## Fragmentation production of doubly heavy baryons

Michael A. Doncheski

Ottawa-Carleton Institute for Physics, Department of Physics, Carleton University, Ottawa, Ontario, Canada K1S 5B6

## J. Steegborn and M. L. Stong

Institut für Theoretische Teilchenphysik, Universität, D-76128 Karlsruhe, Germany

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Baryons with a single heavy quark are being studied experimentally at present. Baryons with two units of heavy flavor will be abundantly produced not only at future colliders, but also at existing facilities. In this paper we study the production via heavy quark fragmentation of baryons containing two heavy quarks at the Fermilab Tevatron, the CERN LHC, DESY HERA, and the NLC. The production rate is woefully small at HERA and at the NLC, but significant at pp and  $p\bar{p}$  machines. We present distributions in various kinematical variables in addition to the integrated cross sections at hadron colliders.

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The constituent quark model has been remarkably successful in describing the observed hadronic states. This includes mesons with one heavy and one light quark (i.e., B and D mesons), baryons with one heavy and two light quarks, and the  $J/\psi$  and  $\Upsilon$  mesons, which contain a heavy quark and heavy antiquark. Recent experiments have made great progress in the observations of the c and b baryons [1, 2] and their decays [3, 4]. The constituent quark model predicts the existence of baryons containing two heavy quarks (cc, bc, or bb) and one light quark (see [5] for a review of the theory and prospects for the discovery of *ccq* baryons), and we are on the verge of obtaining the necessary experimental sensitivity to observe these states. The energies necessary to produce these particles are already reached; the difficulty remaining is in their reconstruction. These states have in general a large number of decay modes, so that their observation and a measurement of their properties will require a large number of them to be produced. This difficulty is increased by the fact that the production rates at  $e^+e^-$  colliders are extremely small, so that the identification of these particles must take place in the messier environment of hadronic collisions.

Were the heavy quark of infinite mass, the two heavy quarks would be bound in a pointlike diquark, and the light degrees of freedom would "see" this diquark as a static antitriplet color source. In this limit, the interactions of the light degrees of freedom with the heavy diquark arc quite similar to interactions of the light degrees of freedom with the  $\bar{b}$  quark in a B meson [6, 7]. For realistic heavy quark masses, this simple picture is not yet valid. In particular, the hyperfine splittings of these baryons are not yet well described by the heavymass limit [8]. The states may nonetheless be accurately treated as a combination of light degrees of freedom and a heavy although nonpointlike diquark. The interactions between the two heavy quarks are analogous to the  $Q\overline{Q}$ system familiar from  $J/\psi$  and  $\Upsilon$  spectroscopy. At short distances, the interaction will be dominated by one-gluon

exchange, although with a color factor 2/3 from the QQ interaction rather than 4/3 from  $Q\overline{Q}$ , and at long distances it will be confining [9].

The production of these states is reliably calculable as a hard process. One takes the full set of Feynman diagrams for the production of two pairs of heavy quarks and heavy antiquarks and requires that the momenta of the two heavy quarks be nearly equal, and then projects out the correct quantum numbers for the relevant  $Q_1 Q_2$ state. This is similar to the calculation of the hard production of heavy quarkonium states [10], though more complicated as an additional QQ pair is needed. On the other hand, because the quarks are heavy, fragmentation functions for a heavy quark to produce a doubly heavy diquark can be predicted in perturbative QCD [11-13]. The fragmentation description of the  $Q_1Q_2$  production process is not valid where the masses of the heavy quarks become important, that is, for small  $p_{\tau}$  or small energies, but for high-energy colliders such as the CERN  $e^+e^-$  collider LEP or Fermilab Tevatron it should be applicable. The calculation is similar to that described in [12] for  $J/\psi$ production. The form of the matrix element is different in the two cases, requiring a somewhat changed method of projection onto the proper spin states. For the production of a  $Q\overline{Q}$  bound state, the standard procedure is to replace the heavy quark production matrix element  $\bar{u}\mathcal{O}v = \text{Tr}(\mathcal{O}v\bar{u})$  with the trace Tr  $(\mathcal{O}P_{SS_z})$ , where  $P_{SS_z}$ is the proper projection onto the spin of the  $Q\overline{Q}$  state [14]. This procedure is, however, not simply applicable to the  $Q_1Q_2$  bound states due to the form of the matrix element, which contains not u and  $\bar{v}$  as in the production of  $Q\overline{Q}$  but  $u_1$  and  $u_2$ . The projection procedure described above is nothing more than a clever method of performing the sum over the quark spins. This sum can also be performed by summing over helicity amplitudes with the proper Clebsch-Gordan coefficients. Although the procedure is somewhat different, the calculations are identical in their essentials, and the fragmentation function is simply proportional to that for a  $Q\overline{Q}$  pair. The

differences reflect the changed color and statistical factors. The fragmentation function for  $Q_1$  to produce a spin-1 diquark  $(Q_1Q_2)$  is

$$D_{Q_1 \to (Q_1 Q_2)}(z, \mu = (m_1 + 2m_2))$$
  
=  $\frac{2N_{12}}{9\pi} \frac{|R_{(Q_1 Q_2)}(0)|^2}{m_2^3} \alpha_s^2(2m_2)F(z),$  (1)

$$F(z) = \frac{rz(1-z)^2}{4(1-z+rz)^6} [2 - 2z(3-2r) + 3z^2(3-2r+4r^2) - 2z^3(1-r)(4-r+2r^2) + z^4(1-r)^2(3-2r+2r^2)],$$
(2)

where  $r = m_2/(m_1 + m_2)$ . The factor  $N_{12} = (1)2$  for (un)equal quark masses, and reflects the presence of identical fermions in the final state. For production of a spin-0 state,  $(Q_1Q_2)'$ , the function F(z) must be replaced by

$$F'(z) = \frac{rz(1-z)^2}{12(1-z+rz)^6} [6 - 18z(1-2r) + z^2(21 - 74r + 68r^2) - 2z^3(1-r)(6 - 19r + 18r^2) + 3z^4(1-r)^2(1-2r+2r^2)].$$
(3)

As in the case of hard quarkonium production, the requirement of nearly equal momenta and the projection onto the proper  $Q_1Q_2$  quantum numbers will modify the  $p_T$  spectrum of the diquark; it will fall more rapidly than that of single heavy quark production. For heavy quarkonium production, the faster falloff in  $p_T$  of the hard production process means that fragmentation production will dominate for sufficiently large  $p_T$  despite the suppression of the fragmentation production by powers of  $\alpha_s$  [15]. For the  $Q_1Q_2$  states, no hard production processes of lower order in  $\alpha_s$  exist. We assume that the doubly heavy diquarks will always hadronize into baryons with two heavy quarks, so that the fragmentation function  $D_{Q_1 \to \text{baryon}} = D_{Q_1 \to (Q_1Q_2)}$ .

A  $Q_1Q_2$  diquark in the ground state has a symmetric spatial wave function. Because the  $Q_1Q_2$  must combine with a light quark to produce a colorless baryon, the diquark must be a color antitriplet (antisymmetric) state. For identical quarks the wave function must satisfy Fermi-Dirac statistics, and thus the cc and bb ground state diquarks have spin 1. The bc states are not restricted by statistics, and we denote the spin singlet (triplet) state by bc'(bc). The spin triplet can hadronize into a spin-1/2 or -3/2 baryon,  $\Xi_{bc}$  and  $\Xi_{bc}^*$ , respectively; the spin singlet can produce only the spin-1/2  $\Xi_{bc}^{\prime}$ . The probabilities for the fragmentation of heavy quarks into these baryons are  $c \to \Xi_{cc}, \Xi_{cc}^*$ , about  $2 \times 10^{-\hat{s}}$ ;  $b \to \Xi_{bc}'$ , about  $4 \times 10^{-5}$ ;  $b \to \Xi_{bc}, \Xi_{bc}^*$ , about  $5 \times 10^{-5}$ . The remaining probabilities are suppressed by  $\sim (m_c/m_b)^3$  and therefore approximately 2 orders of magnitude smaller [11].

In addition to these probabilities, it is desirable to have predictions of the  $p_T$  (or other kinematical variables) distributions for the production of these states. The fragmentation functions given above at low momentum scale  $\mu_0$  must be evolved to higher scales  $\mu$  in order to be of use in calculations for high-energy colliders:

$$d\sigma(A+B\to (Q_1Q_2)+X) = \sum_i \int_0^1 dz \ d\sigma\left(A+B\to i\left(\frac{p_T}{z},\eta\right)+X,\mu^2\right) D_{i\to (Q_1Q_2)}(z,\mu^2).$$
(4)

The evolution occurs via an Altarelli-Parisi equation

$$\mu^{2} \frac{\partial}{\partial \mu^{2}} D_{i \to (Q_{1}Q_{2})}(z,\mu^{2}) = \frac{\alpha_{s}}{2\pi} \sum_{j} \int_{z}^{1} \frac{dy}{y} P_{ij}(z/y) D_{j \to (Q_{1}Q_{2})}(y,\mu^{2}),$$
(5)

which does not change the overall probabilities given above. In the evolution of the fragmentation functions via (5), we include only the  $P_{QQ}$  splitting function. The fragmentation of gluons to heavy diquarks is suppressed relative to that of heavy quarks by  $\alpha_s$ , and may be neglected. A separate numerical evolution of the fragmentation functions to each z and  $\mu$  needed in the course of a numerical simulation would be an extremely computerintensive approach; instead we generate values of the fragmentation function for a reasonably spaced set of  $\mu$ and z values and interpolate to the desired points. The unevolved fragmentation functions depend on  $|R(0)|^2$ , the nonrelativistic radial wave function at the origin, the heavy quark masses, and  $\alpha_s(\mu_0)$ . These input parameters are given in Table I. The  $J/\psi$  and  $\Upsilon$  decays to lepton pairs give the wave functions  $|R(0)|^2$  for these states relatively accurately [18] and independently of phenomenological potential models. The QCD-corrected expression for the leptonic decay of heavy quarkonium states,

$$\Gamma(V \to \ell^+ \ell^-) = \frac{4\alpha^2 e_Q^2}{M_V^2} |R_V(0)|^2 \left[ 1 - \frac{16}{3\pi} \alpha_s(m_Q) \right] ,$$
(6)

is used to extract the  $\Upsilon$  and  $J/\psi$  radial wave function at the origin. There is no information on the  $B_c$  meson to determine the wave function for the  $b\bar{c}$  states, and the annihilation of this state into lepton pairs would not exist in any case to provide the measurement. However, there

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TABLE I. Parameters used to calculate the unevolved fragmentation functions.

	α,	$m_Q$ (GeV)	$ R(0) ^2$
$c \rightarrow (cc)$	0.238	1.5	$(0.65 \text{ GeV})^3$
$c \rightarrow (bc)$	0.173	4.5	$(0.80 \text{ GeV})^3$
c  ightarrow (bc)'	0.173	4.5	$(0.80 \text{ GeV})^3$
$b \rightarrow (bb)$	0.173	4.5	$(1.20 \text{ GeV})^3$
$b \rightarrow (bc)$	0.238	1.5	$(0.80 \text{ GeV})^3$
$b \rightarrow (bc)'$	0.238	1.5	$(0.80 { m GeV})^3$

is an empirical relationship among leptonic decays of the  $\phi$ ,  $J/\psi$ , and  $\Upsilon$  mesons,  $\Gamma(V \to e^+e^-) \simeq 12e_Q^2$  keV, which indicates that  $|R(0)|^2/\mu^2$  is approximately constant.

The one-gluon-exchange contribution for  $Q_1Q_2$  differs from that for  $Q_1\overline{Q}_2$  by a relative color factor 1/2. In a Coulomb potential, therefore, the relation  $|R_{J/\psi}(0)|^2 =$  $8|R_{cc}(0)|^2$  must hold. The heavy quarks are, however, not in a pure Coulomb potential, so that we must rely on phenomenological potentials to better estimate the radial wave function. While many functional forms for the potential provide quite good fits to the heavy quarkonium data, the fitted potentials all have very similar shapes in the region of  $r \sim 1 \text{ GeV}^{-1}$ . We use 1/2 the  $Q_1\overline{Q}_2$  potential for the  $Q_1Q_2$  potential, and find the wave function at the origin using a numerical solution of the Schrödinger equation. We compare in Table II the results of this calculation for two familiar potentials [16,17] with the values extracted from the  $J/\psi$  and  $\Upsilon$  decay rates.

Doubly heavy baryons will be produced at all present and future accelerators where there is sufficient energy. The results presented here include both baryon and antibaryon production. We consider first the  $e^+e^-$  colliders LEP, LEP II, and the Next-Linear Collider (NLC). At LEP, the only subprocess for the production of a heavy quark Q is  $e^+e^- \to Z$ , with the Z decaying to  $Q\overline{Q}$ . The production rates in this case are quite small, a few events per year. For LEP II and NLC the additional processes  $e^+e^- \rightarrow W^+W^-$  where one W decays to a c quark and  $e^+e^- \rightarrow ZZ$  where one Z decays to  $c\bar{c}$  or  $b\bar{b}$  also contribute (depending on the LEP II energy, of course). Despite the additional subprocesses, of which  $W^+W^-$  dominates for NLC, the event rates remain woefully small. At LEP II, only a few events/year can be expected, and a  $60 \text{ fb}^{-1}/\text{yr}$ , 500 GeV linear collider will produce only of

TABLE II. Radial wave functions at the origin,  $|R(0)|^2$ , for  $Q_1 \overline{Q}_2$  and  $Q_1 Q_2$  states for two interquark potentials compared to values extracted from the data; see (6).

	and the second		
	Indiana [16]	Richardson [17]	Data [26]
$c\bar{c}$	$(0.95 \text{ GeV})^3$	$(0.93 \text{ GeV})^3$	$(0.98 \text{ GeV})^3$
cc	$(0.67 { m GeV})^3$	$(0.65 { m GeV})^3$	
bē	$(1.18 \text{ GeV})^3$	$(1.18 \text{ GeV})^3$	
bc	$(0.82 \text{ GeV})^3$	$(0.81 \text{ GeV})^3$	
bb	$(1.72 \text{ GeV})^3$	$(1.88 \text{ GeV})^3$	$(1.96 \text{ GeV})^3$
bb	$(1.15 \text{ GeV})^3$	$(1.23 { m ~GeV})^3$	. , ,

order 30 events per year. The production rates at  $\gamma\gamma$  colliders and  $e\gamma$  colliders were also calculated, including backscattered and Weizsäcker-Williams photons and the contributions from resolved-photon processes. The cross sections for production of these particles at these colliders are also hopelessly small.

The results at the DESY ep collider HERA are similar. The main subprocesses here are  $\gamma g \rightarrow Q\overline{Q}$  and  $\gamma Q \rightarrow gQ$ . Assuming the design luminosity of 200 pb<sup>-1</sup>/yr, we again find that the event rate is rather small, of the order of 30 events/year. The resolved photon processes are likewise negligible.

The situation is considerably more hopeful at hadron colliders. The relevant subprocesses are  $gg \rightarrow Q\overline{Q}$ ,  $q\bar{q} \rightarrow QQ, \ gQ \rightarrow gQ, \ and \ qQ \rightarrow qQ$  (in the latter two subprocesses, q and Q stand for both quarks and antiquarks), although the main contribution is from the gluon-fusion subprocess. For the Tevatron, with an integrated luminosity of 100  $pb^{-1}/yr$ , we anticipate of order  $8 \times 10^4$  events/yr ( $p_T > 5$  GeV,  $|\eta| < 0.5$ ), while the CERN Large Hadron Collider (LHC), with 100 fb<sup>-1</sup>/yr, should produce of order  $1.3 \times 10^8$  events per year ( $p_{\tau} > 10$ GeV,  $|\eta| < 0.5$ ). Total production cross sections per unit rapidity for the various diquarks at the Tevatron and the LHC (operating at both 10 TeV and 14 TeV) are given in Table III. The rates for production of the *bc*-type states are similar to those expected for the  $B_c$  meson, where rates of  $10^4 B_c$ 's have been predicted at the Tevatron  $(p_T > 10 \text{ GeV})$  and  $10^7$  at LHC  $(p_T > 20 \text{ GeV})$  [19, 20]. On the other hand, the production rate for cc-type states is significantly smaller than that for  $J/\psi$  production at the Tevatron which is expected to be  $5 \times 10^6$  $(p_T > 5 \text{ GeV})$  [15].

Some results are given in Figs. 1 and 2. Figure 1 shows  $d\sigma/dp_T/d\eta|_{\eta=0}$  vs  $p_T$ . A kinematical cut of 5 GeV for the baryon  $p_T$  has been imposed here. Figure 2 gives the rapidity distribution of  $Q_1Q_2$  baryons, again with  $p_T > 5$  GeV. The rapidity distributions are rather flat in the central region, so that  $d\sigma/dp_T$  may be estimated from  $d\sigma/dp_T/d\eta|_{\eta=0}$  by multiplying with the  $\eta$  region desired.

Figures 3 and 4 show  $d\sigma/dp_T/d\eta|_{\eta=0}$  vs  $p_T$  for the LHC operating at 14 TeV (Fig. 3) and 10 TeV (Fig. 4). A kinematical cut of  $p_T > 10$  GeV is placed on the  $Q_1Q_2$  baryon. The production rate at the lower center-of-mass energy is slightly lower, but even at the lower energy, the LHC will produce  $Q_1Q_2$  baryons copiously. The rapidity distributions are again relatively flat in the central region  $(|\eta| < 3)$ , and event rates for any specific detector can be estimated by simply multiplying the results in Table III by the  $\eta$  coverage of the detector.

TABLE III. Production cross sections per unit rapidity in the central region.

	Tevatron	LHC (10 TeV)	LHC (14 TeV)
$\overline{\Xi_{**} + \Xi_{**}^*}$	430	330	470
$\Xi_{bc}'$	145	220	330
$\Xi_{bc}^* + \Xi_{bc}^*$	215	350	490
$\Xi_{bb} + \Xi_{bb}^*$	16	27	36



FIG. 1.  $p_T$  distributions (at small rapidity) for doubly heavy baryon production at the Tevatron.

In order to estimate the uncertainties due to the choice of factorization and fragmentation scales, we vary these scales (all chosen to be equal) by a factor of 2 from our nominal choice  $\mu = p_{\tau}$ . The effect of varying the scale on  $d\sigma/dp_{_T}/d\eta|_{\eta=0}$  vs  $p_{_T}$  can be seen in Fig. 5, and the effect on  $d\sigma/d\eta$  vs  $\eta$  can be seen in Fig. 6. The processes shown are representative of all the processes. The behavior seen in Fig. 6 is somewhat counterintuitive in that one expects the rate with  $\mu = p_T/2$  to be larger than that for  $\mu = p_T$ . However, the fragmentation functions are cut off at low  $\mu_{\rm frag}.$  This causes the drop at low  $p_{_T}$  for this choice seen in Fig. 5 and the lower cross sections in Fig. 6. We use Martin-Roberts-Stirling set A (MRSA) [21] parton distribution functions. The effect of changing the parton distribution set choice is negligible — far smaller than the effect of varying the choice of  $\mu$ . Other sources of theoretical uncertainty include the parameters used for the unevolved fragmentation functions, primarily  $|R(0)|^2$ where different phenomenological potentials give about 10% differences in the result, unknown QCD corrections to the parton-level cross sections, the fragmentation function, and the evolution equations, and unknown relativistic corrections to the initial fragmentation functions.

Other potential sources of  $Q_1Q_2$  production must be considered. Production of  $Q\bar{Q}$  in a color octet state was recently put forth as a possible explanation for the excess



FIG. 2. Pseudorapidity distributions for doubly heavy baryon production at the Tevatron.



FIG. 3.  $p_T$  distributions (at small rapidity) for doubly heavy baryon production at the LHC, 14 TeV.

of direct  $J/\psi$  and  $\psi'$  observed at the Tevatron [22]. The color octet  $c\bar{c}$  couples directly to one perturbative gluon while direct color singlet production requires the coupling of the  $c\bar{c}$  to three perturbative gluons. The color octet contribution is thus larger by two powers of  $\alpha_s$ . The lower-order coupling combined with a large probability for octet  $c\bar{c}$  to evolve into the physical  $J/\psi$  and  $\psi'$  makes a substantial contribution to the production rates. Can a similar color octet  $Q_1Q_2$  production mechanism be important? An alternate color combination of the  $Q_1Q_2$ state will yield a process of equal or higher order in  $\alpha_s$ , and so a large increase in the cross section over the color antitriplet mechanism is not to be expected. In addition, no quantitative predictions can yet be made for these processes, as they depend on unknown long-distance matrix elements. For  $J/\psi$  and  $\psi'$  these matrix elements may be extracted from the Tevatron data under the assumption that they explain the large production rates [22]; no data are available for a similar extraction of the doubly heavy baryon matrix elements. There is also the possibility of the hard, perturbative production of  $Q_1Q_2$  states, similar in nature to the processes considered by Baier and Rückl [10] and Gastmans, Troost, and Wu [10]. There is an analogy between the fragmentation production of  $Q_1Q_2$  and that of  $B_c$  mesons.

In the case of  $B_c$  mesons there has been some concern



FIG. 4.  $p_T$  distributions (at small rapidity) for doubly heavy baryon production at the LHC, 10 TeV.



FIG. 5.  $p_T$  distributions (at small rapidity) for bb and bc production at the Tevatron for different choices of scale  $\mu$ .

that the fragmentation approximation may not work as well as anticipated for  $B_c$  production. This assertion is based on a comparison between the full calculation and the fragmentation approximation for  $B_c$  in  $\gamma\gamma$  collisions [23]. There is, of course, a significant difference between  $\gamma\gamma$  and gg initial states, and the convolution of the gluon distributions with the parton-level subprocess can have a large effect. The calculation of  $gg \rightarrow B_c X$  is much more complex than that of  $\gamma \gamma \to B_c X$ , but is necessary to understand the validity of the fragmentation approximation. The authors of [23] went on to study  $B_c$  production at hadron colliders [20]. They found that the fragmentation approximation still differs from the full calculation at the parton level  $(\hat{\sigma}[gg \rightarrow b\bar{b}(\rightarrow B_c X)]$  vs  $\hat{\sigma}[gg \to B_c b\bar{c}])$  except at high  $p_{\tau}$ , but after the convolution of  $\hat{\sigma}$  with the gluon distribution functions, the full calculation and the fragmentation approximation agree quite well for  $p_T > 10$  GeV both at Tevatron and LHC energies. Figure 6 of [20] shows that the agreement between the full calculation and the fragmentation approximation is acceptable down to  $p_T^{\min} = 5$  GeV at the Tevatron and  $p_{m}^{\min} = 10$  GeV at LHC. As the fragmentation functions and the production mechanisms are extremely similar for  $B_c$  and heavy diquark production, our calculation of the production of doubly heavy baryons should also be accurate in this  $p_{\tau}$  range.

The decay modes of doubly charmed baryons have been studied in [24] using SU(3) flavor symmetry. This ap-



FIG. 6. Pseudorapidity distributions for bb and bc production at the Tevatron for different choices of scale  $\mu$ .

proach relates the decays of these particles, and could be used to find relations among the decays of the bb baryons and among the bc baryons. The transitions between the doubly heavy baryons [7] and some decays of the bc baryons [25] have been discussed in the heavy quark limit. The lifetimes of c and b quarks are not very different ( $\tau_{\Lambda_c} \sim 0.2 \times 10^{-12}$  s while  $\tau_{\Lambda_b} \sim 1.07 \times 10^{-12}$  s [26]), so that the weak decays of both b and c are important when considering the decays of the bc baryons. bc baryons will decay weakly (both hadronically and semileptonically) to single b quark baryons as well as doubly charmed baryons. Both of these possibilities have been discussed in the literature [24, 27, 28]. Either a single b baryon or a doubly charmed baryon traced back to a displaced vertex using a vertex detector will be a clear sign of doubly heavy baryon production [29]. In the case of bb baryons, the production of a doubly charmed baryon (possibly with a same sign dilepton), with both b quark decays tagged by a vertex detector, would be an impressive signal. Unfortunately, the bb baryons are produced less frequently than the other doubly heavy baryons, and two semileptonic decays will severely reduce the event rate.

In conclusion, the production via fragmentation of baryons containing two heavy quarks has been calculated in the fragmentation approximation. The fragmentation approximation reproduces well the full calculation of  $B_c$ production at hadron colliders for the  $p_{T}$  studied here [20]; it is expected that the fragmentation approximation will work well for doubly heavy diquark production as well. The production rates of these baryons at  $e^+e^$ colliders and at the ep collider HERA are found to be negligible. The situation is much better at hadron colliders, with approximately  $8 \times 10^4$  events/vr expected at Tevatron and  $1.3 \times 10^8$  events/yr at LHC. In addition to predictions for total production cross sections  $||\eta| < 0.5$ and  $p_T > 5$  (10) GeV at the Tevatron (LHC)], the distributions  $d\sigma/dp_{T}/d\eta|_{\eta=0}$  and  $d\sigma/d\eta$  for  $p_{T} > 5(10)$  GeV at the Tevatron (LHC) are studied. The detection of these particles and measurements of their properties will provide an experimental challenge due to the large number of their decay modes, but will be a rich testing ground, e.g., for heavy quark effective theory (HQET) and QCD potential models.

Shortly after the submission of our manuscript, [30] appeared. The authors of [30] study doubly heavy baryon production both to  $O(\alpha_s^4)$  and in the fragmentation approximation. The  $O(\alpha_s^4)$  calculation is valid to smaller  $p_T$ , where the fragmentation approximation is known to be invalid. They find that, at the Tevatron above  $p_T = 5$  GeV, the fragmentation approximation underestimates the exact result by a small factor (2-3), but the shapes of the  $p_T$  distributions are the same. In view of the remaining theoretical uncertainties in the problem, i.e., the value of the wave function at the origin and the assumption that the doubly heavy baryon, these results are not inconsistent.

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