

Form factors of the semileptonic decays $B(D) \rightarrow \pi + l\nu$ near zero recoil

Hong-Ying Jin

International Centre of Theoretical Physics, P.O. Box 586, Trieste 34100, Italy

(Received 19 June 1995)

We discuss the possible corrections of some recent calculations on the form factors which parametrize the matrix element of the decays $B(D) \rightarrow \pi l\nu$. These calculations are based on the assumption that the $B^*(D^*)$ pole contribution is fully dominant near zero recoil. By using the Bethe-Salpeter approach, we calculate some low-lying state pole terms predicted by the quark model. The numerical results are obtained to the next-leading order of the $1/M$ expansion.

PACS number(s): 12.39.Hg, 13.20.He

Compared with V_{cb} , V_{ub} is much more poorly determined, because studying the weak decay $b \rightarrow u$ is more difficult than $b \rightarrow c$ both theoretically and experimentally. The heavy quark effective theory provides a way to determinate V_{cb} [1], which, however, cannot be used in the decay $b \rightarrow u$. On the other hand, in the experiment, it is only the end-point region of charmless B decays that is not buried in the huge background of the charmed decays. The calculations of V_{ub} from such an end-point region in semileptonic B decays so far suffer from large theoretical uncertainties [2] because of the strong nonperturbative effect [3]. It was suggested in [4] that the semileptonic decays $B \rightarrow \pi l\nu$ and $D \rightarrow \pi l\nu$ may precisely determine the ratio V_{ub}/V_{cd} . The basic idea is that, near zero recoil, the form factors which parametrize the matrix element of the decays $B(D) \rightarrow \pi l\nu$ can be related by the spin-flavor symmetry in the heavy quark limit. The applicability of this idea obviously is dependent on symmetry-breaking corrections. So it is necessary to calculate the form factors near the zero recoil up to the next-leading order of the $1/M$ expansion. For this purpose, some calculations have been done recently under the assumption that the $B^*(D^*)$ pole contribution is fully dominant in the process $B(D) \rightarrow \pi l\nu$ in the soft-pion limit [5]. In B meson decay, it is generally believed that such an approximation is good enough, while the transition $D \rightarrow \pi l\nu$ may be affected by some other low-lying excited state poles. However, in order to estimate such pole contributions, one has to use some nonperturbative approaches. On the other hand, in [5], the form factors have been expressed in terms of some decay constants and coupling constants. These parameters still have to be obtained by some nonperturbative methods before the experimental data are sufficient to determine them.

A formalism by the $1/M$ expansion of the relativistic Bethe-Salpeter (BS) equation for the heavy mesons has been developed in [6–9]. With some assumed kernel, it can be used to calculate some parameters and form factors in the $1/M_Q$ expansion. It has been shown in [8] in the leading-order of $1/M_Q$ expansion that, if the interaction becomes independent of the heavy quark spin as $M_Q \rightarrow \infty$, the number of independent components of the most general form of the BS wave functions for heavy mesons of arbitrary spin-parity is reduced from eight to

two. This greatly simplifies the calculations in the leading order. The simple form of the leading-order wave functions also makes the calculations for corrections in the order $O(1/M_Q)$ easier. In the present paper, we shall employ this formalism to estimate the contributions from some low-lying state poles to the form factors which parametrize the semileptonic decays $B(D) \rightarrow \pi l\nu$ near zero recoil and give the numerical values of some decay constants and coupling constants. Our calculation closely follows that of [5]. By using the current algebra and PCAC (partial conservation of axial-vector current) relation, we express the form factors in terms of decay constants $f_{M^{ex}}$ (M^{ex} is the low-lying excited state of B and D series which are predicted by the quark model) and coupling constants $g_{MM^{ex}\pi}$ which recently can be obtained more reliably. Then the decay constants and coupling constants are numerically obtained by the BS approach. Finally, some discussions are given.

The matrix element of the current responsible for the decay $B(D) \rightarrow \pi$ may be parametrized in terms of two invariant form factors, in Ref. [10], which was defined as

$$\langle \pi(p) | \bar{q} \gamma^\mu Q | M(v) \rangle = f_+(q^2) \left[(m_M v + p)^\mu - \frac{m_M^2 - m_\pi^2}{q^2} q^\mu \right] + f_0(q^2) \frac{m_M^2 - m_\pi^2}{q^2} q^\mu, \quad (1)$$

where $q = m_M v^\mu - p$ and $f_+(q^2)$ is responsible to determine the ratio V_{ub}/V_{cd} [4, 5]. In the following discussion, it is more convenient to work with the velocity of the heavy meson; therefore we define c :

$$\langle \pi(p) | \bar{q} \gamma^\mu Q | M(v) \rangle = f_1(p, v) m_M v^\mu + f_2(p, v) p_t^\mu, \quad (2)$$

where $p_t^\mu = p^\mu - p_l v^\mu$ and $p_l = v \cdot p$.

The two sets of form factors are related by

$$f_+(q^2) = \frac{m_M}{m_M + p_l} f_1 + \frac{m_M - p_l}{m_M + p_l} f_2, \quad (3)$$

$$f_0(q^2) = \frac{m_M}{m_M + p_l} f_1 - \frac{2m_M p_l^2}{(m_M + p_l)(m_M^2 - m_\pi^2)} f_2.$$

In the soft-pion case, using the so-called PCAC relation and Lehmann-Symanzik-Zimmermann (LSZ) reduction formalism, the left-hand side (LHS) of (2) may be expressed as

$$\langle \pi(p) | \bar{q} \gamma^\mu Q | M(v) \rangle = \lim_{p^2 \rightarrow m_\pi^2 \rightarrow 0} \frac{1}{f_\pi} \frac{m_\pi^2 - p^2}{p^2} i \int dx e^{ip \cdot x} \langle 0 | T \{ \partial^\nu A_\nu(x), \bar{q} \gamma^\mu Q \} | M(v) \rangle \quad (4)$$

where f_π is the pion decay constant and $A_\mu = \bar{q}_1 \gamma_\mu \gamma^5 q$ (here we assume π contains q and q_1 quarks).

We rewrite the RHS of (4) as

$$i \int dx e^{ip \cdot x} \langle 0 | T \{ \partial^\nu A_\nu(x), \bar{q} \gamma^\mu Q \} | M(v) \rangle = \int dx e^{ip \cdot x} \langle 0 | T \{ p \cdot A(x), \bar{q} \gamma^\mu Q \} | M(v) \rangle - i \langle 0 | [Q_5, \bar{q} \gamma^\mu Q] | M(v) \rangle, \quad (5)$$

where $Q_5 = \int dx \bar{q}_1(x) \gamma_0 \gamma_5 q(x)$.

The pole contribution is included in $\int dx e^{ip \cdot x} \langle 0 | T \{ \bar{q} \gamma^\mu Q, p \cdot A(x) \} | M(v) \rangle$ which may be expressed as

$$\sum_{M'} \langle 0 | \bar{q} \gamma^\mu Q | M'(v') \rangle \frac{i}{(m_M v - p)^2 - m_{M'}^2} \langle M'(v') | p \cdot A | M(v) \rangle, \quad (6)$$

where $m_{M'} v' = m_M v - p$ and the sum of M' extends over the ground state and all excited states. The polarizations are also taken into account if the meson has spin. In the chiral limit, (6) may be written in terms of the decay constants $f_{M'}$ and the coupling constants $g_{MM'\pi}$, which are defined as

$$\langle 0 | \bar{q} \gamma_\mu Q | M'(v) \rangle = i f_{M'} m_{M'} v_\mu, \quad (7)$$

$$\langle M'(v') | p \cdot A | M(v) \rangle = 2 \sqrt{m_{M'} m_M} g_{MM'\pi} p_i$$

if M' is a scalar meson, and

$$\langle 0 | \bar{q} \gamma_\mu Q | M'(v, \epsilon) \rangle = -i f_{M'} m_{M'} \epsilon_\mu, \quad (8)$$

$$\langle M'(v', \epsilon) | p \cdot A | M(v) \rangle = 2 \sqrt{m_{M'} m_M} g_{MM'\pi} \epsilon \cdot p$$

if M' is a vector meson.

Using (4)–(9), we write f_1 and f_2 in the form

$$f_1 = -f_M / f_\pi + \sum_{M^S} \frac{f_{M^S}}{f_\pi} \frac{2 \sqrt{m_M m_{M^S}} p_i}{(m_M v - p)^2 - m_{M^S}^2} g_{MM^S \pi}, \quad (9)$$

$$f_2 = \sum_{M^V} \frac{f_{M^V}}{f_\pi} \frac{2 \sqrt{m_M m_{M^V}} m_{M^V}}{(m_M v - p)^2 - m_{M^V}^2} g_{MM^V \pi}.$$

Since $v \cdot v' = 1 + O(\frac{1}{M_Q})$, in the actual calculation of $\langle M'(v') | p \cdot A | M(v) \rangle$, we set $v' = v$. Obviously, $v = v'$ is a good approximation if $M' = M^* (m_{M^*} - m_M \approx m_\pi \approx 0)$. From (10) and (4), one may find that the scalar meson's pole term gives a next-leading-order contribution to f_+ in the $1/M$ expansion. Actually, as one will find later, except for the states M^* and M_{2S}^* (or $2S$ state in the language of the nonrelativistic quark model), the vector meson's pole terms with which we are concerned in this paper also only give a next-to-leading-order contribution to f_+ . So one may agree that the approximation we used above is sufficient.

In the Bethe-Salpeter (BS) approach, the matrix elements in (6) are expressed by the integrals of the BS wave functions

$$\langle 0 | \bar{q} \gamma^\mu Q | M'(v') \rangle = \int \frac{d^4 q}{(2\pi)^4} \text{tr} [\gamma^\mu \chi_v^{M'}(q)], \quad (10)$$

$$\langle M'(v) | p \cdot A | M(v) \rangle = \int \frac{d^4 q}{(2\pi)^4} \text{tr} [\chi_v^{M'}(q) \not{p} \chi_v^M(q) \times S_Q^{-1}(m_M v + q)]. \quad (11)$$

Here $q' = \frac{m}{m+m_Q}(m_M v - m_M v) + q - p$. m and m_Q are the mass of the light quark and the heavy quark, respectively (we assume the mass of u quark is equal to the one of d quark), $S_Q(m_M v + q)$ is the propagator of the heavy quark Q , $\chi_v^M(q)$ is the BS wave function of the meson M moving with the velocity v which satisfies the equation

$$\chi_v^M(p) = S_Q(\lambda_1 m_M v + p) \int G(v, p, q) \chi_v^M(q) \frac{d^4 q}{(2\pi)^4} \times S_l(-\lambda_2 m_M v + p), \quad (12)$$

where $\lambda_1 = \frac{m_Q}{m_Q + m}$, $\lambda_2 = \frac{m}{m_Q + m}$, $m_M v$ is the total momentum of the meson, p is the relative momentum, $G(v, p, q)$ is the BS irreducible kernel, and S_l is the propagator of the light quark. For convenience, we shall replace p_l with $p_l - \frac{m}{m_Q + m} M$ in the BS equation.

The $1/M_Q$ expansion in the relativistic BS equation for the heavy meson has been developed in [6–9]. We will not present the details of the derivation, but give a brief review of the main results in [6–9] which will be used below.

We expand the quantities in the BS equation in powers of $1/M$:

$$\chi(p) = \chi_0 + \chi_1^+ + \chi_1^-, \quad (13)$$

where χ_0 is the zeroth order wave function, χ_1^\pm are first-order wave functions satisfying $\not{p} \chi_1^\pm = \pm \chi_1^\pm$. χ_0 satisfies the equation

$$\chi_0(p) = - \frac{1 + \not{p}}{2(p_l + E + m + i\epsilon)} \int G_+(P, p, q) \chi_0(q) \frac{d^4 q}{(2\pi)^4} \times \frac{p_l \not{p} + \not{p}_l + m}{p_l^2 - W^2 + i\epsilon}, \quad (14)$$

where $E = M - m_Q - m$, $W_p = \sqrt{|p_t|^2 + m^2}$ and G_+ may be written in the form

$$-iG_+(v, p, q) = 1 \otimes 1V_1 + 1 \otimes \not{p}V_2. \quad (15)$$

From our numerical calculation, we find that χ_1^+ is much smaller than χ^- , so we shall not consider χ_1^+ . It means the $1/M$ order correction of the coupling constants $g_{MM'\pi}$ will not be taken into account. In [7], our calculation showed the $1/M$ correction of $g_{MM^*\pi}$ is indeed very small. We may expect it is also true for the other cases. However, for the reason we mentioned above, the leading order of $g_{MM'\pi}$ (except the case $M' = M_{2S}^*$) is sufficient. The effect of χ_1^- , which gives a large correction to the decay constants, needs to be taken into account. χ_1^- can be obtained by the equation

$$\begin{aligned} \chi_{0,j(-1)^{j+1},1}(p) &= \sqrt{\frac{2j+1}{2j+2}} \frac{1+\not{p}}{2} \gamma^5 \eta_{\alpha_1 \dots \alpha_j} p_t^{\alpha_2} \dots p_t^{\alpha_j} \left(p_t^{\alpha_1} - \frac{j}{2j+1} \gamma^{\alpha_1} \not{p}_t \right) (\phi_{1j} - \not{p}_t \phi_{2j}), \\ \chi_{0,j+1(-1)^{j+1},1}(p) &= \frac{1}{\sqrt{2}} \frac{1+\not{p}}{2} \eta_{\alpha_1 \dots \alpha_{j+1}} \gamma^{\alpha_1} p_t^{\alpha_2} \dots p_t^{\alpha_{j+1}} (\phi_{1j} - \not{p}_t \phi_{2j}) \end{aligned} \quad (17)$$

and

$$\begin{aligned} \chi_{0,j+1(-1)^j,2}(p) &= \frac{1}{\sqrt{2}} \frac{1+\not{p}}{2} \gamma^5 \eta_{\alpha_1 \dots \alpha_{j+1}} \gamma^{\alpha_1} p_t^{\alpha_2} \dots p_t^{\alpha_{j+1}} (\psi_{1j} - \not{p}_t \psi_{2j}), \\ \chi_{0,j(-1)^j,2}(p) &= \sqrt{\frac{2j+1}{2j+2}} \frac{1+\not{p}}{2} \eta_{\alpha_1 \dots \alpha_j} p_t^{\alpha_2} \dots p_t^{\alpha_j} \left(p_t^{\alpha_1} - \frac{j}{2j+1} \gamma^{\alpha_1} \not{p}_t \right) (\psi_{1j} - \not{p}_t \psi_{2j}). \end{aligned} \quad (18)$$

Equations (17) and (18) correspond to meson doublets of the spin-parity $[j^{(-1)^{j+1}}, (j+1)^{(-1)^{j+1}}]$ and $[j^{(-1)^j}, (j+1)^{(-1)^j}]$, respectively. Here $\eta_{\alpha_1 \dots \alpha_j}$ is the symmetric polarization tensor which satisfies the relation $\eta_{\alpha_1 \dots \alpha_j} v_{\alpha_1} = 0$. The zero-order BS wave functions are normalized by

$$\int \frac{d^4q}{(2\pi)^4} \text{tr}[\bar{\chi}_0^M(q_t) \chi_0(q_t) S_l^{-1}(-mv+q)] = 2m_M. \quad (19)$$

From (17) and (18) one finds that, in the leading order of the $1/M_Q$ expansion, the matrix element $\langle 0 | \bar{q} \gamma^\mu Q | M'(v') \rangle$ vanishes except where M' is the state $1_{1/2}^-$ or 0^+ . It is the reason why we only consider the first order of the matrix elements $\langle M'(v') | p \cdot A | M(v) \rangle$.

To the leading order of the $1/M$ expansion, we can write the decay constants f_{M^*} and f_{M^S} and the coupling constants $g_{MM^*\pi}$ and $g_{MM^S\pi}$ in terms of the scalar functions ϕ_{ij} and ψ_{ij} ($i=1,2$) in (17) and (18) as

$$\begin{aligned} f_M^0 &= \frac{2i}{m_M} \int \frac{d^4q}{(2i\pi)^4} \phi_{10}(q), \\ f_{M^*}^0 &= \frac{2i}{m_{M^*}} \int \frac{d^4q}{(2\pi)^4} \phi_{10}(q), \\ f_{M_{2S}^*}^0 &= \frac{2i}{m_{M_{2S}^*}} \int \frac{d^4q}{(2\pi)^4} \phi_{10}^{2S}(q), \\ f_{M^S}^0 &= \frac{-2i}{m_{M^S}} \int \frac{d^4q}{(2i\pi)^4} \psi_{10}(q), \\ f_{M^{**}}^0 &= 0, \\ g_{MM^*\pi}^0 &= \frac{i}{\sqrt{m_M m_{M^*}}} \int \frac{d^4q}{(2\pi)^4} (E + q_l + m) \\ &\quad \times [|\phi_{10}|^2 + \frac{1}{3} q_t^2 |\phi_{20}|^2], \end{aligned} \quad (20)$$

$$\begin{aligned} \chi_1^-(p) &= \frac{1-\not{p}}{4m_Q} \int G_-(P,p,q) \chi_0(q) \frac{d^4q}{(2\pi)^4} \frac{p_l \not{p} + \not{p}_l + m}{p_l^2 - W^2 + i\epsilon} \\ &\quad + \frac{\not{p}_l}{2m_Q} \chi_0(p), \end{aligned} \quad (16)$$

where G_- is the part of G of which the vertex on the heavy quark line anticommute with \not{p} .

The general solutions of Eq. (14) consist of two series of degenerate doublets of the form

$$\begin{aligned} g_{MM^S\pi}^0 &= \frac{i}{\sqrt{m_M m_{M^S}}} \int \frac{d^4q}{(2\pi)^4} (E + q_l + m) \\ &\quad \times [\phi_{10} \psi_{10}^* + q_t^2 \phi_{20} \psi_{20}^*], \\ g_{MM^{**}\pi} &= \frac{4i}{9\sqrt{m_M m_{M^{**}}}} \int \frac{d^4q}{(2\pi)^4} (E + q_l + m) q_t^2 \phi_{20} \psi_{11}^*, \end{aligned}$$

where M^{**} denotes the state $1_{3/2}^-$.

For calculations of values of physical quantities we must use a specific model. We shall assume that the light quark propagator is that of the free quark with the mass m and that the kernel is the sum of a static confinement term and a gluon exchange term in the Coulomb gauge in the rest frame of the meson. In order to obtain wave functions of moving mesons we generalize the kernel to the covariant form

$$\begin{aligned} V_1 &= \left[\frac{8\pi\kappa}{(|p_t - q_t|^2 + \mu^2)^2} - (2\pi)^3 \delta^3(p_t - q_t) \right. \\ &\quad \left. \times \int \frac{8\pi\kappa}{[k^2 + \mu^2]^2} \frac{d^3k}{(2\pi)^3} \right], \\ V_2 &= -\frac{16\pi\alpha_{\text{seff}}}{3(|p_t - q_t|^2)}, \\ -iG_- &= -\left[\gamma^\mu \otimes \gamma_\mu - \not{p} \otimes \not{p} + \frac{(\not{p}_t - \not{q}_t) \otimes (\not{p}_t - \not{q}_t)}{|p_t - q_t|^2} \right] \\ &\quad \times \frac{16\pi\alpha_{\text{seff}}}{3[|p_t - q_t|^2 - (p_l - q_l)^2 - i\epsilon]}. \end{aligned} \quad (21)$$

μ is taken to approach zero after numerically solving the equation. The running coupling constant α_{seff} is taken as

TABLE I. The masses ([8],[12]), decay constants, and coupling constants.

	D	D^*	D^{0+}	$D_{3/2}^{1-}$	D_{2S}^{*}	B	B^*	B^{0+}	$B_{3/2}^{1-}$	B_{2S}^{*}
m (MeV)	1864	2010	2254	2688	2586	5279	5325	5682	6022	5927
f (MeV)	177	248	95	38	187	163	197	111	14	191
g		0.71	-0.56	-0.35	0.13		0.71	-0.56	-0.35	0.13

$$\alpha_{s\text{eff}}(p_t, q_t) = \frac{12\pi}{27 \ln \frac{\text{Max}(|p_t|^2, |q_t|^2)}{\Lambda_{\text{QCD}}^2}} \quad \text{when } \alpha_{s\text{eff}} < 1, \quad (23)$$

$$\alpha_{s\text{eff}}(p_t, q_t) = 1 \quad \text{otherwise.}$$

Using (21) and (16), we also may obtain the expression of the heavy meson decay constants in terms of the scalar functions ϕ_{ij} and ψ_{ij} ($i = 1, 2$) to the next-to-leading order. Such expressions are lengthy, we do not write them down explicitly.

The following values of the parameters in the BS equation are used in the numerical calculations. They were determined in [8] from fitting the experimental data for the masses of observed heavy meson states:

$$m_\mu = 0.35 \text{ GeV}, \quad \Lambda_{\text{QCD}} = 0.38 \text{ GeV}, \quad \kappa = 0.2 \text{ GeV}^2, \quad (24)$$

$$m_c = 1.59 \text{ GeV}, \quad m_b = 5.02 \text{ GeV}.$$

In the numerical resolutions of the leading-order equation for 0^- and 1^- states, we find the two lowest stable resolutions are $E + M = 0.247$ and $E + M = 0.86$. We assume these correspond to the states $1S$ and $2S$ in the nonrelativistic quark model. Similarly, we obtain the masses of the other states. Since masses of $2P$ states are too large, they have not been taken into account. The $1/M$ order corrections are obtained by the method used in [5]. The integrals in (21) have been cut off at $|q_t| = m_Q$ so that the $1/M$ expansion stays valid inside the integration range, where $m_Q = m_c$ for the series of D mesons, $m_Q = m_b$ for the series of B . From heavy quark effective theory (HQET), we have that the contribution from the region beyond m_Q is negligible. One may notice that, in (14), the quark propagators are that of the free quarks, so the radiative effects have not been taken into account completely. This may be remedied simply by using the renormalization results obtained in the framework of the heavy quark effective theory. In the HQET, the wave function renormalization factors at one-loop order are [11]

$$Z_q = 1 - \frac{\alpha_s}{3\pi\epsilon}, \quad Z_Q = 1 + \frac{2\alpha_s}{3\pi\epsilon}, \quad (25)$$

So the BS wave function in (14) shall obtain a factor

$$\left(\frac{\alpha_s(\Lambda)}{\alpha_s(m_Q)} \right)^a, \quad (26)$$

where $a=2/25$ for mesons of B series and $a=2/27$ for D series. If we set the scale $\Lambda \sim 1 \text{ GeV}$, then the one-loop correction may be taken into account by the replacement

$$\int^{m_Q} d^3 p_t \rightarrow \int^\Lambda d^3 p_t + \int_\Lambda^{m_Q} d^3 p_t \left(\frac{\alpha(p_t)}{\alpha(m_Q)} \right)^a \quad (27)$$

in the calculation of f_M . From (11) and (25) one can know that coupling constants g have no such correction. In Table I, we list the numerical results of the decay constants, form factors, and the masses of the mesons.

Choosing $p_t = m_\pi = 139 \text{ MeV}$, we obtain

$$\begin{aligned} f_1^D &= -1.31 + 0.11, & f_2^D &= -9.53 + 0.27 - 0.55, \\ f_1^B &= -1.18 + 0.12, & f_2^B &= -30.2 + 0.25 - 1.4, \end{aligned} \quad (28)$$

$$\begin{aligned} f_+^D &= -9.5, & f_0^D &= -1.1, \\ f_+^B &= -28, & f_0^B &= -1. \end{aligned}$$

The second term of f_1^M is from M^{0+} pole term, the three terms of f_2^M are from M^* , $M_{3/2}^{1-}$, and M_{2S}^* pole term, respectively.

Except for M^* , the other pole terms give about $\sim 3\%$ of the total contribution to both f_+^B and f_+^D . One may notice that the M_{2S}^* state plays an important role, which gives about $\sim 5\%$ of the total contribution to both f_+^B and f_+^D . This percentage obviously is sensitive to the momentum of the π . If we choose $p_t = 300 \text{ MeV}$, it will increase to 7-8% in both B and D cases. According to the experimental data of the doubly heavy-quark resonance spectrum, the energy of $2S$ state is just about 100 - 200 MeV higher than that of $1P$ state [12]. In our calculation, the mass split between $2S$ and $1^+(2460 \text{ MeV})$ [12] is $\sim 100 \text{ MeV}$. Since the motion of the light quark in $2S$ state is completely relativistic, our calculation is reasonable. From our calculation, one also may find that the pole terms except M^* , which we considered in this paper, cancel each other so that their contributions are not large at zero recoil.

In conclusion, we discuss the possible corrections of some recent calculations on the form factors which parametrize the hadronic matrix element of semileptonic decays $B(D) \rightarrow \pi + l\nu$. Our discussions are based on the assumption that there are some low-lying excited states of the B and D series which are predicted by the quark model. Using the relativistic BS approach, we obtained some numerical results. Our results show that, except the M^* pole term, some other low-lying state poles (not complete) may give around 3% of the total contribution to both f_+^B and f_+^D , which are responsible to determine the ratio V_{ub}/V_{cd} .

The author would like to thank F. Hussain and G. Thompson for very useful discussions. The author also thanks the International Centre for Theoretical Physics for hospitality.

- [1] M.B. Voloshin and M.A. Shifman, *Yad. Fiz.* **45**, 463 (1987) [*Sov. J. Nucl. Phys.* **45**, 292 (1987)]; N. Isgur and M.B. Wise, *Phys. Lett. B* **232**, 237 (1989); **237**, 527 (1990); M. Neubert, *ibid.* **264**, 455 (1991).
- [2] J. Bartelt *et al.*, *Phys. Rev. Lett.* **71**, 4111 (1993); H. Albrecht *et al.*, *Phys. Lett. B* **255**, 297 (1991).
- [3] B. Blok and T. Mannel, *Phys. Rev. D* **51**, 2208 (1995).
- [4] N. Isgur and M.B. Wise, *Phys. Rev. D* **42**, 2388 (1990).
- [5] M.B. Wise, *Phys. Rev. D* **45**, 2188 (1992); G. Burdam, Z. Ligeti, M. Neubert, and Y. Nir, *ibid.* **49**, 2331 (1994).
- [6] Y.B. Dai, C.S. Huang, and H.Y. Jin, *Z. Phys. C* **56**, 707 (1992).
- [7] Y.B. Dai, C.S. Huang, and H.Y. Jin, *Z. Phys. C* **60**, 527 (1993).
- [8] Y.B. Dai, C.S. Huang, and H.Y. Jin, *Phys. Lett. B* **331**, 174 (1994).
- [9] Y.B. Dai, C.S. Huang, and H.Y. Jin, *Z. Phys. C* **65**, 87 (1995).
- [10] X. Wirbel, B. Stech, and M. Bauer, *Z. Phys. C* **29**, 637 (1985).
- [11] M. Neubert, *Phys. Rep.* **245**, 259 (1994).
- [12] Particle Data Group, L. Montanet *et al.*, *Phys. Rev. D* **50**, 1173 (1994).