B\pi$ and the $B^*\pi$ continuum. In Appendix A we shall investigate in detail the role of B^* radial excitations in the region near $q^2 = M^2_{B^*}$. We shall estimate the effect of the background by including the sum of all radial excitations. Of course, this is only an approximate account of the background, but it leads to a well-defined prediction for $G(q^2)$ at $q^2 = M^2_{B^*}$. Furthermore, we shall show that $G(q^2)$ is a function that varies only slowly with q^2. If the pole form (1.2) and the interpolating form (1.8) are compared at $q^2 = 0$ one obtains a relation between $f_+(0)$ and $G(0) = \delta$:

$$(1+\delta) f_+(0) = \sqrt{2} g_{B^*B\pi^+} f_{B^*}/M^2_{B^*} \ . \tag{1.9}$$

We shall take into account the q^2 dependence of $G(q^2)$ in the discussion presented in Sec. IV, but if terms of $O(q^2)$ in $G(q^2)$ are neglected for the sake of illustration, a comparison of the first derivatives at $q^2 = 0$ leads to a relation between the pole mass Λ_1 of Eq. (1.2) and M_{B^*}:

$$(1 + 2\delta)\Lambda_1^2 = (1+\delta) M^2_{B^*} \ . \tag{1.10}$$

Therefore the quantity δ is a measure for the mismatch between the seemingly equivalent forms (1.2) and (1.3). Again, analogous equations hold for $D \to \pi$ transitions. We expect rather large values of δ for $B, D \to \pi$ transitions, and it is interesting to note that already for the electromagnetic form factor of the pion one finds a deviation from the naive VMD prediction which is given by $1 + \delta = g_{\rho\pi\pi} f_\rho / M^2_\rho \simeq 1.20$.

In this work we shall use the relativistic constituent quark model (RQM) of [11]. The RQM is based on the light-front formalism, which provides a simple framework for the determination of hadronic form factors for spacelike momentum transfer. We have demonstrated in [12] that the RQM can predict the electroweak properties of light mesons very reliably, and we expect it to work equally well in the heavy quark sector. The first aim of this paper is to show that these form factors can be continued analytically from spacelike to timelike, i.e., physical, momentum transfer; therefore, hadronic form factors can be calculated for all values of q^2 and we shall use this method to analyze semileptonic and radiative decays of B, B^* and D, D^* mesons. In order to make a quantitative comparison of the theoretical predictions and the experimental results, the parameters of the model have to be determined in terms of a few coupling constants which must be obtained by the analysis of suitable experiments. This has been done for the (u,d,s) quark sector in [12], but not for heavy quarks, since the relevant data are not precise enough or have not yet been obtained.

But even without knowing the values of the heavy-quark parameters one can investigate the behavior of the expression for the form factor $f_+(q^2)$ derived in the RQM in the limit, where the heavy-quark mass m_Q goes to infinity (heavy-quark limit). Of particular interest are the scaling rules for $f_+(q^2)$ at $q^2 \simeq q^2_{\rm max} \sim m_Q^2$ and at $q^2 \simeq 0$. In the heavy-quark limit one obtains, at $q^2 = m_Q^2$,

$$f_+(m_Q^2) \sim m_Q^{1/2} \ . \tag{1.11}$$

This is the heavy-to-light scaling law of QCD. At $q^2 = 0$ the scaling behavior depends on the wave functions for the initial- and final-state mesons. In the RQM Gaussian wave functions are used and one finds

$$f_+(0) \sim \exp(-m_Q^{1/2}) \ . \tag{1.12}$$

The first derivative of $f_+(q^2)$ at $q^2 = 0$ can be expressed in terms of Λ_1 as

$$\frac{f'_+(0)}{f_+(0)} = \frac{1}{\Lambda_1^2} \tag{1.13}$$

and Λ_1 scales as $\Lambda_1 \sim m_Q^{3/4}$. Obviously the pole form (1.2) approximates the form factor well for small values of q^2 but we expect Λ_1 to be smaller than M_V. The scaling properties of $f_+(q^2)$ at large and small values of q^2 are evidently quite different. This observation shows once more that the monopole form factor (1.3), which scales as given in (1.11), cannot be extrapolated to $q^2 = 0$.

The second aim of the paper is to propose an entirely different method for the determination of the heavy-quark parameters, by exploiting the duality of the vector meson dominance picture, in which the form factor for $B \to \pi$ (and similarly $D \to \pi$) transitions is given by Eq. (1.8), and the picture of the bound constituent quarks, where the relevant form factors can be calculated in the framework of the RQM, as indicated above. In fact, we shall show that the parameters for c and b quarks can be determined such that the alternative versions of the form factor match well for all values of q^2. The only external input is the value for the mass of the resonance, namely M_{B^*} or M_{D^*}, while the coupling constants $g_{B^*B\pi}$, $g_{D^*D\pi}$, f_{B^*}, and f_{D^*} are calculated in the RQM. Since the coupling constants depend also on the heavy-quark parameters, obviously a self-consistent procedure must be used, that will be described in greater detail below. Therefore this approach, which requires only that the predictions of the VMD model and the RQM for the transition $B \to \pi$ and $D \to \pi$ are consistent, enables a parameter-free calculation of heavy-quark properties, like coupling constants and form factors.

For charm decays there exists now a set of reliable data and we shall show that the results of our calculation fully agree with these experimental results. Since the success of this concept of duality is perhaps surprising, it might be helpful to demonstrate, as we do in Appendix B, that the calculation of the form factor $f_+(q^2)$ in the light-front formalism automatically takes into account a subset of higher-order gluon exchange diagrams, whose summation would be expected to generate the resonant structures which dominate the form factor $f_+(q^2)$ for large values of q^2. The same calculation performed in the usual instant-form formalism does not have this property, and we expect that results obtained if the light-front and if the instant-form approach is used might be quite different.

In Sec. II we present a brief summary of the general aspects of the light-front formalism for $q\bar{q}$ bound states and derive the procedure for the analytic continuation of hadronic form factors that govern semileptonic decay processes. In Sec. III we make use of a soft-pion theorem to extend the formalism to the treatment of pionic transitions between vector and pseudoscalar mesons, in order to estimate the $B^*B\pi$ and $D^*D\pi$ coupling constants. We illustrate the predictive power of this method by the calculation of the coupling constants for the decays $\rho \to \pi\pi$ and $K^* \to K\pi$. The idea of duality and matching of the vector meson dominance model and the constituent quark model is worked out in Sec. IV and used to determine the heavy-quark parameters. At this stage the RQM is completely defined and Sec. V contains our predictions for the semileptonic and radiative decays of B, B^* and D, D^* mesons and pionic decays of the D^* meson, which we compare with presently available experimental data. We conclude this work in Sec. VI with a summary of the results.

II. GENERAL FORMALISM

We represent the hadronic matrix elements for the decay of a pseudoscalar meson $M' \to M'' e\nu$ in terms of appropriate form factors. If the final meson M'' is a pseudoscalar or a vector particle, we have, respectively,

$$\langle P''|\bar{q}''\gamma_\mu q'|P'\rangle = f_+(q^2)P_\mu + f_-(q^2)q_\mu \tag{2.1}$$

and

$$\langle P''; 1J_3|\bar{q}''\gamma_\mu(1-\gamma_5)q'|P'\rangle = ig(q^2)\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}P^\alpha q^\beta - f(q^2)\varepsilon^*_\mu - a_+(q^2)(\varepsilon^*P)P_\mu - a_-(q^2)(\varepsilon^*P)q_\mu \ , \tag{2.2}$$

where $P = P' + P''$, $q = P' - P''$, and $\varepsilon = \varepsilon(J_3)$ is the polarization vector of the vector meson with $(\varepsilon P'') = 0$.

In the limit where the electron mass is neglected, the form factors f_- and a_- do not contribute to the respective decay rates. In order to be consistent with the usual convention, we rewrite the form factors given in Eq. (2.2) in the following way:

$$V(q^2) = -(M' + M'')g(q^2) \,,$$
$$A_1(q^2) = -(M' + M'')^{-1}f(q^2) \,, \quad (2.3)$$
$$A_2(q^2) = (M' + M'')a_+(q^2) \,,$$

where M' and M'' are the masses of the initial and final mesons, respectively. It was shown in [11] that these form factors can be determined in a consistent relativistic approach, which describes mesons as bound states of a quark and an antiquark, and we shall briefly summarize the main features of the treatment of $q\bar{q}$ bound states in the light-front formalism.

The four-momentum of the meson of mass M in terms of light-front components is $P = (P^-, P^+, P_\perp)$ where $P^\pm = P^0 \pm P^3$ and $P^2 = P^+P^- - P_\perp^2 = M^2$. The appropriate variables for the internal motion of the constituents, whose momenta shall be denoted by k_1 and k_2, are

$$k_1^+ = xP^+, \quad k_2^+ = (1-x)P^+ \,,$$

$$k_{1\perp} = xP_\perp + p_\perp, \quad k_{2\perp} = (1-x)P_\perp - p_\perp \quad (2.4)$$

with $k_1^2 = m_1^2, k_2^2 = m_2^2$ and kinematic invariant mass

$$M_0^2 = (k_1 + k_2)^2 = \frac{p_\perp^2 + m_1^2}{x} + \frac{p_\perp^2 + m_2^2}{1-x} \,, \quad (2.5)$$

where m_1, m_2 are the masses of the constituent quarks. It is sometimes convenient to replace the momentum fraction x by the longitudinal internal momentum p_3 of a vector $p = (p_1, p_2, p_3)$ whose transverse part is $p_\perp$, where p_3 is defined by

$$p_3 = (x - \tfrac{1}{2})M_0 - \frac{m_1^2 - m_2^2}{2M_0} \,. \quad (2.6)$$

In terms of this new variable M_0 is simply given by

$$M_0 = E_1 + E_2 \,, \quad (2.7)$$

where $E_i = \sqrt{m_i^2 + p^2}$.

It is crucial for the computation of form factors to impose the condition $q^+ = 0$, which at this stage of the calculation means that form factors are known only for spacelike momentum transfer $q^2 = -q_\perp^2 \leq 0$. For the sake of illustration we shall consider the form factor $f_+(q^2)$ for the transition between an initial pseudoscalar meson with internal variables and masses of its constituent quarks $(x, p'_\perp, m'_1, m'_2)$ and a final pseudoscalar meson with the corresponding quantities $(x, p''_\perp, m''_1, m''_2)$, where $m'_2 = m''_2 = m_2$. The form factor is given by an overlap integral analogous to the corresponding nonrelativistic formula

$$f_+(q^2) = \frac{N_c}{16\pi^3} \int_0^1 dx \int d^2p'_\perp \frac{\sqrt{M'_0 M''_0}}{x(1-x)} \sum_{\lambda\bar{\lambda}} \psi^\dagger(x, p''_\perp, \lambda\bar{\lambda})\psi(x, p'_\perp, \lambda\bar{\lambda}) \,, \quad (2.8)$$

where $p''_\perp = p'_\perp - (1-x)q_\perp$ and N_c denotes the number of colors. The additional factor in the integrand is a consequence of our normalization of the wave function, Eq. (2.11). The light-front wave function ψ has the same structure as the wave function of nonrelativistic quantum mechanics: it is a product of spin and orbital wave functions and depends only upon the internal variables $x, p_\perp$, the spin variables of the quarks $\lambda, \bar{\lambda} = \pm\frac{1}{2}$, and the spin J of the meson. However, the composition of spin requires a momentum dependent Melosh rotation $\mathcal{R}_M(x, p_\perp)$ of individual spins, which for an S-state meson with spin 0 can be represented in the form [13]

$$\psi(x, p_\perp, \lambda\bar{\lambda}) = \sum_{\mu\bar{\mu}} \langle\lambda|\mathcal{R}_M(x, p_\perp)|\mu\rangle\langle\bar{\lambda}|\mathcal{R}_M(1-x, -p_\perp)|\bar{\mu}\rangle\langle\tfrac{1}{2}\mu, \tfrac{1}{2}\bar{\mu}|00\rangle\Phi(x, p_\perp) \,. \quad (2.9)$$

The orbital wave function is assumed to be a simple function of the invariant mass M_0 of the quarks:

$$\Phi(x, p_\perp) \equiv \Phi(M_0^2) = N \exp(-M_0^2/8\beta^2) \,. \quad (2.10)$$

The factor N is determined by the normalization condition

$$\frac{N_c}{(2\pi)^3} \int d^3p |\Phi(M_0^2)|^2 = 1 \,. \quad (2.11)$$

It is sometimes convenient to change the variables of integration from $x, p'_\perp$ to p'_1, p'_2, p'_3. They are related by Eq. (2.6) and

$$d^3p' = \frac{1}{x(1-x)} \frac{E'_1 E'_2}{M'_0} dx d^2p'_\perp \,,$$

$$4E'_1 E'_2 = M_0'^2 - (m_1'^2 - m_2^2)^2/M_0'^2 \,.$$

In terms of Φ the expression for $f_+(q^2)$ takes the form

$$f_+(q^2) = \frac{N_c}{(2\pi)^3} \int d^3p'\Omega(p'',p') \frac{1}{x(1-x)} \{p'_\perp p''_\perp + [xm_2 + (1-x)m'_1][xm_2 + (1-x)m''_1]\} , \qquad (2.12)$$

$$\Omega(p'',p') = \Phi''(M_0''^2)\Phi'(M_0'^2) \left(\frac{E''_1 E''_2 M'_0}{E'_1 E'_2 M''_0}\right)^{1/2} [M_0''^2 - (m''_1 - m_2)^2]^{-1/2} [M_0'^2 - (m'_1 - m_2)^2]^{-1/2} . \qquad (2.13)$$

The formula (2.12) for $f_+(q^2)$ depends explicitly on $q_\perp$ and is valid for $q_\perp^2 = -q^2 \geq 0$. We shall show now that it can be written as a function that depends explicitly on q^2 and that $F_+(q^2)$ can be analytically continued to physical values $q^2 > 0$.

For this purpose it is useful to substitute the transverse momentum variable in the following manner:

$$p'_\perp \to p_\perp = p'_\perp - (1-x)\frac{\beta'^2}{\beta'^2+\beta''^2} q_\perp \qquad (2.14)$$

and consequently

$$p''_\perp = p_\perp - (1-x)\frac{\beta''^2}{\beta'^2+\beta''^2} q_\perp . \qquad (2.15)$$

The product of initial and final wave functions, expressed in terms of the variable $p_\perp$, becomes a function of $q^2 = -q_\perp^2$:

$$\Phi''(M_0''^2)\Phi'(M_0'^2) = N''N' \exp\left(-\frac{\beta'^2+\beta''^2}{\beta'^2\beta''^2}\frac{K^2}{8x(1-x)}\right) , \qquad (2.16)$$

$$K^2 = p_\perp^2 + xm_2^2 + \frac{1-x}{\beta'^2+\beta''^2}(\beta''^2 m_1'^2 + \beta'^2 m_1''^2) - (1-x)^2 \frac{\beta'^2\beta''^2}{(\beta'^2+\beta''^2)^2} q^2 . \qquad (2.17)$$

The argument of the exponential function is negative definite only for values of q^2 that satisfy the condition

$$q^2 < Q_0^2 \equiv \frac{\beta'^2+\beta''^2}{\beta'^2\beta''^2}(\beta''^2 m_1'^2 + \beta'^2 m_1''^2) . \qquad (2.18)$$

The model cannot deliver the form factor for $q^2 > Q_0^2$. This is a limitation which arises from the simple model chosen for the wave functions of the mesons.

The integrand of Eq. (2.12) for $f_+(q^2)$ contains also square roots whose arguments have terms that are linear in $q_\perp$. The latter can be isolated by means of the identity

$$\sqrt{2}\sqrt{A + (p_\perp q_\perp)} = \sqrt{A + \sqrt{A^2 - (p_\perp q_\perp)^2}} + \frac{(p_\perp q_\perp)}{\sqrt{A + \sqrt{A^2 - (p_\perp q_\perp)^2}}} . \qquad (2.19)$$

We can choose a frame of reference such that $q_\perp = (q_1, 0)$ and $q_1^2 = -q^2$, $(p_\perp q_\perp)^2 = -p_1^2 q^2$, in order to represent $f_+(q^2)$ schematically as

$$f_+(q^2) = \int_0^1 dx \int d^2p_\perp \sum_n a_n(x, p_1^2, p_2^2, q^2)(p_\perp q_\perp)^n . \qquad (2.20)$$

The sum in Eq. (2.20) contains only a few terms that can be constructed by repeated use of the identity (2.19), and all terms with odd n vanish when integrated. The resulting function now depends explicitly on q^2; it has been derived originally for spacelike momentum transfer, but the representation (2.20) permits an analytic continuation of the form factor to timelike momentum transfers satisfying $q^2 \leq Q_0^2$.

An analogous representation can be established for the form factors defined in Eq. (2.2).

We emphasize that the form factors calculated in this manner satisfy the heavy-to-light scaling laws of QCD in the limit where the heavy-quark mass m_Q goes to infinity. If we require that the value of β for the meson composed of a light quark and a heavy antiquark scales as

$$\beta \sim m_Q^{1/2} , \qquad (2.21)$$

we find the following scaling properties for the form factors at $q^2 = m_Q^2$:

$$f_+(q^2 = m_Q^2) \sim m_Q^{1/2} ,$$

$$V(q^2 = m_Q^2) \sim m_Q^{1/2} ,$$

$$A_1(q^2 = m_Q^2) \sim m_Q^{-1/2} ,$$

$$A_2(q^2 = m_Q^2) \sim m_Q^{1/2} .$$

III. PIONIC TRANSITIONS BETWEEN VECTOR AND PSEUDOSCALAR MESONS

Matrix elements for hadronic transitions between a vector meson V and a pseudoscalar meson P can be treated in the framework of the light-front formalism in special cases. We shall investigate the decay $V \to P\pi^0$ whose rate is given by

$$\Gamma = g_{VP\pi}^2 p^3/(6\pi M_V^2) ,$$
$$p^2 = [M_V^2 - (M_P + M_\pi)^2][M_V^2 - (M_P - M_\pi)^2]/(4M_V^2) . \quad (3.1)$$

The matrix element can be expressed in terms of the coupling constant as

$$\langle P''|j_{\pi^0}(0)|P';1J_3\rangle = 2(\varepsilon P'')g_{VP\pi^0} , \quad (3.2)$$

where $\varepsilon = \varepsilon(J_3)$ is the polarization vector of the vector meson V. The matrix element (3.2) can be related to the matrix element of the axial vector current, defined by analogy with Eq. (2.2):

$$\langle P''|A_\mu^3(0)|P';1J_3\rangle = \tfrac{1}{2}\{f(q^2)\varepsilon_\mu + a_+(q^2)(\varepsilon P)P_\mu + a_-(q^2)(\varepsilon P)q_\mu\} \quad (3.3)$$

with

$$A_\mu^3(x) = \tfrac{1}{2}\bar{u}(x)\gamma_\mu\gamma_5 u(x) - \tfrac{1}{2}\bar{d}(x)\gamma_\mu\gamma_5 d(x) .$$

Partial conservation of the axial vector current (PCAC),

$$\partial^\mu A_\mu^3(x) = f_\pi M_\pi^2 \pi^0(x) , \quad (3.4)$$

leads to the following constraint among the form factors, which is originally due to Das, Mathur, and Okubo [14]:

$$4|g_{VP\pi^0}|f_\pi = |f(0) - (M_V^2 - M_P^2)a_+(0)| . \quad (3.5)$$

Note, that if this relation refers to $g_{\rho^+\pi^+\pi^0}$ the factor 4 in Eq. (3.5) must be replaced by the factor 2. In deriving Eq. (3.5) the π^0 has been treated as soft in the sense that $P_{\pi^0}^2 \equiv q^2 \to 0$, and it has been assumed that $g_{VP\pi^0}$ is a gently varying function of $P_{\pi^0}^2$. The combination of form factors on the right side of the soft-pion theorem (3.5) has been calculated in the RQM in [11], and for bound states of a u or d quark with mass m and an antiquark with mass m_Q it is given by

$$f(0) - (M_V^2 - M_P^2)a_+(0) = -2M_V \frac{N_c}{(2\pi)^3}\int d^3p'\Omega(p',p')\frac{1}{x} \times \left\{2xM_0'[xm_Q + (1-x)m] + \frac{(2x-1)M_0' + m - m_Q}{(1-x)(M_0' + m + m_Q)}[p_\perp'^2 + x^2 m_Q^2 - (1-x)^2 m^2]\right\} , \quad (3.6)$$

$$M_0'^2 = \frac{p_\perp'^2 + m^2}{x} + \frac{p_\perp'^2 + m_Q^2}{1-x} , \quad \Omega(p',p') = |\Phi(M_0'^2)|^2[M_0'^2 - (m_Q - m)^2]^{-1} ,$$

and x and p_3' are related by Eq. (2.6). The parameters relevant for the (u,d,s) quark sector have been determined in [12] and are given in Table I. The RQM thus predicts the coupling constants used in Eq. (3.1) for the decays $\rho^+ \to \pi^+\pi^0$ and $K^{*+} \to (K\pi)^+$:

$$g_{\rho\pi\pi} = 5.62 \quad (6.06 \pm 0.01) ,$$

$$g_{K^*K\pi} \equiv \sqrt{3}g_{K^{*+}K^+\pi^0} = 5.05 \quad (5.57 \pm 0.03) ,$$

where we have defined an effective coupling constant $g_{K^*K\pi}$, and used Eq. (3.1) and the isospin symmetry relation

$$g_{K^{*+}K^0\pi^+} = \sqrt{2}g_{K^{*+}K^+\pi^0}$$

in order to determine the experimental coupling constants as given in parentheses. The agreement of the theoretical predictions with the data is reasonable and within the range expected from the effect of an underlying approximate chiral symmetry.

The coupling constants $g_{B^*B\pi}$ and $g_{D^*D\pi}$ will be determined in the next section.

TABLE I. Quark masses m_Q and wave-function parameters β_{meson} for $(q,\bar{Q})$ mesons. The light-quark mass is $m_q = m_{u,d} = 0.25$ GeV.

$(q\bar{Q})$ meson	m_Q (GeV)	β_{meson} (GeV)
π, ρ	0.25	0.3194
K, K^*	0.37	0.3949
D, D^*	1.445	0.4871
B, B^*	4.64	0.6948

IV. A CONSISTENT PARAMETRIZATION OF THE HEAVY QUARK SYSTEM

In our approach we started with a simple ansatz, Eq. (2.10), for the quark wave function, which depends on one parameter $1/\beta_{\text{meson}}$ that essentially determines the confinement scale of the $q\bar{q}$ bound state. The parameters of this model, the constituent masses m_q and the wave-function parameters β_{meson}, can be fixed in terms of measured coupling constants.

Based on the available precise data for the (u,d,s) quark sector, our model has been shown in [12] to give predictions for the electroweak properties of light mesons in excellent agreement with experiment. The data which would determine the parameters of the bound states of a light quark and a heavy antiquark are not precise enough or have not been measured as yet. In this paper we propose an entirely different method for the determination of the heavy quark parameters, by using the duality of the VMD picture and the picture of bound constituent quarks, as presented in Sec. I.

The decay

$$P^0 \to \pi^+ e\bar{\nu}, \quad P^0 = B^0, D^0$$

turns out to be suited best to exploit this idea of duality. For vanishing lepton masses the hadronic matrix element for this transition depends on one form factor $f_+(q^2)$, which has been discussed in the framework of the RQM in Sec. II. We shall denote it in this section by $f_+^{\rm RQM}(q^2)$. An alternative determination of the form factor is possible in the framework of the VMD model in the region near the B^* or D^* poles, and we have argued in Sec. I that we expect Eq. (1.8) to be an interpolating form for all values of q^2. Accordingly we have

$$f_+^{\rm VMD}(q^2) = \frac{\sqrt{2} f_V}{M_V^2 - q^2} \frac{g_{VP\pi^+}}{1 + (1 - q^2/M_V^2) G(q^2)} \tag{4.1}$$

with $V = B^*, D^*$.

The coupling constants that appear in Eq. (4.1) are not known for heavy mesons, but can be calculated in the RQM: $g_{VP\pi^+}$ is given by Eqs. (3.5) and (3.6), and f_V has been calculated in [12]:

$$f_V/M_V = \frac{N_c}{(2\pi)^3} \int d^3p \Phi(M_0^2) \left(\frac{M_0}{EE_Q}\right)^{1/2} \frac{\sqrt{2}}{[M_0^2 - (m_Q - m)^2]^{1/2}} \times \left\{ (1-x)m + x m_Q + \frac{2p_\perp^2}{M_0 + m + m_Q} \right\} \tag{4.2}$$

with $E = \sqrt{m^2 + p^2}$ and $E_Q = \sqrt{m_Q^2 + p^2}$.

We shall use the RQM also to obtain information on the function $G(q^2)$, which we fit by a quadratic approximation

$$G(q^2) = \delta \left(1 - \frac{q^2}{\gamma_1^2} - \frac{q^4}{\gamma_2^4} \right) . \tag{4.3}$$

The expansion coefficients in Eq. (4.3) can be fixed by the requirement that the RQM and the VMD representations of $f_+(q^2)$ and its first and second derivatives agree at $q^2 = 0$. With the following definitions of the quantities Λ_1, Λ_2,

$$\left(\frac{f_+'(0)}{f_+(0)}\right)^{\rm RQM} = \frac{1}{\Lambda_1^2} ,$$

$$\frac{1}{2}\left(\frac{f_+''(0)}{f_+(0)}\right)^{\rm RQM} = \frac{1}{\Lambda_1^4} - \frac{1}{\Lambda_2^4} , \tag{4.4}$$

the parameters $\delta, \gamma_1, \gamma_2$ are determined by the relations

$$(1+\delta) f_+^{\rm RQM}(0) = \sqrt{2} f_V g_{VP\pi^+}/M_V^2 ,$$

$$\frac{1}{\gamma_1^2} = \frac{1+\delta}{\delta}\frac{1}{\Lambda_1^2} - \frac{1+2\delta}{\delta}\frac{1}{M_V^2} , \tag{4.5}$$

$$\frac{1}{\gamma_2^4} = -\frac{1+\delta}{\delta}\frac{1}{\Lambda_2^4} + \frac{2}{\gamma_1^2 M_V^2} + \frac{1}{M_V^4} .$$

Since the light-quark properties are known, both $f_+^{\rm RQM}(q^2)$ and $f_+^{\rm VMD}(q^2)$ depend only on two unknown parameters, the heavy-quark mass m_Q and the wave-function parameter $\beta_P = \beta_V$ (note that we assume identical S-wave functions for singlet and triplet states). We shall show that the parameters of the c quark and the b quark can be chosen such that for the form factors that govern $B \to \pi$ and $D \to \pi$ transitions the two alternative approaches give consistent results for all values of q^2: i.e.,

$$f_+^{\rm VMD}(q^2) \simeq f_+^{\rm RQM}(q^2) . \tag{4.6}$$

In order to find the parameters that satisfy the consistency requirement (4.6) we replace the parameter set (m_Q, β_P) by the set (m_Q, f_P), where the pseudoscalar coupling constant f_P is given by [12]

$$f_P = \frac{N_c}{(2\pi)^3} \int d^3p \Phi(M_0^2) \left(\frac{M_0}{EE_Q}\right)^{1/2} \times \frac{\sqrt{2}}{[M_0^2 - (m_Q - m)^2]^{1/2}} \times \{(1-x)m + x m_Q\} . \tag{4.7}$$

Note that Eq. (4.7) can be used to calculate, e.g., f_π using the parameters given in Table I, with the result $f_\pi = 92.4$ MeV.

For a given value of f_P we find that the consistency relation (4.6) is fulfilled only for a very narrow range of m_Q. As representative examples we have compared the reduced RQM form factor $(M_V^2 - q^2) f_+^{\rm RQM}(q^2)$ with the reduced VMD model form factor $(M_V^2 - q^2)f_+^{\mathrm{VMD}}(q^2)$ for the transition $B \to \pi$ in Fig. 1 and for the transition $D \to \pi$ in Fig. 2. This is the quantity which in the popular pole approximation to the form factors is assumed to be constant, while we find a considerable variation with q^2 as shown in Figs. 1 and 2. We illustrate the results for a fixed value of f_P and for three different values of m_Q, and note that a striking agreement be-

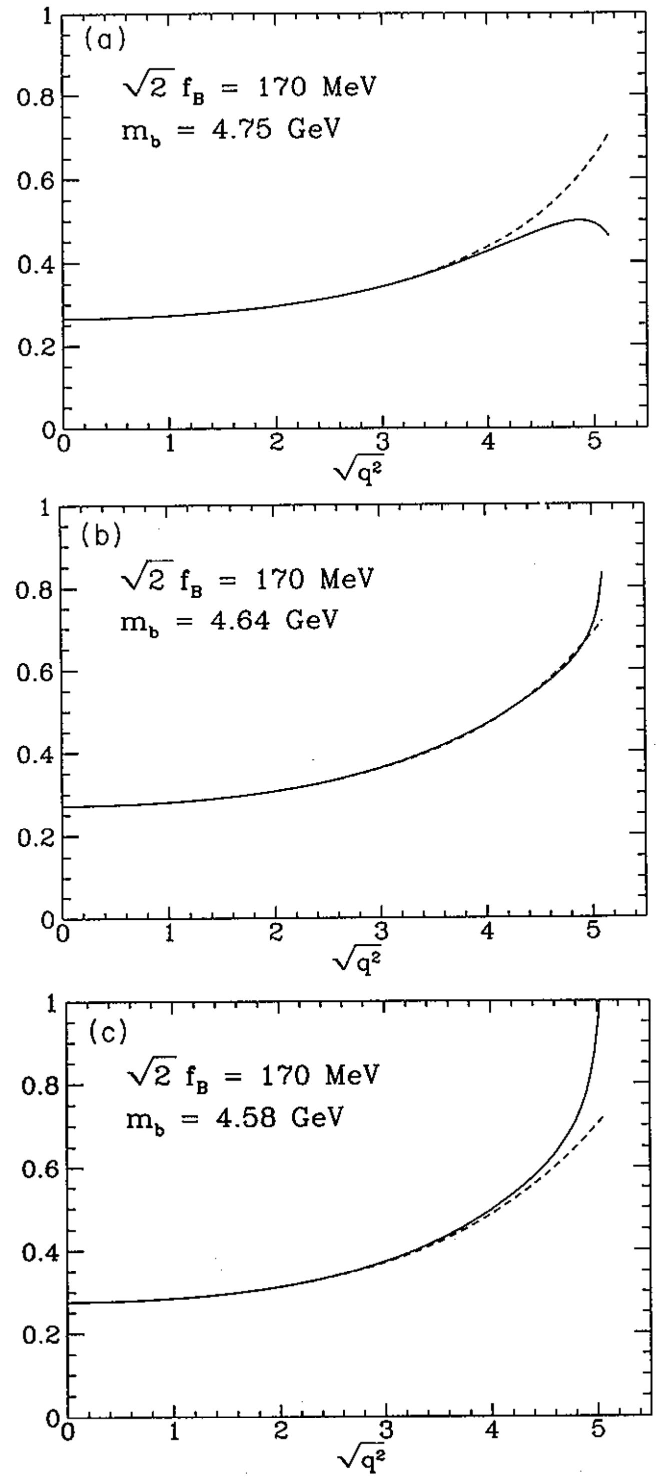

FIG. 1. The reduced form factor $(M_{B^*}^2 - q^2)f_+(q^2)$ for the transition $B \to \pi$. The solid line denotes the result of the RQM and the dashed line the result of the VMD model. Results are given for three different parameter sets: (a) $\sqrt{2}f_B = 170$ MeV, $m_b = 4.75$ GeV; (b) $\sqrt{2}f_B = 170$ MeV, $m_b = 4.64$ GeV; (c) $\sqrt{2}f_B = 170$ MeV, $m_b = 4.58$ GeV.

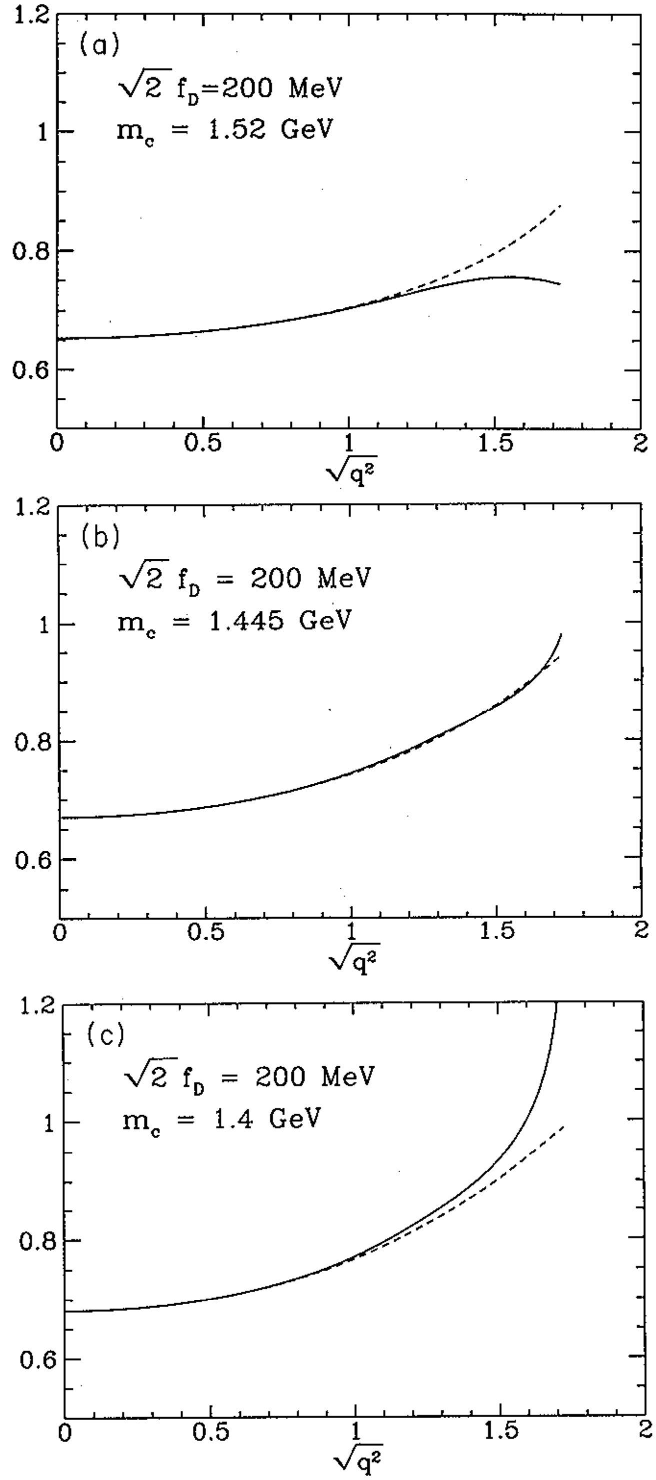

FIG. 2. The reduced form factor $(M_{D^*}^2 - q^2)f_+(q^2)$ for the transition $D \to \pi$. The solid line denotes the result of the RQM and the dashed line the result of the VMD model. Results are given for three difficult parameter sets: (a) $\sqrt{2}f_D = 200$ MeV, $m_c = 1.52$ GeV; (b) $\sqrt{2}f_D = 200$ MeV, $m_c = 1.445$ GeV; (c) $\sqrt{2}f_D = 200$ MeV, $m_c = 1.40$ GeV.

tween the two versions of the reduced form factors can be achieved. For the ranges $140 \leq \sqrt{2}f_B \leq 230$ MeV and $160 \leq \sqrt{2}f_D \leq 230$ MeV we find the heavy-quark masses $m_b = 4.625 \pm 0.015$ GeV and $m_c = 1.42 \pm 0.04$ GeV. The pairs of values of heavy-quark parameters (m_Q, f_P) obtained in this manner can be restricted even further by means of an independent estimate of the function $G(q^2)$. We have presented this analysis in Appendix A, where we generalize the idea of VMD to mean not only the dominance of one pole, but we include also the effect of radially excited states. This mechanism generates a function $G(q^2)$ which agrees with the RQM estimate of Eqs. (4.3)–(4.5) at $q^2 = M_V^2$ only for one pair of parameters.

In this manner we find the following set of parameters for the B, B^* system:

$$m_b = 4.64 \text{ GeV} ,$$

$$\sqrt{2}f_B = 170 \text{ MeV} . \qquad (4.8)$$

The corresponding value of β_B is given in Table I. Additional relations of interest are

$$\sqrt{2}f_{B^*}/M_{B^*} = 185.4 \text{ MeV} ,$$

$$g_{B^*B\pi^0} = g_{B^*B\pi^+}/\sqrt{2} = 16.21 , \qquad (4.9)$$

$$g = g_{B^*B\pi^+}\sqrt{2}f_\pi/M_{B^*} = 0.56 .$$

The parameters that determine the VMD form (4.1) by means of the approximate relation (4.3) are given by

$$\delta = 1.93, \quad \gamma_1 = 11.3 \text{ GeV}, \quad \gamma_2 = 9.5 \text{ GeV} .$$

Evidently the function $G(q^2)$ varies only slowly with q^2.

It is gratifying to see that this approach leads to a value for m_b that is consistent with a recent determination of the heavy-quark mass $m_b = 4.72 \pm 0.05$ GeV in [15]. The value for f_B is within the range established by the lattice calculations of [16], where a high statistics analysis of pseudoscalar decay constants is carried out with the result $\sqrt{2}f_B = 180 \pm 50$ MeV. The coupling constant g has been defined in Eq. (1.6) and is independent of flavor in the heavy-quark limit.

For the D, D^* system we find, in the same manner,

$$m_c = 1.445 \text{ GeV} ,$$

$$\sqrt{2}f_D = 200 \text{ MeV} . \qquad (4.10)$$

The value for β_D is listed in Table I. Additional relations are

$$\sqrt{2}f_{D^*}/M_{D^*} = 248.9 \text{ MeV} ,$$
$$g_{D^*D\pi^0} = g_{D^*D\pi^+}/\sqrt{2} = 6.21 , \qquad (4.11)$$
$$g = g_{D^*D\pi^+}\sqrt{2}f_\pi/M_{D^*} = 0.57 .$$

The function $G(q^2)$ is determined in terms of the parameters

$$\delta = 0.62, \quad \gamma_1 = 30.27 \text{ GeV}, \quad \gamma_2 = 3.51 \text{ GeV} .$$

The value for f_D again is consistent with the results of the lattice calculations in [16]: $\sqrt{2}f_D = 170 \pm 30$ MeV. The value of g given in (4.10) is consistent with the value $|g| = 0.61 \pm 0.22$ obtained by the approach that incorporates both heavy-quark symmetry and chiral symmetry in [3].

The values for γ_1 given above seem to be incompatible with heavy-quark symmetry: The first derivative of the RQM form factor $f_+(q^2)$ calculated at $q^2 = 0$ scales as $\Lambda_1 \sim m_Q^{3/4}$, where m_Q is the mass of the heavy quark. In the heavy-quark limit $(1+\delta)/\delta \to 1$ and $\gamma_1 \to \Lambda_1$, i.e., γ_1 scales also as $\gamma_1 \sim m_Q^{3/4}$. However, γ_1 is defined in Eq. (4.5) as the difference of two terms whose scaling behavior is not very different. This means that a large heavy-quark mass is required to suppress the corrections to the heavy-quark limit $\gamma_1 = \Lambda_1 \sim m_Q^{3/4}$. In fact, for $D \to \pi$ $1/\gamma_1^2$ is close to zero, which leads to the large value for γ_1. For $B \to \pi$ γ_1 gets closer to Λ_1 (note that $\Lambda_1 = 1.71, 3.96$ GeV for $D, B \to \pi$, respectively), but even m_b is too light for scaling to set in. Thus the results found for γ_1 are in accord with the pattern that is expected for values of the heavy-quark mass below the region where scaling rules are meaningful.

In Figs. 1(b) and 2(b) we have plotted the reduced RQM versus the VMD form factor for the parameter sets (4.8) and (4.10), respectively. The two versions of the form factor match well for all values of q^2. The slight deviation for very large values of q^2 is due to the fact that an analytic continuation of the RQM form factor to timelike momentum transfer is possible only for $q^2 \leq Q_0^2$. For the parameters given in Table I we have, according to the definition (2.18),

$$\sqrt{Q_0^2} = \begin{cases} 5.14 \text{ GeV} & \text{for } B \to \pi , \\ 1.79 \text{ GeV} & \text{for } D \to \pi . \end{cases}$$

The close proximity of Q_0^2 to $q^2_{\rm max}$ seems to be related to the need for the (analytically continued) RQM form factors to reproduce the dominant effect of the vector resonance (B^* or D^*) near $q^2_{\rm max}$. Such a feature has been observed also in [17], where the similarity between the free constituent quark model and the VMD model description of certain pion properties was noted.

We did not attempt to obtain a fine-tuned set of parameters, since for values of q^2 very close to Q_0^2 the RQM calculation of $f_+(q^2)$ depends sensitively on the details of the wave function for the $q\bar{q}$ bound state, and we do not expect that our ansatz is reliable there. However, the form factor is predicted there by the VMD model.

V. NUMERICAL RESULTS

A. The semileptonic transitions $D \to \pi, \rho$ and $B \to \pi, \rho$

We have determined the heavy-quark parameters in Sec. IV by the requirement that the predictions of the

RQM and the VMD model for the form factors of the decays $B \to \pi e\nu$ and $D \to \pi e\nu$ are consistent, and at this stage the RQM is completely defined in terms of the parameters given in Table I, or the equivalent parametrization given in Eqs. (4.8) and (4.10). Based on this parametrization we have used the RQM to analyze also the decays $B \to \rho e\nu$ and $D \to \rho e\nu$. Our results for the form factors at $q^2 = 0$ are collected in Tables II and III, and are compared with the results obtained by means of QCD sum rules in [7] and of the popular BSW approach [4]. A more detailed discussion of the predictions of several other models can be found in [7]. The corresponding results for decay rates are given in Tables IV and V.

Our results for the form factors for D decays at $q^2 = 0$ are very similar to the results of the QCD sum-rule approach, but the rates are quite different and reflect the differences in the q^2 dependence of the form factors. In the following we compare our results for the ratios of branching fractions with the data:

$$R_\pi = \frac{\Gamma(D^+ \to \pi^0 l\nu)}{\Gamma(D^+ \to \bar{K}^0 l\nu)} = \begin{cases} 0.042 \ \ \text{(this work)}\ , \\ 0.085 \pm 0.027 \pm 0.014 \ \ \text{(CLEO [19])}\ , \\ 0.055^{+0.03}_{-0.015} \pm 0.05 \ \ \text{(Mark III [18])}\ , \end{cases}$$

and

$$\frac{\Gamma(D^+ \to \rho^0 l\nu)}{\Gamma(D^+ \to \bar{K}^{*0} l\nu)} = \begin{cases} 0.030 \ \ \text{(this work)}\ , \\ 0.044^{+0.031}_{-0.025} \pm 0.014 \ \ \text{(E653 [20])}\ . \end{cases}$$

The predicted value of R_π is somewhat smaller than the recent CLEO data. The obvious remedy would seem to be to increase the D^* signal by increasing $g_{D^*D\pi}$ (which would require smaller values for the parameter f_D). But this choice would increase also the strength of the pionic transition $D^* \to D\pi$, and this would bring us in conflict with the experimental upper bound for $\Gamma_{\rm tot}(D^{*+})$. However, this is a minor point at present, since the data still have rather large errors.

For B decays our results at $q^2 = 0$ resemble those of Bauer, Stech, and Wirbel (BSW) [4]. This is as expected, since BSW work in the infinite momentum frame, which leads to practically the same results as the light-front formalism we used in this work, but BSW employ an approximation for the wave function which agrees with our ansatz only in the heavy-quark limit. Furthermore, BSW postulate the monopole form factor as given in (1.2), which again generates rates different from ours. Our form factors at $q^2 = 0$ for $B \to \rho$ transitions are lower than those obtained by the traditional application of QCD sum rules in [7], but agree with the light-cone QCD sum-rule approach of [21] ([21] contains a comparative discussion of the two QCD sum-rule methods). Furthermore, we find form factors $V(q^2)$, $A_1(q^2)$, $A_2(q^2)$ that increase monotonically with q^2. This result is consistent with that of [21], but partly disagrees with that of [7].

B. The semileptonic transitions $D \to K, K^*$ and $B \to D, D^*$

We have already analyzed semileptonic $D \to K, K^*$ and $B \to D, D^*$ transitions in [11,22]. The parametrization chosen in the present work and the refined treatment of the q^2 dependence of form factors lead to somewhat different results, which we shall present below and compare with the most recent data.

The CLEO Collaboration [23] has measured all four $D \to K(K^*)l\nu$ decays in one experiment, with results that are in excellent agreement with previous measurements. The predictions of our model (we have used the value $|V_{cs}| = 0.9743$) fully agree with these data, as the following comparison shows:

TABLE II. Form factors for semileptonic $D \to \pi, \rho$ transitions at $q^2 = 0$.

	$f_+(0)$	$V(0)$	$A_1(0)$	$A_2(0)$
This work	0.67	0.93	0.58	0.42
Ball [7]	0.5(1)	1.0(2)	0.5(2)	0.4(2)
BSW [4]	0.69	1.23	0.78	0.92

TABLE III. Form factors for semileptonic $B \to \pi, \rho$ transitions at $q^2 = 0$.

	$f_+(0)$	$V(0)$	$A_1(0)$	$A_2(0)$
This work	0.27	0.35	0.26	0.24
Ball [7]	0.26(2)	0.6(2)	0.5(1)	0.4(2)
ABS [19]		0.28(6)	0.24(4)	
BSW [4]	0.33	0.33	0.28	0.28

$$f_+(0) = \begin{cases} 0.78 \ \text{(this work)} , \\ 0.77 \pm 0.04 \ \text{(CLEO)} , \end{cases}$$

$$\Gamma(D \to K l \nu) = \begin{cases} 9.60 \times 10^{10} \ \text{s}^{-1} \ \text{(this work)} , \\ (9.1 \pm 0.3 \pm 0.6) \times 10^{10} \ \text{s}^{-1} \ \text{(CLEO)} , \end{cases}$$

$$\Gamma(D \to K^* l \nu) = \begin{cases} 5.50 \times 10^{10} \ \text{s}^{-1} \ \text{(this work)} , \\ (5.7 \pm 0.7) \times 10^{10} \ \text{s}^{-1} \ \text{(CLEO)} , \end{cases}$$

and

$$R = \frac{\Gamma(D \to K^* l \nu)}{\Gamma(D \to K l \nu)} = \begin{cases} 0.57 \ \text{(this work)} , \\ 0.62 \pm 0.08 \ \text{(CLEO)} . \end{cases}$$

The K^* polarization as defined by the ratio of the longitudinal to the transverse partial widths is

$$\frac{\Gamma_L(D \to K^* l \nu)}{\Gamma_T(D \to K^* l \nu)} = \begin{cases} 1.326 \ \text{(this work)} , \\ 1.18 \pm 0.18 \pm 0.08 \ \text{(E653 [24])} , \\ 1.20 \pm 0.13 \pm 0.13 \ \text{(E687 [25])} . \end{cases}$$

For the sake of completeness we give also our results at $q^2 = 0$ for the form factors that govern $D \to K, K^*$ transitions and compare them with former quark model calculations in Table VI. While our predictions are in agreement with the experimental results, given also in Table VI, they partly disagree considerably with the results of [2,4,5,26]. In particular, the ratio R, which in the older work is about a factor of 2 larger than the experimental value, is predicted correctly in the light-front analysis. We have argued in Appendix B that the light-front formalism which we use in this work is basically different from the usual instant-form formalism. Therefore, we expect that results obtained if the light-front and if the instant-form approach is used can be quite different. Only the BSW model of [4] can be compared at all with our RQM. But BSW calculate the form factors $V(0)$, $A_1(0)$, $A_2(0)$ by taking matrix elements of the transverse components of the currents, which in the infinite momentum frame formalism are "bad" operators, and their determination in the BSW approach is considered even by Bauer and Wirbel [4] to be uncertain. This is the origin of the discrepancy between the results of the two models for $D \to K^*, \rho$ transitions, since we determine all form factors uniquely in the RQM by taking only matrix elements of the plus components of the currents. This subtle point is of only minor importance for $B \to D^*, \rho$ transitions since m_b is large.

The semileptonic $B \to D, D^*$ transitions are used to measure V_{cb}. The CLEO collaboration [28] has recently determined the partial width

$$\Gamma(B \to D^* l \nu) = [29.9 \pm 1.9(\text{stat}) \pm 2.7(\text{syst}) \pm 2.0(\text{lifetime})] \times 10^9 \ \text{s}^{-1} \tag{5.1}$$

based on the average of the LEP and CDF measurements of the lifetimes [29]

$$\tau_{B^+} = 1.68 \pm 0.12 \ \text{ps}, \quad \tau_{B^0} = 1.53 \pm 0.09 \ \text{ps} .$$

CLEO uses the HQET approach of [30] to derive for the CKM matrix element

$$|V_{cb}| = 0.0362 \pm 0.0019(\text{stat}) \pm 0.0020(\text{syst}) \pm 0.0014(\text{model}) \ \text{(HQET)} . \tag{5.2}$$

TABLE IV. Decay rates for semileptonic $D^0 \to \pi^+, \rho^+$ transitions in units 10^{10} s^{-1}. Listed also is the ratio of transverse and longitudinal decay rates Γ_L and Γ_T. The value $|V_{cd}| = 0.220$ has been used to calculate the rates.

	$\Gamma(D^0 \to \pi^+)$	$\Gamma(D^0 \to \rho^+)$	Γ_L/Γ_T
This work	0.80	0.33	1.22
Ball [7]	0.39(3)	0.12 (3)	1.31(11)
BSW [4]	0.68	0.67	0.91

TABLE V. Decay rates for semileptonic $B^0 \to \pi^+, \rho^+$ transitions in units $|V_{ub}|^2 \times 10^{13}$ s^{-1}. Listed also is the ratio of transverse and longitudinal decay rates Γ_L and Γ_T.

	$\Gamma(B^0 \to \pi^+)$	$\Gamma(B^0 \to \rho^+)$	Γ_L/Γ_T
This work	1.00	1.91	0.82
Ball [7]	0.51(11)	1.2(4)	0.06(2)
BSW [4]	0.74	2.6	1.34

TABLE VI. Form factors for semileptonic $D \to K, K^*$ transitions at $q^2 = 0$. Listed also is the ratio $R = \Gamma(D \to K^* l\nu)/\Gamma(D \to K l\nu)$.

	$f_+(0)$	$V(0)$	$A_1(0)$	$A_2(0)$	R
This work	0.78	0.66	0.43	1.04	0.57
ISGW [2]	0.8	0.8	0.8	1.1	1.08
BSW [4]	0.76	0.88	1.2	1.3	1.14
KS [5]	0.7	0.82	0.8	0.8	0.96
AW/GS [26]	0.7	0.8	0.6	1.5	1.34
E687-93 [27]		0.59 ± 0.05	0.46 ± 0.11	1.0 ± 0.3	
Expt. ave. [27]	0.75 ± 0.03	0.56 ± 0.04	0.40 ± 0.08	1.1 ± 0.2	

Our model predicts the rate

$$\Gamma(B \to D^* l\nu) = |V_{cb}|^2 \times 25.33 \times 10^{12}\ \text{s}^{-1} \qquad (5.3)$$

and from the experimental partial width (5.1) we can derive V_{cb}:

$$|V_{cb}| = 0.0344 \pm 0.0023 \ \ \text{(this work)}\ , \qquad (5.4)$$

where the error is a combination of the different uncertainties in (5.1). The values for V_{cb} given in (5.2) and (5.4) are obviously consistent, and the same is true for the predictions of the models of ISGW [2], BSW [4], and KS [5], giving $|V_{cb}| = 0.0348(23), 0.0369(23), 0.0340(23)$, respectively.

The vector to pseudoscalar ratio that we find compares well with the CLEO and ARGUS data

$$\frac{\Gamma(B \to D^* l\nu)}{\Gamma(B \to D l\nu)} = \begin{cases} 2.64 \ \ \text{(this work)}\ , \\ 2.6^{+1.1+1.0}_{-0.6-0.8} \ \ \text{(CLEO) [31])}\ , \\ 2.9^{+1.3+0.5}_{-0.6-0.3} \ \ \text{(ARGUS [32])}\ . \end{cases}$$

Finally we list our values for the form factors that govern $B \to D, D^*$ transitions at $q^2 = 0$:

$$f_+(0) = 0.69, \quad V(0) = 0.81\ ,$$

$$A_1(0) = 0.69, \quad A_2(0) = 0.64\ .$$

In closing we note that the predicted rates for B decays as given above depend very little on the value of the parameter f_B.

C. Radiative and pionic transitions $D^* \to D\gamma$, $D^* \to D\pi$, and $B^* \to B\gamma$

The rate for the decay $V \to P\gamma$ is given by

$$\Gamma = \frac{1}{3}\alpha g_{VP\gamma}^2 \left(\frac{M_V^2 - M_P^2}{2M_V}\right)^3 . \qquad (5.5)$$

We assume that the transition takes place between S-state mesons, which have identical orbital wave functions. The coupling constant $g_{VP\gamma}$ can be expressed in terms of the loop integral $I_3(m, m_Q)$ which has been calculated in [12] and is given by

$$I_3(m, m_Q) = \frac{N_c}{(2\pi)^3} \int d^3p |\Phi(M_0^2)|^2 \frac{2}{M_0^2 - (m_Q - m)^2} \times \frac{1}{x} \left\{(1-x)m + x m_Q + \frac{p_\perp^2}{M_0 + m + m_Q}\right\} . \qquad (5.6)$$

Using the parametrization of Sec. IV we obtain for the respective coupling constants

$$\begin{aligned} g_{D^{*+}D^+\gamma} &= Q_c I_3(m_c, m) + Q_d I_3(m, m_c) \\ &= -0.30\ \text{GeV}^{-1}\ , \\ g_{D^{*0}D^0\gamma} &= Q_c I_3(m_c, m) + Q_u I_3(m, m_c) \\ &= 1.85\ \text{GeV}^{-1}\ , \\ g_{B^{*+}B^+\gamma} &= Q_u I_3(m, m_b) + Q_b I_3(m_b, m) \\ &= 1.40\ \text{GeV}^{-1}\ , \\ g_{B^{*0}B^0\gamma} &= Q_d I_3(m, m_b) + Q_b I_3(m_b, m) \\ &= -0.80\ \text{GeV}^{-1}\ , \end{aligned} \qquad (5.7)$$

where Q_q is the charge of quark q. The rate for the decay $D^* \to D\pi$ is given by Eq. (3.1) and the coupling constant has been derived by means of a soft-pion theorem, with the result given in Eq. (4.10):

$$g_{D^{*+}D^+\pi^0} = g_{D^{*0}D^0\pi^0} = g_{D^{*+}D^0\pi^+}/\sqrt{2} = 6.21\ .$$

Using the equations for the rates we finally obtain the predictions of our model:

$$\Gamma[D^{*+} \to (D\pi)^+] = 91.39\ \text{keV}\ ,$$

$$\Gamma(D^{*+} \to D^+\gamma) = 0.56\ \text{keV}\ ,$$

$$\Gamma_{\text{tot}}(D^{*+}) = \Gamma[D^{*+} \to (D\pi)^+] + \Gamma(D^{*+} \to D^+\gamma) = 91.95\ \text{keV}\ ,$$

$$\Gamma(D^{*0} \to D^0\pi^0) = 43.40\ \text{keV}\ ,$$

$$\Gamma(D^{*0} \to D^0\gamma) = 21.69\ \text{keV}\ ,$$

$$\Gamma_{\text{tot}}(D^{*0}) = \Gamma(D^{*0} \to D^0\pi^0) + \Gamma(D^{*0} \to D^0\gamma) = 65.09\ \text{keV}\ , \qquad (5.8)$$

TABLE VII. Branching ratios in % for radiative and pionic D^* decays.

Decay mode	This work	CLEO [30]
$D^{*+} \to D^+\gamma$	0.61	$1.1 \pm 1.4 \pm 1.6$
$D^{*+} \to D^+\pi^0$	30.97	$30.8 \pm 0.4 \pm 0.8$
$D^{*+} \to D^0\pi^+$	68.42	$68.1 \pm 1.0 \pm 1.3$
$D^{*0} \to D^0\gamma$	33.33	$36.4 \pm 2.3 \pm 3.3$
$D^{*0} \to D^0\pi^0$	66.67	$63.6 \pm 2.3 \pm 3.3$

and

$$\Gamma(B^{*+} \to B^+\gamma) = 0.429 \text{ keV} ,$$

$$\Gamma(B^{*0} \to B^0\gamma) = 0.142 \text{ keV} . \tag{5.9}$$

The D^* lifetimes have not been measured yet, but our predicted D^{*+} lifetime is close to the experimental upper limit $\Gamma_{\text{tot}}(D^{*+}) < 131$ keV [33]. We compare the resulting branching ratios for D^* decays with the respective data from the CLEO Collaboration [34] in Table VII. Again the predictions of our model are in agreement with the data.

VI. CONCLUDING REMARKS

The relativistic quark model, based originally on the light-front formalism, has been extended to the treatment of decay processes with a timelike momentum transfer, and has been combined with a soft pion theorem. This approach enables the calculation of coupling constants and form factors for a great variety of decay processes involving heavy mesons. The crucial step, the determination of the parameters of the RQM, in particular the values of the constituent masses of the c and b quarks, is accomplished by making use of the duality of the VMD and the RQM approaches in the analysis of the transitions $B, D \to \pi$. The q^2 dependence of the form factors provided by the RQM is different from that in the usual pole approximation. We have compared our predictions with a large body of data for decays of D and D^* mesons, and found agreement without exception at the present level of accuracy of the data. It seems that properties of heavy mesons can be analyzed in a simple and consistent manner in the framework provided by this relativistic quark model. We think that such a phenomenological method has significant advantages as an alternative to the heavy-quark effective theory, in particular in view of indications [35] that charmed and even beauty mesons may not be heavy enough for the HQET to permit reliable predictions.

ACKNOWLEDGMENTS

I would like to thank C. Greub, G. Rasche, W. S. Woolcock, and D. Wyler for helpful discussions. In particular I acknowledge the kind assistance of C. Greub in all questions concerning the numerical analysis.

APPENDIX A: GENERALIZATION OF THE VMD MODEL (GVMD)

The form factor $f_+(q^2)$ for the decay $B^0 \to \pi^+ l\nu$ is dominated at $q^2 = q^2_{\text{max}}$ by the B^* pole and has the form given by Eq. (1.3). We have argued that this form has only a limited range of validity. We expect a more extended range of validity if the idea of VMD is generalized by including the effect of B^* radial excitations. This assumption leads to the generalized vector meson dominance (GVMD) formula

$$f_+^{\text{GVMD}}(q^2) = \sum_{n=1} \frac{a_n}{1 - q^2/M_n^2} , \tag{A1}$$

where M_n is the mass of the nS-state meson B_n^* and

$$a_n = \sqrt{2} g_{B_n^* B\pi^+} f_{B_n^*} / M_n^2 . \tag{A2}$$

Analogous equations hold for $D \to \pi$ transitions.

In order to calculate the coupling constants $g_{B_n^* B\pi}$ and $f_{B_n^*}$ from Eqs. (3.5), (3.6), and (4.2), we use harmonic-oscillator wave functions for the orbital nS states, which have the form

$$\Phi_n = N_n H_{2n-1}(z) e^{-z^2/2}/z \tag{A3}$$

for $n = 1, 2, 3, \ldots$, where H_m is a Hermite polynomial and $z^2 = p^2/\beta^2_{\text{HOM}}$. The normalization factor N_n is determined by the normalization condition (2.10). The value of β_{HOM} for $B, B^*(D, D^*)$ mesons and its radial excitations is fixed for a given parameter set (m_Q, f_P). The $n = 1$ S-state wave function of Eq. (A3) is different from the ansatz chosen in Eq. (2.10), except for mesons composed of equal mass quarks; in that case the two representations are equivalent. However, the coupling constants and the values of $f_+(q^2)$ and its derivatives at $q^2 = 0$, calculated for the purpose of this appendix on the basis of Eq. (A3) are only slightly different from those calculated on the basis of Eq. (2.10), and our conclusions are practically independent of the representation chosen for the wave function.

In order to calculate the quantitative consequences of the GVMD model we need at least approximate values for the masses of the radially excited states. In [36] a relativistic quark potential model has been used to describe the mass spectrum of mesons, and the masses of the first radial excitations of B^* and D^* are found to be $M_2(2S) = 5.93$ GeV and $M_2(2S) = 2.68$ GeV, respectively. Once the masses M_1 and M_2 are known, one can estimate the masses of the higher states by means of the mass formula for the relativistic harmonic oscillator

$$M_n = \sqrt{m^2 + \omega_n^2} + \sqrt{m_Q^2 + \omega_n^2} ,$$

$$\omega_{n+1}^2 = \omega_n^2 + \text{const} .$$

We have given the coupling constants and expansion parameters for a fixed value of the parameter set (m_Q, f_P) for B, B^* in Table VIII and for D, D^* in Table IX.

TABLE VIII. Coupling constants $g_{B_n^* B\pi^0}$, $f_{B_n^*}$ and expansion coefficients a_n for nS-state mesons B_n^* for the RQM parameter pair $(m_b, \sqrt{2}f_B) = (4.64$ GeV, 170 MeV).

n	1	2	3	4	5	6
$\sqrt{2}f_{B_n^*}/M_n$ (MeV)	185	202	219	227	235	240
$g_{B_n^* B\pi^0}$	15.06	−4.42	2.23	−1.35	0.90	−0.63
a_n	0.74	−0.21	0.11	−0.06	0.04	−0.03

Although the inclusion of excited states manifests as an effective damping of the monopole form factor, this contribution is not sufficient to account for the behavior of $f_+(q^2)$ for small values of q^2. We expect the GVMD model to work more reliably close to the B^* or D^* pole, where the effect of the generalized form (A1) can be interpreted as a correction of the monopole form (1.3), which is given by

$$f_+^{\rm GVMD}(q^2) = \frac{a_1}{1-q^2/M_1^2}[1-(1-q^2/M_1^2)\kappa + O((1-q^2/M_1^2)^2)] \;, \tag{A4}$$

where

$$\kappa = -\sum_{n=2} \frac{a_n/a_1}{1-M_1^2/M_n^2} \;. \tag{A5}$$

If Eq. (A5) is compared with the interpolating form (4.1) one finds the relation

$$\kappa = G(M_1^2) \;. \tag{A6}$$

We have compared the results obtained for both sides of Eq. (A6) using Eqs. (4.3) and (A5), respectively, for different sets of heavy-quark parameters. For (m_b in GeV, $\sqrt{2}f_B$ in MeV) =(4.625,155), (4.64,170), (4.635,185), (4.635.200) we find κ = 1.1, 1.2, 1.2, 1.3 and $G(M_1^2)$ = 0.8, 1.2, 1.6, 2.0, respectively. For $(m_c, \sqrt{2}f_D)$ = (1.433,190), (1.445,200), (1.452,210) we find κ = 0.43, 0.44, 0.45 and $G(M_1^2)$ = 0.21, 0.47, 0.73, respectively. Evidently the two methods agree only for

$$\sqrt{2}f_B \simeq 170 \text{ MeV} \quad \text{and} \quad \sqrt{2}f_D \simeq 200 \text{ MeV} \;.$$

The precision that is required in order to have the GVMD quantity κ agree with $G(M_1^2)$ and the sharp values for the parameters obtained in this manner might be surprising. However, when κ and $G(M_1^2)$ have been calculated above for different sets of heavy-quark parameters, the values of the meson masses M_n have been kept fixed. We expect that if the dependence of M_n on the heavy-quark parameters would be accounted for correctly then κ and $G(M_1^2)$ would possibly match over a wider range of parameters, and in particular $G(M_1^2)$ would depend less sensitively on the parametrization.

TABLE IX. Coupling constants $g_{D_n^* D\pi^0}$, $f_{D_n^*}$ and expansion coefficients a_n for nS-state mesons D_n^* for the RQM parameter pair $(m_c, \sqrt{2}f_D) = (1.445$ GeV, 200 MeV).

n	1	2	3	4	5	6
$\sqrt{2}f_{D_n^*}/M_n$ (MeV)	248	247	256	255	257	256
$gD_n^* D\pi^0$	6.06	−2.01	1.07	−0.67	0.45	−0.32
a_n	1.06	−0.26	0.12	−0.07	0.04	−0.03

APPENDIX B: WHAT IS SPECIAL ABOUT THE LIGHT-FRONT APPROACH AND THE CONDITION $q^+ = 0$?

The matrix element for the transition between two pseudoscalar mesons is given by

$$\langle P''|\bar{q}''\gamma_\mu q'|P'\rangle = R_\mu \;. \tag{B1}$$

We shall discuss the matrix element (B1) under the condition $q^+ \geq 0$ and in a frame that is defined by

$$P'^+ = M', \quad P'_\perp = 0 \;,$$
$$P''^+ = M' - q^+, \quad P''_\perp = -q_\perp \;. \tag{B2}$$

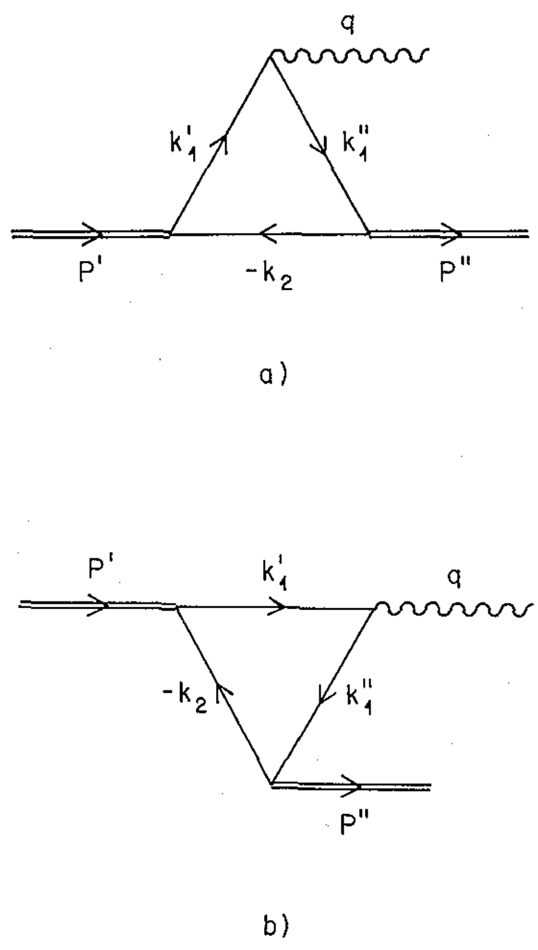

FIG. 3. One-loop diagrams which are ordered with regard to the evolution of the plus components of the momentum lines. They represent the hadronic structure of the amplitude for the weak transition between two mesons.

The matrix element was expressed in Eq. (2.1) in terms of the form factors $f_+(q^2)$ and $f_-(q^2)$ which can be related to its light-front components. If we use

$$R_\perp = Rq_\perp \tag{B3}$$

we find

$$f_+(q^2) = \frac{1}{2M'}(R^+ - Rq^+) ,$$

$$f_-(q^2) = R + \frac{1}{2M'}(R^+ - Rq^+) . \tag{B4}$$

The matrix element (B1) can be represented by the two diagrams of Figs. 3(a) and 3(b), which are ordered with regard to the evolution of the plus components of the momentum lines (from left to right). It is possible to draw four additional diagrams, whose contribution vanishes, however, due to the conservation of the plus components of the momenta at each vertex. The evaluation of the first diagram, Fig. 3(a), is straightforward, since the meson vertices have the usual $q\bar{q}$ structure which can be expressed in terms of the respective wave functions. We choose the following variables for the internal quarks:

$$k_1'^+ = yP'^+, \quad k_{1\perp}' = yP_\perp' + p_\perp' ,$$

$$k_1''^+ = xP''^+, \quad k_{1\perp}'' = xP_\perp'' + p_\perp'' ,$$

$$k_2^+ = (1-x)P''^+, \quad k_{2\perp} = (1-x)P_\perp'' - p_\perp'' ,$$

where

$$p_\perp'' = p_\perp' - (1-y)P_\perp' + (1-x)P_\perp'' ,$$

$$y = 1 - (1-x)P''^+/P'^+ ,$$

and the ranges of the momentum fractions x, y are

$$0 \le x \le 1, \quad y_0 \le y \le 1 ,$$

with $y_0 = q^+/P'^+$. The kinematic invariant masses are given by

$$M_0'^2 = \frac{p_\perp'^2 + m_1'^2}{y} + \frac{p_\perp'^2 + m_2^2}{1-y} ,$$

$$M_0''^2 = \frac{p_\perp''^2 + m_1''^2}{x} + \frac{p_\perp''^2 + m_2^2}{1-x} .$$

The contribution of the first diagram, Fig. 3(a), to the matrix element (B1) is

$$R_\mu(4a) = \frac{N_c}{16\pi^3}\int_{y_0}^1 dy \int d^2p_\perp' \Omega(p'', p') \frac{E_1'E_2'}{M_0'} \frac{S_\mu}{xy(1-y)} , \tag{B5}$$

where $\Omega(p'', p')$ is given by Eq. (2.13) and

$$4E_1'E_2' = M_0'^2 - (m_1'^2 - m_2^2)^2/M_0'^2 .$$

In the notation of [11] the trace S_μ is given by

$$S_\mu = \mathrm{tr}\{\gamma_5(-\gamma k_2 + m_2)\gamma_5(\gamma k_1'' + m_1'')\gamma_\mu(\gamma k_1' + m_1')\}$$

and all quarks are on their respective mass shells. The light-front components of S_μ in the system of reference (B2) are

$$S^+ = 2M'\Bigg\{ y[M_0''^2 - (m_1'' - m_2)^2] + x\frac{P''^+}{M'}[M_0'^2 - (m_1' - m_2)^2]$$

$$+ (1-y)\left[q^2 + \frac{q^+}{M'}(M_0'^2 - M'^2) - \frac{q^+}{P''^+}(M_0''^2 - M''^2) - (m_1' - m_1'')^2\right]\Bigg\} , \tag{B6}$$

$$S_\perp = -2q_\perp[M_0'^2 - (m_1' - m_2)^2] + 2p_\perp'\Bigg[M_0''^2 - (m_1'' - m_2)^2 + M_0'^2 - (m_1' - m_2)^2$$

$$+ (m_1' - m_1'')^2 - q^2 - \frac{q^+}{M'}(M_0'^2 - M'^2) + \frac{q^+}{P''^+}(M_0''^2 - M''^2)\Bigg] . \tag{B7}$$

For $q^+ = 0$ the expression for $f_+(q^2)$ resulting from Eqs. (B4), (B5), and (B6) coincides with Eq. (2.12). Moreover, in this case the contribution of the diagram of Fig. 3(b) vanishes, again due to the conservation of the plus component. Thus $f_+(q^2)$ can be calculated straightforwardly from the diagram of Fig. 3(a) only when $q^+ = 0$ (and therefore $q^2 \leq 0$). This condition is clearly crucial for the calculation of form factors in the light-front formalism.

In order to illustrate the role of the process represented in Fig. 3(b), we consider the transitions $B \to \pi$ and $D \to \pi$, for which $M' \gg M''$, and values of q^2 close to $q^2_{\max} = (M' - M'')^2$. It is commonly believed, that for high positive values of q^2 the form factor $f_+(q^2)$ is dominated by resonant structures. The descriptions of the same process by a valence quark picture and by a VMD picture are quite complementary, but the light-front formalism provides some insight into how the two pictures are linked. At $q^2 = q^2_{\max}$ the contribution of the diagram in Fig. 3(a) to the form factor $f_+(q^2)$ is very small, since $q^+ = M' - M''$ at $q^2 = q^2_{\max}$, and the value of $y_0 = q^+/M'$, the lower limit of the interval over y in Eq. (B5), is then close to 1. For a quantitative comparison we have calculated $f_+(q^2)$ at $q^2 = q^2_{\max}$ for $B^0 \to \pi^+ e\bar{\nu}$, and quote its value together with the result obtained from the interpolating VMD formula (4.1)

$$f_+(q^2_{\max}) = \begin{cases} 0.24 & (\text{diagram } 4a)\ , \\ 10.69 & (\text{VMD})\ . \end{cases}$$

For the numerical evaluation we have used the parameters given in Table I and in (4.8) and (4.10).

Obviously the dominant contribution to the form factors for large values of q^2 must be due to the diagram of Fig. 3(b). Unfortunately its evaluation is hampered by the fact that the vertex for the outgoing meson cannot be expressed directly in terms of a valence quark wave function. One of the quark lines must first be "turned around" by the exchange of one or more gluons. In lowest order this process corresponds to the two-loop diagrams of Fig. 4: the gluon emitted by either the light spectator quark or the heavy-quark creates a $q\bar{q}$ pair such that a wave function can be used at the vertex for the outgoing meson.

However, the treatment of explicit gluon exchange between constituent quarks goes beyond the valence quark picture, which is the basic assumption of the approach used in this paper. Moreover, a perturbative treatment would not be expected to be adequate. In particular the diagram of Fig. 4(b) belongs to the subset of graphs which are characterized by the exchange of gluons between those constituents, which annihilate into the current. The summation of this subset of diagrams would be expected to generate the resonant structures which dominate the form factor $f_+(q^2)$ for large positive values of q^2, according to the generalized VMD relation which we have used.

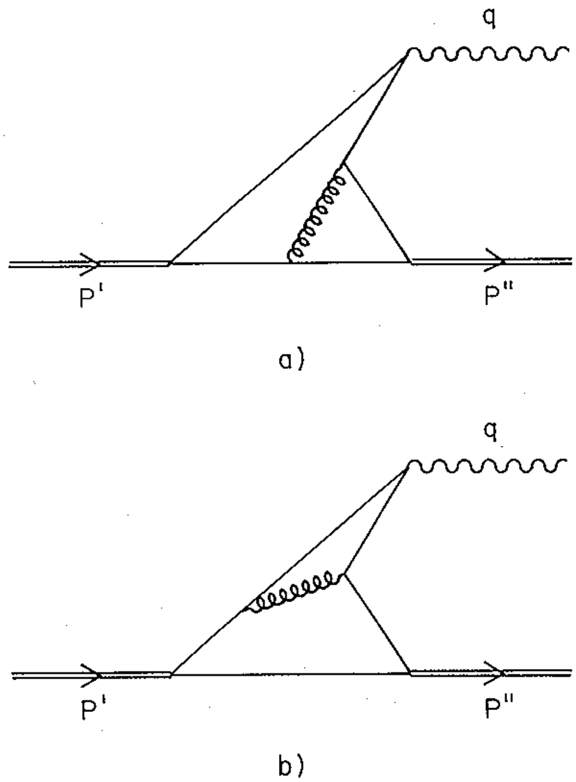

FIG. 4. Two-loop diagrams for the same process as in Fig. 3.

Obviously, the calculation of the form factor $f_+(q^2)$ from the diagram of Fig. 3(a) in the framework of the light-front formalism under the condition $q^+ = 0$ is basically different from the corresponding calculation in the instant-form formalism [in the latter case Fig. 3(a) represents a time-ordered diagram]. A comparison might serve as an illustration: The former is the light-front analog of the nonrelativistic calculation of the photodisintegration of the deuteron using Siegert's theorem [37], while the latter is analogous to the calculation without using Siegert's theorem.

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