Universal and nonuniversal new physics effects in a general four-fermion process: A Z-peak subtracted approach

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We calculate, using a Z-peak subtracted representation of four-fermion processes previously illustrated for the case of electron-positron annihilation into charged lepton-antilepton pairs, the corresponding expressions of the new physics contributions for the case of final quark-antiquark states, allowing the possibility of both universal and nonuniversal effects. We show that, in each case, the main result obtained for the final lepton channel can be generalized, so that every experimentally measurable quantity can be expressed in terms of input parameters *measured* on the Z resonance, of $\alpha(0)$ and of a small number of subtracted one loop expressions. Some examples of models are considered for several c.m. energy values, showing that remarkable simplifications are often introduced by our approach. In particular, for the case of a dimension-six Lagrangian with anomalous gauge couplings, the same reduced number of parameters that would affect the observables of final leptonic states are essentially retained when one moves to final hadronic states. This leads to great simplifications in the elaboration of constraints and, as a gratifying by-product, to the possibility of making the signal from these models clearly distinguishable from those from other (both universal and nonuniversal) competitors.

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I. INTRODUCTION

At the end of this year, the CERN e^+e^- collider LEP 1 will have made its last run. Although the SLAC Linear Collider (SLC) will keep performing for a few more years, with some (potentially, extremely interesting) longitudinal polarization asymmetries [1] still to be investigated, one can conclude that the high precision standard model (SM) test program, based on measurements of electron-positron annihilation into fermions on top of the Z resonance, has essentially been concluded. No deviations from the SM prediction were found (with the only possible remarkable exception of the partial Z decay width into $b\bar{b}$) [2–4] at the achieved precision level of few per mill, and for a large set of experimental variables (partial and total Z widths and asymmetries) that have been extremely carefully analyzed in these years. Stated otherwise, and keeping in mind the previous remark, no virtual effects of new physics have become evident from the several measurements at the few per mill level performed in the considered four-fermion process at $\sqrt{q^2} = M_Z$ $[q^2 = (P_e + P_e')^2]$. Thus the only information achieved on several candidate models comes from a number of bounds that can be, depending on the case, drastic (e.g., for most common technicolor models [5]) or extremely mild (e.g., for the simplest supersymmetric extension [minimal supersymmetric SM (MSSM) [6]] of the SM).

Technicolor and supersymmetry are not the only alternatives to the SM whose virtual effects have been tested by LEP 1 and SLC. In particular, signals of models with anomalous gauge couplings [7] have been also searched for and the negative results have led to some corresponding bounds. But in this case the obtained results are somehow less clean [8]. Leaving aside a number of technical points, one difficulty for these models is also related to the number of involved parameters, essentially too large, even in the presence of the considerable number of LEP 1, SLC high precision measurements.

In a near future, electron-positron annihilation at $\sqrt{q^2} \simeq 2M_Z$ (LEP 2) and (perhaps in the "not too near" future) at $\sqrt{q^2} = 500$ GeV [Next Linear Collider (NLC)] will be measured. For obvious reasons, the relative accuracy of the various measurements will be worse than at LEP 1, SLC, moving from the few per mill to the few percent level. For final fermion-antifermion state it is also likely that the number of measurable experimental variables will decrease. This might lead to the conclusion that the search of virtual effects of new physics in these future processes will be, least to say, tough for a number of potentially interesting models, in particular for those whose effect *on top of Z resonance* are described by a large number of parameters.

In a previous paper [9], we have tried to propose a solution to this problem for the case of final (charged) lepton-antilepton states. Our starting point was the (known) fact that a theoretical analysis of virtual one loop effects can be eased by a proper choice of the "input" parameters. For instance, for the description of physics of electron-positron *on Z resonance*, the introduction of the

Fermi constant G_μ to replace M_W is quite useful. But for a theoretical description of electron-positron annihilation at higher energies, we showed in [9] that G_μ does not seem to be the best choice if an investigation of models of new physics is the theoretical goal. In particular, a self-contained representation of final lepton-antilepton states can be given where G_μ is "traded" and the new input parameters are the Z leptonic width Γ_l and the "effective" $s^2_{\rm eff}(M_Z^2)$ measured by LEP 1 and SLC. Once these quantities [together with the physical electric charge $\alpha_{\rm QED}(0)$] are introduced, the rest of the representation only contains three subtracted quantities (called $\tilde{\Delta}\alpha$, R, and V in Ref. [9]) whose theoretical properties for what concerns the effects of a number of models of new physics appear undeniably, least to say, interesting, in the sense that their actual calculation turns out to be generally much easier [this is simply due to the fact that a number of model's parameters, whose theoretical features might be less pleasant, are often "reabsorbed" in the new Z-peak inputs Γ_l and $s^2_{\rm eff}(M_Z^2)$]. We also showed in Ref. [9] that, at the realistic expected experimental conditions of future e^+e^- colliders, the loss of theoretical accuracy introduced by this "G_μ trading" does not produce any observable effect. We concluded that the proposed "Z-peak subtracted" representation was a good approach to investigate new-physics effects in the final lepton-antilepton channel.

The aim of this paper is that of showing that the same method, with identical conclusions, can be generalized to the case of final quark-antiquark states. The new input parameters will be now those hadronic quantities that are measured on Z resonance [hadronic widths and asymmetries, plus the strong coupling $\alpha_s(M_Z)$]. Again, the use of these inputs will allow us to express the remaining one loop theoretical expression in terms of *subtracted* quantities, that will be the three *universal* corrections $\tilde{\Delta}\alpha$, R, and V already met for the leptonic case and new, nonuniversal terms whose theoretical expression will be given for a number of potentially interesting experimental quantities. This will be done in full detail in Sec. II. In the following sections, we shall try to show that the same remarkable features exhibited by our representation for final leptonic states survive when one moves to hadronic states. With this aim, we shall consider in Sec. III an example of "universal" effects of a model with anomalous gauge couplings, showing that our method would help to solve the problem of "parameters excess" for this case. We shall also illustrate how the possible experimental visible signatures of this model would differ from those of another universal model of technicolor type. In Sec. IV, we shall consider the example of "nonuniversal" effects of a model with one extra Z of the most general nature, and show that it can be formally treated as a special case of our approach. We shall compare the effects of these models on a number of observables and show that it is possible to select a special set of three measurements that, in case a certain "signal" were observed, would be able to indicate to which of the models it did belong. In Sec. V a final discussion will show that for all the new input parameters, the already available LEP 1, SLC accuracy is sufficient to avoid the generation of sensible uncertainties in the theoretical predictions at the expected future experimental conditions. This will then conclude our work.

II. DERIVATION OF THE THEORETICAL EXPRESSIONS

A. General case

The relevant quantity in our description will be the scattering amplitude for the four-fermion process $e^+e^- \to f\bar{f}$ at variable c.m. energy $\sqrt{q^2}$. A very convenient way of writing it at the considered one loop level has been shown in Ref. [9] for the simplified case $f = l$. In this more general paper we shall begin therefore by rewriting the needed expression, that reads

$$\mathcal{A}^{(1)}_{lf}(q^2,\theta) = \mathcal{A}^{\gamma(1)}_{lf}(q^2,\theta) + \mathcal{A}^{Z(1)}_{lf}(q^2,\theta) + \text{"QED"} + \text{"QCD"} \tag{1}$$

with the photon exchange term

$$\mathcal{A}^{\gamma(1)}_{lf}(q^2,\theta) = \mathcal{A}^{\gamma(0)}_{lf}(q^2,\theta)[1 - \tilde{F}^{(lf)}_{\gamma}(q^2,\theta)] \ , \tag{2}$$

$$\mathcal{A}^{\gamma(0)}_{lf}(q^2,\theta) \equiv \frac{i}{q^2} v^{(\gamma)}_{\mu l} v^{(\gamma)\mu f} = \frac{i e_0^2 Q_l Q_f}{q^2} \bar{v}_l \gamma_\mu u_l \bar{u}_f \gamma^\mu v_f \tag{3}$$

($Q_{l,f}$ being the fermion charges in units of $|e|$) and the Z exchange term

$$\mathcal{A}^{Z(1)}_{lf}(q^2,\theta) = \frac{i}{q^2 - M_{Z0}^2}\left(\frac{g_0^2}{4c_0^2}\right)\left[1 - \frac{A^{(lf)}_{Z}(q^2,\theta)}{q^2 - M_{Z0}^2}\right]\bar{v}_l\gamma^\mu \times (g^{(1)}_{Vl} - \gamma^5 g^{(0)}_{Al}) u_l \bar{u}_f \gamma_\mu (g^{(1)}_{Vf} - \gamma^5 g^{(0)}_{Af}) v_f \ , \tag{4}$$

where

$$g^{(1)}_{Vl} = g^{(0)}_{Vl} - 2 s_1 c_1 Q_l \tilde{F}^{(lf)}_{\gamma Z}(q^2,\theta) \ , \tag{5}$$

$$g^{(1)}_{Vf} = g^{(0)}_{Vf} - 2 s_1 c_1 Q_f \tilde{F}^{(lf)}_{Z\gamma}(q^2,\theta) \ , \tag{6}$$

with $s_1^2 = 1 - c_1^2$ and $s_1^2 c_1^2 = \frac{\pi\alpha}{\sqrt{2} G_\mu M_Z^2}$.

Note that in the above equations bare couplings $g_0 = e_0/s_0$; $g^{(0)}_{Al,f} \equiv I^{3L}_{l,f}$; $g^{(0)}_{Vl,f} \equiv I^{3L}_{l,f} - 2Q_{l,f}s_0^2$ and the bare Z mass M_{Z0} are still contained.

The definition of the "generalized" one loop corrections, that are gauge-invariant combinations of self-energies, vertices, and boxes belonging to the independent Lorentz structures of the process (for a full and rigorous discussion about the choice of gauge-invariant combinations, we defer to previous papers by Degrassi and Sirlin [10], to whose conclusions and notations we shall try to stick as much as possible here) is

$$\tilde{F}^{lf}_{\gamma}(q^2,\theta) = F_\gamma(q^2) - (\Gamma^{(\gamma)}_{\mu,l}, v^{(\gamma)}_{\mu,l}) - (\Gamma^{(\gamma)}_{\mu,f}, v^{(\gamma)}_{\mu,f}) + A^{(B)}_{\gamma,lf}(q^2,\theta) \ , \tag{7}$$

$$A_Z^{lf}(q^2,\theta)=A_Z(q^2)-(q^2-M_Z^2)[(\Gamma^{(Z)}_{\mu,l},v^{(Z)}_{\mu,l})+(\Gamma^{(Z)}_{\mu,f},v^{(Z)}_{\mu,f})+^{(B)}_{Z,lf}(q^2,\theta)]\ , \tag{8}$$

$$\begin{aligned}\tilde F^{lf}_{\gamma Z}(q^2,\theta)&\equiv\frac{A^{(lf)}_{\gamma Z}(q^2,\theta)}{q^2}\\&=\frac{A_{\gamma Z}(q^2)}{q^2}-\frac{q^2-M_Z^2}{q^2}(\Gamma^{(\gamma)}_{\mu,f},v^{(Z)}_{\mu,f})-(\Gamma^{(Z)}_{\mu,l},v^{(\gamma)}_{\mu,l})-(q^2-M_Z^2)A^{(B)}_{\gamma Z,lf}(q^2;\theta)\ ,\end{aligned} \tag{9}$$

$$\begin{aligned}\tilde F^{lf}_{Z\gamma}(q^2,\theta)&\equiv\frac{A^{(lf)}_{Z\gamma}(q^2,\theta)}{q^2}\\&=\frac{A_{\gamma Z}(q^2)}{q^2}-\frac{q^2-M_Z^2}{q^2}(\Gamma^{(\gamma)}_{\mu,l},v^{(Z)}_{\mu,l})-(\Gamma^{(Z)}_{\mu,f},v^{(\gamma)}_{\mu,f})-(q^2-M_Z^2)A^{(B)}_{Z\gamma,lf}(q^2,\theta)\ ,\end{aligned} \tag{10}$$

Here F_γ, A_Z, $A_{\gamma Z}$ are the conventional self-energies, for which we shall follow the usual definition:

$$A_i(q^2)\equiv A_i(0)+q^2F_i(q^2) \tag{11}$$

[note that the physical Z mass M_Z now appears in Eqs. (7)–(10)]. The quantities denoted as $(\Gamma^{(\gamma,Z)}_{\mu l,f},v^{(\gamma,Z)}_{\mu l,f})$ are the "components" of the generalized l (or f) vertex along the photon, or Z, Lorentz structure. For instance, we would write for the overall photon vertex correction to a fermion f:

$$\Gamma^{(\gamma)}_{\mu f}\equiv(\Gamma^{(\gamma)}_{\mu f},v^{(\gamma)}_{\mu f})v^{(\gamma)}_{\mu f}+(\Gamma^{(\gamma)}_{\mu f},v^{(Z)}_{\mu f})v^{Z}_{\mu f}\ , \tag{12}$$

where

$$v^{(\gamma)}_{\mu f}=e_0Q_f\bar u_f\gamma_\mu v_f\ , \tag{13}$$

$$v^{(Z)}_{\mu f}=\frac{e_0}{2c_0s_0}\bar u_f\gamma_\mu(g^{(0)}_{Vf}-\gamma^5g^{(0)}_{Af})v_f\ . \tag{14}$$

In our approach we shall need, rather than the previously defined "generalized" corrections, the four "subtracted" quantities defined as

$$\tilde\Delta^{(lf)}\alpha(q^2,\theta)\equiv\tilde F^{(lf)}_\gamma(0,\theta)-\tilde F^{(lf)}_\gamma(q^2,\theta)\ , \tag{15}$$

$$R^{(lf)}(q^2,\theta)\equiv\tilde I^{(lf)}_Z(q^2,\theta)-\tilde I^{(lf)}_Z(M_Z^2,\theta)\ , \tag{16}$$

$$V^{(lf)}_{\gamma Z}(q^2,\theta)\equiv\tilde F^{(lf)}_{\gamma Z}(q^2,\theta)-\tilde F^{(lf)}_{\gamma Z}(M_Z^2,\theta)\ , \tag{17}$$

$$V^{(lf)}_{Z\gamma}(q^2,\theta)\equiv\tilde F^{(lf)}_{Z\gamma}(q^2,\theta)-\tilde F^{(lf)}_{Z\gamma}(M_Z^2,\theta)\ , \tag{18}$$

where the "auxiliary" quantity I_Z is defined as

$$\tilde I^{(lf)}_Z(q^2,\theta)=\frac{q^2}{q^2-M_Z^2}[\tilde F^{(lf)}_Z(q^2,\theta)-\tilde F^{(lf)}_Z(M_Z^2,\theta)]\ . \tag{19}$$

In Eq. (1) the "pure QED" and the "pure QCD" components can be tested separately and will not affect our research, which is only devoted to the investigation on new *electroweak* physics effects to the one-loop perturbative order.

After these (we hope not too long) introductory definitions, that we have given to make this paper as self-contained as possible, we are now in a position to derive our general expressions.

The simplest way to illustrate the philosophy of our procedure is that of showing the standard final form of the pure photonic contribution to the scattering amplitude. Using the conventional definition of the physical electric charge $\alpha\equiv\alpha(0)$ one immediately realizes that

$$\frac{e_0^2}{q^2}[1-\tilde F^{(lf)}_\gamma(q^2,\theta)]\equiv\frac{4\pi\alpha}{q^2}[1+\tilde\Delta^{(lf)}\alpha(q^2,\theta)] \tag{20}$$

showing the known fact that the replacement of the bare charge by the physical charge, *measured at* $q^2=0$, is accompanied by the replacement of the "generalized" correction $\tilde F^{(lf)}_\gamma(q^2)$ with the "photon-peak" subtracted quantity $\tilde F^{(lf)}_\gamma(q^2)-\tilde F^{(lf)}_\gamma(0)$.

Our approach is based on a quite similar attitude for what concerns the "pure Z" and the "$Z-\gamma$ interference" contributions. A typical quantity to be considered corresponds, e.g., in our conventions to the term

$$\frac{ig_0^2}{4c_0^2}\left(\frac{1}{q^2-M_{Z0}^2}\right)\left[1-\frac{A^{(lf)}_Z(q^2,\theta)}{q^2-M_{Z0}^2}\right]\ . \tag{21}$$

Using the tree level identity

$$\frac{g_0^2}{4c_0^2}=\sqrt2G_{\mu,0}M_{Z,0}^2\ , \tag{22}$$

one immediately realizes that the term Eq. (21) becomes exactly

$$\begin{aligned}&\frac{g_0^2}{4c_0^2}\left(\frac{1}{q^2-M_{Z0}^2}\right)\left[1-\frac{A^{(lf)}_Z(q^2,\theta)}{q^2-M_{Z0}^2}\right]\\&\equiv\frac{\sqrt2G_\mu M_Z^2}{q^2-M_Z^2+iM_Z\Gamma_Z}\left[1+\frac{\delta G_\mu}{G_\mu}+\mathrm{Re}\frac{A^{(lf)}_Z(0,\theta)}{M_Z^2}\right.\\&\qquad\left.-\tilde I^{(lf)}_Z(q^2,\theta)\right]\end{aligned} \tag{23}$$

(with the conventional definition of Γ_Z).

In Eq. (23) the physical input is represented by M_Z and G_μ, with a certain "generalized" correction. Our approach consists, essentially, in rewriting this term (and other, similar, ones) by adding and subtracting $\tilde{I}_Z(M_Z^2,\theta)$. In the specific case of Eq. (23), this generates the quantity

$$G_\mu\left[1+\frac{\delta G_\mu}{G_\mu}+\mathrm{Re}\frac{A_Z^{lf}(0,\theta)}{M_Z^2}-\tilde{I}_Z^{(lf)}(M_Z^2,\theta)-\tilde{I}_Z^{(lf)}(q^2,\theta)+\tilde{I}_Z^{(lf)}(M_Z^2,\theta)\right]=G_\mu[1+\epsilon_1^{(lf)}][1-R^{(lf)}(q^2,\theta)]\ , \tag{24}$$

where $R^{(lf)}(q^2,\theta)$ is the "Z-peak" subtracted correction defined by Eq. (16) and

$$\epsilon_1^{(lf)}=\epsilon_1+[(\Gamma_{\mu,f}^{(Z)},v_{\mu,f}^{(Z)})-(\Gamma_{\mu,l}^{(Z)},v_{\mu,l}^{(Z)})]\ . \tag{25}$$

Here ϵ_1 is the Altarelli-Barbieri parameter [11], directly related to the partial Z width into leptons

$$\Gamma_l=\left(\frac{\sqrt{2}G_\mu M_Z^3}{48\pi}\right)[1+\epsilon_1][1+\tilde{v}_l^2(M_Z^2)]\ , \tag{26}$$

where

$$\tilde{v}_l\equiv 1-4s_l^2(M_Z^2) \tag{27}$$

and $s_L^2(M_Z^2)$ is the quantity measured at LEP 1 and SLC. Although a few more steps are still required, one can already understand the final goal of our approach, i.e., that of "trading" G_μ for some physical Z partial width, measured at Z peak. At the same time, this procedure will replace the "generalized" corrections $\tilde{I}_Z^{(lf)}(q^2)$ [and, also, $\tilde{F}_{\gamma Z}^{(lf)}(q^2,\theta)$] with the "$Z$-peak subtracted" quantities $R^{(lf)}(q^2,\theta), V_{\gamma Z}^{(lf)}(q^2,\theta), V_{Z\gamma}^{(lf)}(q^2,\theta)$ defined by Eqs. (16)–(18).

To fully understand the "replacement" mechanism, we shall now write the complete one-loop expression of a representative observable, chosen to be $\sigma_{lf}(q^2)$, the cross section for production of a final $f\bar{f}$ state. This can be done rather easily if one writes the tree-level expression of this quantity, that reads

$$\sigma_{lf}^{(0)}(q^2)=N_f\left(\frac{4\pi q^2}{3}\right)\left\{\frac{\alpha_0^2Q_l^2Q_f^2}{q^4}+\left[\frac{\sqrt{2}G_\mu^{(0)}M_{Z0}^2}{4\pi}\right]^2\frac{(g_{Vl}^{(0)2}+g_{Al}^{(0)2})(g_{Vf}^{(0)2}+g_{Af}^{(0)2})}{(q^2-M_{Z0}^2)^2}+2\frac{\alpha_0Q_lQ_f}{q^2}\left[\frac{\sqrt{2}G_\mu^{(0)}M_{Z0}^2}{4\pi}\right]\frac{g_{Vl}^{(0)}g_{Vf}^{(0)}}{q^2-M_{Z0}^2}\right\} \tag{28}$$

($N_f=3$ for quarks).

The corresponding expression at one loop is written immediately if one uses our starting Eq. (1) and makes the simple and obvious replacements. One then easily derives

$$\sigma_{lf}^{(1)}(q^2)=\sigma_{lf}^{(\gamma)}(q^2)+\sigma_{lf}^{(Z)}(q^2)+\sigma_{lf}^{(\gamma Z)}(q^2)+\text{“QED”}+\text{“QCD”} \tag{29}$$

and finds, for the "pure" Z exchange term,

$$\sigma_{lf}^{(Z)}(q^2)=N_f\left(\frac{4\pi q^2}{3}\right)\left[\frac{\sqrt{2}G_\mu M_Z^2}{16\pi}\right]^2\left(\frac{1}{(q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2}\right)[1+2\epsilon_1^{(lf)}][1+\tilde{v}_l^2][1+\tilde{v}_f^2]\times\left[1-2\tilde{I}_Z^{(lf)}(q^2,\theta)-8s_1c_1\left\{\frac{v_1}{1+v_1^2}V_{\gamma Z}^{(lf)}(q^2)+\frac{v_f|Q_f|}{1+v_f^2}V_{Z\gamma}^{(lf)}(q^2)\right\}\right]\ , \tag{30}$$

where v_1, v_f are defined as

$$v_{1,f}=1-4|Q_{l,f}|s_1^2 \tag{31}$$

and $s_1^2=0.212$ is defined after Eqs. (2) and (3).

We also define the quantity (not to be confused with the one above)

$$\tilde{v}_f=1-4|Q_f|s_f^2(M_Z^2) \tag{32}$$

with

$$s_f^2(M_Z^2)\equiv s_1^2+s_1c_1\tilde{F}_{Z\gamma}^{(lf)}(M_Z^2)\ . \tag{33}$$

From Eq. (30) one recovers the result of Ref. [9] when $f=l$. In that case, $v_1=v_f$, $\epsilon_{1,lf}=\epsilon_1$, $F_{\gamma Z}=F_{Z\gamma}$ and in the "leading" terms $(G_\mu)^2$ has been exactly replaced by the quantity $\left[\frac{\Gamma_l}{M_Z}\right]^2$ (multiplied by a c number), while in the correction the "Z-peak subtracted" quantities $R\equiv R^{ll}$ and $V\equiv V_{\gamma Z}^{ll}\equiv V_{Z\gamma}^{ll}$ appear.

When $l\neq f$, a very similar situation can be reproduced. One only has to introduce the quantity

$$\epsilon_1^{ff} = \epsilon_1 + 2[(\Gamma_{\mu,f}^{(Z)}, v_{\mu,f}^{(Z)}) - (\Gamma_{\mu,l}^{(Z)}, v_{\mu,l}^{(Z)})] \,. \tag{34}$$

This is *exactly* related to the partial width of Z into $f\bar{f}$ by the expression [12]

$$\Gamma_f = N_f^{\rm QCD}\left(\frac{\sqrt{2}G_\mu M_Z^3}{48\pi}\right)[1+\epsilon_1^{ff}][1+\tilde{v}_f^2(M_Z^2)] \,, \tag{35}$$

where

$$N_f^{\rm QCD} \simeq 1 + \frac{\alpha_s(M_Z^2)}{\pi} \,. \tag{36}$$

The final observation is now the exact equality

$$2\epsilon_1^{(lf)} \equiv \epsilon_1 + \epsilon_1^{(ff)} \,. \tag{37}$$

From this equality and from the previous formulas one is then finally led to the relevant expression

$$\sigma_{lf}^{(Z)}(q^2) = N_f\left(\frac{4\pi q^2}{3}\right)\frac{\left[\frac{3\Gamma_l}{M_Z}\right]\left[\frac{3\Gamma_f}{N_f M_Z}\right]}{(q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2}\Bigg[1-2R^{(lf)}(q^2) -8s_1c_1\left\{\frac{v_1}{1+v_1^2}V_{\gamma Z}^{(lf)}(q^2)+\frac{v_f|Q_f|}{1+v_f^2}V_{Z\gamma}^{(lf)}(q^2)\right\}\Bigg] \,. \tag{38}$$

Equation (38) is one of the main results of this paper. It shows that the replacement of G_μ, and the corresponding introduction of "Z-peak subtracted" corrections, can be continued to final hadronic states by introduction of quantities that correspond to those encountered in the leptonic case. Typically, σ_{lf} will contain Γ_l *and* Γ_f, as one would have naively expected, and the strong coupling $\alpha_S(M_Z^2)$ generated by Eq. (35), that only affects the expression if $l \neq f$ and should *not* be considered as a "QCD" correction in the notation of Eq. (1). Note that only quantities that can be exactly defined and (in principle) *measured* on Z resonance have been used to build our "Z-peak modified Born approximation."

The procedure that we have illustrated can now be repeated for the remaining components of σ_{lf} (as well as for the other observables). In fact, there is no need of any trick for the "pure γ" component, that remains given by the expression

$$\sigma_{lf}^{(\gamma)}(q^2) = N_f\left(\frac{4\pi q^2}{3}\right)Q_l^2Q_f^2\frac{\alpha^2(0)}{q^4}[1+2\tilde{\Delta}^{(lf)}\alpha(q^2)] \,. \tag{39}$$

The "$\gamma - Z$" interference can be treated in a straightforward way. To avoid writing too many formulas, we only give here the relevant final expression, that can be easily derived using the previously illustrated procedure:

$$\sigma_{lf}^{(\gamma Z)}(q^2) = N_f\left(\frac{4\pi q^2}{3}\right)2\alpha(0)|Q_f|\frac{q^2-M_Z^2}{q^2(q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2}\left[\frac{3\Gamma_l}{M_Z}\right]^{1/2}\left[\frac{3\Gamma_f}{N_fM_Z}\right]^{1/2} \times\frac{\tilde{v}_l\tilde{v}_f}{(1+\tilde{v}_l^2)^{1/2}(1+\tilde{v}_f^2)^{1/2}}\left[1+\tilde{\Delta}^{(lf)}\alpha(q^2)-R^{(lf)}(q^2)-4s_1c_1\left\{\frac{1}{v_1}V_{\gamma Z}^{(lf)}(q^2)+\frac{|Q_f|}{v_f}V_{Z\gamma}^{(lf)}(q^2)\right\}\right] \,. \tag{40}$$

Note that, besides Γ_l and Γ_f, this expression contains the two parameters $s_l^2(M_Z^2)$ and $s_f^2(M_Z^2)$ (or $\tilde{v}_l$ and $\tilde{v}_f$) defined in Eqs. (27), (32), (33), that are now *not* reabsorbed into Γ_l, Γ_f as in the previous "pure Z" term. This does not represent a problem since $s_l^2(M_Z^2)$ is the quantity measured at LEP 1, SLC. The remaining parameter $s_f^2(M_Z^2)$ is also related to measured (or measurable) quantities at Z peak, more precisely to forward-backward unpolarized asymmetries for b and c (already given by LEP 1) and also, more directly, to the polarized forward-backward asymmetries for b and c, called $A_{b,c}$ in the original proposal [1], to be measured at SLC in the near future [13]. In particular, in terms of $A_{b,c}$ we would have

$$A_{b,c} = \frac{2\tilde{v}_{b,c}}{1+\tilde{v}_{b,c}^2} \tag{41}$$

[the unpolarized asymmetries are essentially given by the product of Eq. (41) with the corresponding leptonic quantity that contains $s_l^2(M_Z^2)$]. In conclusion, also in the case of $\sigma^{\gamma Z}$, the new complete "Born" expression can be given in terms of quantities measured on Z resonance. As we shall show in the final discussion, this will never introduce a relevant "input" uncertainty in the obtained predictions.

To conclude this general part of Sec. II, we still need the derivation of the quantity that appears in the numerator of an unpolarized forward-backward asymmetry. We shall write this observable in the following way:

$$A_{\mathrm{FB},f}(q^2) = \frac{3\sigma_{\mathrm{FB},lf}(q^2)}{4\sigma_{lf}(q^2)} , \tag{42}$$

where σ_{lf} has been previously defined. From the expression (that we do not write explicitly) of $\sigma_{\mathrm{FB},lf}$ at tree level it is immediate to derive, without introducing any other prescription or definition, the final relevant expression:

$$\sigma_{\mathrm{FB},lf}(q^2) \equiv \sigma^{(Z)}_{\mathrm{FB},lf}(q^2) + \sigma^{(\gamma Z)}_{\mathrm{FB},lf}(q^2) , \tag{43}$$

where

$$\sigma^{(Z)}_{\mathrm{FB},lf}(q^2) = N_f \left(\frac{4\pi q^2}{3}\right) \frac{\left[\frac{3\Gamma_l}{M_Z}\right]\left[\frac{3\Gamma_f}{N_f M_Z}\right]}{(q^2-M_Z^2)^2 + M_Z^2\Gamma_Z^2} \times \frac{4\tilde{v}_l\tilde{v}_f}{(1+\tilde{v}_l^2)(1+\tilde{v}_f^2)}\left[1 - 2R^{(lf)}(q^2) - 4s_1c_1\left\{\frac{1}{v_1}V^{(lf)}_{\gamma Z}(q^2) + \frac{|Q_f|}{v_f}V^{(lf)}_{Z\gamma}(q^2)\right\}\right] \tag{44}$$

and

$$\sigma^{(\gamma Z)}_{\mathrm{FB},lf}(q^2) = N_f\left(\frac{4\pi q^2}{3}\right) 2\alpha(0)|Q_f| \frac{q^2 - M_Z^2}{q^2[(q^2-M_Z^2)^2 + M_Z^2\Gamma_Z^2]} \times \left[\frac{3\Gamma_l}{M_Z}\right]^{1/2}\left[\frac{3\Gamma_f}{N_f M_Z}\right]^{1/2} \frac{1}{(1+\tilde{v}_l^2)^{1/2}(1+\tilde{v}_f^2)^{1/2}}[1 + \tilde{\Delta}^{(lf)}\alpha(q^2) - R^{(lf)}(q^2)] . \tag{45}$$

Equations (44) and (45) conclude our general technical introduction. We shall now consider in the next subsection II B the explicit cases of experimental observables that will, or should, be measured in the very near future at LEP 2 and, possibly, in a not too near future at NLC.

B. Application to specific observables

To begin our analysis, we consider the simplest case that might realistically occur, i.e., that of the measurement of the cross section for $b\bar{b}$ production, σ_{lb}. Actually, we should rather consider the (experimentally more accurate) ratio $R_{bl} = \sigma_{lb}/\sigma_{ll}$. Since the theoretical expression of σ_{ll} has been already given in Ref. [9], we shall limit ourselves to deriving and discussing in detail the full expression of the numerator. Then in Secs. III and VI we shall rather use the ratio, whose expression can be easily derived.

When writing the full expression of σ_{lb}, as well as that of the next considered observables, it will be very useful to separate the "universal" contributions of new physics from the "nonuniversal" ones, that depend on properties of the final state that are different from the corresponding ones for leptons (e.g., specific non-SM couplings, or masses). Clearly, the full set of self-energies contributions will belong to the first universal class, while boxes will generally produce nonuniversal effects. For vertices, one can have both cases, as we shall show in the next sections.

After these premises, we can now write the complete expression

$$\sigma_{lb}(q^2) = \sigma^{(\gamma)}_{lb}(q^2) + \sigma^{(Z)}_{lb}(q^2) + \sigma^{(\gamma Z)}_{lb}(q^2) , \tag{46}$$

where the three components are given by Eqs. (39), (44), (45) and, following our previous discussion, we shall express the subtracted corrections in the form

$$\tilde{\Delta}^{(lb)}\alpha(q^2) = \tilde{\Delta}\alpha(q^2) + \delta\tilde{\Delta}^{(lb)}\alpha(q^2) , \tag{47}$$

$$R^{(lb)}(q^2) = R(q^2) + \delta R^{(lb)}(q^2) , \tag{48}$$

$$V^{(lb)}_{\gamma Z}(q^2) = V(q^2) + \delta V^{(lb)}_{\gamma Z}(q^2) , \tag{49}$$

$$V^{(lb)}_{Z\gamma}(q^2) = V(q^2) + \delta V^{(lb)}_{Z\gamma}(q^2) , \tag{50}$$

where the quantities without indices are the universal ones that would appear in the case of final leptonic states treated in [9] and

$$\delta\tilde{\Delta}^{(lb)}\alpha(q^2) = (\Gamma^{(\gamma)}_{\mu,l}(0), v^{(\gamma)}_{\mu,l}) - (\Gamma^{(\gamma)}_{\mu,b}(0), v^{(\gamma)}_{\mu,b}) - [(\Gamma^{(\gamma)}_{\mu,l}(M_Z^2), v^{(\gamma)}_{\mu,l}) - (\Gamma^{(\gamma)}_{\mu,b}(M_Z^2), v^{(\gamma)}_{\mu,b})] + A^{(B)}_{\gamma,ll}(q^2,\theta) - A^{(B)}_{\gamma,lb}(q^2,\theta) , \tag{51}$$

$$\delta R^{(lb)}(q^2) = \mathrm{Re}\{(\Gamma^{(Z)}_{\mu,l}(q^2), v^{(Z)}_{\mu,l}) - (\Gamma^{(Z)}_{\mu,b}(q^2), v^{(Z)}_{\mu,b}) - [(\Gamma^{(Z)}_{\mu,l}(M_Z^2), v^{(Z)}_{\mu,l}) - (\Gamma^{(Z)}_{\mu,b}(M_Z^2), v^{(Z)}_{\mu,b})] + A^{(B)}_{Z,ll}(q^2,\theta) - A^{(B)}_{Z,lb}(q^2,\theta) , \tag{52}$$

$$\delta V_{\gamma Z}^{(lb)}(q^2) = \frac{q^2 - M_Z^2}{q^2}\mathrm{Re}[(\Gamma_{\mu,b}^{(\gamma)}(q^2), v_{\mu,b}^{(Z)}) - (\Gamma_{\mu,l}^{(\gamma)}(q^2), v_{\mu,l}^{(Z)})] + (q^2 - M_Z^2)\mathrm{Re}[A_{\gamma Z,ll}^{(B)}(q^2,\theta) - A_{\gamma Z,lb}^{(B)}(q^2,\theta)] , \tag{53}$$

$$\delta V_{Z\gamma}^{(lb)}(q^2) = \mathrm{Re}\{(\Gamma_{\mu,l}^{(Z)}(q^2), v_{\mu,l}^{(\gamma)}) - (\Gamma_{\mu,b}^{(Z)}(q^2), v_{\mu,b}^{(\gamma)}) - [(\Gamma_{\mu,l}^{(Z)}(M_Z^2), v_{\mu,l}^{(\gamma)}) - (\Gamma_{\mu,b}^{(Z)}(M_Z^2), v_{\mu,b}^{(\gamma)})] + (q^2 - M_Z^2)[A_{Z\gamma,ll}^{(B)}(q^2,\theta) - A_{Z\gamma,lb}^{(B)}(q^2,\theta)]\} . \tag{54}$$

Note that by definition $A_{\gamma,lf}^{(B)}(0,\theta)$ and $A_{Z,lf}^{(B)}(M_Z^2,\theta)$ identically vanish.

The previous expressions and definitions can be easily generalized to the case of the full final hadronic cross section, whose experimental measurement will be statistically favored. After some additions and recombinations we are led to a first general expression that would read

$$\sigma_5(q^2) \equiv \sigma_{l\mathrm{had}}(q^2) = \sigma_5^{(\gamma)}(q^2) + \sigma_5^{(Z)}(q^2) + \sigma_5^{(\gamma Z)}(q^2) , \tag{55}$$

where

$$\sigma_5^{(\gamma)}(q^2) = N\left(\frac{4\pi q^2}{3}\right)\left(\frac{11\alpha^2(0)}{9q^4}\right)[1 + \delta_5^{(\gamma)}] , \tag{56}$$

$$\delta_5^{(\gamma)} = 2\tilde{\Delta}^{(ll)}\alpha(q^2) + \frac{16}{11}\tilde{\Delta}^{(lu)}\alpha(q^2) + \frac{4}{11}\tilde{\Delta}^{(ld)}\alpha(q^2) + \frac{2}{11}\tilde{\Delta}^{(lb)}\alpha(q^2) , \tag{57}$$

where following the attitude explained at the beginning of Sec. II A, we set $N = 3$. For the next term $\sigma_5^{(Z)}$ we find, after a number of elementary steps

$$\sigma_5^{(Z)}(q^2) = N\left(\frac{4\pi q^2}{3}\right)\frac{\left[\frac{3\Gamma_l}{M_Z}\right]\left[\frac{3\Gamma_5}{N_f M_Z}\right]}{(q^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2}[1 + \delta_5^{(Z)}] , \tag{58}$$

where $\Gamma_5 = \Gamma_{\mathrm{had}}$ and

$$\begin{aligned}\delta_5^{(Z)} = & -2R(q^2) - 4s_1c_1p_5V(q^2) \\ & -\frac{2\Gamma_u}{\Gamma_5}\left[2\delta R^{(lu)}(q^2) + \frac{8s_1c_1v_1}{1+v_1^2}\delta V_{\gamma Z}^{(lu)}(q^2) + \frac{16s_1c_1v_u}{3(1+v_u^2)}\delta V_{Z\gamma}^{(lu)}(q^2)\right] \\ & -\frac{2\Gamma_d}{\Gamma_5}\left[2\delta R^{(ld)}(q^2) + \frac{8s_1c_1v_1}{1+v_1^2}\delta V_{\gamma Z}^{(ld)}(q^2) + \frac{8s_1c_1v_d}{3(1+v_d^2)}\delta V_{Z\gamma}^{(ld)}(q^2)\right] \\ & -\frac{\Gamma_b}{\Gamma_5}\left[2\delta R^{(lb)}(q^2) + \frac{8s_1c_1v_1}{1+v_1^2}\delta V_{\gamma Z}^{(lb)}(q^2) + \frac{8s_1c_1v_b}{3(1+v_b^2)}\delta V_{Z\gamma}^{(lb)}(q^2)\right] ,\end{aligned} \tag{59}$$

with

$$p_5 = \frac{v_1}{1+v_1^2} + \frac{4\Gamma_u}{3\Gamma_5}\left(\frac{v_u}{1+v_u^2}\right) + \frac{2\Gamma_d}{3\Gamma_5}\left(\frac{v_d}{1+v_d^2}\right) + \frac{\Gamma_b}{3\Gamma_5}\left(\frac{v_b}{1+v_b^2}\right) . \tag{60}$$

From a glance at Eq. (59), one might have the impression that both in the "universal" and in the "nonuniversal" component of the corrections a number of unwanted (i.e., not directly measured on Z resonance) ratios Γ_q/Γ_5 appear. But this is not a problem at the considered one-loop level since these terms are already multiplied by order (α). Therefore, they must be consistently replaced by expressions that only involve the quantity s_1^2 entering Eq. (33) (note that ϵ_1^{ff} can be neglected for the same reasons). As a consequence we can write, in Eq. (59),

$$\frac{\Gamma_{u,c}}{\Gamma_5} = \frac{1+v_u^2}{2(1+v_u^2)+3(1+v_d^2)} , \tag{61}$$

$$\frac{\Gamma_{d,s,b}}{\Gamma_5} = \frac{1+v_d^2}{2(1+v_u^2)+3(1+v_d^2)} . \tag{62}$$

The same considerations and simplifications strictly valid at one loop can be repeated for the interference component. After some straightforward rearrangements, this leads to the expression

$$\sigma_5^{(\gamma Z)}(q^2) = N\left(\frac{4\pi q^2}{3}\right)\frac{2}{3}\alpha(0)\frac{q^2 - M_Z^2}{q^2[(q^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2]}\left[\frac{3\Gamma_l}{M_Z}\right]^{1/2}\Sigma_5\frac{\tilde{v}_l}{(1+\tilde{v}_l^2)^{1/2}}[1 + \delta_5^{(\gamma Z)}] , \tag{63}$$

$$\delta_5^{(\gamma Z)} = \tilde{\Delta}\alpha(q^2) - R - 4s_1c_z p_5' V$$
$$+\frac{4}{\Sigma_5}\left(\frac{3N_u\Gamma_u}{M_Z}\right)^{1/2}\frac{v_u}{(1+v_u^2)^{1/2}}\left[\delta\tilde{\Delta}^{(lu)}\alpha(q^2) - \delta R^{(lu)}(q^2) - \frac{4s_1c_1}{v_1}\delta V_{\gamma Z}^{(lu)}(q^2) - \frac{8s_1c_1}{3v_u}\delta V_{Z\gamma}^{(lu)}(q^2)\right]$$
$$+\frac{2}{\Sigma_5}\left(\frac{3N_d\Gamma_d}{M_Z}\right)^{1/2}\frac{v_d}{(1+v_d^2)^{1/2}}\left[\delta\tilde{\Delta}^{(ld)}\alpha(q^2) - \delta R^{(ld)}(q^2) - \frac{4s_1c_1}{v_1}\delta V_{\gamma Z}^{(ld)}(q^2) - \frac{4s_1c_1}{3v_d}\delta V_{Z\gamma}^{(ld)}(q^2)\right]$$
$$+\frac{1}{\Sigma_5}\left(\frac{3N_b\Gamma_b}{M_Z}\right)^{1/2}\frac{v_b}{(1+v_b^2)^{1/2}}\left[\delta\tilde{\Delta}^{(lb)}\alpha(q^2) - \delta R^{(lb)}(q^2) - \frac{4s_1c_1}{v_1}\delta V_{\gamma Z}^{(lb)}(q^2) - \frac{4s_1c_1}{3v_b}\delta V_{Z\gamma}^{(lb)}(q^2)\right], \quad (64)$$

with

$$\Sigma_5 = 4\left(\frac{3N_u\Gamma_u}{M_Z}\right)^{(1/2)}\frac{\tilde{v}_u}{(1+\tilde{v}_u^2)^{1/2}} + 2\left(\frac{3N_d\Gamma_d}{M_Z}\right)^{(1/2)}\frac{\tilde{v}_d}{(1+\tilde{v}_d^2)^{1/2}} + \left(\frac{3N_b\Gamma_b}{M_Z}\right)^{(1/2)}\frac{\tilde{v}_b}{(1+\tilde{v}_b^2)^{1/2}}, \quad (65)$$

$$p_5' = \frac{1}{v_1} + \frac{8}{3(1+v_u^2)^{1/2}\Sigma_5}\left(\frac{3N_u\Gamma_u}{M_Z}\right)^{1/2} + \frac{2}{3(1+v_d^2)^{1/2}\Sigma_5}\left(\frac{3N_d\Gamma_d}{M_Z}\right)^{1/2} + \frac{1}{3(1+v_b^2)^{1/2}\Sigma_5}\left(\frac{3N_b\Gamma_b}{M_Z}\right)^{1/2}, \quad (66)$$

and $N_{u,d,b} = 3$.

Similarly to the case of Eqs. (61), (62), the quantities $\Gamma_f^{1/2}/\Sigma_5$ can be safely evaluated in terms of s_1^2 only. The quantity Σ_5 Eq. (65) requires a separate discussion, that will be given in the concluding remarks, to show that it can be safely neglected (or approximated).

To conclude our review, we still have to consider the separation of the quantities Eqs. (44), (45) that give the numerator of the forward-backward asymmetry for $b\bar{b}$ production. This can be done in the way that we have illustrated, and leads to the expressions

$$\sigma_{\mathrm{FB},lb}^{(\gamma Z)}(q^2) = N\left(\frac{4\pi q^2}{3}\right)2\alpha(0)|Q_b|\frac{q^2 - M_Z^2}{q^2[(q^2-M_Z^2)^2 + M_Z^2\Gamma_Z^2]}$$
$$\times\left[\frac{3\Gamma_l}{M_Z}\right]^{1/2}\left[\frac{3\Gamma_b}{N_bM_Z}\right]^{1/2}\left(\frac{1}{(1+\tilde{v}_l^2)^{1/2}(1+\tilde{v}_b^2)^{1/2}}\right)$$
$$\times[1 + \tilde{\Delta}\alpha(q^2) - R(q^2) + \delta\tilde{\Delta}^{(lb)}\alpha(q^2) - \delta R^{(lb)}(q^2)], \quad (67)$$

$$\sigma_{\mathrm{FB},lb}^{(Z)}(q^2) = N\left(\frac{4\pi q^2}{3}\right)\frac{\left[\frac{3\Gamma_l}{M_Z}\right]\left[\frac{3\Gamma_f}{N_fM_Z}\right]}{[(q^2-M_Z^2)^2 + M_Z^2\Gamma_Z^2]}\left(\frac{4\tilde{v}_l\tilde{v}_b}{(1+\tilde{v}_l^2)(1+\tilde{v}_b^2)}\right)$$
$$\times\left[1 - 2R(q^2) - 4s_1c_1\left[\frac{1}{v_1} + \frac{|Q_b|}{v_b}\right]V(q^2)\right.$$
$$\left. -2\delta R^{(lb)}(q^2) - 4s_1c_1\left\{\frac{1}{v_1}\delta V_{\gamma Z}^{(lb)}(q^2) + \frac{|Q_b|}{v_b}\delta V_{Z\gamma}^{(lf)}(q^2)\right\}\right]. \quad (68)$$

Equations (67), (68) conclude this long section. We are now in a position to calculate, using the leptonic formulas or Ref. [9], the contributions of new physics of both universal and nonuniversal type to the full set on experimental quantities that will be measured at LEP 2 and NLC (without the extra facility of longitudinal initial electron polarization in the latter case). In particular, we shall consider on top of the leptonic observables previously considered in Ref. [9], i.e., σ_μ, $A_{\mathrm{FB},\mu}$ and A_τ (the final τ polarization), the *ratios*

$$R_5 = \frac{\sigma_5}{\sigma_\mu}, \quad (69)$$

$$R_b = \frac{\sigma_{lb}}{\sigma_\mu}, \quad (70)$$

and $A_{\mathrm{FB},b}$. The relevant expressions can be derived from Sec. II and from Ref. [9]. We shall give them explicitly in Secs. III and IV for two orthogonal situations of models with universal and nonuniversal type of effects.

III. A MODEL WITH ANOMALOUS GAUGE COUPLINGS

As a first example of application of our approach, we shall consider the case of a model of new physics in which anomalous gauge couplings (AGC's) [7] are generated by an effective Lagrangian. Although the discussion could be much more general, we shall first stick to the dimension six, CP-conserving Lagrangian proposed by Hagiwara, Ishihara, Szalapski, and Zeppenfeld [8]. This contains, in principle, eleven parameters of which nine would affect the WWV couplings. In particular, the most general four fermion process at the one loop level would be affected by four "renormalized" parameters denoted in Ref. [8] as f^r_{DW}, f^r_{DB}, $f^r_{\phi 1}$, and f^r_{BW} for $f \neq b$. If final $b\bar{b}$ production is considered, one should, in principle, include the order (m_t^2) contributions generated by f_W, f_B that have been recently shown to appear in the partial width of Z into $b\bar{b}$ [4].

The calculation of this type of effects has been already performed for the purely leptonic case in Ref. [9]. The main feature that appears is that only *two* independent parameters, i.e., f^r_{DW}, f^r_{DB} survive in the full set of leptonic observables. This is due to the fact that in the contribution of the model to the subtracted corrections $\tilde{\Delta}\alpha$, R, and V the terms proportional to $f_{\phi 1}$, f_{BW}, that carry no sufficient powers of q^2, are fully reabsorbed into the subtraction constant, i.e., into the trading of G_μ by Γ_l and $s_l^2(M_Z^2)$. This leads to

$$\tilde{\Delta}^{(\mathrm{AGC})}\alpha(q^2) = -q^2\left(\frac{2e^2}{\Lambda^2}\right)(f^r_{DW} + f^r_{DB}) \ , \tag{71}$$

$$R^{(\mathrm{AGC})}(q^2) = (q^2 - M_Z^2)\left(\frac{2e^2}{s_1^2 c_1^2 \Lambda^2}\right)(f^r_{DW}c_1^4 + f^r_{DB}s_1^4) \ , \tag{72}$$

$$V^{(\mathrm{AGC})}(q^2) = (q^2 - M_Z^2)\left(\frac{2e^2}{s_1 c_1 \Lambda^2}\right)(f^r_{DW}c_1^2 + f^r_{DB}s_1^2) \ . \tag{73}$$

We now perform the same calculation for R_5, R_b, and $A_{\mathrm{FB},b}$. In principle, we might expect the appearance of the extra parameters f_W, f_B in the expression of the final b contribution. In fact, the rigorous expression for R_5 would read (neglecting numerically irrelevant contributions)

$$\begin{aligned}\frac{\delta R_5^{(\mathrm{AGC})}}{R_5} \simeq\ & C_\alpha(q^2)\tilde{\Delta}^{(\mathrm{AGC})}\alpha(q^2) \\ & + C_R(q^2)R^{(\mathrm{AGC})}(q^2) + C_V(q^2)V^{(\mathrm{AGC})}(q^2) \\ & + C_b(q^2)\delta R_{lb}^{(\mathrm{AGC})}(q^2)\end{aligned} \tag{74}$$

and $C_\alpha, C_R, C_V(q^2)$ are certain kinematical functions whose numerical value at the "reference" points $q^2 = 4M_Z^2$ (LEP 2) and $q^2 = (500\ \mathrm{GeV})^2$ are

$$C_\alpha(4M_Z^2) = -0.77, \quad C_\alpha[(500\ \mathrm{GeV})^2] = -0.67 \ , \tag{75}$$

$$C_R(4M_Z^2) = -0.77, \quad C_R[(500\ \mathrm{GeV})^2] = -0.67 \ , \tag{76}$$

$$C_V(4M_Z^2) = -0.81, \quad C_V[(500\ \mathrm{GeV})^2] = -0.75 \ . \tag{77}$$

The last term in Eq. (77) contains a kinematical coefficient C_b such that

$$C_b(4M_Z^2) = -0.25, \quad C_b[(500\ \mathrm{GeV})^2] = -0.20 \tag{78}$$

and a "nonuniversal" contribution, typical of the final $Zb\bar{b}$ couplings. In terms of parameters of the model, one gets after a straightforward calculation whose main points have been illustrated in a previous reference [4]:

$$\begin{aligned}\delta R_{lb}^{(\mathrm{AGC})} = & 2\left(\frac{q^2 - M_Z^2}{M_Z^2}\right)\left(\frac{\alpha m_t^2}{64\pi s_1^2\Lambda^2}\right)\left(f_W - f_B\frac{s_1^2}{c_1^2}\right) \\ & \times \ln\left(\frac{\Lambda^2}{M_Z^2}\right) \ .\end{aligned} \tag{79}$$

In fact, in Ref. [4] a bound for a different combination of f_W, f_B was calculated, assuming that the still conceivable small discrepancy between the experimental values of $\Gamma_{b\bar{b}}$ at resonance and the SM prediction was originated by this type of new physics. From that calculation one sees however that, even pushing the bound to the extreme value, we would not affect the relative R_5 shift by more than a fraction of a percent, hardly visible at realistic experimental conditions. For this reason, and keeping in mind that the complete correction to R_5 contains in principle such nonuniversal terms, we have in fact neglected them in the coming considerations. This has the welcome consequence that another experimental variable can be added to the previous leptonic set without increasing the overall number of parameters to be fitted, or bounded. More precisely, we would have now, at LEP 2 energy

$$\frac{\delta R_5^{(\mathrm{AGC})}}{R_5} = 1.87\left(\frac{M_Z^2}{\Lambda^2}\right)f^r_{DW} + 0.68\left(\frac{M_Z^2}{\Lambda^2}\right)f^r_{DB} \ . \tag{80}$$

The previous considerations can be exactly repeated for R_b. Leaving aside a more general discussion, we would find in this case in the configuration $q^2 = 4M_Z^2$:

$$\begin{aligned}\frac{\delta R_b^{(\mathrm{AGC})}}{R_b} = & -1.13[\tilde{\Delta}^{(\mathrm{AGC})}\alpha(q^2) + R^{(\mathrm{AGC})}(q^2)] \\ & -0.94V^{(\mathrm{AGC})}(q^2) - 1.53\delta R_{lb}^{(\mathrm{AGC})}(q^2) \ .\end{aligned} \tag{81}$$

Again, the conceivable contribution from the nonuniversal term would be, at most, of a few (two–three) relative percent, that should be realistically below the observability limits. Neglecting again this contribution would lead us to the approximate expression

$$\frac{\delta R_b^{(\mathrm{AGC})}}{R_b} = -4.14\left(\frac{M_Z^2}{\Lambda^2}\right)f^r_{DW} - 1.26\left(\frac{M_Z^2}{\Lambda^2}\right)f^r_{DB} \ . \tag{82}$$

To conclude this illustration, we have calculated the contribution to the forward-backward b asymmetry. This quantity, unlike the two previous cases, does not receive in practice appreciable contributions from the nonuniversal part, which is essentially of left-handed type. The rigorous expression at LEP 2 energies would therefore read

$$\frac{\delta A_{\mathrm{FB},b}^{(\mathrm{AGC})}}{A_{\mathrm{FB},b}} = 0.40[\tilde{\Delta}^{(\mathrm{AGC})}\alpha(q^2) + R^{(\mathrm{AGC})}(q^2)] -0.42V^{(\mathrm{AGC})}(q^2)\ . \qquad (83)$$

In conclusion, we have now at our disposal six experimental variables ($\sigma_\mu, A_{\mathrm{FB},\mu}, A_\tau, R_5, R_b$, and $A_{\mathrm{FB},b}$) that only depend on *two* parameters (and that, at most, would contain *one* extra third combination of f_W and f_B). This represents, in our opinion, an interesting alternative to the conventional analyses [8], where the full set of six parameters should enter in the previous observables. In fact a rigorous calculation, that fully takes into account the effects of QED radiation, is at the moment being performed and will be shown in a separate dedicated paper. Here we can give a qualitative hint looking, e.g., at the particular effect on R_5, Eq. (64). In correspondence to a typical couple of values that would still be allowed [14] by the available low-energy constraints, i.e., $f_{DW} = -1, f_{DB} = 4$, we would find a relative positive shift of approximately six percent in R_5, that would lead to a spectacular visible signal.

As a final byproduct of our approach, in which the number of parameters for this specific model is drastically reduced and in practice only two independent quantities remain, we shall obtain the (pleasant) result that, for any chosen triplet of observables, there will be a linear relationship between the separate effects that will correspond to a plane in the three-dimensional space of the observables. Drawing these planes for various choices of variables is rather easy. Here we want to show two particular examples related to the choices of $(\sigma_\mu, A_{\mathrm{FB},\mu}, A_\tau)$ and $(\sigma_\mu, A_{\mathrm{FB},\mu}, R_5)$ as "coordinate axes." The corresponding regions are shown in Figs. 1 and 2 in the simple approximation that corresponds to our approximate equations (a more rigorous derivation, with a full QED convolution of effects, will be given, as we preannounced, in a forthcoming paper). To make a meaningful statement, we have shown in these figures the "dead" region where a signal would not be distinguishable, corresponding to a relative experimental error of 1.5 percent for the various cross sections and forward-backward asymmetry and 15 percent for the τ polarization (these values assume an integrated luminosity of 500 pb^{-1} at $\sqrt{q^2} = 2M_Z$, and correspond to a muon cross section of 4.4 pb). Therefore, *if* a signal of new physics were seen in some of the aforementioned observables, one would be able to decide whether the signal belongs to the considered model, or not. In fact, one might even hope to find a sort of one-to-one correspondence between models and regions of a certain three-dimensional space of observables.

Although we cannot prove this statement in general, we have found an encouraging manifestation of this possibility considering the case of a technicolor-type model with a couple of strong vector resonances. The full details of this model have been already discussed in two previous references [15,9], and we shall not repeat them here. The only thing that we will show are the characteristic regions of the model, that is essentially describable by two parameters. As one can see in Figs. 1 and 2 the

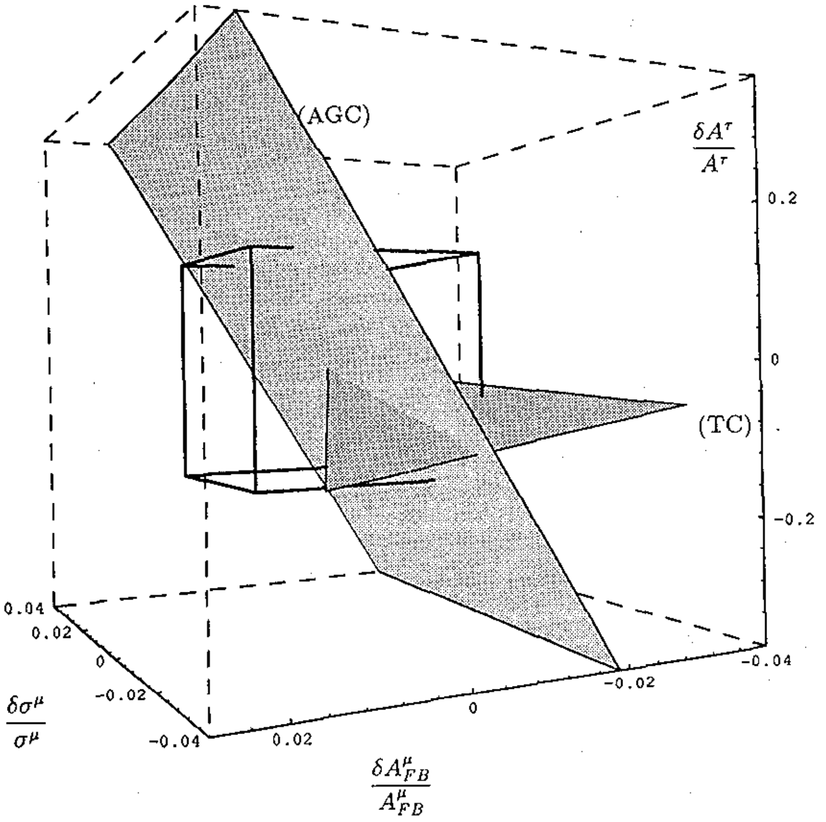

FIG. 1. Trajectories in the three-dimensional space of relative departures from SM for leptonic observables $\sigma_\mu, A_{\mathrm{FB},\mu}, A_\tau$ at a LEP 2 energy of 175 GeV for AGC models and technicolor (TC) models. The box represents the unobservable domain corresponding to a relative accuracy of 1.5 percent for $\sigma_\mu, A_{\mathrm{FB},\mu}$ and 15 percent for A_τ.

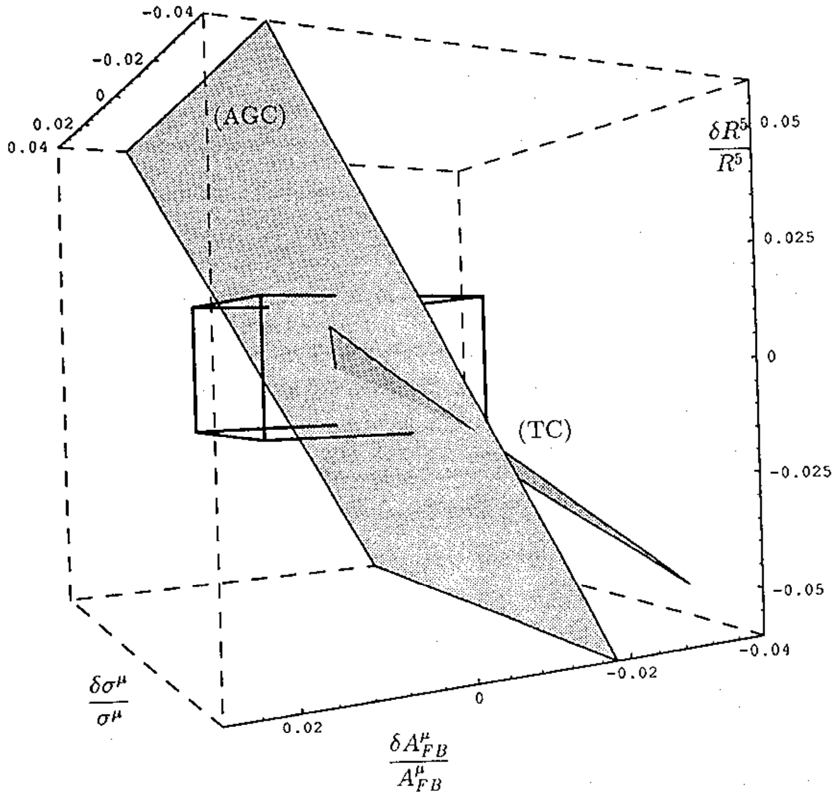

FIG. 2. Trajectories in the three-dimensional space of relative departures from SM for leptonic and hadronic observables $\sigma_\mu, A_{\mathrm{FB},\mu}, R_5$ at a LEP 2 energy of 175 GeV for AGC models and TC models. The box represents the unobservable domain corresponding to a relative accuracy of 1.5 percent for all three observables.

visible regions (where the size of the effect is larger than that of the realistic experimental error [2]) of the AGC and of the TC models are indeed well separated, and no confusion between these two models would possibly arise.

Having illustrated, we hope in a clear way, the main features of our approach for a specific type of (almost) universal new physics effects, we shall devote the next and last section to the discussion of a "typically" nonuniversal kind of effects, generated by the presence of one extra (and of the most general type) Z.

IV. A MODEL WITH ONE GENERAL EXTRA Z

As a possibly rewarding unconventional application of our method, we illustrate the treatment of a model where one extra Z (generically denoted Z'), with the most general type of vector and axial couplings to leptons and quarks, is supposed to exist. All the popular "canonical" models (E_6, LR symmetry, composite models, ...) will be then recovered by adjusting the couplings to the corresponding values.

The effect of a heavy Z', of a mass not smaller than $\simeq$ 400–500 GeV, as *suggested* form the available Collider Detector at Fermilab (CDF) limits [16], is usually treated at "Z'-tree level," i.e., only adding to the full amplitude the graph with the Z' exchange, where both its couplings to fermions and its mass are identified with the physical ones. This leads to a modification of the Born amplitude of the form

$$\mathcal{A}^{\gamma,Z,Z'}_{lf} = \mathcal{A}^{\gamma,Z}_{lf} + \frac{i}{q^2 - M^2_{Z'}} v^{(Z')}_{\mu,l} v^{(Z')}_{\mu,f} \ , \tag{84}$$

with the general $Z'ff$ couplings

$$v^{(Z')}_{\mu,f} = \left(\frac{e}{2c_1 s_1}\right) \bar{u}_f \gamma_\mu (g'_{Vf} - \gamma^5 g'_{Af}) v_f \ . \tag{85}$$

From a formal point of view, that will be particularly suited for our approach, it is possible to rewrite the Z' effect as a modification of our "generalized" subtracted corrections. This effect, that would correspond exactly to a "box-type" modification of completely nonuniversal type, can be described as

$$\tilde{\Delta}^{(lf)(Z')}\alpha(q^2) = \frac{q^2}{q^2 - M^2_{Z'}} \left(\frac{1}{4s_1^2 c_1^2}\right) \left(\frac{g_{Vl} g_{Vf}}{Q_l Q_f}\right) [(\xi_{Vl} - \xi_{Al})(\xi_{Vf} - \xi_{Af})] \ , \tag{86}$$

$$R^{(lf)(Z')}(q^2) = -\left(\frac{q^2 - M^2_Z}{q^2 - M^2_{Z'}}\right) \xi_{Al} \xi_{Af} \ , \tag{87}$$

$$V_{\gamma Z}^{(lf)(Z')}(q^2) = -\left(\frac{q^2 - M_Z^2}{q^2 - M_{Z'}^2}\right)\left(\frac{g_{Vl}}{2s_1c_1Q_l}\right)\xi_{Af}(\xi_{Vl} - \xi_{Al}) \ , \tag{88}$$

$$V_{Z\gamma}^{(lf)(Z')}(q^2) = -\left(\frac{q^2 - M_Z^2}{q^2 - M_{Z'}^2}\right)\left(\frac{g_{Vf}}{2s_1c_1Q_f}\right)\xi_{Al}(\xi_{Vf} - \xi_{Af}) \ , \tag{89}$$

where we have used the definitions

$$\xi_{Vl,f} \equiv \frac{g'_{Vl,f}}{g_{Vl,f}} \ , \tag{90}$$

$$\xi_{Al,f} \equiv \frac{g'_{Al,f}}{g_{Al,f}} \ , \tag{91}$$

$$g_{Al,f} \equiv I_{l,f}^{3L} \ , \tag{92}$$

$$g_{Vl,f} \equiv I_{l,f}^{3L} - 2Q_{l,f}s_1^2 \ . \tag{93}$$

One sees from Eqs. (90)–(92) that the most general Z' effect at e^+e^- colliders is parametrizable via *six* independent effective couplings, that could be chosen as, e.g., $\xi_{V,A,i}\frac{M_Z}{\sqrt{M_{Z'}^2-q^2}}$ $(i = l,u,d)$. Therefore, with one experiment at fixed q^2 it would never be possible to disentangle $\xi_{V,A}$ from $M_{Z'}$, so that the normal attitude would be to derive (in case of negative searches) bound for $M_{Z'}$ for given $\xi_{V,A}$. In fact, this will be done in another specific dedicated paper in preparation. Here we want to show that, in full analogy with the final example of the previous section, it would be possible to draw a region in a three-dimensional space of observables that would be typical of the *most general* Z'. To achieve this goal, one must necessarily choose three purely leptonic observables.

At LEP 2, this might be obtained by combining the measurements of σ_μ and $A_{\mathrm{FB},\mu}$ with that of the final τ polarization. At NLC, the role of the final τ polarization would be played by the (theoretically equivalent) longitudinal polarization asymmetry for leptons. The general Z' contribution to these quantities will actually take the form of Eqs. (71)–(73) with $\tilde{\Delta}^{(\mathrm{AGC})}\alpha(q^2)$, $R^{(\mathrm{AGC})}(q^2)$, $V^{(\mathrm{AGC})}(q^2)$, respectively, replaced by $\tilde{\Delta}^{(Z')}$, $R^{(Z')}(q^2)$, and $V^{(Z')}(q^2)$ given in Eqs. (93)–(95) for $f = l$.

Eliminating the two effective leptonic parameters gives then rise to a relationship between the shifts of σ_μ, $A_{\mathrm{FB},\mu}$, and A_τ that would lead, at LEP 2 energies, to a certain three-dimensional region characteristic of this model and represented in Fig. 3 (we assumed the same experimental errors as in the previous figures). Note that, with this procedures, all residual "intrinsic" Z' ambiguities, e.g., in the normalization of g'_V, g'_A disappear.

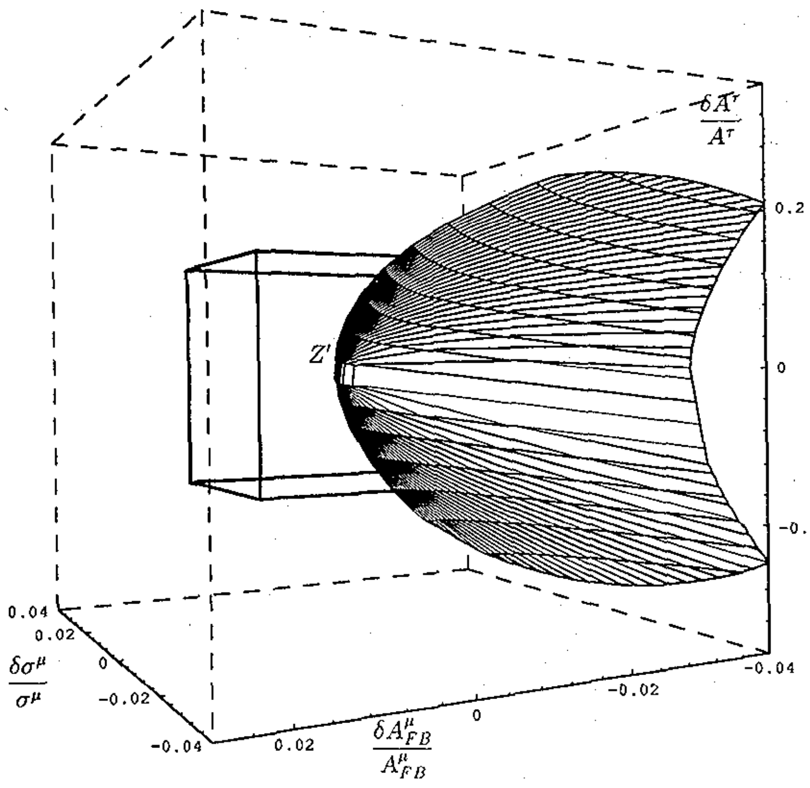

FIG. 3. Trajectories in the three-dimensional space of relative departures from SM for leptonic observables $\sigma_\mu, A_{\mathrm{FB},\mu}, A_\tau$ at a LEP 2 energy of 175 GeV for general Z' models. The box has the same meaning as in Fig. 1.

A warning is necessary at this point since this figure, as well as the previous ones, have been drawn in "first approximation," i.e., without calculating the fully QED convoluted effects (this is, in fact, in preparation at the moment). We can, though, claim that, as a general feature of such more realistic calculations, the "first approximation" results are quite reasonably reproduced provided that a suitable cut is enforced on the hard photon spectrum. In this spirit, we believe that it makes sense to compare Fig. 3 for the Z' model with the corresponding Fig. 1 for the AGC and TC models and conclude that, at least in this orientative picture, the three regions corresponding to these theoretically "orthogonal" models are completely (i.e., in the physically reasonable region where a statistical meaning can be attributed to the signal) separated.

V. CONCLUDING REMARKS

We have shown in this paper that the calculation of new physics effects in a general four-fermion process is facilitated if the procedure of "trading" G_μ by quantities measured on Z peak is generalized from the case of final leptonic states to that of final hadronic states. The new relevant quantities that enter the modified Born approximation are the Z hadronic widths Γ_5 and Γ_b and, to a much smaller extent the charm width Γ_c and the two forward-backward asymmetries $A_{\mathrm{FB},c}, A_{\mathrm{FB},b}$ on Z resonance, if we only consider the measurements of σ_5, σ_b, and $A_{\mathrm{FB},b}$ at variable q^2. We want to conclude this paper by making this statement more quantitative.

Consider σ_5 first. Here the leading terms at Born level are the pure photon and the pure Z contributions. In our modified expression, the only Born term that changes is that corresponding to Z exchange, whose numerical weight is roughly of the same size as that of the photon. The net effect of the change is that of replacing here G_μ^2 by the product of Γ_l and Γ_5. The corresponding relative experimental error thus introduced is a fraction of a percent [2], much below the experimental reach at any future e^+e^- collider. The same conclusion applies to the term $N_f = 3\left(1+\frac{\alpha_S(M_Z^2)}{\pi}\right)$ that divides Γ_5 and that generates an error of a few per mill at most. Note that the same relative error will affect the contribution that we called "QCD," since $\alpha_s(q^2)$ should be known with the same accuracy as $\alpha_s(M_Z^2)$. The remaining new input quantities that enter σ_5 are Γ_c and $s_{b,c}^2(M_Z^2)$ defined by Eq. (33). But even without discussing this point in full detail, as one could easily do, one sees immediately that these new parameters only contribution to the interference $\gamma - Z$. The latter is, already at the starting Born level, completely negligible with respect to the dominant pure photon and Z ones. Therefore, a discussion on the effect of "small" changes in this term is, indeed, completely academic and we shall not give it here.

In the case of σ_b, the same situation is almost identically reproduced, with the only replacement of Γ_5 by Γ_b in the Z Born expression. The error on Γ_b is in fact slightly larger, of a relative one percent [2], but also the experimental accuracy for σ_b will be certainly larger than one percent, and the same conclusions as in the case of σ_5 still apply.

The last case to be discussed is that of $A_{\mathrm{FB},b}$. Here the situation is quite different since the $\gamma - Z$ term is now largely dominating. This term contains Γ_l, Γ_b that will introduce errors of negligible size (i.e., at the relative level of less than one percent) and a term containing $s_b^2(M_Z^2)$ as one sees from Eq. (67). In fact, the relevant quantity to be considered is

$$\frac{1}{(1+\tilde{v}_b^2)^{1/2}} \tag{94}$$

that is directly related to the forward-backward asymmetry on Z resonance $A_{\mathrm{FB},b}(M_Z^2)$ [2]. From the 4 percent uncertainty on this quantity given in Ref. [2] one can derive the relative error on the term in Eq. (94) that generates a 3 percent uncertainty on the prediction for $A_{\mathrm{FB},b}$. This is also weaker than the experimental uncertainty expected at LEP 2.

In conclusion, all the replacements in the Born approximation are completely harmless for the considered process. Therefore, the gain that we obtained in the corresponding simplifications of the "subtracted" corrections seems to us rather remarkable. We would say that the full and rigorous exploitation of the high precision measurements of electroweak physics at $q^2 = M_Z^2$ allows us to perform calculations of virtual new physics effects at LEP 2 (and, possibly, at NLC) in a way that seems to us simpler and cleaner than the conventional one where G_μ, the high precision electroweak measurement at $q^2 = 0$, is used. We are now in the process of applying the method to other possibly interesting models of new physics for which calculations of virtual effects might be relevant at future e^+e^- colliders.

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