QCD corrections to the spin-dependent Drell-Yan process and a global subtraction scheme

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(Received 18 August 1995)

We present QCD corrections to the Drell-Yan process in the transversely polarized, longitudinally polarized, and unpolarized cases. The analytical results are presented in a form valid for all n-dimensional regularization schemes. A universal mass factorization scheme is presented in which the results reduce to those of dimensional reduction. The connection between the parton distributions and fragmentation functions of dimensional reduction and those of dimensional regularization is elucidated in a simple manner. Numerical results are presented for proton-proton collisions at energies relevant to the BNL Relativistic Heavy Ion Collider. The perturbative stability of the transverse and longitudinal asymmetries is investigated.

PACS number(s): 12.38.Bx, 13.75.Cs, 13.85.Qk, 13.88.+e

I. INTRODUCTION

The unpolarized Drell-Yan process has been studied rather extensively in the literature, including $O(\alpha_s)$ [1, 2] and $O(\alpha_s^2)$ [3] corrections. As well, the $O(\alpha_s)$ corrections to the corresponding longitudinally polarized [4] and transversely polarized processes [5,6] have been studied. What was still lacking was a unified picture for dealing with the polarized processes. The basic problem is the ambiguity associated with defining the γ_5 matrix, or $\varepsilon^{\mu\nu\lambda\rho}$ tensor, in n dimensions; both of these objects arise in polarized processes. For unpolarized QCD processes, *dimensional regularization* (DREG) preserves all the necessary invariances and symmetries for doing calculations to any order in α_s. Hence DREG is the most commonly used regularization for QCD. The ambiguity associated with the continuation of the γ_5 matrix makes it impossible to uniquely define higher order corrections (HOC's) for polarized processes using DREG. Various prescriptions are available, but problems with either mathematical or physical consistency generally arise. As a result, another n-dimensional scheme, *dimensional reduction* (DRED) may be used. This scheme avoids the γ_5 problem, although it requires certain ultraviolet (UV) counterterms which are the same in both unpolarized and polarized processes and may be unambiguously determined.

In this paper, we present analytical results for the unpolarized and (both longitudinally and transversely) polarized Drell-Yan process in a form valid for all n-dimensional schemes. For the polarized case, the ambiguity (or scheme dependence) in the DREG results is parametrized by the ambiguity in the polarized n-dimensional split functions. As well, we present numerical results for p-p collisions relevant to the BNL Relativistic Heavy Ion Collider (RHIC).

We go further to show that for a wide class of subprocesses, including the one-loop corrections to Drell-Yan and deep-inelastic scattering, DRED is simply equivalent to a particular mass factorization scheme in DREG. We call this scheme the ε modified minimal subtraction ($\overline{\text{MS}}_\varepsilon$) [or ε *minimal subtraction scheme* (MS_ε)] since it involves subtracting the ε-dimensional part of the n-dimensional Altarelli-Parisi split functions, where $n = 4-2\varepsilon$. As a consequence, the final results in the $\overline{\text{MS}}_\varepsilon$ scheme are regularization scheme independent within the n-dimensional schemes. The final result is equivalent to that obtained in DRED and all ambiguities associated with the continuation of the γ_5 matrix are subtracted via the n-dimensional split functions.

We will also show the connection between the DRED parton distributions and fragmentation functions and those of DREG in a simple manner. More specifically, we show how to convert existing DREG distributions into ones suitable for use with cross sections determined using DRED. This is important since DRED is equivalent to four-dimensional helicity amplitude techniques which considerably simplify perturbative calculations. We may thus calculate new unpolarized cross sections using DRED or helicity amplitudes and then simply convolute them with the DRED distributions obtained from well-known unpolarized DREG parton distributions and fragmentation functions.

Similar conclusions (for unpolarized processes) may be obtained in the approach of [7], which converts DRED cross sections into DREG ones by considering differences in the Lagrangians and using fictitious ε scalars to calculate the differences in the cross sections. Transition rules between the two schemes are also given in [8]. Here, we take a simpler and more phenomenological approach, investigating how the scheme dependences arise in the Feynman graphs. The connection between the two schemes is simply the relation between the distributions of the respective schemes. This allows for easy interpretation and extension to a wide class of processes. We also explicitly consider polarized observables, unlike [7, 8].

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II. n-DIMENSIONAL REGULARIZATION SCHEMES

There are two parts to the dimensional continuation: the continuation of the *momenta* and the continuation of all other *tensor structures* (i.e., γ matrices). The continuation of the momenta is unique, but there are various methods for continuing the tensors. The choice of the latter defines which dimensional method is being used.

Continuation of the momenta

For the continuation of the momenta, all momenta and phase spaces are continued to n dimensions [9, 10]. The phase space integrals are continued by generalizing integer-dimensional integrals to noninteger dimensions. Consequently, all loop integrals can be reduced, using Feynman parameters, to the fundamental integral

$$\int \frac{d^n q}{(2\pi)^n} \frac{(q^2)^r}{(q^2 - C)^m} = \frac{i(-1)^{r-m}}{(4\pi)^{n/2}\Gamma(n/2)} C^{r-m+n/2} \times B(r+n/2, m-r-n/2) \qquad (1)$$

(see, for example, [11]), with $m > 0$, $r \geq 0$, and B the Euler beta function. As well, defining $n = 4 - 2\varepsilon$ ($n' = 4 - 2\varepsilon'$) with $\varepsilon < 0$ ($\varepsilon' > 0$), we see that ε' is required for UV-divergent integrals and ε for infrared- (IR-) divergent ones, initially. Then we must continue to

$$\varepsilon' = \varepsilon \qquad (2)$$

since we can only work in one dimension at any time. From (1) it follows that massless self-energy insertions on massless external lines vanish. This means that on-shell wave function renormalization is trivial when all particles are massless. Hence, in the absence of coupling renormalization (i.e., gluon self-energies), effectively no UV renormalization is required.

Continuation of the tensors (DREG)

In DREG, one continues the metric tensor and the gamma matrices to n dimensions. Letting $g_n^{\mu\nu}$ denote the n-dimensional metric tensor, we have the relations

$$g_n^{\mu\nu} g_{\mu\nu}^n = n, \quad \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g_n^{\mu\nu}. \qquad (3)$$

As well, the usual convention is to take $2 - 2\varepsilon = n - 2$ helicity states when averaging over initial gluons or photons. This is related to the continuation of the helicity sum rule

$$\sum_\lambda A^\mu(p,\lambda) A^{*\nu}(p,\lambda) \to -g_n^{\mu\nu}. \qquad (4)$$

Here, $A^\mu(p,\lambda)$ is the gluon or photon polarization vector for gluon or photon momentum p and helicity λ. Different conventions simply amount to finite renormalizations of the parton distributions (which, we will see, arise from differences in the n-dimensional split functions).

There exist two popular methods for continuing the γ_5 matrix ($\varepsilon^{\mu\nu\lambda\rho}$) tensor to n dimensions: the anticommuting-γ_5 scheme [12] (see also [13] concerning $\varepsilon^{\mu\nu\lambda\rho}$) and the 't Hooft–Veltman–Breitenlohner–Maison (HVBM) scheme [9, 14].

In the anticommuting-γ_5 scheme, we use the relations

$$\gamma_5\gamma_\mu = -\gamma_\mu\gamma_5, \qquad \gamma_5^2 = -1. \qquad (5)$$

If traces with only one γ_5 occur though, there are known mathematical inconsistencies [14].

In the HVBM scheme, we formally take $n > 4$ (with regards to the tensor algebra) and keep the γ_5 and $\varepsilon^{\mu\nu\lambda\rho}$ in four dimensions so that

$$\{\gamma_5, \gamma_\mu\} = 0 : \mu \leq 4, \quad [\gamma_5, \gamma_\mu] = 0 : \mu > 4, \qquad (6)$$

which follow from the definition

$$\gamma_5 = \frac{i}{4!}\varepsilon_{\mu_1\mu_2\mu_3\mu_4}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}, \qquad (7)$$

where

$$\varepsilon_{\mu_1\mu_2\mu_3\mu_4} = 0, \qquad \mu_i > 4; \qquad (8)$$

otherwise, it is the usual Levi-Civita tensor.

This scheme is mathematically consistent, but cumbersome. Physically, it has the problem that the nonanticommuting γ_5 leads to nonconservation of helicity of massless fermions in a minimal subtraction scheme such as $\overline{\text{MS}}$ [15].

Dimensional reduction

Dimensional reduction [16] is perhaps the simplest of all the dimensional methods. It was originally introduced because DREG violates supersymmetry. As will become obvious, it is also manifestly mathematically consistent. The idea is simple; all γ matrices and tensors are taken to be four-dimensional, and formally $n < 4$. This implies that the components of all momenta between n and 4 must vanish. We have the contraction identities

$$g^{\mu\nu}g_{\mu\nu} = 4, \qquad g_n^{\mu\nu}g_{\mu\nu}^n = g^{\mu\nu}g_{\mu\nu}^n = n \qquad (9)$$

and the usual four-dimensional relations such as

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}. \qquad (10)$$

It is also useful to define

$$\gamma_\varepsilon^\mu \equiv \gamma_\nu(g^{\mu\nu} - g_n^{\mu\nu}). \qquad (11)$$

This method is particularly simple for the calculation of tree graphs (i.e., graphs not involving loops) since the traces are equal to their four-dimensional counterparts, implying gauge invariance. One may thus use four-dimensional helicity amplitude methods, for instance. Then the phase space integrals are carried out in n dimensions, providing an IR regulator. As well, the anticommuting γ_5 implies helicity conservation of massless fermions.

The only subtlety comes from the fact that the virtual momentum integrations generate the tensor $g_n^{\mu\nu}$, which is generally contracted with four-dimensional γ matrices. This can lead to a term $\sim \gamma_\varepsilon^\mu$ which gives the incorrect Lorentz structure and must be removed by a counterterm. In [17] the counterterm for the quark-$\gamma(Z)$ vertex was presented. It is (working in the Feynman gauge)

$$\gamma^\mu \to -C_F \frac{g^2}{(4\pi)^2}\frac{1}{\varepsilon}\gamma^\mu_\varepsilon, \tag{12}$$

with $C_F = 4/3$ (i.e., the Feynman rule for the counterterm is obtained by making the above substitution in the usual rule). For the lepton-$\gamma(Z)$ vertex, we use (12) with

$$C_F g^2 \to e^2. \tag{13}$$

Throughout, we consider (12) to be a Feynman rule for DRED. For the type of processes considered here, (12) is the only counterterm required to make DRED physically consistent.

III. ANALYTICAL RESULTS FOR THE DRELL-YAN PROCESS

We will first consider the unpolarized and longitudinally polarized cases, then the transversely polarized case. We have the general process

$$A(P_1,\lambda_A) + B(P_2,\lambda_B) \to l^-(p_3) + l^+(p_4) + X, \tag{14}$$

where λ_A, λ_B denote the helicities of hadrons A, B. The unpolarized and longitudinally polarized cross sections are defined, respectively, by

$$\sigma \equiv \frac{1}{2}[\sigma(+,+)+\sigma(+,-)], \qquad \Delta\sigma \equiv \frac{1}{2}[\sigma(+,+)-\sigma(+,-)] \tag{15}$$

in the notation $\sigma(\lambda_A,\lambda_B)$.

The general $2\to 2$ $[2\to 3]$ subprocess contributing to (14) has the form

$$a(p_1,\lambda_1) + b(p_2,\lambda_2) \to \gamma^*(q) + [c(k)] \to l^-(p_3) + l^+(p_4) + [c(k)] \tag{16}$$

for general partons a, b, c.

First, we define the process-level invariants

$$S = (P_1+P_2)^2, \qquad M^2 = (p_3+p_4)^2, \qquad \tau = M^2/S. \tag{17}$$

In the parton model, we have

$$p_1 = x_a P_1, \qquad p_2 = x_b P_2. \tag{18}$$

Hence, we may define the subprocess invariants

$$s = (p_1+p_2)^2 = x_a x_b S, \qquad w = \frac{M^2}{s} = \frac{M^2}{S x_a x_b} = \frac{\tau}{x_a x_b}. \tag{19}$$

The unpolarized [polarized] momentum distributions are given by

$$[\Delta]F_{i/I}(x_i, M_f^2) = x_i [\Delta] f_{i/I}(x_i, M_f^2)\ , \tag{20}$$

where the $[\Delta]f_{i/I}$ are the unpolarized [polarized] parton densities for parton i in hadron I, evaluated at factorization energy scale M_f^2.

The parton model expression for the Drell-Yan cross section corresponding to (14) is then

$$[\Delta]\frac{d\sigma_{AB}}{dM^2} = \sum_{ab}\int_\tau^1 \frac{dx_a}{x_a}\int_{w_1}^1 \frac{dw}{w}[\Delta]F_{a/A}(x_a,M_f^2)[\Delta]F_{b/B}(x_b,M_f^2)[\Delta]\frac{d\hat\sigma_{ab}}{dM^2}\ , \tag{21}$$

where

$$w_1 = \tau/x_a, \qquad x_b = w_1/w\ , \tag{22}$$

and $[\Delta]\hat\sigma_{ab}$ is the unpolarized [polarized] subprocess cross section corresponding to (16). We must consider the subprocesses $a=q$, $b=\bar q$, $c=g$ and $a=q$, $b=g$, $c=q$, which are symmetric under $a\leftrightarrow b$ and $q\leftrightarrow\bar q$ as far as $[\Delta]d\hat\sigma_{ab}/dM^2$ is concerned.

We define the unintegrated leptonic tensor as

$$L^{\alpha\beta}_{\rm DRED} = \mu^{2\varepsilon}e^2[p_3^\alpha p_4^\beta + p_4^\alpha p_3^\beta - (M^2/2)g^{\alpha\beta}]\ ,$$
$$L^{\alpha\beta}_{\rm DREG} = \mu^{2\varepsilon}e^2[p_3^\alpha p_4^\beta + p_4^\alpha p_3^\beta - (M^2/2)g_n^{\alpha\beta}]\ , \tag{23}$$

where the arbitrary mass scale $\mu^{2\varepsilon}$ arises from the n-dimensional coupling $e^2\to e^2\mu^{2\varepsilon}$. Furthermore, we define the integrated leptonic tensor as

$$\mathcal{L}^{\alpha\beta} = \int \frac{d^{n-1}p_3}{(2\pi)^{n-1}2p_{3,0}}\frac{\delta[(q-p_3)^2]}{M^2}L^{\alpha\beta}. \tag{24}$$

One finds

$$\mathcal{L}^{\alpha\beta}_{\rm DRED} = \mu^{2\varepsilon}e^2\frac{\pi^\varepsilon}{2^n}\frac{(q^2)^{-\varepsilon}}{\pi^2}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\frac{1}{(3-2\varepsilon)(1-2\varepsilon)} \times\left[(1-\varepsilon)\frac{q^\alpha q^\beta}{q^2} + \frac{g_n^{\alpha\beta}}{2} - (3-2\varepsilon)\frac{g^{\alpha\beta}}{2}\right]. \tag{25}$$

The corresponding DREG tensor is obtained by replacing $g^{\alpha\beta}\to g_n^{\alpha\beta}$. This gives

$$\mathcal{L}^{\alpha\beta}_{\rm DREG} = \mu^{2\varepsilon}e^2\frac{\pi^\varepsilon}{2^n}\frac{(q^2)^{-\varepsilon}}{\pi^2}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\frac{(1-\varepsilon)}{(3-2\varepsilon)(1-2\varepsilon)} \times\left[\frac{q^\alpha q^\beta}{q^2} - g_n^{\alpha\beta}\right]. \tag{26}$$

The part $\sim q^\alpha q^\beta$ does not contribute to the cross section, as can be seen from gauge invariance. Hence the integrated leptonic tensor is effectively a constant. Nonetheless, we keep all the terms for completeness.

We may then define the unpolarized [polarized] subprocess hadronic tensor $[\Delta]W^{\alpha\beta}_{ab}$ through the unpolarized [polarized] subprocess squared Feynman amplitude

$$[\Delta]|M|^2_{ab} \equiv \frac{1}{M^4}L_{\alpha\beta}[\Delta]W^{\alpha\beta}_{ab}, \tag{27}$$

where $|M|^2_{ab}$, $\Delta|M|^2_{ab}$ are defined analogously to (15). Having done so, we may write the $2\to 2$ phase space as

$$[\Delta]\frac{d\hat\sigma_{ab,2\to 2}}{dM^2} = \frac{1}{M^4}\left(16\pi\frac{\delta(1-w)}{M^2}[\Delta]W^{\alpha\beta}_{ab,2\to 2}\right)\mathcal{L}_{\alpha\beta}. \tag{28}$$

Similarly, for the $2 \to 3$ phase space,

$$[\Delta]\frac{d\hat{\sigma}_{ab,2\to 3}}{dM^2} = \frac{1}{M^4}\left[\frac{2w}{\pi}\frac{\pi^\varepsilon M^{-2\varepsilon}}{2^{1-2\varepsilon}}\frac{w^\varepsilon(1-w)^{1-2\varepsilon}}{\Gamma(1-\varepsilon)} \times \int_0^1 dy y^{-\varepsilon}(1-y)^{-\varepsilon}[\Delta]W^{\alpha\beta}_{ab,2\to 3}\mathcal{L}_{\alpha\beta}\right], \quad (29)$$

where $y = (1+\cos\theta)/2$ and θ is the angle between p_1 and k in the p_1, p_2 c.m. This is all we need to calculate $[\Delta]d\sigma_{AB}/dM^2$.

In order to present $[\Delta]d\hat{\sigma}_{ab}/dM^2$ in a form valid for all n-dimensional schemes, we must first give the general form of the n-dimensional split functions $P^n_{ij}(z)$, related to the probability of parton j splitting into a collinear parton i having momentum fraction z, plus an arbitrary final state carrying the rest of the momentum.

We may write

$$P^n_{ij}(z,\varepsilon) = P^<_{ij}(z,\varepsilon) + \delta(1-z)P^\delta_{ij}(\varepsilon), \quad (30)$$

with

$$P^<_{ij}(z,\varepsilon) = P^{<,4}_{ij}(z) + \varepsilon P^{<,\varepsilon}_{ij}(z) \quad (31)$$

and

$$P^\delta_{ij}(\varepsilon) = P^{\delta,4}_{ij} + \varepsilon P^{\delta,\varepsilon}_{ij}. \quad (32)$$

In DRED, $P^{<,\varepsilon}_{ij}(z)$ and $P^{\delta,\varepsilon}_{ij}$ are zero. In other words

$$P^{\rm DRED}_{ij}(z) = P^4_{ij}(z), \quad (33)$$

where $P^4_{ij}(z)$ is the usual four-dimensional split function.

One might wonder how to determine the $P^n_{ij}(z,\varepsilon)$. It is done in the same way as for the four-dimensional case, but keeping the terms of $O(\varepsilon)$.

We can make this clearer by considering the $2 \to m$ process (all particles massless)

$$a(p_a) + b(p_b) \to c_1(k_1) + c_2(k_2) + \cdots + c_m(k_m). \quad (34)$$

When k_1 is collinear with p_a, we have (at one loop)

$$[\Delta]|M|^2_{2\to m}(ab \to c_1\cdots c_m) \sim \sum_d \frac{[\Delta]P^<_{da}(z,\varepsilon)}{p_a\cdot k_1}[\Delta]|M|^2_{2\to m-1}(db \to c_2\cdots c_m), \quad (35)$$

where $a \to d + c_1$ and

$$k_1 \approx (1-z)p_a \to p_d \approx zp_a, \quad z<1, \quad (36)$$

with ΔP_{ij} being the corresponding polarized split function. Then P^δ_{ij} is determined using probability and momentum conservation, and it only appears when $i=j$. Also, by definition $\Delta P^\delta_{ij} = P^\delta_{ij}$.

In this paper, we will need $[\Delta]P_{qq}(z)$ and $[\Delta]P_{qg}(z)$. In four dimensions [18]

$$\Delta P^4_{qq}(z) = P^4_{qq}(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right] = C_F\left[\frac{2}{(1-z)_+} - 1 - z + \frac{3}{2}\delta(1-z)\right],$$

$$P^4_{qg}(z) = \frac{1}{2}(1-2z+2z^2), \qquad \Delta P^4_{qg}(z) = z - \frac{1}{2}. \quad (37)$$

For the unpolarized case in DREG, the ε-dimensional parts are unique [19]:

$$P^{<,\varepsilon}_{qq}(z) = -C_F(1-z), \qquad P^{\delta,\varepsilon}_{qq} = \frac{C_F}{2},$$

$$P^{<,\varepsilon}_{qg}(z) = z^2 - z, \qquad P^{\delta,\varepsilon}_{qg} = 0, \quad (38)$$

except that $P^{<,\varepsilon}_{qg}$ depends on the convention used for averaging over initial gluon states. In the anticommuting-γ_5 scheme, one has

$$P^n_{qq}(z) = \Delta P^n_{qq}(z) \equiv P^n_{q+q+}(z) - \underbrace{P^n_{q-q+}(z)}_{0} \quad (39)$$

(with the $\pm$'s denoting helicities) as a consequence of helicity conservation (i.e., an anticommuting γ_5). This is not true in the HVBM scheme due to (6), which violates helicity conservation of massless fermions (see [15] concerning the polarized split functions in the HVBM scheme). We will show that this problem is overcome in the $\overline{\rm MS}_\varepsilon$ factorization scheme, which we shall now define.

Factorization of the mass singularities is equivalent to expressing the bare parton distributions (and fragmentation functions) in terms of the renormalized ones. In the $\overline{\rm MS}$ ($\overline{\rm MS}_\varepsilon$) scheme, this is done via

$$[\Delta]f^0_{i/A}(x) = [\Delta]f^{\overline{\rm MS}_{(\varepsilon)}}_{i/A}(x,M_f^2) + \frac{c(\varepsilon)}{\varepsilon}\sum_j \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}[\Delta]f^{\overline{\rm MS}_{(\varepsilon)}}_{j/A}(y,M_f^2)[\Delta]P^{(n)}_{ij}(x/y), \quad (40)$$

where

$$\frac{c(\varepsilon)}{\varepsilon} = \frac{1}{\varepsilon}\left(\frac{4\pi\mu^2}{M_f^2}\right)^\varepsilon \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} = \left(\frac{\mu^2}{M_f^2}\right)^\varepsilon\left(\frac{1}{\varepsilon} - \gamma_{\rm E} + \ln 4\pi\right) + O(\varepsilon). \quad (41)$$

The conventional factor $(\mu^2/M_f^2)^\varepsilon$ is not necessary, but it allows for a distinction between coupling renormalization and mass factorization energy scales. We will take $M_f = \mu$ in our calculations. In other processes though, this distinction might be necessary in order to avoid large logarithms.

For the fragmentation functions $\mathcal{D}_{A/i}$, representing the probability for quark i to split into hadron A, the corresponding renormalization is

$$[\Delta]\mathcal{D}^0_{A/i}(z) = [\Delta]\mathcal{D}^{\overline{\rm MS}(\epsilon)}_{A/i}(z, M_f^2) + \frac{c(\varepsilon)}{\varepsilon}\sum_j \frac{\alpha_j}{2\pi}\int_z^1 \frac{dy}{y}[\Delta]\mathcal{D}^{\overline{\rm MS}(\epsilon)}_{A/j}(y, M_f^2) \times [\Delta]P^{(n)}_{ji}(z/y), \quad (42)$$

where $\alpha_j = \alpha_s$, unless $j\ (=A) = \gamma$, in which case $\alpha_j = \alpha$.

It is clear that the $\overline{\rm MS}_\varepsilon$ scheme is just the $\overline{\rm MS}$ scheme, extended so as to subtract off the entire n-dimensional split function. In DRED, $\overline{\rm MS}_\varepsilon$ is equivalent to $\overline{\rm MS}$ since there is no ε-dimensional part of the split function.

We are now in a position to write down the results for $[\Delta]d\hat{\sigma}_{ab}/dM^2$ in a form valid for all n-dimensional schemes. We start with the $q\bar{q}$ subprocess. The Born term is given by

$$[\Delta]\frac{d\hat{\sigma}_{q\bar{q}}}{dM^2} = [\Delta]\chi_B(\varepsilon)\delta(1-w) \quad (43)$$

with

$$\chi_B^{\rm DRED}(\varepsilon) = -\Delta\chi_B^{\rm DRED}(\varepsilon) = \frac{\alpha^2}{N_c}\frac{e_q^2}{2^{-2\varepsilon}}\frac{2\pi^{1+\varepsilon}}{M^{4+2\varepsilon}}\mu^{4\varepsilon}\frac{(2-\varepsilon)}{(3-2\varepsilon)(1-2\varepsilon)} \times \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \quad (44)$$

and

$$\chi_B^{\rm DREG}(\varepsilon) = 2\frac{(1-\varepsilon)^2}{(2-\varepsilon)}\chi_B^{\rm DRED}(\varepsilon). \quad (45)$$

Here, $N_c = 3$ and e_q is the quark fractional charge. In the anticommuting-γ_5 scheme

$$\Delta\chi_B^{\rm AC}(\varepsilon) = -\chi_B^{\rm DREG}(\varepsilon). \quad (46)$$

This is not true in the HVBM scheme though. Of course, in the limit $\varepsilon \to 0$, helicity conservation is restored in all schemes so long as there are no $1/\varepsilon$ poles arising from mass singularities. If there are such $1/\varepsilon$ poles, then one needs a scheme such as $\overline{\rm MS}_\varepsilon$, as we will see.

The factorization counterterm in the $\overline{\rm MS}$ ($\overline{\rm MS}_\varepsilon$) scheme is

$$[\Delta]\frac{d\hat{\sigma}^{\rm ct}_{q\bar{q}}}{dM^2} = \frac{2}{\varepsilon}[\Delta]\chi_B(\varepsilon)\frac{\alpha_s}{2\pi}w^{1+\varepsilon}[\Delta]P^{(n)}_{qq}(w) \times \left(\frac{4\pi\mu^2}{M^2}\right)^\varepsilon \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(\frac{s}{M_f^2}\right)^\varepsilon. \quad (47)$$

The gluonic bremsstrahlung contribution is

$$[\Delta]\frac{d\hat{\sigma}^{\rm Br}_{q\bar{q}}}{dM^2} = [\Delta]\chi_B(\varepsilon)\frac{\alpha_s}{2\pi}w^{1+\varepsilon}C_F\left(\frac{4\pi\mu^2}{M^2}\right)^\varepsilon\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left[\frac{2}{\varepsilon^2}\delta(1-w) - \frac{2}{\varepsilon}\frac{[\Delta]P^<_{qq}(w)}{C_F} + 8\left(\frac{\ln(1-w)}{1-w}\right)_+ -4(1+w)\ln(1-w) - 2(1-w)\right]. \quad (48)$$

The virtual contribution is

$$[\Delta]\frac{d\hat{\sigma}^{\rm V}_{q\bar{q}}}{dM^2} = [\Delta]\chi_B(\varepsilon)\delta(1-w)C_F\frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu^2}{M^2}\right)^\varepsilon\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left[-\frac{2}{\varepsilon^2} - 7 + \frac{2\pi^2}{3} - \frac{2}{\varepsilon}\frac{P^\delta_{qq}}{C_F}\right]. \quad (49)$$

So, adding (43) and (47) – (49), we obtain the total result

$$[\Delta]\frac{d\hat{\sigma}_{q\bar{q}}}{dM^2} = [\Delta]\chi_B(0)\left[\delta(1-w) + C_F\frac{\alpha_s}{2\pi}w\left\{\left(\frac{2\pi^2}{3} - 7\right)\delta(1-w) + 8\left(\frac{\ln(1-w)}{1-w}\right)_+ + 2\frac{P^4_{qq}(w)}{C_F}\ln\frac{s}{M_f^2} -4(1+w)\ln(1-w) - 2(1-w) + [\Delta]d_{q\bar{q}}\right\}\right], \quad (50)$$

where

$$[\Delta]d^{\overline{\rm MS}_\varepsilon}_{q\bar{q}} = 0, \qquad [\Delta]d^{\overline{\rm MS}}_{q\bar{q}} = -\frac{2}{C_F}\{[\Delta]P^{<,\varepsilon}_{qq} + P^{\delta,\varepsilon}_{qq}\delta(1-w)\}. \quad (51)$$

We see that the $O(\varepsilon)$ scheme dependence of the Born term cancels with the $1/\varepsilon$ ($1/\varepsilon^2$) divergences multiplying it. Also, we see that all the n-dimensional regularization schemes give the same answer in the $\overline{\rm MS}_\varepsilon$ scheme and it corresponds to the DRED $\overline{\rm MS}$ answer.

We now consider $[\Delta]d\hat{\sigma}_{qg}/dM^2$. There is no $O(1)$ term (in α_s). At $O(\alpha_s)$ there is a factorization counterterm contribution which is given in the $\overline{\rm MS}$ ($\overline{\rm MS}_\varepsilon$) scheme by

$$[\Delta]\frac{d\hat{\sigma}^{\rm ct}_{qg}}{dM^2} = \frac{1}{\varepsilon}[\Delta]\chi_B(\varepsilon)\frac{\alpha_s}{2\pi}w^{1+\varepsilon}[\Delta]P^{(n)}_{qg}(w) \times \left(\frac{4\pi\mu^2}{M^2}\right)^\varepsilon\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(\frac{s}{M_f^2}\right)^\varepsilon. \quad (52)$$

The bremsstrahlung contribution is

$$[\Delta]\frac{d\hat{\sigma}^{\rm Br}_{qg}}{dM^2} = [\Delta]\chi_B(\varepsilon)\frac{\alpha_s}{2\pi}w^{1+\varepsilon}\left(\frac{4\pi\mu^2}{M^2}\right)^{\varepsilon}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left[-\frac{1}{\varepsilon}[\Delta]P^{<}_{qg}(w) + 2[\Delta]P^4_{qg}(w)\ln(1-w) + \frac{(1-w)^2}{4} + w(1-w)\right]. \quad (53)$$

Adding (52) and (53), we obtain the total result

$$[\Delta]\frac{d\hat{\sigma}_{qg}}{dM^2} = [\Delta]\chi_B(0)\frac{\alpha_s}{2\pi}w\left\{[\Delta]P^4_{qg}(w)\left[\ln\frac{s}{M_f^2} + 2\ln(1-w)\right] + \frac{(1-w)}{4}(1+3w) + [\Delta]d_{qg}\right\}, \quad (54)$$

where

$$[\Delta]d^{\overline{\rm MS}_\varepsilon}_{qg} = 0, \qquad [\Delta]d^{\overline{\rm MS}}_{qg} = -[\Delta]P^{<,\varepsilon}_{qg}(w). \quad (55)$$

In both the $q\bar{q}$ and qg cases, we verify that the unpolarized DREG $\overline{\rm MS}$ result agrees exactly with that previously determined (see, for example, [3]). Since $P^n_{qq} \neq \Delta P^n_{qq}$ in the HVBM scheme, we see from (50), (51) that $d\hat{\sigma}_{q\bar{q}}/dM^2 \neq -\Delta d\hat{\sigma}_{q\bar{q}}/dM^2$ in the HVBM scheme if one uses $\overline{\rm MS}$. But this is a physical requirement. Hence, if one uses HVBM regularization, then it is necessary to use a subtraction scheme such as $\overline{\rm MS}_\varepsilon$ or one which subtracts *at least* the helicity nonconserving part, $\Delta P^{<,\varepsilon}_{qq} - P^{<,\varepsilon}_{qq}$ as well (see also [15]) in the polarized case. Of course, it makes more sense to subtract the entire ΔP^n_{qq} since this leads to regularization scheme-independent results.

Now let us consider the Drell-Yan process with transversely polarized hadrons (*transverse Drell-Yan process*). The general process is

$$A(P_1,S_1) + B(P_2,\pm S_2) \to l^-(p_3) + l^+(p_4) + X\ , \quad (56)$$

where the S_i are reference spin vectors satisfying

$$S_i^2 = -1, \quad S_i\cdot P_j = 0, \quad i,j = 1,2, \quad (57)$$

implying that S_1 and S_2 lie in the plane transverse to the beam axis. Now, letting $\uparrow$ denote polarization in the direction of the spin axis and letting $\downarrow$ denote polarization opposite to the spin axis, we may define the transversely polarized cross section as

$$\Delta_T\sigma \equiv \frac{1}{2}[\sigma(\uparrow,\uparrow) - \sigma(\uparrow,\downarrow)], \quad (58)$$

in the notation $\sigma(S_1, \pm S_2)$.

The general $2\to 2$ $[2\to 3]$ subprocess contributing to (56) is

$$q(p_1,s_1) + \bar{q}(p_2,\pm s_2) \to \gamma^*(q) + [g(k)] \to l^-(p_3) + l^+(p_4) + [g(k)] \quad (59)$$

with

$$s_1 = S_1, \qquad s_2 = S_2, \quad (60)$$

and $\Delta_T\hat{\sigma}$ defined analogously to (58). There is no qg subprocess since gluons cannot be transversely polarized. We may define the transversity distribution

$$\Delta_T f_{q/A}(x,M_f^2) \equiv \Delta_T F_{q/A}(x,M_f^2)/x = f_{q\uparrow/A\uparrow}(x,M_f^2) - f_{q\downarrow/A\uparrow}(x,M_f^2), \quad (61)$$

which is often denoted as $h_1^q(x,M_f^2)$.

Let ϕ_3 denote the azimuthal angle of p_3 about the beam axis (with respect to some reference axis) and let $\hat{\theta}_3$ denote the angle between p_1 and p_3 in the p_1, p_2 center-of-momentum frame. Then, the quantity we are interested to calculate is $\Delta_T d\hat{\sigma}/dM^2 d\phi_3$. This was done in [6] using DRED, where $\Delta_T d\hat{\sigma}/dM^2 d\hat{\Omega}_3$ was first determined, with $\hat{\Omega}_3$ representing the solid angle of p_3 in the c.m. of p_1, p_2. Then $\Delta_T d\hat{\sigma}/dM^2 d\phi_3$ was obtained via

$$\frac{\Delta_T d\hat{\sigma}}{dM^2 d\phi_3} = \int_{-1}^{1} d(\cos\hat{\theta}_3)\frac{\Delta_T d\hat{\sigma}}{dM^2 d\hat{\Omega}_3}. \quad (62)$$

Of course, if one was only interested in $\Delta_T d\hat{\sigma}/dM^2 d\phi_3$, then one could integrate over $\hat{\theta}_3$ separately for the Born term, loops, bremsstrahlung, and factorization counterterm, then add all the different parts to get a finite result for $\Delta_T d\hat{\sigma}/dM^2 d\phi_3$. Either way, the expression for $\Delta_T\sigma_{AB}/dM^2 d\phi_3$ is obtained from (21) by replacing $[\Delta] \to \Delta_T$ and differentiating with respect to ϕ_3.

From the form of the unpolarized and longitudinally polarized results, it is straightforward just to take the final DRED result of [6] and put it in a form valid for all regularization schemes. The result is (with ϕ_1, ϕ_2 the azimuthal angles of s_1, s_2)

$$\frac{\Delta_T d\hat{\sigma}}{dM^2 d\phi_3} = \Delta_T\chi_B\left[\delta(1-w) + \frac{\alpha_s}{2\pi}C_F w\left\{\left(\frac{2\pi^2}{3} - 7\right)\delta(1-w) + 8\left(\frac{\ln(1-w)}{1-w}\right)_+ + 2\frac{\Delta_T P^4_{qq}(w)}{C_F}\ln\frac{s}{M_f^2} - 8\ln(1-w) - 6w\frac{\ln^2 w}{1-w} + 4(1-w) + \Delta_T d\right\}\right], \quad (63)$$

with

$$\Delta_T\chi_B = \cos(\phi_1+\phi_2-2\phi_3)\frac{\alpha^2}{3N_c}\frac{e_q^2}{M^4}, \tag{64}$$

and

$$\Delta_T d_{\overline{\rm MS}_\varepsilon} = 0, \quad \Delta_T d_{\overline{\rm MS}} = -\frac{2}{C_F}[\Delta_T P_{qq}^{<,\varepsilon} + P_{qq}^{\delta,\varepsilon}\delta(1-w)]. \tag{65}$$

The transversity split function $\Delta_T P_{qq}^n$ is obtained via (35) with $[\Delta]\to\Delta_T$. In four dimensions, $\Delta_T P_{qq}^4$ is given by [20]

$$\Delta_T P_{qq}^4(z) = C_F\left[\frac{2}{(1-z)_+} - 2 + \frac{3}{2}\delta(1-z)\right]. \tag{66}$$

In the anticommuting-γ_5 scheme, it is straightforward to obtain

$$\Delta_T P_{qq}^{<,\varepsilon}(z) = -C_F(1-z). \tag{67}$$

The transversity renormalizations corresponding to (40), (42) are obtained by replacing $[\Delta]\to\Delta_T$. We verified explicitly that the form of (40) does indeed hold for the transverse case.

IV. CONNECTION BETWEEN n-DIMENSIONAL SCHEMES

Here we will show how to convert results of one n-dimensional scheme to those of another in a straightforward manner. We do this by examining the origin of the scheme-dependent parts. Strictly speaking, this only applies to processes not requiring coupling constant renormalization, since other differences may arise from the UV sector. On the other hand, it has been shown [21, 22] that the UV sectors of DREG and DRED in QCD can be related via a finite $O(\alpha_s^2)$ renormalization of the coupling. Namely,

$$\alpha_s^{\rm DRED} = \alpha_s^{\overline{\rm MS}}\left(1+\frac{\alpha_s^{\overline{\rm MS}}}{2\pi}\frac{N_c}{6}\right) \tag{68}$$

(see, for example, [8]). In other words, one may go from one scheme to the other by simply expressing the coupling of one scheme in terms of that in the other. We will assume this has been done so that the only differences may arise from the IR sector. Then all the following argumentation can be seen to apply to all one-loop QCD processes.

As an example, we will consider the $q\bar{q}$ subprocess in the unpolarized Drell-Yan process, to show the origin of the scheme dependences. Then we will show that the same argumentation holds for all one-loop processes.

In order to extract the scheme-dependent parts, we need only consider terms which give rise to $1/\varepsilon$ poles. This is because the scheme dependences come from $\frac{1}{\varepsilon}\cdot O(\varepsilon)$ terms, where the $O(\varepsilon)$ terms are in general scheme dependent. We therefore consider the contribution to $d\hat{\sigma}_{q\bar{q}}/dM^2$ when k is collinear with one of the initial partons, say p_1. From (35) we see that

$$|M|^2_{2\to3} \sim \frac{P_{qq}^<(w,\varepsilon)}{p_1\cdot k}|M|^2_{\rm Born}, \quad w<1 \tag{69}$$

with

$$k \approx (1-w)p_1. \tag{70}$$

After phase space integrations, this will yield a contribution to $d\hat{\sigma}_{q\bar{q}}/dM^2$:

$$\begin{aligned}\frac{d\hat{\sigma}^{\rm coll}_{q\bar{q}}}{dM^2} &\sim \frac{1}{\varepsilon}\frac{\chi_B(\varepsilon)}{(1-w)^{1+2\varepsilon}}[(1-w)P_{qq}^<(w,\varepsilon)] \\ &= \frac{\chi_B(\varepsilon)}{\varepsilon}\left[-\frac{1}{2\varepsilon}\delta(1-w)+\frac{1}{(1-w)_+}\right. \\ &\quad\left. -2\varepsilon\left(\frac{\ln(1-w)}{1-w}\right)_+\right] \\ &\quad\times[(1-w)P_{qq}^<(w,\varepsilon)].\end{aligned} \tag{71}$$

Hence [noting that $P_{qq}^{<,\varepsilon}(w)\sim 1-w$], we get a scheme-dependent part from the bremsstrahlung,

$$\frac{d\hat{\sigma}^{\rm SD}_{\rm br}}{dM^2} \sim \chi_B(0)P_{qq}^{<,\varepsilon}(w) \tag{72}$$

as well as an equal term arising from $k\approx(1-w)p_2$. The soft divergent terms $\sim[\frac{1}{\varepsilon}\delta(1-w)]\cdot\frac{1}{\varepsilon}$ cancel exactly with the loops, having the same overall factor, and hence do not lead to scheme dependences.

As well, there is a scheme-dependent term coming from the loops. By definition of P_{qq}^δ, there must be a term proportional to

$$\frac{d\hat{\sigma}_V}{dM^2} \sim \chi_B(\varepsilon)\frac{P_{qq}^\delta}{\varepsilon}\delta(1-w). \tag{73}$$

This will lead to a scheme-dependent part

$$\frac{d\hat{\sigma}_V^{\rm SD}}{dM^2} \sim \chi_B(0)P_{qq}^{\delta,\varepsilon}\delta(1-w). \tag{74}$$

Hence, the entire scheme dependence can be traced back to the process-independent n-dimensional split functions (their ε-dimensional part). We also see explicitly why the $\overline{\rm MS}_\varepsilon$ factorization scheme will lead to regularization scheme-independent results; all the regularization scheme-dependent parts are subtracted. Of course, if one has longitudinal or transverse polarization, all the above holds with

$$\chi_B(\varepsilon)\to\Delta_{[T]}\chi_B(\varepsilon), \quad P_{qq}(w,\varepsilon)\to\Delta_{[T]}P_{qq}(w,\varepsilon). \tag{75}$$

For conciseness, we will drop the $\Delta_{[T]}$'s with the understanding that the same argumentation holds for the polarized cases.

It is now clear how to convert results calculated in DRED to the corresponding DREG $\overline{\rm MS}$ results, if desired. One simply replaces [defining $P_{ij}^\varepsilon(z)\equiv P_{ij}^{<,\varepsilon}(z)+P_{ij}^{\delta,\varepsilon}\delta(1-z)$]

$$P_{ij}^4(z)\to P_{ij}^4(z)-\varepsilon P_{ij}^\varepsilon(z) \tag{76}$$

in the factorization counterterm.

Or, to convert from DREG to DRED (i.e., $\overline{\rm MS}_\varepsilon$), simply replace

$$P^4_{ij}(z) \to P^n_{ij}(z,\varepsilon) = P^4_{ij}(z) + \varepsilon P^\varepsilon_{ij}(z) \tag{77}$$

in the factorization counterterm. Hence, one can go from any n-dimensional scheme to any other using (76) and (77). From the previous argumentation, it is clear that polarization poses no difficulty in this approach. This procedure is equivalent to expressing the parton distributions and fragmentation functions of one scheme in terms of those in the other, as will be shown in the next section.

Up until now, we have used the Drell-Yan process as an example. We now show that the argumentation here applies to all QCD processes at one loop once the UV sectors have been made to agree (if coupling constant renormalization is required). The renormalization of the parton distributions is process independent, since the same collinear configurations occur in all processes. This is because they are related to hadronic emissions (or fragmentation into hadrons) which occur in a process-independent manner and can be constructed from a universal set of subdiagrams containing the configurations having collinear divergences with respect to the particular parton. The only difference then, from one process to the next, is the soft singularity structure. But the soft singularities cancel via the Bloch-Nordsieck mechanism [23] [or Kinoshita-Lee-Nauenberg (KLN) theorem [24]] such that the singular cross section for soft emissions is proportional to the Born term with (minus) the same overall factor as that coming from the loops. Hence, the scheme dependences of $O(\varepsilon)$ which multiply the $1/\varepsilon$ singular terms, cancel exactly between the loops and the soft bremsstrahlung contributions.

Thus, the only scheme dependence may come from the noncancelling mass singularities, whose structure is process independent. Hence, all the argumentation here applies to all QCD processes at one loop; extension to higher orders should be analogous. Also, the same conclusions concerning the regularization scheme independence in the $\overline{\rm MS}_\varepsilon$ scheme apply to all one-loop QCD processes.

V. DRED PARTON DISTRIBUTIONS AND FRAGMENTATION FUNCTIONS

It is straightforward to show how to relate the DRED parton distributions and fragmentation functions to those in DREG ($\overline{\rm MS}$). This is useful if one wishes to work strictly in DRED, but make use of the abundant sets of DREG parton distributions and fragmentation functions. First, we will consider the parton distributions, again dropping the $\Delta_{[T]}$'s for conciseness.

We will make use of the fact that DRED is equivalent to the $\overline{\rm MS}_\varepsilon$ scheme of DREG. Noting that the bare parton distribution, $f^0_{i/A}(x)$, on the left-hand side (LHS) of (40) is factorization scheme independent, we obtain

$$f^{\rm DRED}_{i/A}(x) = f^{\overline{\rm MS}_\varepsilon}_{i/A}(x) = f^{\overline{\rm MS}}_{i/A}(x) + \sum_j \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\frac{c(\varepsilon)}{\varepsilon} \times [f^{\overline{\rm MS}}_{j/A}(y)P^4_{ij}(x/y) - f^{\overline{\rm MS}_\varepsilon}_{j/A}(y)P^n_{ij}(x/y)] \tag{78}$$

or

$$f^{\rm DRED}_{i/A}(x) = f^{\overline{\rm MS}_\varepsilon}_{i/A}(x) = f^{\overline{\rm MS}}_{i/A}(x) - \sum_j \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} f^{\overline{\rm MS}}_{j/A}(y)P^\varepsilon_{ij}(x/y) + O(\alpha_s^2). \tag{79}$$

From (42), we may immediately write for the fragmentation functions

$$\mathcal{D}^{\rm DRED}_{A/i}(z) = \mathcal{D}^{\overline{\rm MS}_\varepsilon}_{A/i}(z) = \mathcal{D}^{\overline{\rm MS}}_{A/i}(z) - \sum_j \frac{\alpha_j}{2\pi}\int_z^1 \frac{dy}{y}\mathcal{D}^{\overline{\rm MS}}_{A/j}(y)P^\varepsilon_{ji}(z/y) + O(\alpha_s^2). \tag{80}$$

Noting (76) and (77), we see explicitly that going from DRED to DREG $\overline{\rm MS}$ simply amounts to expressing the DRED (or $\overline{\rm MS}_\varepsilon$) parton distributions and fragmentation functions in terms of the DREG $\overline{\rm MS}$ ones, and vice-versa from series inversion.

VI. NUMERICAL RESULTS

Here we present asymmetries and cross sections for the Drell-Yan process in p-p collisions at energies relevant to RHIC. In general, we use the two-loop $\overline{\rm MS}$ expression for $\alpha_s(\mu^2)$, with four flavors and $\Lambda = 0.2$ GeV, except in the transversely polarized cross sections where we use the one-loop expression in order to be consistent with [6]. Also, we take $\mu^2 = M_f^2 = M^2$. For the unpolarized cross sections, we use the DREG subprocess cross section convoluted with the unpolarized parton distributions of [25] (Set S-$\overline{\rm MS}$). For the longitudinally polarized case, we use the $\overline{\rm MS}_\varepsilon$ (or DRED) subprocess cross sections, since they are physically consistent (and regularization scheme independent), convoluted with the longitudinally polarized parton distributions of [26] [Set 1, SU(3) symmetric sea] which fit well the recent DIS data [27] except at very low x not covered for the kinematics considered here.

For the transversely polarized subprocess cross sections, we again use the $\overline{\rm MS}_\varepsilon$ result. For the transversity distributions, we choose for the valence distributions (at $Q_0^2 = 4$ GeV^2)

$$\Delta_T F_{u_v/p}(x,Q_0^2) = 2.1\,x^{0.8}(1-x)^{2.4}, \tag{81}$$

$$\Delta_T F_{d_v/p}(x,Q_0^2) = -0.76\,x^{0.8}(1-x)^{3.4}, \tag{82}$$

and for the sea-quark distributions

$$\Delta_T F_{\bar q/p}(x,Q_0^2) = -0.12\,x^{0.1}(1-x)^{9.5}, \quad q = u,d,s \tag{83}$$

(one-half the value used in [6,28]). These satisfy the upper bound proposed by Soffer [29]. As well, the valence distributions are consistent with bag model predictions. The small- and large-x behavior is consistent with the longitudinal and unpolarized cases. There are no other definite constraints we may impose. Q^2-dependent parametrizations for the longitudinal and transversity distributions are given in [28]. We use one-loop evolutions, as two-loop polarized split functions do not yet exist.

We define the asymmetries

$$A_T = \frac{\Delta_T d\sigma/dM^2 d\phi_3}{d\sigma/dM^2 d\phi_3} \tag{84}$$

and

$$A_L = \frac{\Delta d\sigma/dM^2}{d\sigma/dM^2}, \tag{85}$$

noting that

$$\frac{d\sigma}{dM^2 d\phi_3} = \frac{1}{2\pi}\frac{d\sigma}{dM^2}. \tag{86}$$

Figure 1(a) presents A_T for $\sqrt{S} = 100$ GeV, in the range $0.05 \le \sqrt{\tau} \le 0.5$. The largest value for $-A_T$ is 8%. We notice that for the $q\bar{q}$ subprocess, A_T is reasonably perturbatively stable, unlike the cross sections which increase by 50 – 100% under HOC [6]. Inclusion of the qg subprocess makes the asymmetry somewhat more negative since the qg subprocess contributes negatively as noticed in [1, 2].

Figure 1(b) presents the corresponding polarized cross section, $-\Delta_T d\sigma/dM^2 d\phi_3$. The sharp dropoff with increasing $\sqrt{\tau}$ is a combined result of the softness of $\Delta_T F_{\bar{q}/p}$, the $1/M^4$ behavior of the cross section, and the decreasing integration region with increasing $\sqrt{\tau}$.

Figure 2 presents the corresponding quantities for $\sqrt{S} = 200$ GeV and $0.05 \le \sqrt{\tau} \le 0.25$ (away from the Z-exchange region). Similar features hold, except the cross sections are somewhat smaller because of the $1/M^4$ suppression (which amounts to $2/M^3$ in $\Delta_T d\sigma/dM d\phi_3$).

In Figs. 3 and 4 we present the corresponding plots for A_L and $-\Delta d\sigma/dM^2$. The largest value for $-A_L$ is 16%. As expected from helicity conservation, the $q\bar{q}$ subprocess exhibits great perturbative stability. Inclusion of the qg subprocess, however, upsets this stability since it contributes with sign opposite to that of the $q\bar{q}$ subprocess (in both the polarized and unpolarized cases) and is relatively large in the polarized case. Hence, the net asymmetry becomes somewhat smaller in magnitude. This

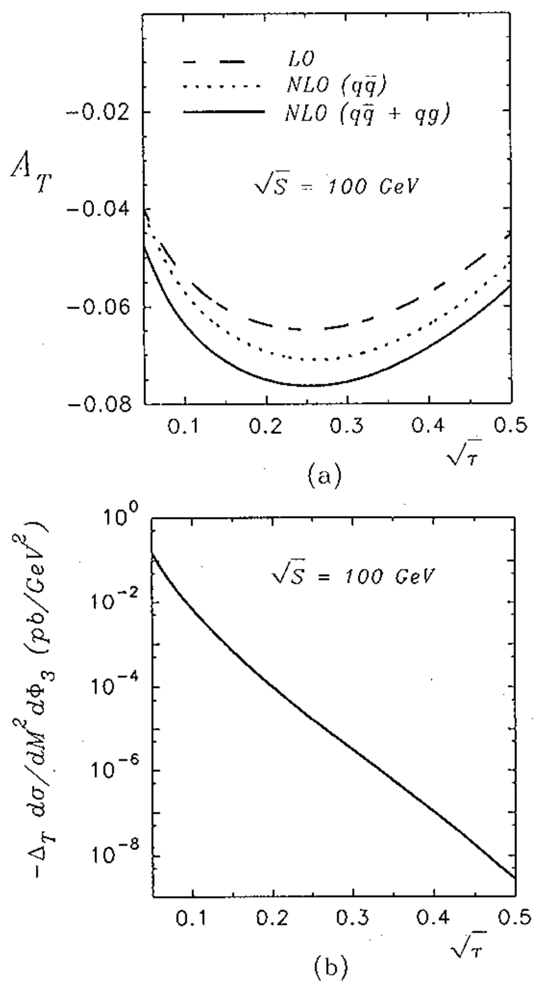

FIG. 1. (a) Transverse asymmetry, A_T, in leading order and in next-to-leading order; (b) corresponding next-to-leading order polarized cross section versus $\sqrt{\tau}$ at $\sqrt{S} =$ 100 GeV.

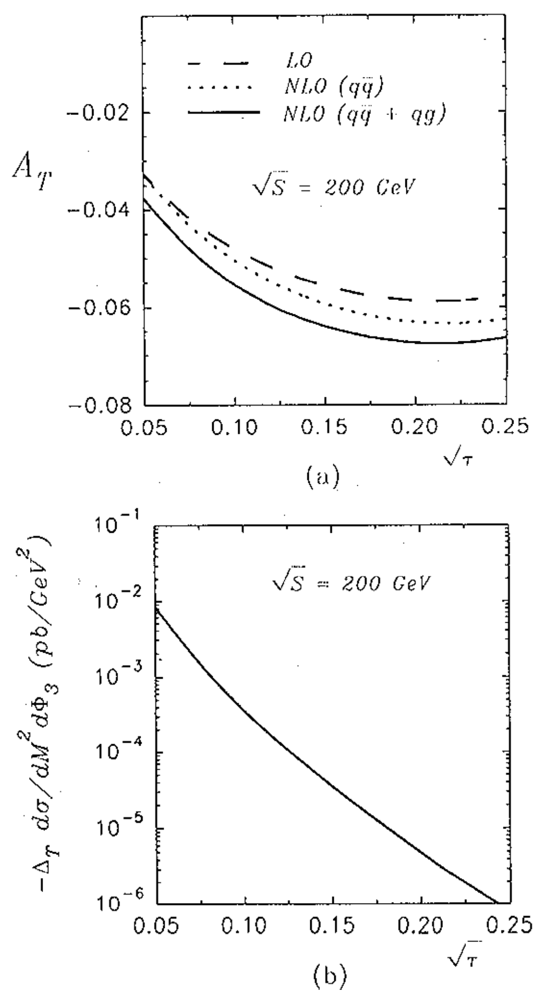

FIG. 2. (a) Transverse asymmetry, A_T, in leading order and in next-to-leading order; (b) corresponding next-to-leading order polarized cross section versus $\sqrt{\tau}$ at $\sqrt{S} =$ 200 GeV.

would not be a problem in p-$\bar{p}$ collisions, where one is probing valence-valence distributions. For p-p collisions, the smallness of $\Delta F_{\bar{q}}$ makes the qg subprocess more significant. A smaller polarized gluon distribution and/or a larger polarized sea-quark distribution would reduce this effect. Still, for the larger $\sqrt{\tau}$, measurable asymmetries are obtained.

For a discussion of the perturbative stability of the longitudinal asymmetry in p-p direct photon production, see [30, 28]. There it was noted that the asymmetry is perturbatively stable if the polarized gluon distribution is sufficiently large so that the qg subprocess dominates. This contrasts with the p-p Drell-Yan process, where a large polarized gluon distribution destabilizes the asymmetry.

VII. CONCLUSIONS

We have presented complete next-to-leading order analytical results for the Drell-Yan process with unpolarized, longitudinally polarized, and transversely polarized hadrons. These results are in a form valid for all n-dimensional schemes. It was shown how one can easily convert from results obtained in one scheme to those of another, regardless of the polarization, for one-loop QCD processes. This procedure simply amounts to expressing the parton distributions and fragmentation functions in one scheme in terms of those in the other. As well, the origin of the scheme dependences was elucidated. A mass factorization scheme, which we call $\overline{\text{MS}}_\epsilon$, was introduced. It was shown that in this factorization scheme, the final results are regularization scheme independent and coincide with those of DRED $\overline{\text{MS}}$. A simple method for converting parton distributions and fragmentation functions from DREG to DRED was given.

For p-p collisions at energies relevant to RHIC, asymmetries and cross sections for transversely and longitudinally polarized collisions were presented. For the transverse case, the asymmetries reached -8% and exhibited reasonable perturbative stability. For the longitudinal case, the asymmetries reached -16% and the $q\bar{q}$ subprocess exhibited great perturbative stability. Inclusion of the qg subprocess somewhat lessened the longitudinal asymmetries, however. Still, p-p collisions serve as the best probe for the polarized antiquark distributions in the proton, and they may be extracted with sufficient experimental statistics.

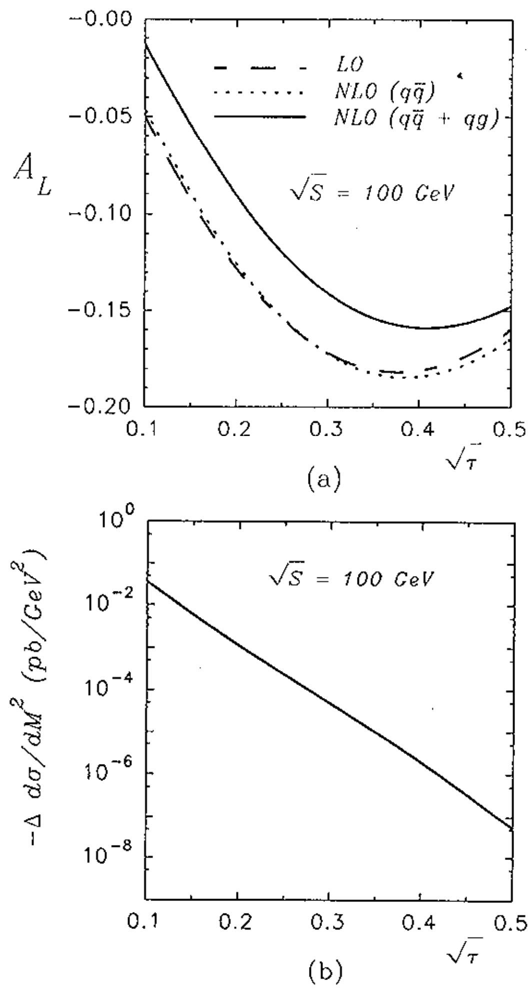

FIG. 3. (a) Longitudinal asymmetry, A_L, in leading order and in next-to-leading order; (b) corresponding next-to-leading order polarized cross section versus $\sqrt{\tau}$ at $\sqrt{S}$ = 100 GeV.

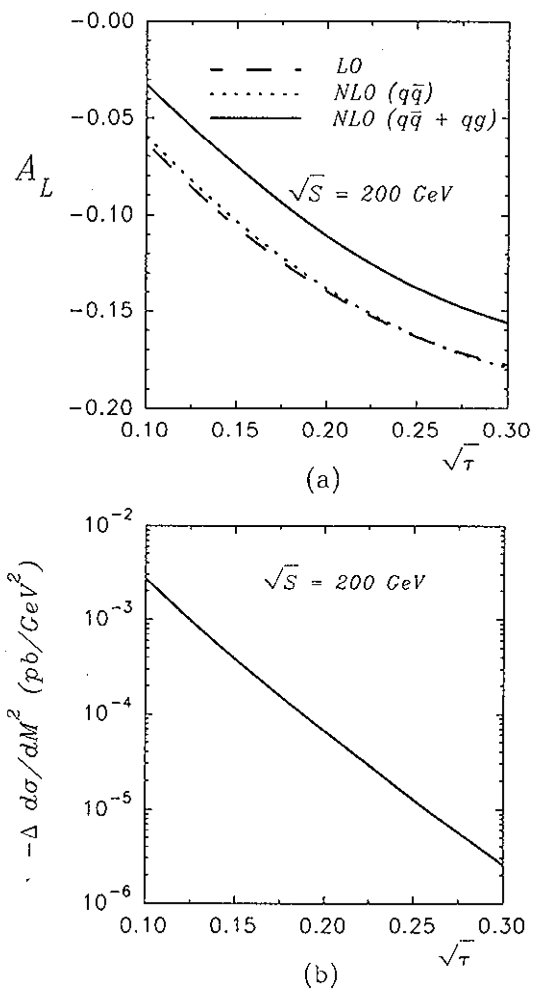

FIG. 4. (a) Longitudinal asymmetry, A_L, in leading order and in next-to-leading order; (b) corresponding next-to-leading order polarized cross section versus $\sqrt{\tau}$ at $\sqrt{S}$ = 200 GeV.

ACKNOWLEDGMENTS

The author thanks A.P. Contogouris and Z. Merebashvili for enlightening discussions as well as G. Bunce for useful conversations. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada and by the Québec Department of Education.

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