Quark fragmentation into ^3P_J quarkonium

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We calculate the functions of parton fragmentation into ^3P_J quarkonium at order α_s^2, where the parton can be a heavy or light quark. The obtained functions explicitly satisfy the Altarelli-Parisi equation and they are divergent, behaving as z^{-1} near $z = 0$. However, if one chooses the renormalization scale as twice the heavy quark mass, the fragmentation functions are regular over the whole range of z.

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I. INTRODUCTION

Fragmentation functions are in general nonperturbative objects in the context of the factorization theorem in QCD [1]. This makes it hard to study them by starting directly from QCD. However, if partons, i.e., quarks and glouns, undergo fragmentation into a quarkonium, fragmentation functions can be factorized—they are sums of products of constants and coefficient functions. The constants represent the nonperturbative effects and may be defined as matrix elements in nonrelativistic QCD (NRQCD) or are related to a wave function, while the coefficients functions can be calculated with perturbative theory. With this factorization, fragmentation functions for various quarkonia were studied in [2–9].

A quarkonium contains a heavy quark Q and its antiquark $\bar{Q}$, which move with a small velocity v in the quarkonium rest frame. Because of the small velocity, the bound-state effect, i.e., the nonperturbative effect in the quarkonium, can be well described by employing NRQCD. Recently, such an approach has been established [10]. The approach is basically distinct from earlier treatments within the color-singlet model (for a nice review see [11]). In the color-singlet model one treats a quarkonium system simply as a bound state of Q and $\bar{Q}$, where the $Q\bar{Q}$ pair is in a color-singlet state, and the nonperturbative effect is contained in the wave function of the bound state. Then an expansion in the small parameter v is made and to leading order only the wave function at the origin or its derivative at the origin is involved. This model has serious problems. First, it does not tell us how to handle the Coulomb singularities. Since an expansion in v is made, this type of singularity must appear because of massless photons and massless gluons. The effect related to the Coulomb singularities is nonperturbative. In the color-singlet model these singularities were absorbed into the wave function without solid reasons from theories. Second, infrared (IR) singularities appear when a P-wave quarkonium is involved and appear even at the leading order of α_s. In the model the IR singularities were regularized as divergences in the limit of the zero binding energy. Such IR singularities clearly indicate that the wave function of a P-wave quarkonium can not contain all the nonperturbative effects. In the approach of [10], quarkonium systems are analyzed in the framework of QCD. A systematic expansion in v can be made and the nonperturbative effect is represented by matrix elements in NRQCD. With these matrix elements one can show that Coulomb singularities are factorized into these matrix elements. From the point of view of a relativistic quantum theory a quarkonium system consists not only of a $Q\bar{Q}$ color singlet but also many other components such as $|Q\bar{Q}G\rangle$, etc. In the approach of [10] the effect of these other components is also included in the systematic expansion in v. It should be emphasized [10,12] that a $Q\bar{Q}$ color-octet state is as important as a $Q\bar{Q}$ color-singlet state for a P-wave quarkonium. Therefore one should take both states into account. It was already shown that results at the one-loop level for gluon fragmentation into ^3P_J quarkonium [9] are free from IR singularities and from Coloumb singularities.

In this paper we will study the quark fragmentation into a ^3P_J $Q\bar{Q}$ quarkonium at leading order α_s. We will show that because of the contribution of the color-octet $Q\bar{Q}$ pair, a light quark q can also fragment into the quarkonium at the same order of α_s as a heavy quark Q. The heavy quark fragmentation into P-wave quarkonium was originally studied in [8]. As pointed out in [9] the results for charmonium cannot be correct, because the fragmentation functions obtained there do not satisfy the Altarelli-Parisi equation.

The paper is organized as follows. In Sec. II we introduce the definition [13] of renormalized quark fragmentation functions and the factorization [10] forms for quark fragmentation into ^3P_J quarkonium. Further details may be found in [10,13]. In Sec. III we start from the definition to calculate the heavy quark fragmentation functions. In Sec. IV we calculate the light quark fragmentation function. Finally we discuss and summarize the results of our work in Sec. V. Throughout this paper we always assume that the polarization of the quarkonium is not observed. We will use dimensional regularization and work at leading order in the expansion of v.

II. DEFINITION OF THE QUARK FRAGMENTATION FUNCTION AND THE FACTORIZATION FORM FOR ^3P_J QUARKONIUM

As we will use dimensional regularization we give the definition of the quark fragmentation function in d dimensions. To give the definitions for a fragmentation function it is convenient to work in the light-cone coordinate system. In this coordinate system a d vector p is expressed as $p^\mu = (p^+, p^-, \mathbf{p}_T)$, with $p^+ = (p^0 + p^{d-1})/\sqrt{2}, p^- = (p^0 - p^{d-1})/\sqrt{2}$. Introducing a vector n with $n^\mu = (0, 1, 0, \dots, 0) = (0, 1, \mathbf{0}_T)$, the fragmentation function for a spinless hadron H or for a hadron without observing its polarization is defined as [13]

$$\begin{aligned} D^{(0)}_{H/q}(z) = \frac{z^{d-3}}{4\pi} \int dx^- e^{-iP^+x^-/z} \frac{1}{3}\mathrm{Tr}_{\mathrm{color}} \frac{1}{2}\mathrm{Tr}_{\mathrm{Dirac}}\{n\cdot\gamma\langle 0|q(0) \\ \times \bar{P}\exp\left\{-ig_s \int_0^{\infty d} \lambda n \cdot G^T(\lambda n^\mu)\right\} a^\dagger_H(P^+, \mathbf{0}_T) a_H(P^+, \mathbf{0}_T) \\ \times P\exp\left\{ig_s \int_{x^-}^{\infty} d\lambda n \cdot G^T(\lambda n^\mu)\right\} \bar{q}(0, x^-, \mathbf{0}_T)|0\rangle, \end{aligned} \tag{2.1}$$

where $G_\mu(x) = G^a_\mu(x)T^a$, $G^a_\mu(x)$ is the gluon field and T^a $(a = 1, \dots, 8)$ are the SU(3)-color matrices. The subscript T denotes the transpose. $a^\dagger_H(\mathbf{P})$ is the creation operator for the hadron H. For hadrons with nonzero spin the summation over the spin is understood. The definition is a unrenormalized version. Ultraviolet divergences will appear in $D^{(0)}_{H/q}(z)$ and call for renormalization. Following [13] the renormalized quark fragmentation function can be defined as

$$D_{H/Q}(z) = D^{(0)}_{H/Q}(z) + \sum_{a=G,q} \int_z^1 \frac{dy}{y} L_a\left(\frac{y}{z}\right) D^{(0)}_{H/a}(y). \tag{2.2}$$

Here the summation over all possible partons is understood. The function $L_a(z)$ is chosen so as to cancel the UV divergences. In the minimal subtraction (MS) scheme $L_a(z)$ takes the form

$$L_a(z) = \sum_N \frac{1}{\epsilon^N} L^{(N)}_a(z), \tag{2.3}$$

where $\epsilon = 4 - d$. From Eq. (2.2) one can derive the Altarelli-Parisi-type evolution equation for the fragmentation function. We will use the modified MS scheme, where $L_a(z)$ is chosen to cancel the terms with $N_\epsilon = \frac{2}{\epsilon} - \gamma + \ln(4\pi)$. The function $D_{H/q}(z)$ is interpreted as the probability of a quark q with momentum k to decay into the hadron H with momentum component $P^+ = zk^+$; it is gauge invariant by definition. Further, it is also invariant under a Lorentz boost along the moving direction of the hadron and under a rotation with the direction as the rotate axis.

If the hadron is a ^3P_J quarkonium, a factorized form for the fragmentation function can be taken. We will use the notation χ_J for the ^3P_J quarkonium. At the leading order of v, $D_{H/q}(z)$ can be written according to [10] as

$$D_{\chi J/q}(z) = \frac{\hat{D}_1(z, J)}{M^5}\langle 0|O^{\chi J}_1(^3P_J)|0\rangle + \frac{\hat{D}_8(z)}{M^3}\langle 0|O^{\chi J}_8(^3S_1)|0\rangle, \tag{2.4}$$

where $D_1(z, J)$ and $\hat{D}_8(z)$ are dimensionless and $\hat{D}_8(z)$ is same for all J. The operators $O^{\chi J}_1(^3P_J)$ and $O^{\chi J}_8(^3P_1)$ are given by

$$\begin{aligned} O^H_8(^3S_1) &= \chi^\dagger\sigma_i T^a\psi(a^\dagger_H a_H)\psi^\dagger\sigma_i T^a\chi, \\ 0^H_1(^3P_0) &= \frac{1}{3}\chi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\sigma\right)\psi(a^\dagger_H a_H)\psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\sigma\right)\chi, \\ O^H_1(^3P_1) &= \frac{1}{2}\chi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\times\sigma\right)_i\psi(a^\dagger_H a_H)\psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\times\sigma\right)_i\chi, \\ O^H_1(^3P_2) &= \chi^\dagger\left(-\frac{i}{2}\overleftrightarrow{D}_{\{i}\sigma_{j\}}\right)\psi(a^\dagger_H a_H)\psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{D}_{\{i}\sigma_{j\}}\right)\chi, \end{aligned} \tag{2.5}$$

where $\mathbf{D}$ is the space part of the covariant derivative D^μ and σ_i $(i = 1, 2, 3)$ is the Pauli matrix. The notation $\{ij\}$ means only the symmetric and traceless part of a tensor is taken. In Eq. (2.5), ψ and $\chi^\dagger$ are fields with two components for the heavy quark Q and its antiquark $\bar{Q}$ in NRQCD. M is the mass of the heavy quark. $a_H^\dagger$ is the creation operator for the hadron in its rest frame. The matrix elements on Eq. (2.4) are defined in NRQCD. In Eq. (2.4) the part with $\hat{D}_8$ is the contribution from a color-octet $Q\bar{Q}$ pair in a $^3S_1$ state and the part with $\hat{D}_1(z, J)$ is the contribution from a color-singlet $Q\bar{Q}$ pair in a ^3P_J state. We will call them the color-octet and color-singlet components, respectively. The matrix elements represent the nonperturbative effect, while $\hat{D}_1$ and $\hat{D}_8$ can be calculated perturbatively and they should be free from IR singularities.

A good method to calculate $\hat{D}_1$ and $\hat{D}_8$ is to use wave functions to project out different states from a general $Q\bar{Q}$ pair. At the leading order of v the projection can easily be worked out; details can be found in [14]. We will use a radial wave function $R_1(r)$ to project the ^3P_J color-singlet $Q\bar{Q}$ state and an octet radial wave function $R_8^{(a)}(r)$ to project the $^3S_1$ color-octet $Q\bar{Q}$ state. Calculating with these wave functions the left-hand side (LHS) of Eq. (2.4) and the matrix elements on the RHS of Eq. (2.4) we can extract the functions $\hat{D}_1(z, J)$ and $\hat{D}_8(z)$. The results for $\hat{D}_1(z, J)$ and $\hat{D}_8(z)$ are independent of these wave functions. At the order of α_s we consider, only the tree-level results for the matrix elements are needed; they are

$$\begin{aligned} \langle 0|O_1^{\chi_J}(^3P_J)|0\rangle &= \frac{9(2J+1)}{2\pi}|R_1'(0)|^2, \\ \langle 0|O_8^{\chi_J}(^3S_1)|0\rangle &= \frac{3}{8\pi}\sum_c |R_8^{(c)}(0)|^2, \end{aligned} \tag{2.6}$$

where $R_1'(0)$ is the first derivative of $R_1(r)$ at the origin.

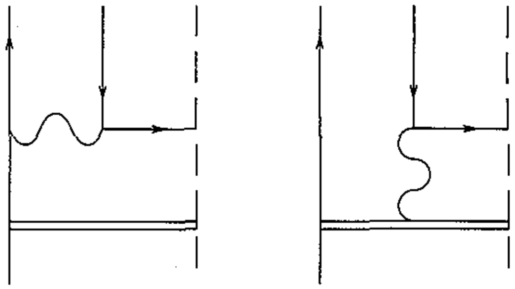

FIG. 1. The Feynman diagrams for the color-singlet component of the heavy quark fragmentation. The line is for the heavy quark, and the wavy line is for gluons. The double line represents the line operator in Eq. (2.1).

III. HEAVY QUARK FRAGMENTATION FUNCTION

From the definition in Eq. (2.1) we can always decompose the fragmentation function by sandwiching the operator $\sum_X |X\rangle\langle X|$ between $a_H^\dagger$ and a_H as

$$D_{H/Q}^{(0)}(z) \sim \sum_X \mathrm{Tr}\{n\cdot\gamma T_H^\dagger T_H\}, \tag{3.1}$$

where T_H may be called the fragmentation amplitude for $Q \to H + X$. Here we have the conservation of total momentum only in the + direction.

A. Color-singlet component

The color-singlet component receives nonzero contributions at order α_s^2. The Feynman diagrams for T_H are given in Fig. 1. Because the $Q\bar{Q}$ pair is in a color-singlet state, there are two gluon lines attached to the quark line. Here, there are no divergences, and so renormalization is not required. The calculation is complicated. Because of the summation over intermediate states, we encountered integrals of the type

$$\int \left(\frac{dq_T}{2\pi}\right)^{d-2} \left(q_T^2 + \frac{(2-z)^2M^2}{z^2}\right)^{-n} \quad \text{for } n = 2, 3, 4, 5. \tag{3.2}$$

Here $\mathbf{q}_T$ is the transversal momentum of the quark as the intermediate state in Fig. 1. The integrals are finite and after performing the integrations we can extract

$$\begin{aligned} \hat{D}_1(z, J=0) &= \frac{16}{729}\alpha_s^2(\mu)\frac{z(1-z)^2}{(2-z)^8}(192 + 384z + 528z^2 - 1376z^3 + 1060z^4 - 376z^5 + 59z^6), \\ \hat{D}_1(z, J=1) &= \frac{64}{729}\alpha_s^2(\mu)\frac{z(1-z)^2}{(2-z)^8}(96 - 288z + 496z^2 - 408z^3 + 202z^4 - 54z^5 + 7z^6), \\ \hat{D}_1(z, J=2) &= \frac{128}{3645}\alpha_s^2(\mu)\frac{z(1-z)^2}{(2-z)^8}(48 - 192z + 480z^2 - 668z^3 + 541z^4 - 184z^5 + 23z^6). \end{aligned} \tag{3.3}$$

These results agree with those in [8]. It is interesting to note that there is a common factor $z(1-z)^2(2-z)^{-8}$ for all J, whereas there is a common factor $z(1-z)^2(2-z)^{-6}$ for heavy quark fragmentation into S-wave quarkonium [6]. Note that the same diagrams in Fig. 1 contribute to heavy quark fragmentation into S-wave quarkonium. The difference between these two factors is because for P-wave quarkonium the derivative of the fragmentation

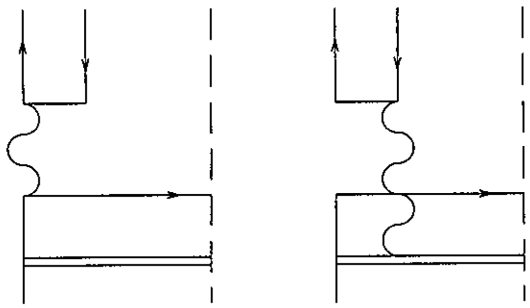

FIG. 2. The Feynman diagrams for the color-octet component of the heavy and light quark fragmentation.

amplitude with the relative momentum between Q and $\bar{Q}$ is involved whereas for S-wave quarkonium only the amplitude itself is involved. The appearance of these common factors can be roughly understood by counting the denominators due to the quark and gluon propagators in the amplitudes and factors from phase space. A successful model for heavy quark fragmentation was obtained in this way [15]. However, we will see in the next subsection that such a counting rule will be violated due to renormalization.

B. Color-octet component

For the color-octet component, not only the diagrams in Fig. 1, but also the diagrams in Fig. 2 will contribute. The two diagrams in Fig. 2 were missing in [8] and they lead to divergences. Instead of integrals in Eq. (3.2), we have

$$\int \left(\frac{dq_T}{2\pi}\right)^{d-2} \left(q_T^2 + \frac{(2-z)M}{z}\right)^{-n} \quad \text{for } n = 1,2,3,4. \tag{3.4}$$

The integral with $n = 1$ is ultraviolet divergent, requiring renormalization. For the renormalization we note that the function of gluon fragmentation into χ_J quarkonium is nonzero at order α_s [9,6,7]:

$$D_{\chi_J/G}(z) = \frac{\pi}{24}\alpha_s \delta(1-z)\frac{1}{M^3}\langle 0|O_8^{\chi_J}(^3S_1)|0\rangle + O(\alpha_s^2). \tag{3.5}$$

Substituting this into Eq. (2.2) for $H = \chi_J$ we can easily choose the function $L_G(y)$ to cancel the divergence. Finally, we obtain the renormalized function $\hat{D}_8(z)$:

$$\hat{D}_8(z) = \frac{1}{36}\alpha_s^2(\mu)\Bigg\{\frac{1}{z}[1+(1-z)^2]\left[\ln\left(\frac{\mu^2}{4M^2}\right) - 2\ln\left(1-\frac{z}{2}\right)\right] - z + \frac{2(1-z)}{9(2-z)^6}(192 - 1184z + 2016z^2 - 1360z^3 + 352z^4 - 14z^5 - 5z^6)\Bigg\}. \tag{3.6}$$

Here there is no common factor like $z(1-z)^2(1-z)^{-6}$. In Eq. (3.6) the results in the second line are not only from the contribution of Fig. 1 for the fragmentation amplitude, but also from the contribution of Fig. 2. With the results in Eqs. (3.6) and (3.3) we complete the heavy quark fragmentation function at order α_s^2. This function should in general satisfy its evolution equation

$$\mu\frac{\partial D_{H/Q}(z,\mu)}{\partial\mu} = \sum_q^{N_f}\int_z^1 \frac{dy}{y}P_{Q\to q}(z/y,\mu)D_{H/q}(y,\mu) + \int_z^1 \frac{dy}{y}P_{Q\to G}(z/y,\mu)D_{H/G}(y,\mu). \tag{3.7}$$

The splitting functions $P_{Q\to q}(y,\mu)$ and $P_{Q\to G}(y,\mu)$ are in the one-loop approximation, the same as those for parton distributions. Using this fact and the result in Eq. (3.5) we obtain the evolution equation for the quark fragmentation at order of α_s^2:

$$\mu\frac{\partial D_{\chi_J}/Q(z,\mu)}{\partial\mu} = \frac{1}{18}\alpha_S^2(\mu)\frac{1}{M^3}\langle 0|O_8^{\chi_J}(^3S_1)|0\rangle\frac{1+(1-z)^2}{z}. \tag{3.8}$$

Substituting our results into the LHS in Eq. (3.8) one can check that our results are in agreement with this equation. From our result the fragmentation function is divergent as z^{-1} when $z \to 0$. However, this singularity disappears if we choose the renormalization scale μ as twice the mass M. The same was also found in the gluon fragmentation in [9]. This property is important for possible applications of our result. In practical applications one solves the evolution equation at μ numerically, where one needs the moments of our result for the fragmentation function at some initial scale μ_0 as the boundary condition. To ensure that the perturbative result is a good approximation, one should choose $\mu_0 \sim M$ to avoid large logarithm terms at higher order. Our result tells us that one should choose $\mu_0 = 2M$ to avoid these terms at higher order and also to safely calculate the moments.

IV. LIGHT QUARK FRAGMENTATION FUNCTION

Since a color-octet $Q\bar{Q}$ pair will lead to a contribution to P-wave quarkonium production at the leading order of

v, a light quark q can undergo fragmentation into χ_J by generating a color octet $Q\bar{Q}$ through emission of a virtual gluon. Such a process happens at the same order of α_s as the heavy quark fragmentation. The Feynman diagrams for T_H are those in Fig. 2, where the quark line attached by the double line is for the light quark q. At the leading order of α_s and v the light quark fragmentation function $D_{\chi_J/q}(z)$ can be written

$$D_{\chi_J/q}(z) = \frac{\hat{D}_{8,q}(z)}{M^3}\langle 0|O_8^{\chi_J}(^3S_1)|0\rangle. \tag{4.1}$$

The color-singlet component only becomes nonzero at higher order than α_s^2. The calculation is similar to the previous section. We introduce the notation $y = m_q/M$, where m_q is the mass of q. The result for $\hat{D}_{8,q}(z)$ is

$$\hat{D}_{8,q}(z) = \frac{1}{36}\alpha_s^2(\mu)\left\{\frac{1}{z}[1+(1-z)^2]\left[\ln\left(\frac{\mu^2}{4M^2}\right) - \ln\left((1-z)+\frac{1}{4}y^2z^2\right)\right] - z - 2(1-z)(2+y^2)\frac{z}{4(1-z)+y^2z^2}\right\}. \tag{4.2}$$

Again the light quark fragmentation function must satisfy its evolution equation. At order α_s^2 this equation is the same in Eq. (3.8). It is easy to check that the function in Eqs. (4.1) and (4.2) satisfies the evolution equation. The light quark fragmentation function has the same property near $z = 0$ as the heavy one; i.e., it is divergent as z^{-1} at any renormalization scale μ except when $\mu = 2M$. The light quark mass m_q can be safely neglected. With $m_q = 0$ the function in (4.2) becomes

$$\hat{D}_{8,q}(z) = \frac{1}{36}\alpha_s^2(\mu)\left\{\frac{1}{z}[1+(1-z)^2]\left[\ln\left(\frac{\mu^2}{4M^2}\right) - \ln(1-z)\right] - 2z\right\}. \tag{4.3}$$

For the convenience of later discussion we introduce here some relations between the various matrix elements in Eq. (2.5). In principle these matrix elements have series expansions in v and the leading order is v^2. Since we only work at the leading order, the higher order corrections can be neglected. In this case, the matrix elements in Eq. (2.5) are related to each other with a spin factor of χ_J. We introduce two parameters H_1 and H_8' as in [10], and the relations can be expressed as

$$\langle 0|O_1^{\chi_J}(^3P_J)|0\rangle \approx (2J+1)M^4H_1, \tag{4.4}$$

$$\langle 0|O_8^{\chi_J}(^3S_1)|0\rangle \approx (2J+1)M^2H_8'.$$

With these relations the whole set of the quark fragmentation functions contains only two unknown parameters, which can only be computed nonperturbatively or extracted from experiments.

V. DISCUSSION AND SUMMARY

Some useful quantities of parton fragmentation functions are their first moments. These moments allow one to roughly estimate a single hadron production rate through fragmentation, where the rate may be taken as product of a parton production rate and the corresponding first moment, where the summation over different partons is understood. We will give results of the first moments of our fragmentation functions. We denote the first moment as $M(q_f \to \chi_J)$, where q_f stands for Q or q. Taking $\mu = 2M$, we obtain

$$M(q \to \chi_J) \approx 0.029(2J+1)\alpha_s^2(2M)\frac{H_8'}{M},$$

$$M(Q \to \chi_0) \approx 0.024\alpha_s^2(2M)\frac{H_8'}{M} + 0.035\alpha_s^2(2M)\frac{H_1}{M},$$

$$M(Q \to \chi_1) \approx 0.072\alpha_s^2(2M)\frac{H_8'}{M} + 0.039\alpha_s^2(2M)\frac{H_2}{M}, \tag{5.1}$$

$$M(Q \to \chi_2) \approx 0.12\alpha_s^2(2M)\frac{H_8'}{M} + 0.015\alpha_s^2(2M)\frac{H_1}{M}.$$

Here we neglect the mass of the light quark. We take the c quark as an example to give some value for the moments. For the value of H_1 and H_8' we use $H_1 \approx 15$ MeV [12] and $H_8' \sim 1.4$ MeV [16]. Taking $M = m_c = 1.5$ GeV and $\alpha_s(2m_c) = 0.26$, the value for $M(q \to \chi_{cJ})$ is $1.8 \times 10^{-6}(2J+1)$ and the value for $M(Q \to \chi_{cJ})$ is 2.4×10^{-5}, 2.9×10^{-5}, and 1.7×10^{-5} for χ_{c0}, χ_{c1}, and χ_{c2}, respectively. The contribution from the color-octet component is not negligible. For χ_{c2} the contribution from the color-octet component is roughly 70% of the heavy quark moment. From these values one can see that the moments for the light quark fragmentation are roughly one order of magnitude smaller than those for the heavy quark. But the contribution from light quarks should not be neglected, especially in a hadron reaction, since the production rate of light quarks as partons may be larger than the production rate of a heavy quark and hence a substantial contribution from light quark fragmentation to the χ_J production is possible.

With the results here and those in [9,7] the functions of all possible parton fragmentation into ^3P_J quarkonium are calculated at order α_s^2. Only two parameters, which represent the nonperturbative effect at the leading order of v, are not known precisely. The functions have the general feature that they are divergent as z^{-1} when $z \to 0$. But at $\mu = 2M$ they are regular distributions over the whole range of z. The functions also satisfy the Altarelli-Parisi equation, as expected.

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