

## High energy hadron-nucleus cross sections and their extrapolation to cosmic ray energies

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Old models of the scattering of composite systems based on the Glauber model of multiple diffraction are applied to hadron-nucleus scattering. We obtain an excellent fit with only two free parameters to the highest energy hadron-nucleus data available. Because of the quality of the fit and the simplicity of the model it is argued that it should continue to be reliable up to the highest cosmic ray energies. Logarithmic extrapolations of  $p$ - $p$  and  $\bar{p}$ - $p$  data are used to calculate the proton-air cross sections at very high energy. Finally, it is observed that if the exponential behavior of the  $\bar{p}$ - $p$  diffraction peak continues into the few TeV energy range it will violate partial wave unitarity. We propose a simple modification that will guarantee unitarity throughout the cosmic ray energy region.

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### I. INTRODUCTION

The highest energy hadronic interactions currently observable occur when a primary cosmic ray strikes an air nucleus. For primary protons with energy of  $10^{18}$  eV, which are easily observed in the Utah "Flys's Eye" detector, the  $\sqrt{s}$  exceeds that of the recently canceled Superconducting Super Collider (SSC). The phenomenology of the extensive air showers created by the hadronic cascade in the atmosphere depends critically on the hadron-nucleus and the nucleus-nucleus cross sections at extremely high energy. Furthermore, extracting any information about the basic hadronic interactions requires some model that can reliably relate the nuclear cross section to the nucleon cross section. The purpose of this paper is to develop such a model and check it with experimental data in the energy range in which both the hadron-nucleus and hadron-nucleon cross sections are known.

At high energies, hadron-nucleon scattering amplitudes become very simple. The amplitude is diffractive, being dominated by the imaginary part with rather weak (logarithmic) energy dependence. The momentum transfer dependence in the dominant part of the diffraction peak is a pure exponential. This is to be contrasted with the situation at lower energies where resonance phenomena produce rapid energy dependence and complicated angular dependence. In this paper we have resurrected an old model of Fishbane and Trefil, and of Franco [1] and show that this simple application of the Glauber multiple diffraction model [2] can be used to provide reliable hadron-nucleus cross sections from the basic hadron-nucleon interaction. We further argue that since no energy dependence is required in this model in fitting the data, i.e., the distribution of nucleons in the nucleus is independent of the momentum of the incident hadrons, this method should continue to be applicable to scattering at the highest cosmic ray energies. One important reason that the nuclear corrections are simple at high energy is

that for large  $A$ , the nucleus is mostly black and only the peripheral shell of the nucleus is sensitive to the details of the hadron-nucleon interaction; thus the nuclear cross sections depend only weakly on the nucleon density distributions and hadron-nucleon cross sections. It should be kept in mind, however, that Glauber's approximation is only valid for small scattering angles.

In the next section we will review the Glauber multiple diffraction model for hadron-nucleus and nucleus-nucleus scattering, assuming a Gaussian distribution of nucleons in the nucleus. In Sec. III, we determine the input parameters, the hadron-nucleon cross sections, and slope parameters for  $p$ - $p$ ,  $\bar{p}$ - $p$ ,  $\pi^+$ - $p$ ,  $\pi^-$ - $p$ ,  $K^+$ - $p$ , and  $K^-$ - $p$ . The two parameters of the nuclear model are then adjusted to fit the corresponding nuclear cross sections as measured by Carroll *et al.* [3].

In Sec. IV we obtain simple fits to the energy dependence of the hadron-nucleon parameters and propose a simple model which satisfies partial wave unitarity and can be used to extrapolate these quantities to very high energy. These extrapolations are then used in Sec. V to calculate the hadron-air and nucleus-air cross sections in the energy range needed in the phenomenology of extensive air showers produced by cosmic rays. In the Conclusion we summarize our results and point out that more hadron-nucleon data are needed to improve our understanding and to aid the analysis of cosmic ray showers.

### II. THEORETICAL FORMULATION OF HADRON-NUCLEUS AND NUCLEUS-NUCLEUS SCATTERING

Many authors, including those listed in [1], have used the Glauber multiple diffraction approximation [2] to relate hadron-nucleus scattering and nucleus-nucleus scattering to the basic hadronic interaction. In the interest of making this paper self contained we will review this procedure.

The amplitude for hadron-nucleus elastic scattering can be written in the impact parameter representation as

$$F_{h-A}(q^2) = \frac{ik}{2\pi} \int d^2\vec{b} e^{i\vec{q}\cdot\vec{b}} (1 - e^{i\chi_{h-A}}), \quad (2.1)$$

where  $\chi_{h-A}$  is the hadron-nuclear phase shift. The quantity  $\Gamma_{h-A} = (1 - e^{i\chi_{h-A}})$  is the scattering amplitude in impact parameter space, often referred to as the nuclear profile function, and is bounded by unitarity because the imaginary part of  $\chi_{h-A}$  is positive. In this treatment we will ignore spin and treat neutrons and protons alike as nucleons. The essential approximation of the Glauber model is that the scattering is predominantly forward and there is no momentum transfer to the individual nucleons. In this limit the nuclear phase shift for a particular nucleon configuration is given by the sum of the phase shifts for the collisions of the hadron with the individual nucleons in the nucleus. We believe that this should give good results for the total cross section given by the imaginary part of the forward amplitude and for the elastic scattering which is dominated by the forward diffraction peak. On the other hand, the fact that the nuclear amplitude contains no nuclear recoil makes it unlikely that this approximation will give good results for quasielastic processes in which the nucleus ends up either in an excited state or fragmented. For cosmic ray applications there are a number of processes that fall into the general category of target fragmentation that reduce the beam momentum, but do not contribute to shower formation. Since these must be treated in the simulation, we see no particular reason to use the Glauber model calculation of the quasielastic scattering to remove an ill-defined, and perhaps incorrect, fraction of these events.

If correlations between the nucleons are ignored and we average over the position of the ( $A$ ) nucleons relative to the center-of-mass of the nucleus, the overall hadron-nucleus phase shift is

$$F_{A'-A}(q^2) = \frac{ik}{2\pi} \int d^2\vec{b} e^{i\vec{q}\cdot\vec{b}} \left\{ 1 - \left[ 1 - \int d^3\vec{r} \int d^3\vec{r}' \rho_A(\vec{r}) \Gamma_{N-N}(\vec{b} + \vec{r}_\perp - \vec{r}'_\perp) \rho_{A'}(\vec{r}') \right]^{AA'} \right\}. \quad (2.5)$$

In practice, the optical limit is a good approximation for  $A, A' \geq 10$  [8]. From this expression, using the optical theorem, we obtain the total cross section

$$\sigma_{A-A'}^T = \frac{4\pi}{k} \text{Im} F_{A'-A}(0) \quad (2.6)$$

and the elastic cross section

$$\sigma_{A-A'}^{\text{el}} = \int d\Omega_k |F_{A'-A}(q^2)|^2. \quad (2.7)$$

In this work we will use a Gaussian single nucleon density

$$\rho_A(\vec{r}) = \frac{R(A)^3}{(2\pi)^{\frac{3}{2}}} e^{-\frac{r^2}{R(A)^2}}. \quad (2.8)$$

$$e^{i\chi_{h-A}(\vec{b})} = \int \prod_{i=1}^A d^3\vec{r}_i \rho(\vec{r}_i) \exp \left( i \sum_{j=1}^A \chi_{h-N}(\vec{b} - \vec{r}_{j\perp}) \right). \quad (2.2)$$

The contribution of each nucleon to  $\Gamma_{h-A}$  factorizes and we obtain

$$F_{h-A}(q^2) = \frac{ik}{2\pi} \int d^2\vec{b} e^{i\vec{q}\cdot\vec{b}} \times \left\{ 1 - \left[ 1 - \int d^3\vec{r} \Gamma_{h-N}(\vec{b} - \vec{r}_\perp) \rho(\vec{r}) \right]^A \right\} \quad (2.3)$$

where  $\rho(\vec{r})$  is the single nucleon density,  $\vec{r}_\perp$  is the component of  $\vec{r}$  in the impact parameter plane, and  $\Gamma_{h-N}$  is the hadron-nucleon profile function which is just the Bessel transform of the hadron-nucleon scattering amplitude:

$$\Gamma_{h-N}(\vec{b}) = \frac{1}{2\pi ik} \int d^2\vec{q} e^{-i\vec{q}\cdot\vec{b}} F_{h-N}(\vec{q}). \quad (2.4)$$

All of the scattering processes are at a fixed incident hadron momentum. Being able to neglect the nuclear Fermi momentum is another simplification that is possible at high energy.

The multiple diffraction approximation can also be applied to nucleus-nucleus scattering [8] (see also Refs. [4,7]). In this case the overall phase shift is, according to the Glauber assumption, simply the sum of the phase shifts of  $A \times A'$  individual nucleon-nucleon interactions. In this case we must average over the positions of the nucleons in both nuclei. Unlike the hadron-nucleus case, however, the integrals do not factorize and lead to very complicated expressions even for the  $2 \times 3$  case. It can be shown [8] that in the optical limit, that is both  $A, A' \gg 1$ , many terms can be neglected and the elastic scattering amplitude can be approximated by

This is of course only valid for  $A > 1$  and for  $A = 1$  the density should be a  $\delta$  function. While other distributions such as Saxon-Woods are probably more realistic for large  $A$  and shell wave functions are more appropriate for small  $A$ , this form seems to work very well. When the density given in Eq. (2.8) is combined with the observed form for the hadron-nucleon scattering amplitudes, Eqs. (2.6) and (2.7) can be evaluated analytically. We assume that  $R(A)$  has the form

$$R(A) = R_0 A^\gamma. \quad (2.9)$$

If we use experiment to determine  $\Gamma_{h-N}$ , the only free parameters in this model are  $R_0$  and  $\gamma$ . These could of course depend both on energy as well as particle type,

although the simplest picture is one in which the density only reflects the average positions of the nucleons in the nucleus.

The hadron-nucleon elastic amplitudes at small momentum transfer are well fit by a Gaussian form. More complicated dependence appears at larger angles, but it is the simple small angle region that makes the dominant contribution to the elastic cross sections. This remains the case up to the highest experimental energies available at the Tevatron collider. The form of the scattering amplitude is then

$$F_{h-N} = \frac{k\sigma_{h-N}^T}{4\pi}(\rho + i)e^{\frac{Bt}{2}}, \quad (2.10)$$

where  $B$  is the slope parameter ( $t = -q^2$ ) and  $\rho$  is the ratio of the real to imaginary part of the amplitude. Here we have assumed that for small  $q^2$ ,  $\rho$  is a constant. At high energy  $\rho$  is quite small, typically 0.10–0.14, and we take it to be zero in our work. The resulting hadron-nucleus absorptive cross section is

$$\sigma_{h-A}^{\text{abs}} = \beta \int_0^D dx \frac{(1+x)^{2A} - 1}{x} = \beta \sum_{n=1}^{2A} \binom{2A}{n} \frac{D^n}{n}, \quad (2.11)$$

and the total cross section is

$$\sigma_{h-A}^T = 2\beta \int_0^D dx \frac{(1+x)^A - 1}{x} = 2\beta \sum_{n=1}^A \binom{A}{n} \frac{D^n}{n}, \quad (2.12)$$

where

$$\beta = -\pi[2B + R^2(A)] \quad (2.13)$$

and

$$D = \frac{\sigma_{h-N}^T}{2\beta}. \quad (2.14)$$

Finally, we include the prediction for the quasielastic scattering. While we are doubtful about its reliability, it turns out to be small and we will be forced to use it in our fit to accelerator data:

$$\sigma_{h-A}^{\text{qe}} = \beta \int_0^1 dx \frac{\left(1 + 2Dx + \frac{D^2(1+\gamma)^2}{(1+2\gamma)} x^{\frac{2+2\gamma}{1+2\gamma}}\right)^A - 1}{x} - \sigma_{h-A}^{\text{abs}}, \quad (2.15)$$

where  $\gamma = \frac{R^2}{2B}$ .

The nucleus-nucleus cross sections can be obtained from Eqs. (2.11) and (2.12) by letting  $A \rightarrow AA'$  and using

$$\beta = -\pi[2B + R(A)^2 + R(A')^2] \quad (2.16)$$

and

$$D = \frac{\sigma_{N-N}^T}{2\beta}. \quad (2.17)$$

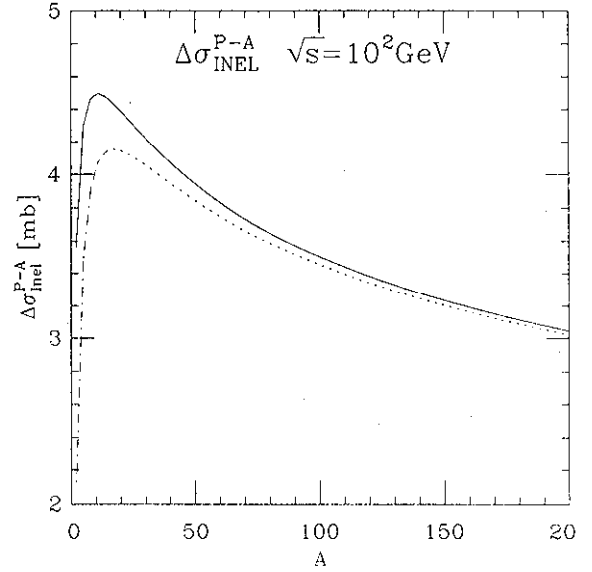


FIG. 1. Correction to the  $p-A$  inelastic cross section because of the center-of-mass constraint as a function of  $A$  at  $\sqrt{s} = 10^2$  GeV. The dotted line is the large  $A$  approximation.

The identity [1]

$$\lim_{M \rightarrow \infty} \int_0^D dx \frac{(1+x)^M - 1}{x} = \text{Ei}(MD) - \ln(-MD) - C, \quad (2.18)$$

where  $C$  is Euler's constant, can be used to evaluate these cross sections for large  $A$  or  $AA'$ . Up to this point we have neglected position correlations of the nucleons in the nucleus. Indeed, the nucleon coordinates  $\vec{r}_i$  are subject to the center-of-mass constraint  $\frac{1}{A} \sum_{i=1}^A \vec{r}_i = \vec{r}_{\text{c.m.}}$ . It can be easily shown, by transforming to the c.m. coordinates  $\vec{r}_i^{\text{c.m.}} = \vec{r}_i - \frac{1}{A} \sum_{j=1}^A \vec{r}_j$ , that the effect of the c.m. constraint on the elastic scattering amplitude, for the case of Gaussian nucleon density distributions, is simply

$$F_{h-A}^{\text{c.m.}}(q) = e^{q^2 \frac{R(A)^2}{4A}} F_{p-A}(q), \quad (2.19)$$

$$F_{A-A'}^{\text{c.m.}}(q) = e^{q^2 \left( \frac{R(A)^2}{4A} + \frac{R(A')^2}{4A'} \right)} F_{A-A'}(q). \quad (2.20)$$

Thus the total cross sections are unaffected by the c.m. constraint. The inelastic and elastic cross sections, however, are modified, although as shown in Fig. 1, the correction is at most a few mb and decreases as  $\sim A^{-1/3}$ . We find

$$\Delta\sigma_{p-A}^{\text{inel}} = -\frac{2\beta}{A} \sum_{N=1}^A \sum_{M=1}^A \binom{A}{N} \binom{A}{M} D^{M+N} \times \frac{MN}{(M+N) \left[ (M+N) \left( \frac{2B+R(A)^2}{R(A)^2} \right) - \frac{2MN}{A} \right]} \quad (2.21)$$

and for  $A \gg 1$

$$\Delta\sigma_{p-A}^{\text{inel}} \approx \frac{1}{2A} [\sigma_{p-A}^{\text{inel}} + \beta(1+D)^{2A}]. \quad (2.22)$$

A recent model developed by Engel, Gaisser, Lipari, and Stanev (hereafter the EGLS model) [4], and discussed in more detail by Fletcher *et al.* [5], is based in part on the work of Durand and Pi [6], which is a semiclassical treatment motivated by the parton model. The EGLS model is a probabilistic approach to hadron-nucleus and nucleus-nucleus inelastic scattering, treating the individual hadron-nucleon interactions incoherently, and summing probabilities of interaction instead of amplitudes. While this treatment is somewhat different from ours, for the special case of purely imaginary nuclear phase shift the results are formally identical. However, our nucleon profile function, the nucleon density distribution, and the extrapolations of the hadron-nucleon cross section and nuclear slope are all different from those of EGLS. Thus one would not expect the same predictions from these two models.

### III. CROSS SECTION FITS

The input parameters of Glauber's model are the hadron-nucleon cross sections  $\sigma_{h-N}^T$  and nuclear slopes  $B_{h-N}$ . We determine these at energies of 60, 200, and 280 GeV. The total cross sections are found in a compilation of data [9]. Some sample fits are ( $s$  is in  $\text{GeV}^2$  and  $\sigma^T$  is in mb)

$$\sigma_{\pi^+-p}^T = 23.1 + 0.525(\ln s/112)^2 \quad (3.1)$$

and

$$\sigma_{\pi^--p}^T = 24.0 + 0.582(\ln s/170)^2, \quad (3.2)$$

with a  $\chi^2 = \frac{1.12}{N_{\text{DF}}}$  and  $\chi^2 = \frac{1.30}{N_{\text{DF}}}$ , correspondingly. There are some values of the slope parameters in the literature [10-12] but we obtain most values by fitting directly the elastic differential cross section data (at low momentum transfer) with an exponential form

$$\frac{d\sigma_{h-N}^{\text{el}}}{dt} = Ae^{Bt}. \quad (3.3)$$

Sample fits of these values are ( $s$  is in  $\text{GeV}^2$  and  $B$  is in  $\text{GeV}^{-2}$ )

$$B_{\pi^--p} = 9.53 - 0.705 \ln s + 0.136(\ln s)^2, \quad (3.4)$$

$$B_{\pi^+-p} = 7.90 - 0.155 \ln s + 0.076(\ln s)^2. \quad (3.5)$$

Ayres *et al.* [13] have measured  $\pi-p$  and  $K-p$ , differential elastic cross sections at 50, 70, 100, 140, 175, and 200 GeV. There are some data for  $\pi^- - p$  at 360 GeV [14] and for  $\pi^- - p$  at 200 GeV [15]. By interpolating and extrapolating in the case of  $K-p$ , we obtain  $B_{h-p}$  at the required energies. The only free parameters in our model are  $R_0$  and  $\gamma$  that appear in the nucleon den-

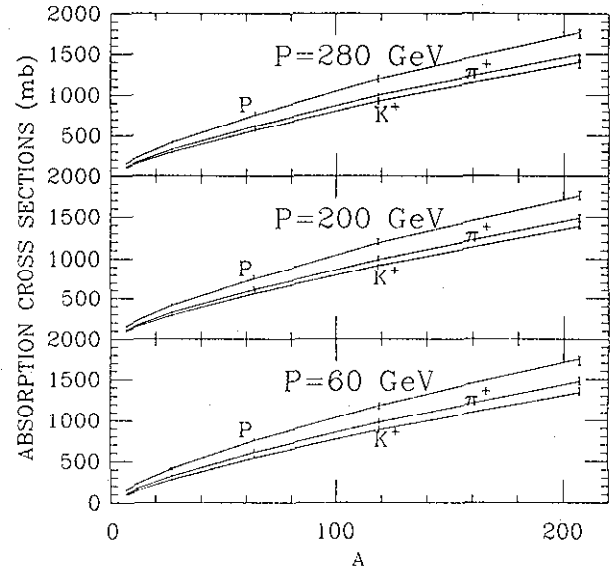


FIG. 2. Fits to  $p-A$ ,  $\pi^+-A$ , and  $K^+-A$  absorption cross sections at incident hadron momenta of 60, 200, and 280 GeV/c. Shown are the data by Carroll *et al.* and our fitted values connected by straight line segments.

sity distribution. The most recent hadron-nucleus data is from Carroll *et al.* [3]. The comparison of our model with their data is complicated by the fact that they removed the quasielastic cross section from their absorption cross section. While this is a small correction to the absorption data, it was done in the process of extrapolating the data to zero momentum transfer and hence we cannot obtain the total absorption cross sections from their published data. As a result, we have two courses of comparison. First, we can simply ignore these small corrections and

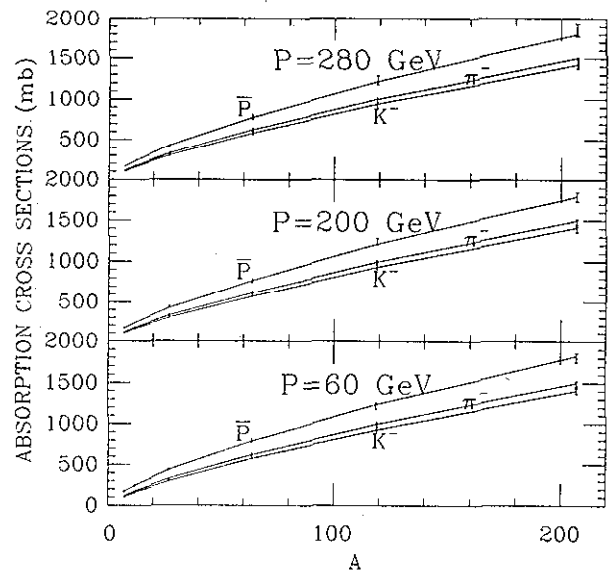


FIG. 3. Fits to  $\bar{p}-A$ ,  $\pi^- - A$ , and  $K^- - A$  absorption cross sections at incident hadron momenta of 60, 200, and 280 GeV/c. Shown are the data by Carroll *et al.* and our fitted values connected by straight line segments.

fit their data with our model. The alternative is to use the Glauber model to calculate the quasielastic cross sections. This has the disadvantage of subtracting a small but unreliable term from the quantities that we believe can be calculated accurately.

We first ignore the quasielastic cross sections and simply fit the 108 data points of Carroll *et al.* [3] for scattering of  $p, \bar{p}, K^+, K^-, \pi^+, \pi^-$  off Li, C, Al, Cu, Sn, and Pb targets at 60, 200, and 280 GeV incident hadron energy with our two free parameters. These results are shown in Figs. 2 and 3. We obtain

$$R_0 = 3.89 \text{ GeV}^{-1}, \quad (3.6)$$

$$\gamma = 0.31, \quad (3.7)$$

with  $\chi^2/N_{\text{DF}} = 0.25$ . This  $\chi^2$  was calculated using the quoted error of the data which includes an estimated systematic error of 3%. The small value of our  $\chi^2$  is an indication that our fit is not sensitive to the type of systematic error that exists in the data. If we use only the statistical error of 1% as quoted by Carroll *et al.* [3] to calculate  $\chi^2$ , we obtain a  $\chi^2 \sim 2$  per degree of freedom.

If we use the Glauber model to remove the quasielastic scattering from our fit, we obtain

$$R_0 = 4.74 \text{ GeV}^{-1}, \quad (3.8)$$

$$\gamma = 0.28, \quad (3.9)$$

with  $\chi^2/N_{\text{DF}} = 0.34$ . While this is not as good a fit as the one described above, it is quite acceptable. Note that there is a noticeable change in the parameters.

#### IV. UNITARITY CONSTRAINT AT HIGH ENERGY

The data for the total nucleon-nucleon cross section  $\sigma^T$  and nuclear slope  $B$  up to Tevatron energies [16] can be fit with quadratic logarithmic forms and at infinite energy they grow as  $\ln(s)^2$ . Our fits, shown in Figs. 4 and 5, are ( $s$  is in  $\text{GeV}^2$ ,  $\sigma^T$  in mb, and  $B$  in  $\text{GeV}^{-2}$ )

$$\sigma_{p-p}^T = 38.46 + 0.41[\ln(s/118)]^2 \quad (4.1)$$

and

$$B_{p-p} = 9.25 + 0.37 \ln(s) + 0.01[\ln(s)]^2, \quad (4.2)$$

where we have assumed that at  $p-p$  and  $p-\bar{p}$  are identical at collider energies. Note that for  $s \rightarrow \infty$ ,  $\Gamma(0) = \frac{\sigma^T}{4\pi B} \rightarrow 8.35$ . In fact, the unitarity limit is saturated at  $\sqrt{s} \simeq 2.5$  TeV (see Fig. 6). To prevent unitarity violation we suggest that at high energies the elastic scattering amplitude deviates from the pure exponential form in  $t$  that is commonly used at small scattering angles. We set

$$f(t, s) = \frac{ik}{4\pi} \sigma^T(s) e^{\frac{B(s)t}{2}} J_0[\alpha(s)\sqrt{-t}]. \quad (4.3)$$

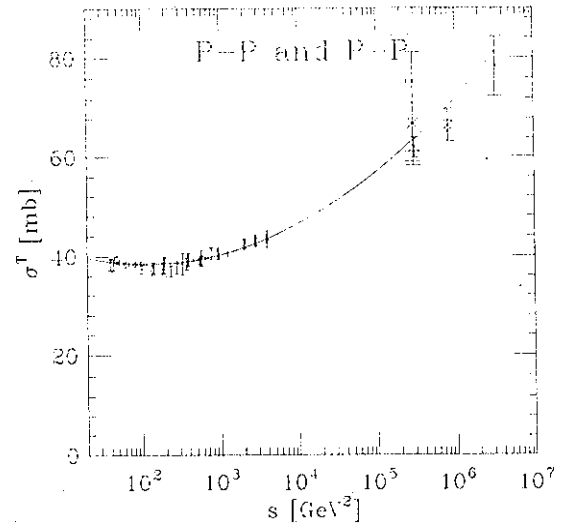


FIG. 4. The  $p-p$  total cross section as a function of  $s$ . The solid line is our fit of Eq. (4.1), and included are the high energy  $p-\bar{p}$  collider data which we assume are equal to  $p-p$  at these energies.

Since  $dJ_0(x)/dx|_{x=0} = 0$ ,  $B$  is still defined as the nuclear slope of the elastic amplitude. The profile function that follows from Eq. (4.3) becomes smeared with a modified Bessel function (see Fig. 6),

$$\Gamma(b) = \frac{\sigma^T}{4\pi B} e^{\frac{\alpha^2 + b^2}{2B}} I_0\left(\frac{\alpha b}{B}\right), \quad (4.4)$$

and  $\alpha(s)$  is chosen such that  $\Gamma(b, s) \leq 1$  or  $\alpha \geq [2B \ln(\frac{\sigma^T}{4\pi B})]^{1/2}$ . For  $\sqrt{s} \leq 2.5$  TeV, we set  $\alpha = 0$  so that our cross section fits are not modified since they are well below this energy. At higher energies, the above

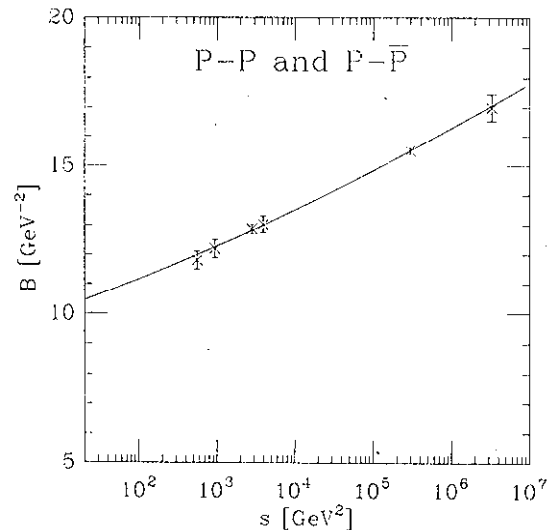


FIG. 5. The  $p-p$  nuclear slope  $B$  as a function of  $s$ . The solid line is our fit of Eq. (4.2), and included are the high energy  $p-\bar{p}$  collider data which we assume are equal to  $p-p$  at these energies.

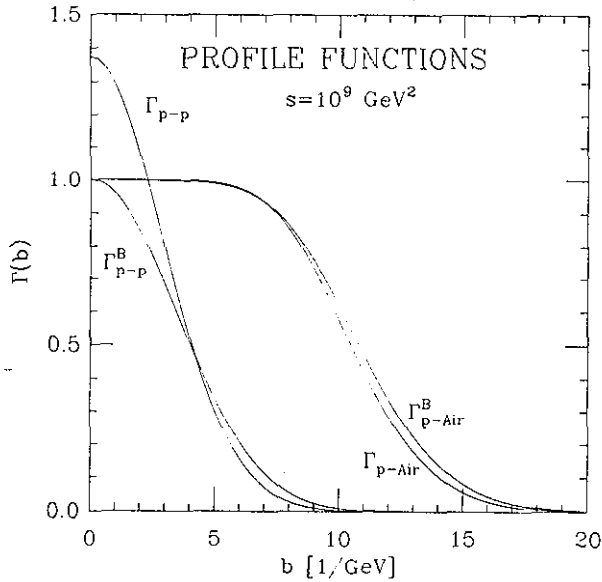


FIG. 6. Profile functions for  $p$ - $p$  and  $p$ -air scattering at  $s = 10^9 \text{ GeV}^2$ . Shown are  $\Gamma_{p-p}$  the unitarity violating profile, the Bessel-modified profile  $\Gamma_{p-p}^B$ , and the  $p$ -air profiles that follow from them using Glauber's approximation.

profile function leads to

$$\sigma_{h-A}^T = -2\beta \int_0^\infty dx \{1 - [1 + D'e^{-x} I_0(\lambda\sqrt{x})]^A\}, \quad (4.5)$$

$$\sigma_{h-A}^{\text{inel}} = -\beta \int_0^\infty dx \{1 - [1 + D'e^{-x} I_0(\lambda\sqrt{x})]^{2A}\}, \quad (4.6)$$

where  $\beta = -\pi[2B + R(A)^2]$ ,  $\lambda = 2\alpha\sqrt{\pi/\beta}$ , and

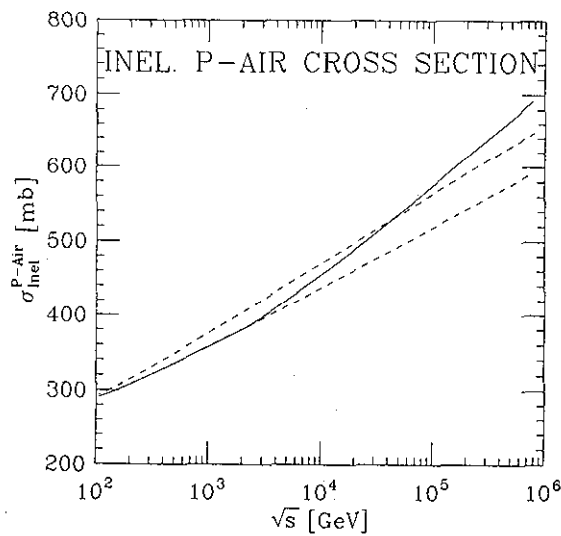


FIG. 7. Inelastic  $p$ -air cross sections as a function of c.m. energy. The solid line is the Bessel-modified cross section which is higher than the unphysical unitarity violating cross section above  $\sim 2.5 \text{ TeV}$  (dashed segment). Also shown for comparison as a dashed line, is the Block and Cahn parametrization of Eq. (5.2).

$D' = De^{\lambda^2/4}$ . The nucleus-nucleus case can be obtained as before, in the optical limit, by substituting  $A \rightarrow AA'$  and  $\beta = -\pi[2B + R(A)^2 + R(A')^2]$ . As shown in Fig. 6 for  $A = \text{air}$ , the  $p$ - $A$  profile function remains unchanged for small impact parameter (saturating the unitarity limit for large  $A$ ) but increases at large  $b$ , because of the smearing of the nucleon-nucleon profile function. This results in an increase of the total and inelastic  $p$ - $A$  cross sections, as shown in Fig. 7. In these plots we have used the minimum value of  $\alpha(s)$  that satisfies unitarity.

Our model of course is by no means unique since there may be other ways to preserve unitarity. The choice of the Bessel function is not essential and other modifications, such as exponentials, etc., should produce the same qualitative results.

## V. EXTRAPOLATION TO COSMIC RAY ENERGIES

The development of cosmic ray showers in the atmosphere depends strongly on the size of the relevant hadron-nucleus and nucleus-nucleus cross sections at very high energies  $E \sim 10^{18} \text{ eV}$ ,  $\sqrt{s} \sim 40 \text{ TeV}$ . Both the interaction length in the atmosphere and the number of nucleons in the nucleus that participate in a collision are a function of the absorption cross sections.

In the previous sections we have shown that the hadron-nucleus cross sections can be calculated reliably from the basic hadron-nucleon interaction. Since Glauber's analysis does not depend explicitly on the energy scale at which the scattering takes place, we expect that it is still applicable at very high energy. On the other hand, only for  $p$ - $\bar{p}$  does experimental data approach modest cosmic ray energies. Even this data at  $\sqrt{s} = 1.8 \text{ TeV}$  ( $E_{\text{lab}} \simeq 1.7 \times 10^6 \text{ GeV}$ ) must be extrapolated over three decades in energy to reach the highest cosmic ray energies of  $E_p > 10^9 \text{ GeV}$ . Because of the observed logarithmic behavior of the cross section and slope parameter, this procedure should be fairly reliable and data that will eventually come from the CERN Large Hadron Collider (LHC) should provide direct input for all but the very highest energies. The situation for the  $K^\pm - N$  and  $\pi^\pm - N$  interactions is much less satisfactory. Data for these reactions exist only up to a few hundred GeV lab energy with little hope of future measurements above the TeV range even with the LHC. While external beam experiments up to the Tevatron's maximum energy would improve our understanding of these reactions, it is unlikely that energy extrapolations over seven or eight orders of magnitude can be trusted, which means that the forms given in Eqs. (3.1)–(3.4) cannot be used at cosmic-ray energies. It appears that the best method for determining  $K^\pm - N$  and  $\pi^\pm - N$  scattering at cosmic ray energies is to use the independent quark model result

$$\sigma_{\pi-N}^T = \sigma_{K-N}^T = \frac{2}{3} \sigma_{N-N}^T. \quad (5.1)$$

Although this is only approximately true at low energy, it is expected to improve at very high energy. The hadronic

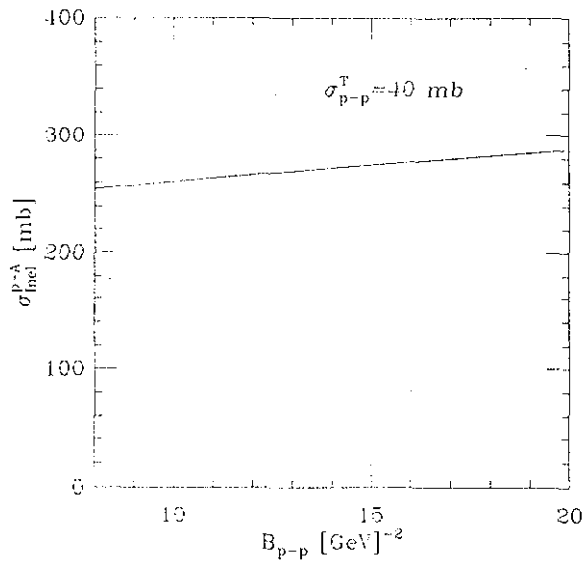


FIG. 8. Dependence of the  $p$ -air inelastic cross section on the nuclear slope of the proton-proton elastic scattering, keeping the  $p$ - $p$  cross section constant at 40 mb.

nuclear slopes are taken equal. These approximations are probably adequate considering the fact that the hadron-nucleus cross sections are relatively insensitive to the precise value of  $B_{h-N}$  and  $\sigma_{h-N}^T$  as shown in Figs. 8 and 9 for  $p$ -air. In Fig. 7 we plot our prediction for the  $p$ -air cross section as a function of center-of-mass energy and compare with a fit by Block and Cahn [17]. Based on theoretical arguments, they assumed that the total proton-proton cross section has the form

$$\sigma_{p-p}^T = A + \beta \left[ \ln^2 \left( \frac{s}{s_0} \right) - \frac{\pi^2}{4} \right] + c \sin \left( \frac{\pi\mu}{2} \right) s^{\mu-1} + D \cos \left( \frac{\pi\alpha}{2} \right) s^{\alpha-1}. \quad (5.2)$$

The parameters from their fit are  $A = 41.74$  mb,  $\beta = 0.66$  mb,  $s_0 = 338.5$  GeV<sup>2</sup>,  $c = 0$ ,  $D = -39.37$  mb GeV<sup>2(1- $\alpha$ )</sup>, and  $\alpha = 0.48$ . At  $\sqrt{s} = 10^5$  GeV, our result for  $\sigma_{\pi\text{-air}}$  is 460 mb. We view this prediction as less reliable than the  $p$ -air results because of the extra assumptions needed for this calculation.

The nucleus-nucleus cross sections can also be calculated at cosmic ray energies based on the nucleon-nucleon interaction using Eqs. (2.11) and (2.12) with the appropriate substitutions. However, there are not enough experimental data on nucleus-nucleus cross sections to check whether the Glauber model works as well in the hadron-nucleus case, especially since for practical purposes one is forced to use the optical limit approximation.

## VI. CONCLUSION

We have shown that Glauber's model of multiple diffraction accurately predicts hadron-nucleus cross sec-

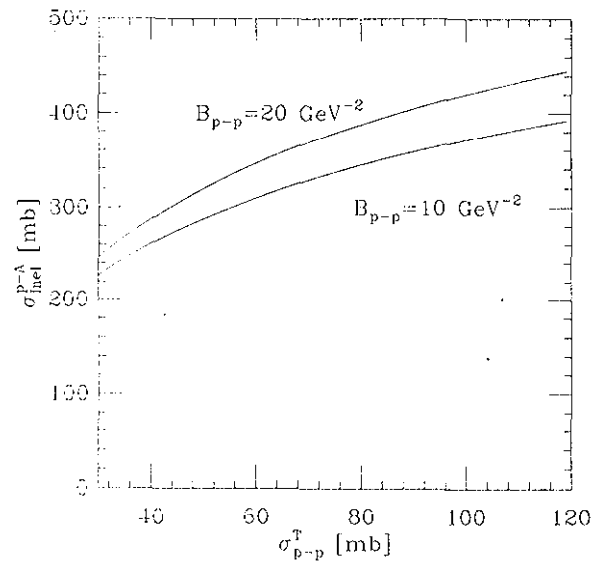


FIG. 9. Dependence of the  $p$ -air inelastic cross section on the proton-proton cross section, keeping the  $p$ - $p$  nuclear slope constant at 10 and 20 GeV<sup>-2</sup>.

tions from the basic underlying hadron-nucleon interaction. We have obtained excellent agreement with the experimental data for a variety of incident hadrons and target nuclei in the energy range of 60–280 GeV with just two free parameters. External beam experiments at 800 GeV on nuclear targets would provide a useful check of the validity of these methods over a much broader range of energies. Since Glauber's analysis does not depend explicitly on the energy scale of the scattering process and it is unlikely that the distribution of nucleons inside a nucleus would change with energy, we expect this method to continue to be applicable at very high energies. The  $p$ - $\bar{p}$  data extend to a large energy range and can be extrapolated to cosmic ray energies. Using Glauber's model we can then obtain the relevant nucleon-nucleus cross sections at these energies. We can use the independent quark model to estimate the  $\pi$ - $N$  and  $K$ - $N$  cross sections, but have not yet developed a model for the nuclear slope.

Finally, we would like to emphasize that extrapolating our fits to available data on  $\sigma_{p-p}^T$  and  $B_{p-p}$  to energy  $\sqrt{s}$  greater than a few TeV violates unitarity. This means that above the current Tevatron energies and the energy available at the LHC we should expect some change in the shape of the nucleon diffraction peak and its related profile function. This is to be contrasted with the behavior at lower energies where the Gaussian profile function height and radius have grown logarithmically with  $s$ .

We propose a simple model in which the  $p$ - $p$  elastic scattering amplitude deviates from a pure exponential in the momentum transfer and satisfies unitarity. The resulting absorption cross sections are increased appreciably at cosmic ray energies. How nature actually chooses to satisfy unitarity awaits experiments in the few TeV energy range.

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