Hadron helicity violation in exclusive processes: Quantitative calculations in leading order QCD

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We study a new mechanism for hadronic helicity flip in high energy hard exclusive reactions. The mechanism proceeds in the limit of perfect chiral symmetry, namely, without any need to flip a quark helicity. The fundamental feature of the new mechanism is the breaking of rotational symmetry of the hard collision by a scattering plane in processes involving independent quark scattering. We show that in the impulse approximation there is no evidence for strong suppression of the helicity-violating process as the energy or momentum transfer Q^2 is increased over the region 1 $\mathrm{GeV}^2 < Q^2 < 100$ GeV^2. In the asymptotic region $Q^2 > 1000$ GeV^2, a saddle-point approximation yields suppression by a fraction of a power of Q^2. "Chirally odd" exclusive wave functions which carry a nonzero orbital angular momentum, and yet are leading order in the high energy limit, play an important role.

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I. INTRODUCTION

The theory of hard exclusive hadronic scattering in quantum chromodynamics (QCD) has evolved considerably over many years of work. Currently there exist two self-consistent perturbative descriptions, each with a specific factorization method for separating the hard scattering from nonperturbative wave functions. A well-known procedure using the "quark-counting" diagrams has been given by Lepage and Brodsky [1]. A consequence, and direct test, of the factorization defining this mechanism is the hadron helicity conservation rule [2]

$$h_A + h_B = h_C + h_D \ , \tag{1.1}$$

where the h_j's are the helicities of the participating hadrons in the reaction $A+B \to C+D$. The fact that this rule is badly violated in almost every case tested suggests two alternatives. One possibility, advocated by Isgur and Llewellyn-Smith [3], is that the energy and momentum transfer (Q^2) in the data are not large enough for the formalism to apply. However, the data also show power-law dependence on Q^2, indicating that hard scattering of a few pointlike quarks is being observed. The apparent contradiction has led to much discussion, and has even caused some authors to suggest that perturbative QCD itself might be wrong.

The second alternative is that another power-behaved process causing helicity flip is present. In fact the "independent scattering" subprocess, introduced by Landshoff [4], is actually the leading process at very high energies [5]. But it has been assumed that hadron helicity conservation would be the same in the independent scattering process as in the quark-counting one, since both involve exchange of hard gluons with large Q^2. In general, terms proportional to a quark mass, m_q, for example, have been understood to cause helicity flip in either model, but with amplitude suppressed by a power of m_q/Q relative to the leading term. Such terms seem to be quite small and are probably not a believable explanation of the persistent pattern of large violations of the helicity conservation rule.

Here we show that the independent scattering mechanism predicts high energy helicity *nonconservation* at a substantial rate. The calculations in momentum space are sufficiently complicated that this phenomenon has been overlooked for almost twenty years. Adopting a transverse position space formalism introduced by Botts and Sterman [6], we show that the details rest on nonperturbative wave functions that should be *measured* rather than calculated. These wave functions measure nonzero orbital angular momentum not taken into account by short distance expansions. We argue that the novel factorization properties of independent scattering processes cannot practically be reduced to the same ingredients used in the quark-counting scattering. In any case, it is not necessary to flip a quark helicity: the new mechanism proceeds unimpeded in the limit of arbitrarily small quark mass and perfect chiral symmetry in the hard scattering.

This paper is organized as follows. In Sec. II we review the derivation of helicity conservation in the short-distance model. In Sec. III, we present the independent scattering mechanism with special emphasis on nonzero orbital angular momentum wave functions. We compute the contribution of these components to a helicity-conserving reaction in Sec. IV, and to a helicity-violating reaction in Sec. V. These two sections contain our most important results at asymptotic and at accessible energies. Section VI is an experimental outlook.

II. HELICITY CONSERVATION IN SHORT-DISTANCE-DOMINATED PROCESSES

First we review the conventional derivation of hadron helicity conservation [2]. The quark-counting factorization introduces the distribution amplitude $\varphi(x,Q^2)$ [7], an integral over the transverse momentum variables of the wave function for quarks to be found carrying momentum fraction x in the hadron. For simplicity of presentation we specialize to a single pair of quarks, the meson case. Let $\psi(x,\mathbf{k}_T)$ be the light cone wave function to find quarks with light cone momentum fractions x and $1-x$ and relative transverse momentum $\mathbf{k}_T$ and $-\mathbf{k}_T$. In terms of the Fourier conjugate transverse space variable $\mathbf{b}_T$ separating the quarks, then

$$\begin{aligned}\varphi(x,Q^2) &= \int_0^Q d^2\mathbf{k}_T\;\psi(x,\mathbf{k}_T)\\ &= \int_0^Q d^2\mathbf{k}_T\int_0^\infty \frac{d^2\mathbf{b}_T}{(2\pi)^2}e^{i\mathbf{b}_T\cdot\mathbf{k}_T}\\ &\quad\times\sum_m e^{im\varphi}\mathcal{P}_m(x,|\mathbf{b}_T|). \end{aligned}\tag{2.1}$$

In the second line we have expanded the wave function $\mathcal{P}(x,\mathbf{b}_T)$ to exhibit the SO(2) orbital angular momentum eigenvalues m, using the hadron momentum as the "z" axis. Suppose the distribution amplitude $\varphi(x,Q^2)$ is assumed to be a good description of a process. Then, whatever the angular momentum content of the wave function, evaluating the integrals reveals that the $m=0$ element is the sole surviving term in Eq. (2.1):

$$\varphi(x,Q^2) = \int_0^\infty d|\mathbf{b}_T|Q\,\frac{J_1(|\mathbf{b}_T|Q)}{2\pi}\mathcal{P}_0(x,|\mathbf{b}_T|).\tag{2.2}$$

This shows that use of $\varphi(x,Q^2)$ imposes two things: as $Q^2\to\infty$ both the scattering region is "small," since the region $\mathbf{b}_T^2<1/Q^2$ dominates in the Bessel function, and the scattering is "round," i.e., cylindrically symmetric. In the absence of orbital angular momentum, the hadron helicity becomes the sum of the quark helicities. The quark helicities being conserved up to $O(m_q^2/Q^2)$ corrections, the total hadron helicities are conserved. The hadron helicity conservation rule (1.1) therefore represents an exact *symmetry* of the quark-counting factorization in the limit of massless quarks. Crucial questions are the following. Does this symmetry of the model represent a property of the entire perturbative theory? Or can we simply assume "s wave" SO(2) wave functions to be the main contribution as in a nonrelativistic picture?

The answer to both questions is *no*. In general, quark wave functions themselves are not particularly restricted in orbital angular momentum content, even in the high energy limit. For example, in the pion (pseudoscalar meson) case the light cone wave function may be expanded in terms of four Dirac tensors allowed by parity symmetry as

$$\begin{aligned}&\mathcal{P}_{\alpha\beta}(x,\mathbf{b}_T;p)\\ &\quad=\frac{1}{4}\gamma_5\left\{\mathcal{P}_{0\pi}\not{p}+\mathcal{P}'_{0\pi}+\mathcal{P}_{1\pi}[\not{p},\not{b}_T]+\mathcal{P}'_{1\pi}\not{b}_T\right\}_{\alpha\beta},\end{aligned}\tag{2.3}$$

where p is the pion momentum. The $\mathcal{P}$'s are functions of the light cone fraction x and the transverse separation $\mathbf{b}_T$. (For the moment we do not discuss dependence on gauge fixing and conventions used to make the wave function gauge invariant.) The $\mathcal{P}_{1\pi}$ term carries one unit of orbital angular momentum and yet scales with the same power of the "big" momentum p^+ as the $\mathcal{P}_{0\pi}$ term, which is s wave. Since the $\mathcal{P}_{1\pi}$ term has a $\mathbf{b}_T$ factor, which can be written in terms of $b_{T,1}\pm ib_{T,2}$, this term represents one unit of orbital angular momentum. In terms of power counting, then, the $m=0$ and $m\neq0$ amplitudes can be equally large.

We also note that wave functions are not objects to be derived in perturbation theory, but instead represent the nonperturbative long-time evolution proceeding inside a hadron. The nonperturbative Hamiltonian of QCD does not conserve spin and orbital angular momentum separately, but instead generates mixing between orbital and spin angular momentum. Finally, there is no simple relation between "s-wave" nonrelativistic models of constituent quarks, and the pointlike quarks resolved in large-Q^2 collisions; no statement can reliably be made about quark angular momentum content of hadrons. Thus *if* a nonzero orbital angular momentum component somehow enters the hard scattering—and this is a crucial point—*then* the long-time evolution before or after the scattering can convert this angular momentum into the observed hadron spin. It is not necessary to flip a quark spin in the hard interaction, because the asymptotic hadron spin fails to equal the sum of the quark spins. Such a mechanism is totally consistent with the impulse approximation of perturbative QCD.

The challenge in high energy hadron scattering is therefore to find those large-Q^2 processes in which nonzero orbital angular momentum enters, or, in other words, to find those which are not "round." It turns out that in any treatment relevant to current energies the independent scattering process is not "round" but instead "cigar shaped" (Fig. 1). The subprocess is highly asymmetric, showing an extreme dependence on the scattering plane.

The origin of the asymmetry is kinematic. Let us separate hadron "center of mass" variables from the internal coordinates. The center of mass variables are handled by overall momentum conservation. The internal variables measure more dynamical information about the relative positions of quarks in the subprocesses. Let us consider the internal variables in coordinate space, and focus on directions transverse to different hadron directions. In the hadron center of momentum frame, let us construct an orthonormal basis with initial state hadron momenta along the $\hat{3}$ axis, $\hat{1}$ in the scattering plane, and $\hat{2}$ perpendicular to the scattering plane [Fig. 1(a)]. Quark transverse separations are limited by their wave functions in the hadrons, which are about 1 fm. In the independent scattering mechanism, let one pair of quarks scatter defining a scattering plane [the plane indicated by dashed lines, top subprocess, Fig. 1(a)]. Since large momentum, $O(Q)$, is exchanged in the plane ($\hat{1}$ and $\hat{3}$ directions), the distances of quarks' closest approach in the plane are order $1/Q$. For meson-meson scattering, let another, unrelated pair of quarks scatter independently [bottom subprocess, Fig. 1(a)]. Again, large Q will lead to short distance of the in-plane variables for the bottom plane. Finally, if the outgoing quarks are to have a good overlap to form a hadron, then the plane determined by the second scattering must be parallel to the first, up to a small (typically 300 MeV) relative fluctuation of quarks to fit into the same bound state. The power counting of independent scattering comes by estimating the phase space for these conditions to occur. Demanding coincidence of momenta of the second scattering with the first can be represented by three δ functions of large momentum (energy and components along $\hat{1}$ and $\hat{3}$). Each δ function scales like $1/Q$ in the fixed angle limit; three such functions scale like Q^{-3}. But the allowed out-of-plane (along $\hat{2}$) variation is set by the bound state wave functions, and is kinematically unrelated to Q. The corresponding integration in coordinate space over the conjugate transverse variable b is weighted by coordinate space wave functions and will produce a spatial scale we can call $\langle b^2\rangle^{1/2}$. The b variable measures the separation between the planes [Fig. 1(a)], which ranges from zero to the maximum the bound states allow. The overall phase space for meson-meson scattering thus scales as $\langle b^2\rangle^{1/2}Q^{-3}$. By repeating the argument, proton-proton scattering just adds another plane and will scale as $(\langle b^2\rangle^{1/2}Q^{-3})^2$. The power counting is well known [4], but the coordinate space picture is not. The crucial point is that the out-of-scattering-plane direction is preferentially selected, creating a kinematic asymmetry. Allowing for the Lorentz contracted pancake nature of the fast hadrons in real space, the hard scattering occurs in an oriented cigar-shaped region, with three space dimensions of order $\langle b^2\rangle^{1/2} \times 1/Q \times 1/Q$. This fact is quite hard to see in covariant perturbative calculations in momentum space, explaining why it has been overlooked.

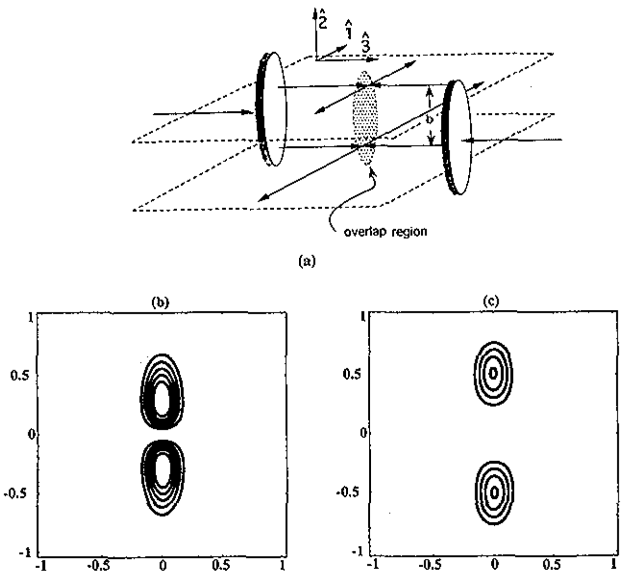

FIG. 1. Coordinate space picture of meson-meson independent scattering. (a) Scattering planes (dashed) and directions of quarks (arrows) are independent, but to make a final state hadron the important region is the case where they are parallel. The intersection of the Lorentz contracted hadrons (initial state pancakes are shown) and the region of integration over the transverse separation between quarks, $\mathbf{b}_T$, inside the hadrons is a cigar-shaped overlap region. Pancake wave functions of outgoing hadrons have been omitted. (b) Contour map of the cigar-shaped region, an integrand made from the Sudakov $U(x,Q,b)$ factors, defined in Sec. III, in units of fermis. The out-of-plane variable b and in-plane variable $\mathbf{b}_T\cdot\hat{1}$ are transverse space coordinates defined relative to each hadron (Sec. III). The regions which are not strongly suppressed by coherence of gluon emission correspond to small b. The "hole" as $b\to 0$ is an unimportant artifact of the standard Sudakov expression (see Sec. III D). (c) The same as (b) but multiplied by a polynomial representing quark orbital angular momentum $m=2$. Because of the scattering plane asymmetry, the integrand has no symmetries to suppress the overlap of the $m\neq 0$ case with the $m=0$ case, meaning that hadron helicity conservation generally fails to be true. Actual calculations also use bound-state wave functions (Sec. III) not shown in the contour plots. Both (b) and (c) are multiplied by a Gaussian in $\mathbf{b}_T\cdot\hat{1}$ for the hard scattering in the plane, and drawn to scale at $Q^2 = 2$ GeV^2; higher Q^2 flattens the $\mathbf{b}_T\cdot\hat{1}$ dependence without rapidly changing the b dependence.

The kinematic violation of hadron helicity conservation by independent scattering raises several new questions. It is clear that the usual association of leading twist (short distance) and large Q^2 either breaks down or hinges on delicate dynamical details. Our approach will exploit the fact that leading approximations to any kinematically distinct amplitude are always perturbatively calculable. For example, corrections of nonleading twist type in the distribution amplitude formalism cannot violate the hadron helicity conservation symmetry and will not affect our approach. The first nonvanishing contributions to helicity-violating amplitudes in the short-distance formalism involve extra partons. A gluon embedded in the hard scattering, for example, could transfer spin to an outgoing hadron. We need not consider such processes, because, as demonstrated later, they are subleading by a power of Q^2 and perturbatively small since such gluons are "hard." It remains to be shown, of course, that helicity violation from independent scattering is not suppressed by the same order. That is the main technical task of this paper.

III. INDEPENDENT SCATTERING: FORMALISM

A. Kinematical analysis

Botts and Sterman have considered [6] the generic "elastic" reaction $M_1 + M_2 \to M_3 + M_4$, where the M_i's are light pseudoscalar mesons, at high center of mass energy $\sqrt{s}$ and large scattering angle $\theta = \arccos(1 + 2t/s)$. Their study has shown that the reaction is dominated by the two independent scatterings of the valence constituents with a kinematical configuration depicted in Fig. 2. One has two scattering planes separated at the collision point by a transverse distance b.

To define coordinates, let us consider different light cone bases (v_i, v_i', ξ_i, η) attached to each meson M_i, and chosen so that, in the center of mass frame where $\hat{\mathbf{p}}_1 = \hat{3}$ and $\hat{\mathbf{p}}_3 = \cos\theta\,\hat{3} + \sin\theta\,\hat{1}$,

$$v_1 = v_2' = \frac{1}{\sqrt{2}}(\hat{0} + \hat{3}), \quad v_1' = v_2 = \frac{1}{\sqrt{2}}(\hat{0} - \hat{3}),$$

$$\xi_1 = \xi_2 = \hat{1}, \quad \eta = \hat{2},$$

$$v_3 = v_4' = \frac{1}{\sqrt{2}}(\hat{0} + \sin\theta\,\hat{1} + \cos\theta\,\hat{3}),$$

$$v_3' = v_4 = \frac{1}{\sqrt{2}}(\hat{0} - \sin\theta\,\hat{1} - \cos\theta\,\hat{3}),$$

$$\xi_3 = \xi_4 = \cos\theta\,\hat{1} - \sin\theta\,\hat{3}.$$

In the c.m. frame, neglecting meson masses, one has $p_i = Qv_i$, defining $Q = \sqrt{s/2}$.

The scattering amplitude $A(s,t)$ is written as a convolution

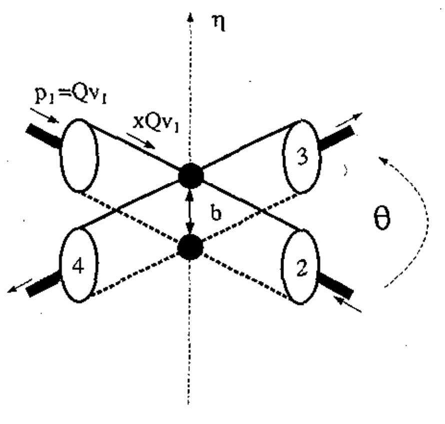

FIG. 2. Kinematics of the independent scattering mechanism. The two scattering planes are separated at the collision point by a transverse distance b. In the c.m. frame, we choose direction $\hat{3}$ along the direction of flight of M_1 and $\hat{2} \equiv \eta$ transverse to the scattering plane. A light cone basis (v_i, v_i', ξ_i, η) is attached to each meson M_i (see text). Neglecting meson masses, one has $p_i = Qv_i$ $(Q = \sqrt{s/2})$.

$$A(s,t) = H * H' * \prod_{i=1}^{4} X_i,$$

where H and H' are hard scattering subprocess amplitudes which depend on quark momenta k_i, and X_i are Bethe-Salpeter amplitudes

$$X_i(k_i, p_i) = \int \frac{d^4y}{(2\pi)^4} e^{ik_i\cdot y} \langle 0|T[\psi_\alpha(y)\bar{\psi}_\beta(0)]|\pi(p_i)\rangle.$$

The X_i contain the information on "soft" bound-state dynamics. To avoid overcounting of the hard momentum transfer regions, they are defined so that $q\bar{q}$ configurations are eliminated when their relative transverse momentum (along ξ_i and η) is $O(Q)$. The relative minus momentum (along v_i') is of $O(M^2/Q)$ with M some typical hadronic scale. Then a simple kinematical analysis shows that momentum transfers in H or H' are dominated by large invariants built with x_iQ terms. A first approximation is to neglect all but these components of the quark or antiquark momenta in the hard amplitudes H and H'. This is the impulse approximation.

A second observation follows from kinematics. Although momentum conservation at the hard scattering H relates the internal momentum dependence of the X's, a variation of one such momentum k_i in its ξ_i or v_i' direction induces negligible modifications in the three other X's. Consequently, momentum components of k_i along ξ_i or v_i' only appear as relevant variables in the wave function X_i and integrations over these components can be carried out. The components along η, denoted l_i, represent transverse momentum out of the scattering plane and do not share the same property. Thus the vertex δ function may be reexpressed as

$$\delta^4(k_1 + k_2 - k_3 - k_4)$$

$$= \frac{\sqrt{2}}{|\sin\theta|Q^3} \prod_{i=2}^{4} \delta(x_1 - x_i)\,\delta(l_1 + l_2 - l_3 - l_4),$$

which indicates that the four constituents which enter or leave each hard scattering carry the same light cone fraction ($x_i = x$ or $1 - x$). We let x be the common light cone fraction of the four constituents in H and $\bar{x} = 1 - x$ be the one in H'. Introducing the "out-of-plane" impact parameter b through

$$2\pi\delta(l_1 + l_2 - l_3 - l_4) = \int_{-\infty}^{+\infty} db\, e^{i(l_1 + l_2 - l_3 - l_4)b},$$

one may write the amplitude of the process $M_1 + M_2 \to M_3 + M_4$ [6] as

$$A(s,t) = \frac{\sqrt{2}Q}{2\pi|\sin\theta|} \times \int_0^1 dx\,(2\pi)^4 H(\{xQv\})\,H'(\{\bar{x}Qv\})\Big|_{\{\alpha\beta\}} \times \int_{-\infty}^{+\infty} db \prod_{i=1}^{4} \frac{\mathcal{P}_{\alpha_i\beta_i}(x, b; Qv_i)}{Q}, \qquad (3.1)$$

where

$$\mathcal{P}_{\alpha\beta}(x,b;Qv) = \int \frac{dy^-}{2\pi}\, e^{ixQy^-} \times \langle 0|\, T\left[q_\alpha(y)\,\bar{q}_\beta(0)\right] |\pi(Qv)\rangle \Big|_{y=y^- v' + b\eta}, \tag{3.2}$$

with Dirac indices $\alpha\beta$. Color indices are suppressed in Eq. (3.1) and sums over repeated indices are understood. We consider unflavored quarks; the effects of flavor are implemented by setting to 0 some of the graphs we are going to consider. H and H' are Feynman amplitudes (a sum over allowed diagrams is understood) computed with standard perturbative QCD vertices and internal propagators. At lowest order in the coupling constant, H and H' consist of one-gluon exchange or $q\bar{q}$ annihilation for each quark pair. Note that $\mathcal{P}$ and H are not individually gauge invariant.

If the short-distance $b \to 0$ limit is assumed, then the four general wave functions in Eq. (2.3) can be reduced to the $\mathcal{P}_{0\pi}$ term, by taking the trace

$$\mathcal{P}_{0\pi}(x,0) = \mathcal{P}_{\alpha\beta}(x,0;Qv)\frac{\not{v}'\gamma^5_{\beta\alpha}}{Q}.$$

Then, the zeroth moment of $\mathcal{P}_0$ is related to the decay constant of the corresponding meson

$$\int_0^1 dx\, \mathcal{P}_0(x, b=0) = f_M$$

where, e.g., for the pion, $f_\pi = 133$ MeV. This zero-distance quantity contains no information on the interesting dependence on the transverse variable b.

The selection of $\mathcal{P}_{0\pi}$ to compute the leading-twist component of helicity-conserving amplitudes was shown to be consistent by Botts and Sterman [6]. Here we are concerned with the leading order description of helicity-violating terms. Thus, we will consider $\mathcal{P}_{0\pi}$-type and $\mathcal{P}_{1\pi}$-type amplitudes on an equal footing, and make no *a priori* assumption that the region $b \to 0$ dominates.

B. Gauge invariance

The development so far has been sufficient to isolate the kinematic region of interest, which as we have already noted is characterized by finite separation between the participating quarks in the out-of-scattering-plane direction. The amplitude is thus a strong function of the spatial dependence of the wave function. The Bethe-Salpeter wave function is a bilocal matrix element and is not gauge invariant. However, we will now discuss how gauge invariance of the description can be obtained.

The key is in how the perturbation theory is rearranged. In the Sterman and Botts factorization certain "soft" corrections are put into the wave functions, leaving other parts of Feynman diagrams to go into the hard scattering kernel. In dressing the wave function in this way, it is no longer a quark correlation (the Bethe-Salpeter wave function), but the matrix elements of operators determined by the types of diagrams put in. The operators chosen in [6] are path ordered exponentials (POE's), shown by Collins and Soper [8] to be the generators of eikonal approximations to the gluon attachments. The POE's are gauge covariant, leading to a gauge-invariant amplitude.

This is partly forced by physics, and partly a convention. As a convention for the perturbation theory, subsequent diagrams must be evaluated with subtractions to avoid double counting. More generally, any operator functional of the A fields which transforms properly could serve in place of the POE's, with the effect of creating a different subtraction procedure. Let us extract what we can that is independent of convention.

Let the operator in the definition of the wave functions be called $U(A;x)$; we will call it a gauge-dressing operator. Under a gauge transformation at the position x, we require $U(A;x)$ to transform like an antiquark. Then products such as $U(A;x)\,\psi(x)$ are gauge invariant. That is, we have gauge-invariant matrix elements to find a dressed quark

$$\langle 0|T[U(A;y)\psi(y)\,U^\dagger(A;x)\bar{\psi}(x)]|\pi(p)\rangle.$$

It is obvious that this requirement does not determine $U(A;x)$ uniquely, because one could always attach a factor which is gauge invariant without changing the gauge transformation properties. The particular choice of what to attach is a prescription, i.e., a definition of what parts of the amplitude will be put in the wave function and what in the hard scattering, and it cannot be determined by gauge invariance alone. However, due to gauge transformations one must attach some kind of gauge-dressing operator to have well-defined matrix elements.

C. Path-independent dressing

Although the standard way to do gauge dressing with the POE is path dependent, no path dependence generally need be associated with $U(A;x)$ and in particular *the observable process does not determine or favor any path.* This important point can be seen with an elementary example from QED, where the U(1) gauge invariance is easier to control. The straightforward QED analogue of our process involves equal time (not light cone time) correlation functions in the gauge $A^0 = 0$. This gauge choice eliminates a mode, but there still remains a lack of definition of the A coordinates due to time-independent gauge transformations $\theta(x)$:

$$\mathbf{A}(x) \to \mathbf{A}'(x) = \mathbf{A}(x) + \nabla\theta(x),\ \partial^0\theta = 0.$$

This produces a change in the longitudinal modes. These modes are sometimes also called unphysical, a very unfortunate choice of terminology. In free space and in the absence of coupling a gauge theory has two transverse degrees of freedom and the third would be called unphysical. However, we are interested in the case that matter fields exist (and the non-Abelian coupling is turned on) in which case the third mode is real, but special inasmuch as being determined in terms of the other variables by gauge invariance. To see this, note that we can decompose into transverse and longitudinal parts,

$$\mathbf{A} = \mathbf{A}_T + \mathbf{A}_L = \mathbf{A}_T + \nabla\phi, \tag{3.3}$$

with the transforming part

$$\phi = \frac{1}{\nabla^2}\left[\nabla\cdot\mathbf{A}\right]$$

and invariant part $\mathbf{A}_T = \mathbf{A} - \nabla\phi$. Here $1/\nabla^2$ is the Green function defined so that the $\nabla_x^2(1/\nabla_{x,x'}^2) = \delta^3(x-x')$.

Since it is there for gauge transformations, the longitudinal part ϕ is not free to be varied in arbitrary dynamical ways, but must accompany the matter field in a prescribed, unique functional. At time $t=0$ this operator is

$$U_c(\mathbf{A};x) = e^{ig\phi(x)} \tag{3.4}$$

from which one can verify that, under a gauge transformation,

$$\begin{aligned}\phi(x) &\to \phi(x) + \theta(x),\\ \psi(x) &\to e^{-ig\theta(x)}\psi(x),\\ U_c(\mathbf{A};x) &\to e^{ig\theta(x)}U_c(\mathbf{A};x),\\ U_c(\mathbf{A};x)\psi(x) &\to U_c(\mathbf{A};x)\psi(x).\end{aligned} \tag{3.5}$$

We will call $U_c(\mathbf{A};x)$ the Coulomb dressing operator because it creates a classical Coulomb field around the matter particle, as the reader can check by calculation. Since the Hamiltonian commutes with the gauge transformation operator once $A^0 = 0$ has been set, the time evolution will maintain the invariance of the combination $U_c(\mathbf{A};x)\psi(x)$. However, as noted already, one is not forced to accept this as the unique answer, but can opt for $U_c(\mathbf{A};x)\,f(\mathbf{A}_T)$, which will transform in the same way for any $f(\mathbf{A}_T)$.

The reader may still be curious to know the relation between Coulomb dressing and the POE approach. This can be very simply exhibited by noting that

$$\psi(y)e^{ig\phi(y)}e^{-ig\phi(x)}\bar{\psi}(x) = \psi(y)e^{ig\int_x^y d\mathbf{z}\cdot\nabla(\phi)(\mathbf{z})}\bar{\psi}(x). \tag{3.6}$$

This expression is still path independent. This is the choice $f(\mathbf{A}_T) = 1$. A different choice is the path-dependent one

$$f(\mathbf{A}_T) = \exp\left(ig\int_x^y d\mathbf{z}\cdot\mathbf{A}_T(\mathbf{z})\right), \tag{3.7}$$

in which case we have

$$U_c(\mathbf{A};y)f(\mathbf{A}_T)U_c^\dagger(\mathbf{A};x) = \exp\left(ig\int_x^y d\mathbf{z}\cdot\mathbf{A}(\mathbf{z})\right), \tag{3.8}$$

which is the standard path-dependent POE. Both procedures are equally acceptable, as far as satisfying gauge invariance. An important difference is that the path ordered exponential can create a line of physical transverse gauge field particles between the charged matter fields, depending on how the path is oriented. Such a line of gluons, existing only along the chosen path, can be interpreted as an arbitrary model for the transverse gauge field inside the state of interest. Similarly, if one boosts the Coulomb dressed definition with $f(\mathbf{A}_T) = 1$, the boost also creates a blast of transverse gauge fields as seen by the "equivalent photon" approximation, and this is path independent.

In perturbation theory, the lowest order approximation to non-Abelian dressing is the Abelian case. It is possible in the non-Abelian theory to write down expressions analogous to the Coulomb dressing but care must be used to keep track of the color indices. It is equally valid to use POE's, which definitely transform properly, as building blocks to generate an infinite number of different ways to dress the quarks. The different choices are not relevant for a leading order calculation to which we restrict this study.

D. Factorization

The next step is to elaborate a factorized form for the amplitude, whose prototype is Eq. (3.1), regarding radiative corrections. Generalizing the results of [6] to the case of the helicity-violating Dirac projections, a leading approximation to the soft region rearranges these corrections to obtain the expression

$$\begin{aligned}A(s,t) = \frac{\sqrt{2}Q}{2\pi|\sin\theta|}\int_0^1 dx\,(2\pi)^4 H(\{xQv\})\,H'(\{\bar{x}Qv\})\\ \times\int_{-1/\Lambda}^{+1/\Lambda} db\,U(x,b,Q)\prod_{i=1}^{4}\frac{\mathcal{P}_i^{(S)}(x,b;Qv_i)}{Q},\end{aligned} \tag{3.9}$$

where H and H' are evaluated at respective scales xQ and $(1-x)Q$ which are assumed to be large (Dirac indices are understood), and $\Lambda = \Lambda_{\rm QCD}$, the strong coupling scale in a particular scheme.

Large logarithmic corrections to the process, with the coexistence of the two scales Q and $1/b$, are resummed in the Sudakov factor U. At leading order, we approximate it by its dominant expression at large Q [6]:

$$\begin{aligned}U(x,b,Q) \approx \exp\Big[-c\ln\frac{xQ}{\Lambda}\big(-\ln u(xQ,b)\\ -1 + u(xQ,b)\big)\Big]\ \exp\Big[(x\to\bar{x})\Big]\end{aligned} \tag{3.10}$$

with

$$u(xQ,b) = \left(-\frac{\ln b\Lambda}{\ln xQ/\Lambda}\right) \text{ and } c = 4\,\frac{4}{3}\,\frac{2}{11-2n_f/3} = \frac{32}{27}$$

for $n_f = 3$. The notation $(x\to\bar{x})$ in Eq. (3.10) indicates writing the same function in the exponent with this substitution. We have introduced the variable $u(xQ,b)$ which turns out to be the relevant one to describe the Sudakov unsuppressed region in the (b,Q) plane. For

$u(xQ,b) = 1$ $[u(\bar{x}Q,b) = 1]$ there is no suppression from the first [second] exponential in Eq. (3.10). For $u(xQ,b) \ll 1$, then b is much larger than $1/xQ$, and one gets a strong suppression due to the large $\ln xQ$ factor. The Sudakov suppression in the region $u(xQ,b) > 1$ is inessential (as we explicitly checked numerically), because the independent scattering mechanism does not receive much contribution from this short-distance domain. After perturbative resummation, the QCD scale Λ turns out to be the natural bound for the integral over the impact parameter ($U = 0$ for $b \geq \Lambda^{-1}$).

Because U includes the logarithmic corrections from Q down to $1/b$, each $\mathcal{P}_i^{(S)}$ is a soft object which does not include perturbative corrections harder than $1/b$. In the short-distance regime, one can relate the s wave $\mathcal{P}_0^{(S)}(x, b \to 1/Q; Qv_i)$ to the distribution amplitude $\varphi(x,Q)$ [6]. There is no such correspondence for $\mathcal{P}_1^{(S)}$ which is an entirely new object.

End points in the x integral where hard subprocesses would become soft may look problematic. Both the distribution amplitude and wave function approaches used here become self-consistent given end-point zeros, e.g., $\varphi(x \to 0) \sim x^k$, where $k > 0$ should occur independently of spin projection.

IV. CONTRIBUTION FROM NONZERO ORBITAL MOMENTUM COMPONENTS OF THE WAVE FUNCTION IN $\pi\pi \to \pi\pi$

Before analyzing helicity-violating processes let us examine the leading contribution from the various components in Eq. (2.3) to a standard helicity-conserving reaction such as $\pi\pi \to \pi\pi$.

A. Computation of amplitudes

In their study, Botts and Sterman were interested in identifying the asymptotic behavior of the amplitude $A(s,t)$ ($s \to +\infty$, t/s fixed). Asymptotically, the Sudakov mechanism contained in $U(x,b,Q)$ results in a suppression of the large-b region in the integral of Eq. (3.9). In this limit one can forget about tensorial components of $\mathcal{P}_{\alpha\beta}(x,b;Qv) \propto (\cdots \not{b} \cdots)_{\alpha\beta}$ and only the component $\frac{Q}{4}\mathcal{P}_0\gamma_5\not{v}|_{\alpha\beta}$ of $\mathcal{P}_{\alpha\beta}$ survives.

In the intermediate-Q^2 regime, configurations of the $q\bar{q}$ pair sitting in a light meson with transverse separation smaller than the meson charge radius are not strongly affected by the Sudakov mechanism [10]. As anticipated in Sec. II, any $m = 1$ components of the wave function which form large invariants in H (as large as the s-wave term) may give sizable contributions to the interaction amplitude. For contributions with leading power behavior in the pseudoscalar case, we must keep the tensorial decomposition

$$\mathcal{P}_{\alpha\beta}(x,b;Qv) = \frac{Q}{4}\gamma_5 \left\{\mathcal{P}_0(x,b)\not{v} + \mathcal{P}_1(x,b)[\not{v}, \not{b}_T]\right\}_{\alpha\beta}. \quad (4.1)$$

We now explore the calculation with this assumption. In Eq. (3.9), one has to carry out the projection of the hard part H and H' onto the Dirac and color matrices coming from each wave function. This is done in Appendix A. One finds the hard amplitudes a_i labeled by the power of b entering:

$$\begin{aligned} a_0 &= \left(\frac{\pi}{6}\right)^4 \frac{256g^4}{x^2\bar{x}^2s^2} \frac{s^4(s^2-3tu)+t^2u^2(s^2-tu)}{s^2t^2u^2}, \\ a_2 &= b^2 \left(\frac{\pi}{6}\right)^4 \frac{2048g^4}{x^2\bar{x}^2s^2} \frac{s^4(s^2-3tu)-t^3u^3}{s^2t^2u^2}, \\ a_4 &= b^4 \left(\frac{\pi}{6}\right)^4 \frac{2048g^4}{x^2\bar{x}^2s^2} \frac{s^4(s^2-3tu)+t^2u^2(s^2-tu)}{s^2t^2u^2}. \end{aligned} \quad (4.2)$$

These hard parts are then multiplied by the four accompanying soft wave function components and by the Sudakov factor, Eq. (3.10), and integrated over b and x:

$$\begin{aligned} A_0 &= \frac{\sqrt{2}Q}{2\pi|\sin\theta|} \\ &\quad \times \int_0^1 dx \int_{-1/\Lambda}^{+1/\Lambda} db\, a_0 U(x,b,Q) \left(\mathcal{P}_0^{(S)}(x,b)\right)^4, \\ A_2 &= \frac{\sqrt{2}Q}{2\pi\sin\theta} \int_0^1 dx \int_{-1/\Lambda}^{1/\Lambda} db\, a_2 U(x,b,Q) \\ &\quad \times \left(\mathcal{P}_0^{(S)}(x,b)\right)^2 \left(\mathcal{P}_1^{(S)}(x,b)\right)^2, \\ A_4 &= \frac{\sqrt{2}Q}{2\pi\sin\theta} \\ &\quad \times \int_0^1 dx \int_{-1/\Lambda}^{1/\Lambda} db\, a_4 U(x,b,Q) \left(\mathcal{P}_1^{(S)}(x,b)\right)^4. \end{aligned} \quad (4.3)$$

One then adds the three terms to obtain the amplitude $A(s,t) = A_0 + A_2 + A_4$ in Eq. (3.9). Let us note that the angular dependences are different for a_0, a_2, and a_4 and that within the approximations these dependences are not affected by the integrations. This remark is important for phenomenological analysis: the p-wave contributions should be more important in some kinematic domain.

We now limit ourselves to the study of the relative energy dependence. At 90°, the hard parts are

$$\begin{aligned} a_0(90^\circ) &= 19 \left(\frac{\pi}{6}\right)^4 \frac{64g^4}{x^2\bar{x}^2s^2}, \\ a_2(90^\circ) &= 120b^2 \left(\frac{\pi}{6}\right)^4 \frac{64g^4}{x^2\bar{x}^2s^2}, \\ a_4(90^\circ) &= 304b^4 \left(\frac{\pi}{6}\right)^4 \frac{64g^4}{x^2\bar{x}^2s^2}. \end{aligned} \quad (4.4)$$

Inserting these in Eq. (4.3), we denote the ratios of amplitudes

$$R_i(Q) = A_i(Q,90^\circ)/A_0(Q,90^\circ).$$

B. Model wave functions

For an asymptotic estimate assuming short distance, the soft wave function $\mathcal{P}_0^{(S)}$ may be approximated to its value at small b, approaching the distribution amplitude [6]: $\mathcal{P}_0^{(S)}(x,b) \approx \varphi(x, 1/|b|)$. In this limit one has models for the x dependence:

$$\varphi_{\rm as}(x) = 6 f_\pi x(1-x), \qquad (4.5)$$

$$\varphi_{\rm CZ}(x, \mu \sim 500 \text{ MeV}) = 5(2x-1)^2 \varphi_{\rm as}(x), \qquad (4.6)$$

which are standard choices for the pion. The asymptotic form Eq. (4.5) is derived in [1,7]; it has no evolution with μ and is indeed the limit as $\mu \to \infty$ of all distributions. Because evolution with μ is slow, the effective distribution at the nonasymptotic regime may be very different from $\varphi_{\rm as}$. Chernyak and Zhitnisky [11] have built from QCD sum rules the above CZ form Eq. (4.6). This evolves with the scale, but at a rate which is quite negligible. We will ignore these effects in the following.

The existence of the additional component $\mathcal{P}_1$ complicates the problem. Next to nothing is known about it, but a reasonable ansatz is to adopt a form similar to $\mathcal{P}_0$. One notices that $\mathcal{P}_0$ has a mass dimension set by f_π. In the case of $\mathcal{P}_1$, the dimension is a squared mass and the normalization constant is unknown. We will adjust the normalization in a way described below, and assume the same x dependence for $\mathcal{P}_0$ as $\mathcal{P}_1$.

C. Asymptotic behavior

To get the asymptotic behavior, we analytically evaluate the b integral

$$\int_0^{\Lambda^{-1}} db\, b^n U(b,x,Q)$$

with a saddle-point approximation, using the change of variable $u = -\ln b / \ln \sqrt{x\bar{x}}Q$. One has a maximum of the integrand at $u_0 = \frac{2c}{2c+n+1}$ and finds [9]

$$\int_0^{\Lambda^{-1}} db\, b^n U(b,x,Q) \approx u_0 \sqrt{\frac{\pi \ln Q}{c}} (x\bar{x}Q^2)^{c \ln u_0}. \quad (4.7)$$

Defining the x integrals

$$I(\varphi, n) = \int_0^1 dx \varphi(x)^4 \left[x(1-x)\right]^{-2+c\ln\frac{2c}{2c+n+1}},$$

an asymptotic expression is found for the ratio

$$R_n(Q) = \frac{2c+1}{2c+n+1}\left(\frac{Q}{\Lambda}\right)^{2c\ln\frac{2c+1}{2c+n+1}} \frac{I(\varphi,n)}{I(\varphi,0)}, \quad (4.8)$$

from which one deduces

$$R_2 \propto Q^{-1.10},\; R_4 \propto Q^{-1.85}.$$

The power suppression for each power of b^2 is intermediate between the case of no suppression (naive independent scattering) and the short-distance expectation $1/Q^2$. Note that the asymptotic power does not depend on the model for the x dependence.

D. Intermediate behavior

At accessible energies, we expect deviations from the result given in Eq. (4.8). We have numerically evaluated the amplitude Eq. (3.9) with U given by Eq. (3.10). Results for our computation of the ratio of amplitudes are displayed in Fig. 3 with CZ distribution (solid line). We get similar results for the asymptotic distribution amplitude (Fig. 4). To fix the normalization, we choose here and in the following to set arbitrarily the ratios to 1 at $\sqrt{s} = 2$ GeV. We observe that R_2 decreases by a factor of around 7 from $\sqrt{s} = 2$ to 20 GeV. This is a much milder suppression than the naively expected $1/Q^2$ factor (dotted curve). R_4 drops more drastically by a factor of around 20 in the same energy interval. A numerical study shows that logarithmic corrections ignored within the saddle-point approximation are not negligible in the

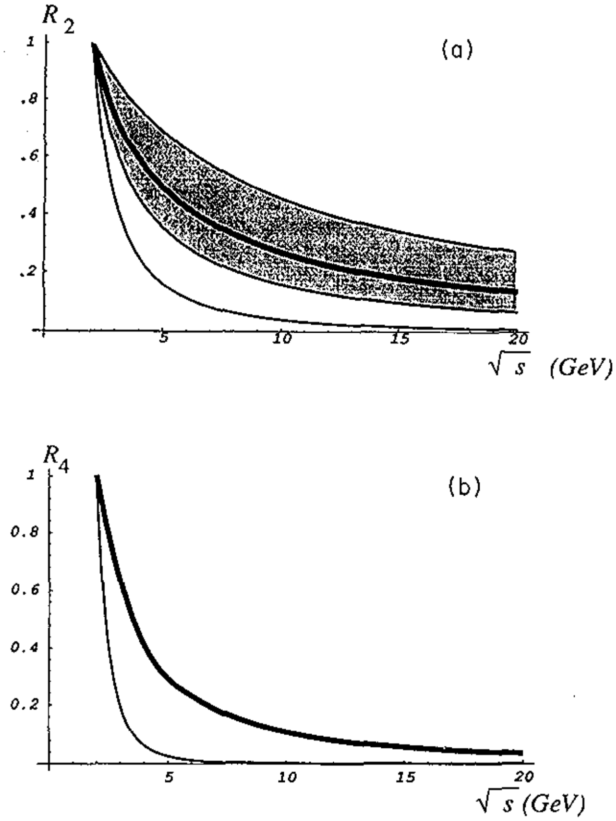

FIG. 3. The energy dependence of the R_2 (a) and of the R_4 (b) ratios with CZ distribution amplitude [11] (thick lines). Also shown is naive behavior postulated in short-distance models (thin lines), respectively $1/Q^2$ and $1/Q^4$. The shaded area in (a) indicates an estimated uncertainty from neglected subleading logarithms. The ratios are normalized to 1 at $\sqrt{s}$ =2 GeV.

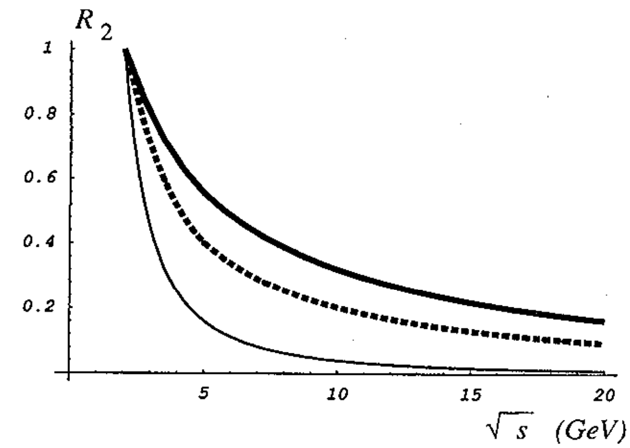

FIG. 4. The energy dependence of the R_2 ratio calculated using the asymptotic distribution amplitude with (solid thick line) and without (dashed line) intrinsic b dependence. The thin line is as in Fig. 3.

accessible range of energies. At larger values of the energy, $\sqrt{s} > 20$ GeV, the approximated result of Eq. (4.8) becomes accurate.

Secondly, Eq. (3.10) should be supplemented by nonleading terms which are known in the s-wave case [6] but presumably differ in the p-wave case. The neglected logarithms in the expression of U [Eq. (3.10)] may modify somewhat the ratio over an intermediate range of energy. To model such an effect, we add a simple x, b-independent factor

$$\exp\left(K \ln\ln \frac{Q}{\Lambda}\right)$$

in the expression of R_2 with K some constant. The ratio modified by such a factor is shown in Fig. 3(a) as a shaded area limited by the curves corresponding to $K = 1$ and $K = -1$; this measures in some way the theoretical uncertainty on the p-wave contribution. Further theoretical progress in the computation of these subleading terms might be possible.

A third effect may come from the intrinsic transverse dependence of the wave function. While the replacement of $\mathcal{P}^{(0)}(x, b; 1/b)$ by $\varphi(x; 1/b)$ discussed in Sec. IV B is reasonable at large Q^2 (> 100 GeV^2), it is more questionable at intermediate values. In that case, long-range physics may be accounted for by including some intrinsic b dependence [10] as

$$\mathcal{P}_0(x, b) = 4\pi\mathcal{N}\varphi(x) \exp\left(-\frac{\alpha^2}{x\bar{x}} - \frac{x\bar{x}b^2}{4\beta^2}\right). \quad (4.9)$$

Following [10] we use parameters $\alpha^2 = 0.096$, $\beta^2 = 0.88\,\text{GeV}^2$, and $\mathcal{N} = 1.68$ with the asymptotic distribution amplitude. The results are depicted in Fig. 4, where the curves from this wave function and from the asymptotic distribution amplitude are shown for comparison. We remark that the phenomenology may also be modified considerably by studying further model variations.

These three effects show that the powerlike decrement Eq. (4.8) of the ratio is diluted at intermediate energies and consequently the amplitudes A_0 and A_2 are likely to compete over a rather large interval of s, say 1 GeV^2 < s <100 GeV^2.

V. HELICITY-VIOLATING PROCESSES IN $\pi\pi \to \rho\rho$

A. ρ wave function

To begin, let us find the possible tensorial decomposition for the quark-antiquark wave functions of the ρ meson. A one-particle state is specified by the momentum p_i ($i = 3, 4$), written $p_i = Qv_i + \frac{m^2}{2Q}v_i'$ (m is the ρ mass which we do not neglect for the moment), and by the helicity $h_i \in \{1, 0, -1\}$. The three-helicity states are described in a covariant way with the help of the three-vectors $\varepsilon_h^\mu(p_i)$, satisfying $\varepsilon_h \cdot p_i = 0$ and $\varepsilon_h \cdot \varepsilon_{h'}^* = -\delta_{hh'}$. For $\rho(p_3)$, the set $(\vec{p}_3, \vec{\xi}_3, \vec{\eta})$ forms a right-handed orthogonal basis and we choose polarization vectors

$$
\begin{aligned}
&(h = +1) \qquad && \varepsilon_R = -\frac{1}{\sqrt{2}}(\xi_3 + i\eta), \\
&(h = \ \ 0) \qquad && \varepsilon_0 = \frac{Q}{m}v_3 - \frac{m}{2Q}v_3', \qquad (5.1) \\
&(h = -1) \qquad && \varepsilon_L = \frac{1}{\sqrt{2}}(\xi_3 - i\eta).
\end{aligned}
$$

A consistent choice for $\rho(p_4)$ is

$$
\begin{aligned}
&(h = +1) \qquad && \varepsilon_R = \frac{1}{\sqrt{2}}(\xi_3 - i\eta), \\
&(h = \ \ 0) \qquad && \varepsilon_0 = \frac{Q}{m}v_4 - \frac{m}{2Q}v_4', \qquad (5.2) \\
&(h = -1) \qquad && \varepsilon_L = -\frac{1}{\sqrt{2}}(\xi_3 + i\eta).
\end{aligned}
$$

Thus, demanding a parity "$-$" state, the Bethe-Salpeter amplitude has the most general Dirac-matrix expansion

$$
\begin{aligned}
\mathcal{P}(x, \mathbf{b}_T; p, h) = \frac{1}{4}\Big(&\mathcal{P}_0\,[\not\varepsilon_h, \not p] + \mathcal{P}_0'\not\varepsilon_h + \mathcal{P}_1\,\varepsilon_h\cdot\mathbf{b}_T\not p \\
&+\mathcal{P}_1'\,\varepsilon_h\cdot\mathbf{b}_T + \tilde{\mathcal{P}}_1\,[\not\varepsilon_h, \not p]\not\mathbf{b}_T \\
&+\tilde{\mathcal{P}}_1'\,[\not\varepsilon_h, \not\mathbf{b}_T] + \mathcal{P}_2\,\varepsilon_h\cdot\mathbf{b}_T\,[\not p, \not\mathbf{b}_T] \\
&+\mathcal{P}_2'\,\varepsilon_h\cdot\mathbf{b}_T\not\mathbf{b}_T\Big). \qquad (5.3)
\end{aligned}
$$

One can then extract the relevant components for the study of independent scattering processes. That is, we isolate leading high energy tensors, which contain one power of the large scale Q. [We have already set the relative "x^+" coordinates to zero in Eq. (5.3), but this does not affect the counting of terms.] We get, for a longitudinally polarized ρ,

$$\mathcal{P}(x,\mathbf{b}_T;p,h=0)=\frac{Q}{4m}\left(\mathcal{P}_0'\not p+\tilde{\mathcal{P}}_1'[\not p,\not b_T]\right), \quad (5.4)$$

and, for a transversely polarized one with $\varepsilon_h=\varepsilon_R$ or ε_L,

$$\mathcal{P}(x,\mathbf{b}_T;p,|h|=1)$$

$$=\frac{Q}{4}\Big(\mathcal{P}_0\,[\not\varepsilon_h,\not p]+\mathcal{P}_1\,\varepsilon_h\cdot\mathbf{b}_T\not p+\tilde{\mathcal{P}}_1\,[\not\varepsilon_h,\not p]\not b_T$$
$$+\mathcal{P}_2\,\varepsilon_h\cdot\mathbf{b}_T\,[\not p,\not b_T]\Big). \quad (5.5)$$

B. The double-flip rule

Before computing a helicity-violating process, let us exhibit what we call the *double-flip* rule. The rule summarizes the consequences of angular momentum and chiral conservation in perturbation theory. Let each term in a meson wave function be classified by its chirality. The chirality will be "+" if the term anticommutes with γ_5, and "−" if it commutes. In the short-distance limit, a chirally "+" state has quark-antiquark spins canceling [as in the s-wave contribution to the pion, i.e., the $\gamma_5\not p$ term in Eq. (2.3)]; a chirally "−" state has the spins aligned. There is a selection rule due to the hard scattering (which acts like an overall factor of 1) conserving chirality: for each connected quark loop, the product of all the chiralities of the wave functions must be "+." Otherwise the Dirac traces will vanish due to chiral symmetry.

Consider next a nonzero amplitude for a meson scattering process calculated using the s-wave wave functions. Let us compare this to the same process computed with non-s-wave terms. Adding a unit of orbital angular momentum is done with a factor of $\not b_T$ in the wave function. Keeping the meson's overall angular momentum fixed, this is compensated by flipping the sign of the chirality. To keep the product of chiralities around a loop "+" using such a wave function, there must be another such term elsewhere on the same loop. This is the double-flip rule: terms with $\mathbf{b}_T$ in the wave function occur in pairs on each connected quark loop. The rule can be checked in our earlier example of $\pi\pi$ scattering.

The power suppression of short-distance amplitudes with extra transverse gluons is affected by the same considerations. By power counting, each extra gluon embedded in a hard scattering creates a suppression by a power of $1/Q$. However, the chirality of the wave function attached is flipped by one "−" for angular momentum conservation. There is no point in adding one transverse gluon, so gluons (like $\mathbf{b}_T$) must be inserted in pairs. When orbital angular momentum and gluon components are compared, orbital angular momentum dominates: from Sec. IV, we found an asymptotic suppression of $Q^{-0.55}$ for one "p wave" compared to Q^{-1} for a gluon. At finite energies the dominance of orbital angular momentum over extra Fock components is even greater, because the p-wave contribution is even less suppressed.

Here is a further consequence of the rule. Consider that the s-wave function for a meson state (spin 0 or spin 1) with net helicity zero is chirally "−," while the s-wave part of a helicity 1 meson wave function must be chirally "+." Within the s-wave calculation, one concludes that a process must have an even number of zero meson helicity legs, or it will vanish. Adding orbital angular momentum adds an even number of $\mathbf{b}_T$'s on the same internal loop. This does not change the counting; the previous conclusion remains generally true. For a two-to two-meson scattering process, we find that only those processes which violate hadron helicity conservation by two units are allowed. For example, $\pi\pi\to\rho_0\rho_T$ vanishes. $\pi\pi\to\rho_R\rho_R$ is allowed, and will be studied next.

C. $\pi\pi\to\rho_R\rho_R$

It is easy to verify that the $\pi\pi\to\rho_R\rho_R$ vanishes when calculated using the s-wave components of the external mesons (see Appendix B). By the double-flip rule the first nonzero term is a b^2 amplitude $M_2(x,b)$. Its computation, within the approximation of Sec. IV A, leads to

$$M_2\left(\pi\pi\to\rho_R\rho_R\right)=\left(\frac{\pi}{6}\right)^4\frac{128g^4}{x^2\bar{x}^2t^2u^2}b^2U(x,b,Q)\bigg\{\frac{16(3s^2-7tu)}{3}\mathcal{P}_{1\pi}^2\mathcal{P}_{0\rho}^2-\frac{t^3u^3}{s^4}\mathcal{P}_{0\pi}^2\mathcal{P}_{1\rho}^2$$
$$+8\frac{t^3u^3}{s^4}\mathcal{P}_{0\pi}^2\mathcal{P}_{0\rho}\mathcal{P}_{2\rho}-16(s^2-3tu)\mathcal{P}_{0\pi}\mathcal{P}_{1\pi}\mathcal{P}_{0\rho}\tilde{\mathcal{P}}_{1\rho}$$
$$+4\frac{t^3u^3}{s^4}\mathcal{P}_{0\pi}^2\mathcal{P}_{1\rho}\tilde{\mathcal{P}}_{1\rho}+4\left(s^2-3tu+\frac{t^2u^2}{s^2}-2\frac{t^3u^3}{s^4}\right)\mathcal{P}_{0\pi}^2\tilde{\mathcal{P}}_{1\rho}^2\bigg\}. \quad (5.6)$$

This amplitude has to be integrated over b and x. Although it involves several unknown objects, one notices that the angular dependence varies from one component to another and is rather different from the one obtained in $\pi\pi$ elastic scattering. It may be possible to analyze the contribution to helicity violation processes from different wave functions and use this information to deduce properties of the wave functions.

The numerical study of Sec. IV can be used to understand the energy dependence of double-helicity-violating processes at accessible energies. As explicitly shown in Figs. 3(a) and 4, the naive $1/Q^2$ factor is replaced by a milder suppression. This is primarily due to the specificity of the independent scattering mechanism supplemented by Sudakov effects. Even at very large energies the $Q^{-1.10}$ ratio of Eq. (4.8) looks to be quite a weak suppression.

VI. REALISTIC PROCESSES AND EXPERIMENTAL OUTLOOK

Studying meson-meson scattering is an interesting but unrealistic simplification. Including baryons is a necessary but quite intricate further step. A high number of Feynman diagrams and internal degrees of freedom to be integrated over then occur. We can, however, still draw some conclusions from our analysis, leaving baryons to future work. The mechanism we have explored occurs in several experimentally accessible circumstances. Indeed, there is a host of reactions involving hadronic helicity violation from which we could learn about the interface of perturbative and nonperturbative QCD.

The helicity density matrix of the ρ meson produced in $\pi p \to \rho p$ at 90° is a nice measure of helicity-violating components. Experimental data [12] yield $\rho_{1-1} = 0.32 \pm 0.10$, at $s = 20.8$ GeV^2, $\theta_{\rm c.m.} = 90°$, for the nondiagonal helicity-violating matrix element. Without entering a detailed phenomenological analysis, we may use the results of Sec. V through the following line of reasoning. Assume that the presence of the third valence quark, which is not subject to a third independent scattering, does not much alter the results. Then one may view ρ_{1-1} as coming from the interference of a helicity-conserving amplitude like $\pi\pi \to \rho_L\rho_R$ with a double-helicity-flip amplitude like $\pi\pi \to \rho_R\rho_R$. We then predict a mild energy dependence of this matrix element, i.e., $Q^{-1.10}$ [Eq. (4.8)] at asymptotic energies, or behavior as shown in Fig. 3(a) and Fig. 4 at accessible energies. This is at variance with the picture emerging from the diquark model [13]. Measuring the energy dependence of this effect would be highly interesting.

The most well-known example of hadron helicity violation occurs in $pp \to pp$ scattering [14]. Our demonstration of helicity-violating contributions to meson-meson scattering has a bearing on this, because generalized meson scattering is embedded in the diagrams for proton scattering. Without needing to make any dynamical assumption of "diquarks," the perturbative QCD diagrams for $pp \to pp$ scattering contain numerous diquark regions, convolved with scattering of an extra quark. There is no known selection rule which would prevent the scattering of such subprocesses from causing helicity flip in pp elastic scattering. This does not exhaust the possibilities, because there are other channels of momentum flow and color combinatorics which might have different interpretations. The data for $pp \to pp$ also reveal large oscillations about power-law behavior, a second piece of evidence that the short-distance picture is inadequate. Elsewhere [15] we have identified these oscillations as a sign of independent scattering. Given the theoretical [16] and experimental evidence, we therefore find no evidence that hadronic helicity conservation is a general feature of perturbative QCD, and we believe that independent scattering is a main contender in explaining the observations.

Since reactions of baryons are extremely complicated, and next to nothing is known about the various wave functions in the proton, a productive approach to the question is to ask for experimental circumstances in which the general mechanism we have outlined could be tested without requiring too much detail. We believe that progress here will come from using nuclear targets, and studying the phenomenon of color transparency in hard (as opposed to diffractive) reactions. This program has been outlined elsewhere [17]; it suffices to mention here that suppression of large-b^2 regions is expected in reactions involving large nuclei. It follows that helicity conservation should be obtained in the same circumstances. Thus the mechanism we have outlined is testable. We believe that a multitude of phenomena involving spin, color transparency, and detailed hadron structure will play a major role in the future.

ACKNOWLEDGMENTS

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APPENDIX A: COMPUTATION OF HARD AMPLITUDES

In this Appendix, we explain how to carry out the computation of hard amplitudes for the process $\pi\pi \to \pi\pi$.

Following Eq. (3.1) or (3.9), one forms the projection, denoted as t and t', of the hard amplitudes H and H' on the Dirac tensors $(\frac{1}{4}\gamma_5 \not{p}_i|_{\alpha_i\beta_i}$ and $\frac{1}{4}\gamma_5[\not{p}_i, \not{b}]_{\alpha_i\beta_i})$ coming from each wave function. We follow the Botts-Sterman classification of graphs, with three fermionic flows for H and two gluonic channels each (see Fig. 5):

f	$H(M_1M_2 \to M_3M_4)$	gluon	$c^f_{1,\{a_i\}}$	$c^f_{2,\{a_i\}}$
1	$1\bar{2} \to \bar{3}4$	u, s	$\delta_{a_1a_2}\delta_{a_3a_4}$	$\delta_{a_1a_4}\delta_{a_2a_3}$
2	$1\bar{2} \to 3\bar{4}$	t, s	$\delta_{a_1a_2}\delta_{a_3a_4}$	$\delta_{a_1a_3}\delta_{a_2a_4}$
3	$12 \to 34$	t, u	$\delta_{a_1a_4}\delta_{a_2a_3}$	$\delta_{a_1a_3}\delta_{a_2a_4}$

Color flow in this problem is simplified by noting that one-gluon exchange between two quark lines gives a color tensor

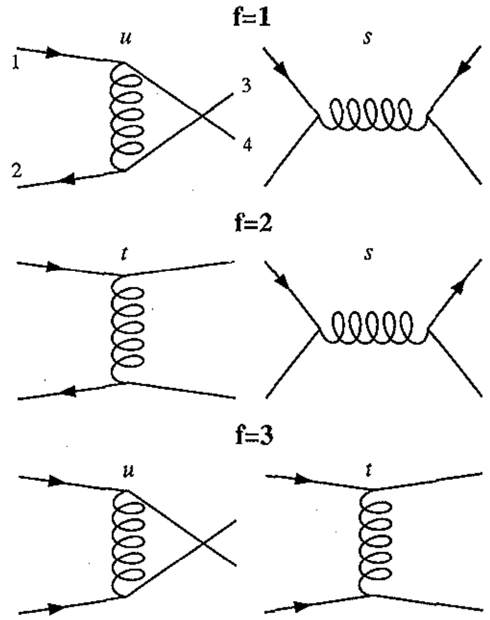

FIG. 5. Feynman graphs for the lowest order hard amplitude H; for H' reverse the arrows.

$$\sum_c T^c_{ij}T^c_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{kj} - \frac{1}{3}\delta_{ij}\delta_{kl}\right),$$

which we may reexpress with the color tensors listed above, in the form

$$[t \,\text{or}\, u1]_{\{a_i\}} = \sum_{I=1}^{2} C_I c^f_{I\{a_i\}}, \quad [s \,\text{or}\, u3]_{\{a_i\}} = \sum_{I=1}^{2} \tilde{C}_I c^f_{I\{a_i\}},$$

with

$$C_1 = \tilde{C}_2 = \frac{1}{2}, \; C_2 = \tilde{C}_1 = -\frac{1}{6}.$$

With this notation, one finds for the hard amplitude containing no b factor (as indicated by the subscript 0),

$$\begin{aligned}
(t_I t'_J)^{(1)}_0 &= C_I C_J \frac{s^2+t^2}{u^2} + (\tilde{C}_I C_J + C_I \tilde{C}_J)\frac{t^2}{su} \\
&\quad + \tilde{C}_I \tilde{C}_J \frac{t^2+u^2}{s^2}, \\
(t_I t'_J)^{(2)}_0 &= C_I C_J \frac{s^2+u^2}{t^2} + (\tilde{C}_I C_J + C_I \tilde{C}_J)\frac{u^2}{st} \\
&\quad + \tilde{C}_I \tilde{C}_J \frac{t^2+u^2}{s^2}, \qquad \text{(A1)} \\
(t_I t'_J)^{(3)}_0 &= C_I C_J \frac{s^2+u^2}{t^2} + (\tilde{C}_I C_J + C_I \tilde{C}_J)\frac{s^2}{tu} \\
&\quad + \tilde{C}_I \tilde{C}_J \frac{s^2+t^2}{u^2},
\end{aligned}$$

times an overall common factor

$$\left(\frac{\pi}{2}\right)^4 \frac{32g^4}{x^2\bar{x}^2s^2},$$

where g is the QCD coupling constant which appears in Feynman rules. We have already indicated that the whole amplitude Eq. (3.9) can be properly defined regarding renormalization and factorization, so that g^4 stands for $(4\pi)^2\alpha_S(xQ)\alpha_S(\bar{x}Q)$.

There is no term with an odd power of b, due to the corresponding odd number of γ matrices; this is a consequence of chiral symmetry. The second term consists therefore in three hard amplitudes containing b^2 (as indicated by the subscript 2), which are found to be

$$\begin{aligned}
(t_I t'_J)^{(1)}_2 &= C_I C_J \frac{2st}{u^2} + (\tilde{C}_I C_J + C_I \tilde{C}_J)\frac{su-t^2}{su} \\
&\quad + \tilde{C}_I \tilde{C}_J \frac{2tu}{s^2}, \\
(t_I t'_J)^{(2)}_2 &= C_I C_J \frac{2su}{t^2} + (\tilde{C}_I C_J + C_I \tilde{C}_J)\frac{st-u^2}{st} \\
&\quad + \tilde{C}_I \tilde{C}_J \frac{2tu}{s^2}, \qquad \text{(A2)} \\
(t_I t'_J)^{(3)}_2 &= C_I C_J \frac{2su}{t^2} + (\tilde{C}_I C_J + C_I \tilde{C}_J)\frac{tu-s^2}{tu} \\
&\quad + \tilde{C}_I \tilde{C}_J \frac{2st}{u^2},
\end{aligned}$$

with a common factor

$$-\left(\frac{\pi}{2}\right)^4 \frac{256g^4}{x^2\bar{x}^2s^2}b^2.$$

The third hard amplitude is the same combination as Eq. (A1), but with an overall factor

$$\left(\frac{\pi}{2}\right)^4 \frac{512g^4}{x^2\bar{x}^2s^2}b^4.$$

A check of the above expressions (and a trick to reduce the number of graphs one has to compute) is provided by symmetries under meson exchange. Starting from the expression one gets with two-u-gluon exchange and fermionic flow $f = 1$ which we label $uu1$ (Fig. 5), one can generate

Exchange	channel	kinematic	color
$2 \leftrightarrow 4$	$ss1$	$u \leftrightarrow s$	$C \leftrightarrow \tilde{C}$
$3 \leftrightarrow 4$	$tt2$	$u \leftrightarrow t$	$C \leftrightarrow C$
$2 \leftrightarrow 3$	$uu3$	$s \leftrightarrow t$	$C \leftrightarrow \tilde{C}$.

The reader will easily find the channels obtained from another starting point, say $us1$, and the combination of exchanges needed to determine graphs which do not appear in the above array, thus completing the whole amplitude.

In the leading-logarithm approximation, U in Eq. (3.10) is a scalar in color space and one easily performs the color traces. The color matrix coming from the ith wave function is $\frac{1}{3}\delta_{a_ib_i}$ and one has

$$c_{I\{a\}}c_{J\{b\}} \prod_{i=1}^{4} \frac{\delta_{a_ib_i}}{3} = \left(\frac{1}{3}\right)^4 \begin{pmatrix} 9 & 3 \\ 3 & 9 \end{pmatrix}.$$

With

$$C_I C_J = \frac{1}{36}\begin{pmatrix} 9 & -3 \\ -3 & 1 \end{pmatrix}, \; C_I \tilde{C}_J = \frac{1}{36}\begin{pmatrix} -3 & 9 \\ 1 & -3 \end{pmatrix},$$

$$\tilde{C}_I \tilde{C}_J = \frac{1}{36}\begin{pmatrix} 1 & -3 \\ -3 & 9 \end{pmatrix},$$

one then obtains the following hard amplitudes, labeled by the power of b entering:

$$\begin{aligned} a_0 &= \left(\frac{\pi}{6}\right)^4 \frac{256 g^4}{x^2 \bar{x}^2 s^2} \frac{s^4(s^2-3tu)+t^2u^2(s^2-tu)}{s^2t^2u^2}, \\ a_2 &= b^2 \left(\frac{\pi}{6}\right)^4 \frac{2048 g^4}{x^2 \bar{x}^2 s^2} \frac{s^4(s^2-3tu)-t^3u^3}{s^2t^2u^2}, \\ a_4 &= b^4 \left(\frac{\pi}{6}\right)^4 \frac{2048 g^4}{x^2 \bar{x}^2 s^2} \frac{s^4(s^2-3tu)+t^2u^2(s^2-tu)}{s^2t^2u^2}. \end{aligned} \tag{A3}$$

At next-to-leading-logarithm order, the corrections are color matrices [15], and $i\pi \ln(Q^2)$ terms can produce oscillating amplitudes as Q^2 is varied. Such terms may be phenomenologically important but are not studied here.

APPENDIX B: VANISHING OF THE HELICITY-VIOLATING AMPLITUDE WITH s-WAVE WAVE FUNCTIONS

Let us verify explicitly the vanishing of the hard amplitude using the s-wave components of the external mesons. For this purpose, it is instructive to examine the connection between quark helicities and the s-wave Dirac tensors we have used until now. This is accomplished in the following way. One considers the free massless spinors of a quark and an antiquark moving in the same direction, the quark having a momentum xp and the antiquark a momentum $\bar{x}p$, so that the compound system has a momentum p. Then one constructs the four possible helicity states of the system with solutions of the Dirac equation and finds

$$\begin{aligned} &(\pi) && \frac{1}{\sqrt{2}}\left(u_\alpha(xp,\uparrow)\bar{v}_\beta(\bar{x}p,\downarrow) - u_\alpha(xp,\downarrow)\bar{v}_\beta(\bar{x}p,\uparrow)\right) = -\sqrt{\frac{x\bar{x}}{2}}\,\gamma_5 \not{p}|_{\alpha\beta} \\ &(\rho_0) && \frac{1}{\sqrt{2}}\left(u_\alpha(xp,\uparrow)\bar{v}_\beta(\bar{x}p,\downarrow) + u_\alpha(xp,\downarrow)\bar{v}_\beta(\bar{x}p,\uparrow)\right) = -\sqrt{\frac{x\bar{x}}{2}}\,\not{p}|_{\alpha\beta} \\ &(\rho_R) && u_\alpha(xp,\uparrow)\bar{v}_\beta(\bar{x}p,\uparrow) = \sqrt{\frac{x\bar{x}}{2}}\,\not{\varepsilon}_R \not{p}|_{\alpha\beta} \\ &(\rho_L) && u_\alpha(xp,\downarrow)\bar{v}_\beta(\bar{x}p,\downarrow) = \sqrt{\frac{x\bar{x}}{2}}\,\not{\varepsilon}_L \not{p}|_{\alpha\beta}, \end{aligned} \tag{B1}$$

The helicity conservation rule is then easily verified with these combinations of spinors when one chooses the chiral representation [18]. In this representation, the two diagonal blocks of each γ^μ are equal to the null 2×2 matrix. A Feynman-graph fermion line, with vector (or axial) couplings and massless propagator, is an even number of γ matrices between two spinors:

$$\psi'^\dagger(p',h')\gamma_0\gamma^{\mu_1}\not{k}_1\cdots\not{k}_n\gamma^{\mu_{n+1}}\psi(p,h).$$

Inclusions of γ_5, which is diagonal in this representation, do not modify this property. Then, since to order m/p the chirality of a spinor corresponds to the helicity of the state and, in this representation, chiral eigenstate spinors have either their two first components or their two last equal to 0, states of different helicity always give a null product.

This property is algebraic and therefore independent of the representation chosen. However, it is more difficult to observe its effects in the trace formalism. Let us examine how it works in the case of the reaction considered. Among the twelve graphs discussed in Appendix A, each tt and uu graph is 0 because they contain traces over the product of an odd number of γ^μ matrices. For the eight remaining graphs, the sequence

$$\not{\varepsilon}_{3R}\not{p}_3\{\gamma^{\mu_1}\cdots\gamma^{\mu_{2n+1}}\}\not{\varepsilon}_{4R}\not{p}_4\cdots$$

occurs (the anticommutation to the left of every γ_5 is understood and does not modify the reasoning). The product of an odd number of γ^μ being a linear combination of γ^μ and $\gamma_5\gamma^\mu$, one is left with the evaluation of

$$\not{\varepsilon}_R\not{p}\gamma^\mu\not{\varepsilon}_L\not{p}'\cdots \quad \text{and} \; \not{\varepsilon}_R\not{p}\gamma_5\gamma^\mu\not{\varepsilon}_L\not{p}'\cdots,$$

where $v_4 = v_3'$ and $\varepsilon_{4R} = \varepsilon_{3L}$ have been used and the index 3 dropped. One can decompose each γ^μ onto ($\not{p}$, $\not{p}'$, $\not{\varepsilon}_R$, $\not{\varepsilon}_L$), which are such that their square is 0 and their anticommutation rules are

$$\{\not{p},\not{\varepsilon}_R\} = \{\not{p},\not{\varepsilon}_L\}\{\not{p}',\not{\varepsilon}_R\} = \{\not{p}',\not{\varepsilon}_L\} = 0,$$

to conclude that all graphs effectively vanish.

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