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Intercommutation of Z-boson string loops violates baryon number

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We show that delinking of Z-boson string loops changes the helicity and thus violates baryon number. The key point is that an unlinked vortex loop cannot be twisted. The helicity of an eventual magnetic twist when averaged in time is zero.

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Recently there has been wide interest in baryon-number-violating processes in the standard electroweak theory. The first indication of such nonperturbative processes was the discovery of the electroweak sphaleron [1,2]. Another type of solution which at first sight is completely different is the electroweak string [3]. The Z-boson string can also be interpreted as a two-dimensional type of sphaleron [4]. The Weinberg-Salam (WS) model possesses all the necessary ingredients to explain matter-antimatter asymmetry such as C and CP violation and also there is a place for baryon-number-violating processes as one can deduce from the anomaly equation

$$\partial_\mu j^\mu_B = \frac{N_F}{32\pi^2}(-g^2 W^a_{\mu\nu}\tilde{W}^{a\mu\nu} + g'^2 Y_{\mu\nu}\tilde{Y}^{\mu\nu}), \tag{1}$$

where $\tilde{F}_{\mu\nu} = \frac{1}{2!}\varepsilon_{\mu\nu\rho\gamma}F^{\rho\gamma}$. The right-hand side (RHS) of the equation can be rewritten as a total divergence and if we assume that there is no baryon flux through boundaries we can relate a change of the baryon number to a change of the Chern-Simons (CS) index of the fields

$$\triangle Q_B = N_F \triangle (N_{\rm CS} - n_{\rm CS}). \tag{2}$$

The non-Abelian $\mathrm{SU}(2)_L$ Chern-Simons number is

$$N_{\rm CS} = \frac{g^2}{32\pi^2}\int d^3x \;\; \varepsilon_{ijk}\left(W^{aij}W^{ak} - \frac{g}{3}\,\varepsilon_{abc}W^{ai}W^{bj}W^{ck}\right), \tag{3}$$

while its Abelian $\mathrm{U}(1)_Y$ counterpart reads

$$n_{\rm CS} = \frac{g'^2}{32\pi^2}\int d^3x \;\; \varepsilon_{ijk}(Y^{ij}Y^k). \tag{4}$$

The Chern-Simons numbers themselves are not gauge invariant but once the gauge is fixed there is a direct relation between the change of these indices and the change of the baryon number [Eq. (2)].

In this paper similarly as in [5,6] we will regard the bosonic part of the WS model as a massive classical background for fermionic degrees of freedom. In the semiclassical framework we will restrict ourselves to configurations with $W^1_\mu = W^2_\mu = 0$. After orthogonal transformation

$$Z_\mu = \cos\theta_W W^3_\mu - \sin\theta_W Y_\mu,$$
$$A_\mu = \sin\theta_W W^3_\mu + \cos\theta_W Y_\mu \tag{5}$$

we will make a further restriction to configurations with $A_\mu = 0$ and a one-component Higgs field $\phi_1 = 0$, $\phi_2 = \psi$. It is a straightforward calculation to check that if such restrictions are imposed as initial conditions on the fields and their time derivatives they are satisfied all through the time evolution of the system. Equation (2) now takes a simple form in terms of helicity H_Z:

$$\triangle Q_B = \frac{N_F}{16\pi^2}\,\alpha^2\cos(2\theta_W)\triangle H_Z,$$
$$H_Z = \int d^3x \;\; \vec{B}_Z\cdot\vec{Z}, \tag{6}$$

where $\alpha^2 = g^2 + g'^2$. In this way we have restricted ourselves to the Abelian Higgs model

$$L = -\tfrac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + D_\mu\psi^\star D^\mu\psi - V(\psi^\star\psi), \tag{7}$$

where $D_\mu\psi = \partial_\mu\psi + i\frac{\alpha}{2}Z_\mu\psi$ with the coupling constant $\alpha = g\cos\theta_W + g'\sin\theta_W$. The potential is $V(\psi^\star\psi) = -\mu^2\psi^\star\psi + \lambda(\psi^\star\psi)^2$.

By restriction to the Abelian Higgs model we are not able to say anything conclusive about generic non-Abelian configurations but our goal is to show that at least within this framework helicity is not conserved during delinking of string loops. This conclusion cannot be obtained without a careful treatment of the topology of the complex Higgs field. For a configuration of the Higgs field to be well defined the phase of the field has to be single valued everywhere except at the lines of vortices themselves. It is a basic condition both for classical time-dependent solutions and for off-shell configurations contributing to a path integral. To proceed further we need a rather plausible dynamical assumption. Namely, we assume that outside of the finite-width vortex core the modulus of the Higgs field approaches exponentially its vacuum expectation value with some characteristic length which is small as compared to actual intervortex separations. At the same time we assume that also the covariant derivative of the Higgs field approaches exponentially zero outside of the core. The phase ω and the gauge potential are related by $Z_\mu = -\frac{2}{\alpha}\partial_\mu\omega$. In other words we assume there is a finite-width core of a vortex and outside of the core the energy

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density is approximately zero. These are characteristic fixtures of a straight Nielsen-Olesen vortex but also known exact or approximate time-dependent solutions confirm this expectation [7,8,13,9].

Now we can look on vortex networks on a larger scale such that cores are negligibly thin as compared to interstring distances. A discussion of core effects is postponed until later. Under our assumption outside of the cores the only relevant field variable is the phase ω while the gauge potential is related to this phase by a "pure gauge" condition. As already mentioned ω has to be single valued everywhere except at string lines. At every moment of time we can find surfaces of a constant phase. The surfaces can terminate only on string (defect) lines or at infinity. For a Z string along the z axis,

$$\psi(r,\theta)=f(r)e^{i\theta}, \quad Z_\theta=\frac{-a(r)}{\frac{\alpha}{2}r}, \tag{8}$$

with boundary conditions $f(0)=0$, $f(\infty)=\sqrt{(\mu^2/2\lambda)}$ and $a(0)=0$, $a(\infty)=1$, the constant phase surfaces are semiplanes of constant θ terminating on the string line and at spatial infinity. One can perform a U(1) gauge transformation on the fields:

$$\psi'=\psi e^{i\chi z}, \quad Z'_\theta=Z_\theta, \quad Z'_3=-\frac{\chi}{\frac{\alpha}{2}}. \tag{9}$$

Now the surfaces are twisted. The helicity per unit length of such a string is $8\pi\chi/\alpha^2$. The twist of the gauge field is connected with a twist in the phase of the Higgs field.

Now we can try to construct a p-fold twisted string loop. Let the string configuration at time t be $\vec{X}(t,\sigma)$. It is a vortex so the phase winds around this line by 2π. We chose a closed line $\vec{X}'(t,\sigma)$ close to the string line but not linked with the string. What is more we demand that on a strip spanned by these two lines the phase is single valued. As such a line is followed in the direction of the twist the phase rises by $2\pi p$. The above is just a definition of what we mean by a twisted string loop. On the closed line $\vec{X}'(t,\sigma)$ we can span a smooth surface S. As the circulation of the phase along its edge is nontrivial for a nonzero twist p there must exist at least $|p|$ points in its interior where the phase is not single valued. Circulation of the phase is concentrated in these points. Points with k-fold circulation around them are counted k times. Now the surface can be slightly deformed and once again we have at least $|p|$ singular points. By continuously varying the surface we can construct lines of defects. They can be admitted provided the moduli of the Higgs field vanishes on these lines. In other words the lines are nothing else but vortices. Thus a p-fold twisted vortex loop can exist but only if it is stuck on a bundle of $|p|$ vortices. In a world where the finiteness of energy condition admits only vortex loops but not infinite strings the necessary condition for a loop to be p-fold twisted is that it is linked with at least $|p|$ other loops with the same orientation. A string loop which is not linked with other loops cannot be twisted.

Now let us consider a special case of two loops linked once. Each of them must be one fold twisted. The question is whether the twist of a phase along a given string is necessarily related to helicity [Eq. (6)]. The positive answer is suggested by the minimal energy configuration in Eq. (9). It is indeed so as can be shown with first principles. A one-fold twisted string loop must be stuck on another vortex. The total magnetic flux through a surface spanned on the loop $\vec{X}'(t,\sigma)$ is equal to $\pm\frac{4\pi}{\alpha}$ with the sign dependent on relative orientation. Inside the core of the vortex loop we can follow the lines of magnetic field. As the magnetic field is confined to the core each of its lines is stuck on the other vortex. For a chosen line the magnetic field can be approximated by

$$\delta\vec{B}=\delta\Phi\,\vec{t}\,\delta[\vec{x}-\vec{Y}(t,\sigma)], \tag{10}$$

where $\vec{t}$ is a unit vector tangent to the magnetic flux line $\vec{Y}$ and $\delta\Phi$ is a part of the total flux in a given line. The contribution to helicity (6) from the line is

$$\delta H=\delta B\int dl\;\vec{t}\cdot\vec{Z}=\delta B\left(\frac{4\pi}{\alpha}+\phi\right). \tag{11}$$

ϕ is a total flux of magnetic lines being linked with a given line. The first contribution is due to the vortex the loop is stuck on. When contributions from all magnetic field lines are put together the total helicity from a region around the loop due to the linking with the other loop is $\pm(\frac{4\pi}{\alpha})^2$. Under the plausible restrictions discussed at the beginning this part of helicity relies merely on topology of the vortex network.

If two string loops intercommute they form a single vortex loop. Now the topological contribution to helicity is zero. Lo [6] uses a magnetohydrodynamical (MHD) analogy to decide on how the magnetic field lines are reconnected during intercommutation. His argument suggests that the initial topological helicity is transformed into helicity associated with a twist of magnetic field lines. Now we will show that helicity of such a magnetic twist when averaged in time is zero.

Small fluctuations around the background of the Nielsen-Olesen vortex (8) were analyzed in [10]. The deformations of the fields were taken as

$$A^\alpha=W^\alpha(t,z)u(x,y), \tag{12}$$

where greek indices mean t or z. Other field components are unchanged to leading order. Field equations linearized in the above fluctuations are

$$\partial_\alpha W^\alpha=0,$$

$$-\partial_\beta\partial^\beta W^\alpha=m^2W^\alpha,$$

$$-\triangle u+M_Z^2f^2(r)u=m^2u, \tag{13}$$

where m^2 is a separation constant and $\triangle$ is a Laplacian in x,y. The second equation is the planar Schrödinger equation which is well known to have at least one bound state with an eigenvalue $0<m^2<M_Z^2$, which can be interpreted as a mass of the gauge field trapped within the core. With such an excitation the total helicity is

$$H(t)=\int dx\;dy\;dz\;(B^3Z^3+B^\theta Z^\theta), \tag{14}$$

where both Z^3 and B^θ come only from the excitation while other components are those of the background. For a localized magnetic twist [11] the integral along the z axis is convergent. With a use of the second equation in the set (13) and one integration by parts one easily obtains an equation of motion for the total helicity

$$\frac{d^2}{dt^2}H(t)=-m^2H(t). \tag{15}$$

Helicity oscillates around zero $H(t)=H_0\cos(mt)$ with H_0 being its initial value. The time dependence is nontrivial because of nonzero m^2; even the gauge field trapped within the vortex core has a nonzero mass. This result contradicts MHD analogy as such so it may also be dubious if there is any nonzero H_0 right after intercommutation. In MHD magnetic flux simply drifts together with the fluid and here it has its own fully relativistic dynamics. Even if the initial value of H is nonzero its time average vanishes $\langle H\rangle=0$. Thus the internal magnetic helicity, even if oscillations are not dumped by some dissipation process, is not related to the net baryon number. The net baryon number is related to and only to the topological helicity or in other words linking of string loops. Thus delinking of a pair of string loops by intercommutation changes the average helicity from its initial value given by topology to zero. The net baryon number is violated in this process. Intercommutation of Z-string loops violates baryon number.

To analyze changes of the topological helicity it is convenient to represent vortices as ribbons. One edge of a ribbon should be identified with a line of vanishing Higgs field. The rest of the ribbon should coincide with a surface of constant phase $\omega=\pi$ say. A string loop is p-fold twisted if the two edges of the ribbon are linked p times. An isolated and one-fold twisted vortex ring would be represented by a Mobius strip. It is not possible to span a surface of constant, say, zero phase on one edge of a Mobius strip without the other edge cutting it but if it cuts it will not any longer be a surface of constant phase. This is a new formulation of our previous argument that an unlinked vortex loop cannot be twisted.

The initially linked ribbons have to be twisted. The directions of the twists have to be correlated with the orientation of the link. One can construct a loop after intercommutation in the following way. Staple the ribbons together in an antiparallel fashion—strings do rearrange in this way just before intercommutation [14]. The ribbons coincide with a constant phase thus the edges of zero Higgs field have to be put together. Now cut the stapled part in the middle—real antiparallel string segments annihilate [14]. The result is a single untwisted ribbon as it should be according to our discussion. The initial twists have undone one another during intercommutation.

Turning this around one can take two separate ribbon loops which have to be untwisted. One can staple them together in an antiparallel fashion, cut, and reconnect the ends. What one obtains is an untwisted single string loop. In this case intercommutation does not change the topological helicity. Conversely if strings colliding with relativistic velocities just pass one through another local twists by 2π should appear to be in agreement with our discussion. The strings must reconnect in such a way that the surfaces of constant phase change continuously. This explains the usefulness of the Christmas ribbons toy model, which has been originally applied to reconnections of magnetic field lines in magnetohydrodynamics [15].

Now we will consider an example which strongly suggests that there is a family of solutions which continuously interpolates between two linked loops and two separate vortex loops. With the passage from the initial configuration to the final one helicity smoothly changes from its initial value to zero. Let us consider the Bogomol'nyi limit [12] of the Abelian Higgs model in dimensionless units:

$$L=-\tfrac{1}{4}Z_{\mu\nu}Z^{\mu\nu}+\tfrac{1}{2}D_\mu\psi^\star D^\mu\psi-\tfrac{1}{8}(\psi^\star\psi-1)^2, \tag{16}$$

with $D_\mu=\partial_\mu+iZ_\mu$. The model admits static planar two-vortex solutions

$$\psi=\psi(x,y,\lambda_A], \quad Z_\alpha=Z_\alpha(x,y,\lambda_A], \quad Z_k=0, \tag{17}$$

where $\alpha,\beta,\ldots$ mean 1,2 while $k,l,\ldots$ take values 0,3. λ's are a set of four real parameters defining positions of vortices. It was shown in [13] that for a coincident two-vortex configuration there are exact splitting modes in a form of traveling waves. The nonvanishing fields in Eq. (17) are modified by introducing time dependence through the parameters $\lambda_A=\lambda_A(t-z)$. The 0 and 3 components of gauge potential take the form

$$Z_k=\sum_A F_A(x,y,\lambda_A]\partial_k\lambda_A, \tag{18}$$

where the profile functions satisfy

$$\left(\frac{\partial}{\partial x^\alpha}\frac{\partial}{\partial x^\alpha}-\psi^\star\psi\right)F_A=-(\psi^\star\psi)\frac{\partial\omega}{\partial\lambda_A}, \tag{19}$$

with ω being an actual phase.

Let us consider the vortex configuration

$$\psi=z(z+X)W(z,z^\star), \quad X=X^1+iX^2, \tag{20}$$

where $W(z,z^\star)$ is a real positive function and traveling wave defined by $X=R\exp(ik_ix^i)$ with $k_ik^i\equiv k_0^2-k_z^2=0$. Such a solution is a vortex helix of radius R winding around a straight linear vortex and moving up the z-axis with the speed of light.

Let us consider first a situation of large R as compared to length scales of the model. An approximate solution to Eq. (19) is [13] $F_{(X^\beta)}=Z_\beta(\vec{x}+\vec{X})$ with $Z_\beta(\vec{x})$ being the potential of the single vortex solution (8). The z-component contribution to helicity per unit length is

$$\int d^2x\ Z^3B^3=-\int d^2x\ [B(|\vec{x}|)+B(|\vec{x}+\vec{X}|)]\times\sum_\beta Z_\beta(\vec{x}+\vec{X})\frac{\partial X^\beta}{\partial z}, \tag{21}$$

where $B(|\vec{x}|)$ is a magnetic field of the single vortex (9). Because of the symmetries of the fields a contribution to the above integral from around $-\vec{X}$ vanishes. For large R we can approximate under the integral $B(|\vec{x}|)\approx-2\pi\delta^{(2)}(\vec{x})$ and then perform the integration with the result $2\pi\omega$, where $\omega = k_0$. Induced components of magnetic field are $B^\alpha = -\Sigma_{\beta\gamma}\varepsilon_{\alpha\beta}Z_{\beta\gamma}(\partial X^\gamma/\partial z)$. Their contribution to helicity is

$$\sum_\alpha \int d^2x \;\; Z^\alpha B^\alpha = \sum_{\alpha\beta\gamma} \int d^2x \;\; [Z_\alpha(\vec{x}) + Z_\alpha(\vec{x}+\vec{X})] \times \varepsilon_{\alpha\beta} Z_{\beta\gamma}(\vec{x}+\vec{X}) \frac{\partial X^\gamma}{\partial z}. \qquad (22)$$

Once again we can use symmetry properties of the fields and then approximate $Z_{\beta\gamma}(\vec{x}+\vec{X}) \approx 2\pi\varepsilon_{\beta\gamma}\delta(\vec{x}+\vec{X})$. The result is $2\pi\omega$. Thus the total helicity per unit length is $2\times 2\pi\omega$. If we take into account that the straight linear vortex and the helix are linked once on a distance of $\frac{2\pi}{\omega}$ we conclude that the helicity per one link amounts to $2(-2\pi)^2$ what is nothing else than the expected $2\Phi^2$.

We have calculated helicity for large R. As the helix radius R is turned to zero the helicity per unit length also smoothly diminishes to zero. For $R=0$ we have just a single vortex with a winding number 2. Now the vortex can be split into two unit vortices which can be moved apart. In this way we have constructed a family of solutions which interpolates between a linked pair of vortices and straight vortices standing far apart. The helicity changed continuously from its initial value to zero. We have considered only infinitely long vortices but the result should be qualitatively the same for very large vortex loops. Two loops which were initially linked can be delinked into two fairly separated loops with a continuous change in helicity. For this way of delinking one would require much more restrictive initial conditions than for intercommutation of loop segments so we think intercommutation to be the dominant process.

Let us summarize the scenario of baryon number violation which has been clarified in this paper. The initial configuration is a pair of linked Z-string loops. Linking of the loops enforces them to be twisted. There are two channels of decay. The first is a decay of the linked loops into twisted string segments terminated by monopoles. Contrary to the statement in [5] this process can take place before delinking. If the segments untwist and shrink helicity then the baryon number will be violated. The final configuration is a set of sphalerons and antisphalerons their number and orientation dependent on details of string breaking and shrinking. The CS number of a sphaleron configuration is $\frac{1}{2}$ but only in a gauge which is unitary at infinity [2,16]. Without such a gauge the helicity of a parity-odd configuration is zero.

The other channel is through delinking of the loops by intercommutation or just passing one through another. In both cases we get a single loop or two separate loops which are untwisted. Delinking has changed helicity. Now the loops can decay but this time segments are untwisted and there is no further helicity violation. Thus both channels of decay lead to violation of baryon number. In collisions of infinite strings intercommutation would not change helicity but super-relativistic collisions in which strings pass one through another would introduce local twists and violate baryon number.

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