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Scale factor duality and hidden supersymmetry in scalar-tensor cosmology

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It is shown that spatially flat, isotropic cosmologies derived from the Brans-Dicke gravity action exhibit a scale factor duality invariance. This classical duality is then associated with a hidden $N=2$ supersymmetry at the quantum level and the supersymmetric quantum constraints are solved exactly. These symmetries also apply to a dimensionally reduced version of vacuum Einstein gravity.

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The possibility that scalar-tensor gravity theories may be relevant in the arena of the very early Universe has been widely investigated in recent years [1–5]. The defining feature of these theories is the nonminimal coupling of a scalar field to the space-time curvature. These couplings arise naturally in the low energy limit of various unified field theories such as superstring theory [4,6]. Theories containing higher-order terms in the Ricci scalar may also be expressed in a scalar-tensor form by means of a suitable conformal transformation [7]. Furthermore, the dimensional reduction of higher-dimensional gravity also results in an effective scalar-tensor theory [8].

The study of scalar-tensor theories is therefore well motivated. In this paper we consider the D-dimensional vacuum theory

$$S=\int d^Dx\sqrt{-g}e^{-\Phi}[R-\omega(\nabla\Phi)^2-2\Lambda], \qquad (1)$$

where Φ represents the dilaton field, g is the determinant of the space-time metric $\mathscr{G}$, ω is a space-time constant, and Λ is the effective cosmological constant. This theory represents one of the simplest extensions to Einstein gravity and is formally equivalent to the standard Brans-Dicke theory when $\Lambda=0$ [9]. It represents the genus-zero effective action for the bosonic string when the antisymmetric tensor field $B_{\mu\nu}$ vanishes, $\omega=-1$ and $\Lambda=(D-26)/3\alpha'$, where α' is the inverse string tension [6]. The effective action for the bosonic sector of the closed superstring also has this form, but with Λ proportional to $(D-10)$.

It is known that the reduced string action corresponding to cosmologies that are spatially flat and homogeneous exhibits a symmetry known as *scale factor duality* [10]. The symmetry group is Z_2^{D-1} and it is generated by inverting the scale factor and shifting the value of the dilaton. Scale factor duality is a special case of a more general $O(d,d)$ symmetry of the theory and it relates expanding dimensions to contracting ones [11]. It also forms the basis for the pre-big bang cosmological model developed by Gasperini and Veneziano [5].

In general, it is important to search for symmetries in a given theory because they can provide valuable insight into the dynamics of the Universe. They form the basis for selection rules that forbid the existence of certain states and processes. They also allow one to generate new, inequivalent solutions to the field equations from a given solution. Moreover, the conserved quantity associated with a given symmetry represents an integrability condition on the field equations. In some cases, this may allow more general solutions to be found.

In this paper we find the analogue of scale factor duality in theory (1) when $\omega\neq-1$. We show that a duality invariance exists for spatially flat, isotropic cosmologies of arbitrary dimension $D\geqslant2$ when $\omega\neq-D/(D-1)$. We then consider the quantization of this family of Universes by mapping the system onto the constrained oscillator-ghost-oscillator model. This procedure uncovers a hidden supersymmetry in the theory that exists at the quantum level. The origin of this supersymmetry may be traced to the scale factor duality invariance of the classical action. The superquantum constraints, corresponding to the Dirac-type square root of the Wheeler-DeWitt equation, are then solved exactly in closed form.

We express the space-time metric as $\mathscr{G}=\mathrm{diag}[-N(t),G(t)]$, where t represents cosmic time, $N(t)$ is the lapse function, and the $(D-1)\times(D-1)$ matrix $G(t)$ is the metric on the spatial hypersurfaces. We further assume that the dilaton field is constant on these surfaces and that $\omega>-(D-1)/(D-2)$. It then follows that action (1) reduces to

$$S=\int dte^{\ln\sqrt{\det G}-\Phi}\left(\frac{1}{N}[2\partial_t^2\ln\sqrt{\det G}+(\partial_t\ln\sqrt{\det G})^2-\tfrac{1}{4}\mathrm{Tr}(\partial_tG\partial_tG^{-1})+\omega(\partial_t\Phi)^2]-2N\Lambda\right), \qquad (2)$$

where $\partial_t\equiv\partial/\partial t$. The comoving volume of the Universe has been normalized to unity in this expression without loss of generality. For the class of spatially flat, isotropic and homogeneous Universes, the action simplifies further to

$$S=\int dte^{(D-1)\alpha-\Phi}\left(\frac{1}{N}[-(D-1)(D-2)\dot{\alpha}^2+2(D-1)\dot{\alpha}\dot{\Phi}+\omega\dot{\Phi}^2]-2N\Lambda\right), \qquad (3)$$

where $e^{\alpha(t)}$ represents the scale factor of the Universe, an overdot denotes differentiation with respect to t, and a boundary term has been neglected. The field equations derived from this action take the form

$$2\Lambda=2(D-2)\ddot{\alpha}-2\ddot{\Phi}+(D-1)(D-2)\dot{\alpha}^2+(2+\omega)\dot{\Phi}^2-2(D-2)\dot{\alpha}\dot{\Phi},$$

$$2\Lambda=2(D-1)\ddot{\alpha}+2\omega\ddot{\Phi}+D(D-1)\dot{\alpha}^2-\omega\dot{\Phi}^2+2(D-1)\omega\dot{\alpha}\dot{\Phi},$$

$$2\Lambda=(D-1)(D-2)\dot{\alpha}^2-\omega\dot{\Phi}^2-2(D-1)\dot{\alpha}\dot{\Phi}, \quad (4)$$

in the gauge $N=1$, where the third expression represents the Hamiltonian constraint.

If we equate the first two expressions in Eq. (4) and integrate, we find that

$$F\equiv e^{(D-1)\alpha-\Phi}[\dot{\alpha}+(1+\omega)\dot{\Phi}] \quad (5)$$

is a constant of motion, i.e., $\dot{F}=0$. The conservation of this quantity is associated with a nontrivial symmetry of the action (3). It is known that this action is invariant under time reversal $t=-\tilde{t}$. However, it is also symmetric under the simultaneous transformation

$$\alpha=\left[\frac{(D-2)+(D-1)\omega}{D+(D-1)\omega}\right]\tilde{\alpha}-\left[\frac{2(1+\omega)}{D+(D-1)\omega}\right]\tilde{\Phi},$$

$$\Phi=-\left[\frac{2(D-1)}{D+(D-1)\omega}\right]\tilde{\alpha}-\left[\frac{(D-2)+(D-1)\omega}{D+(D-1)\omega}\right]\tilde{\Phi}, \quad (6)$$

for all $D\geq 2$ and all $\omega\neq -D/(D-1)$. It is straightforward to verify that the form of F remains invariant under these simultaneous interchanges. The two-dimensional version of this symmetry has been discussed recently by Cadoni and Cavaglià [12] and Eq. (6) reduces to the scale factor duality invariance of the string effective action when $\omega=-1$. In this case, $\alpha=-\tilde{\alpha}$ and $\Phi=-2(D-1)\tilde{\alpha}+\tilde{\Phi}$ [10].

When $\omega\neq -D/(D-1)$, Eqs. (4) admit the exact solution $\Phi=vt$ and $\alpha=Ht$, where $H=-(1+\omega)v$ and

$$v^2=\frac{2\Lambda}{[D-1+(D-2)\omega][D+(D-1)\omega]}. \quad (7)$$

This solution applies in the regime $\omega>-D/(D-1)$ when $\Lambda>0$ and $\omega<-D/(D-1)$ when $\Lambda<0$. It is an attractor to the general solution of Eqs. (4) in the limit $t\rightarrow+\infty$ [2]. Application of Eq. (6) maps this attractor onto itself, so the solution is "self-dual." It is interesting that the symmetry (6) does not hold for $\omega=-D/(D-1)$ and this feature appears to be related to the form of the attractor. Consequently, we shall not consider this particular value further.

Before quantizing this model it proves convenient to express Eq. (3) in a manifestly canonical form by means of an appropriate change of variables. For $D\geq 3$ we define

$$x\equiv\exp\left[\left(\frac{D-1}{2}+\frac{\gamma}{2}\right)\left(\alpha+\frac{1}{D-2}\left(\frac{1}{\gamma}-1\right)\Phi\right)\right],$$

$$y\equiv\exp\left[\left(\frac{D-1}{2}-\frac{\gamma}{2}\right)\left(\alpha-\frac{1}{D-2}\left(\frac{1}{\gamma}+1\right)\Phi\right)\right], \quad (8)$$

where

$$\gamma\equiv\left[\frac{D-1}{D-1+(D-2)\omega}\right]^{1/2}. \quad (9)$$

On the other hand, we define

$$x\equiv e^{\alpha+\omega\Phi/2}, \quad y\equiv e^{-(\omega+2)\Phi/2}, \quad (10)$$

if $D=2$. Action (3) is therefore given by

$$S=\int dt\left[-\frac{4}{N}\left(\frac{D-1+(D-2)\omega}{D+(D-1)\omega}\right)\dot{x}\dot{y}-2N\Lambda xy\right] \quad (11)$$

for all $D\geq 2$.

A second coordinate pair

$$w\equiv\epsilon^{1/2}\left[\frac{D-1+(D-2)\omega}{D+(D-1)\omega}\right]^{1/2}(x-y),$$

$$z\equiv\epsilon^{1/2}\left[\frac{D-1+(D-2)\omega}{D+(D-1)\omega}\right]^{1/2}(x+y), \quad (12)$$

may now be introduced, where $\epsilon=+1$ if $\omega>-D/(D-1)$ and $\epsilon=-1$ if $\omega<-D/(D-1)$. It follows that action (11) transforms to

$$S=\frac{1}{\epsilon}\int dt\left[\frac{1}{N}(\dot{w}^2-\dot{z}^2)-\frac{\lambda}{4}(w^2-z^2)N\right], \quad (13)$$

where

$$\lambda\equiv-2\Lambda\left[\frac{D+(D-1)\omega}{D-1+(D-2)\omega}\right]. \quad (14)$$

This is the action for the constrained oscillator-ghost-oscillator pair when $\lambda>0$ [13]. The pair oscillate with identical frequency, but have equal and opposite energy. The general solution to the classical field equations of this model may be represented by a family of ellipses in the (w,z) plane [14]. The major axis of the trajectories is given by $w=\pm z$ and the eccentricities are determined by the integration constants. Physical solutions for the scalar-tensor action (3) are restricted to the $|w|\leq z$ sector of the plane.

Rewriting the system in terms of these new variables is useful because the scale factor duality invariance of action (3) becomes more apparent. Substitution of Eq. (6) into Eqs. (8) and (10) implies that the duality transformation is formally equivalent to the simultaneous interchange of the canonical variables $x\leftrightarrow y$. Action (11) is clearly symmetric under this interchange. Moreover, it then follows directly from the definition (12) that the transformation (6) may also be generated by $w\rightarrow -w$ and $z\rightarrow z$.

The classical Hamiltonian for this system is given by

$$2H_0=G^{\mu\nu}p_\mu p_\nu+W(q^\mu)=0, \quad (15)$$

where $\mu,\nu=0,1$, and $G^{\mu\nu}=\mathrm{diag}[-1/2,1/2]$ is proportional to the inverse of the metric over the configuration space. This space is spanned by the "timelike" coordinate $q^0=w$ and "spacelike" coordinate $q^1=z$. The momenta conjugate to these variables are $p_0=\partial S/\partial q^0=2\dot{w}/N$ and $p_1=\partial S/\partial q^1=-2\dot{z}/N$, respectively, and the potential is given by

$$W=-\lambda(w^2-z^2)/2. \tag{16}$$

In the standard approach to quantum cosmology, one views H_0 as an operator that annihilates the state vector Ψ of the Universe, i.e., $H_0\Psi=0$. This is the Wheeler-DeWitt equation [15]. To quantize the model one imposes the algebra $[q^\mu,p^\nu]_-=i\delta^{\mu\nu}$ and this is realized by identifying $p^\mu=-i\partial_\mu$ ($\hbar=1$). If we neglect any ambiguities that may arise due to factor ordering, the Wheeler-DeWitt equation takes the form

$$\left[\frac{\partial^2}{\partial w^2}-\frac{\partial^2}{\partial z^2}-\lambda(w^2-z^2)\right]\Psi=0 \tag{17}$$

and admits the family of solutions

$$\Psi_n=H_n(\lambda^{1/4}w)H_n(\lambda^{1/4}z)e^{-\sqrt{\lambda}(w^2+z^2)/2}, \tag{18}$$

where H_n is the Hermite polynomial of order n. When $\lambda>0$, these solutions form a discrete basis for any bounded wave function $\Psi=\Sigma c_n\Psi_n$, where c_n are complex coefficients [14,16,17]. The ground state corresponds to $n=0$ and excited states to $n>0$. The ground state is symmetric under the action of Eq. (6). These states do not oscillate and therefore represent Euclidean geometries [18]. Oscillating wave functions representing Lorentzian geometries may be found, however, when the c_n take appropriate values [14].

Further insight may be gained by introducing new variables $u=\sqrt{\lambda}(w+z)^2/4$ and $v=\sqrt{\lambda}(w-z)^2/4$. This transforms the Wheeler-DeWitt equation (17) into the unit-mass Klein-Gordon equation. One family of solutions to this equation is given by $\Psi_b=e^{-bu-v/b}$, where b is an arbitrary, complex constant and $|\Psi_b|$ is bounded for Re$b>0$. This family represents the basis for a continuous spectrum of states of the form $\Psi=\int_C dbM(b)\Psi_b$, where $M(b)$ is an arbitrary function of b and C is some contour in the Re$b>0$ sector of the complex b plane [16]. Since the $\Psi_{b=1}$ solution coincides with the ground state $\Psi_{n=0}$ of the discrete spectrum (18), it may be viewed as the ground state of this continuous spectrum. The excited states therefore correspond to $b\neq 1$. Moreover, the transformation (6) is equivalent to the simultaneous interchange $u\leftrightarrow v$. Thus, the ground state of the continuous spectrum respects the symmetry of the action (3), but the excited states do not. In this sense, therefore, the ground state represents the maximally symmetric, self-dual solution.

We shall now illustrate how the invariance of action (3) under Eq. (6) implies that the classical Hamiltonian H_0 is the bosonic component of a supersymmetric Hamiltonian when $\lambda>0$. In general, there exists a hidden supersymmetry in the theory if there is a solution to the Euclidean Hamilton-Jacobi equation that respects the same symmetries as H_0 [19]. The Euclidean Hamilton-Jacobi equation has the form

$$G^{\mu\nu}\frac{\partial I}{\partial q^\mu}\frac{\partial I}{\partial q^\nu}=W, \tag{19}$$

where $I=I(q^\mu)$ and, for the model we are considering here, admits the exact solution

$$I=\sqrt{\lambda}(w^2+z^2)/2. \tag{20}$$

This solution is manifestly invariant under the duality transformation $w\to -w$ and $z\to z$.

The system with Hamiltonian (15) may now be quantized by considering the quantum Hamiltonian H defined by

$$2H=[Q,\bar{Q}]_+,\quad Q^2=\bar{Q}^2=0, \tag{21}$$

where Q is a non-Hermitian supersymmetry charge with adjoint $\bar{Q}$. It follows that $[H,Q]_-=[H,\bar{Q}]_-=0$ and Eq. (21) represents the algebra for an $N=2$ supersymmetry [19,20]. The supercharges are linear operators of the form

$$Q\equiv\varphi^\mu\left(p_\mu+i\frac{\partial I}{\partial q^\mu}\right),\quad \bar{Q}\equiv\bar{\varphi}^\mu\left(p_\mu-i\frac{\partial I}{\partial q^\mu}\right), \tag{22}$$

where the fermionic degrees of freedom $\varphi^\mu,\bar{\varphi}^\nu$ satisfy the spinor algebra

$$[\varphi^\mu,\varphi^\nu]_+=[\bar{\varphi}^\mu,\bar{\varphi}^\nu]_+=0,\quad [\varphi^\mu,\bar{\varphi}^\nu]_+=G^{\mu\nu}, \tag{23}$$

and the bosonic degrees of freedom satisfy the usual anticommutation relation. It follows, therefore, that the quantum Hamiltonian has the form

$$H=H_0+\frac{\hbar}{2}\frac{\partial^2 I}{\partial q^\mu\partial q^\nu}[\bar{\varphi}^\mu,\varphi^\nu]_- \tag{24}$$

and reduces to the bosonic Hamiltonian (15) in the classical limit. However, it acquires an additional spin term at the quantum level whose form suggests that imaginary solutions to Eq. (19) will be inappropriate. In view of this we restrict the analysis to the region of parameter space where $\lambda>0$. This corresponds to $\omega<-D/(D-1)$ when $\Lambda>0$ and $\omega>-D/(D-1)$ if $\Lambda<0$. We note that $\lambda>0$ for the effective bosonic (super-) string action if $D<26$ ($D<10$). The existence of a hidden supersymmetry in the two-dimensional bosonic string cosmology has recently been discussed in Ref. [21].

A suitable representation for the fermionic sector of the supersymmetric algebra is given in terms of the Grassmann variables θ^μ and their derivatives, i.e., $\bar{\varphi}^\mu=\theta^\mu$ and $\varphi^\mu=G^{\mu\rho}\partial/\partial\theta^\rho$. The general form of the supersymmetric wave function may then be expanded in terms of these variables such that $\Psi=A_++B_0\theta^0+B_1\theta^1+A_-\theta^0\theta^1$, where the coefficients $A_\pm$, B_0, and B_1 are functions of (w,z) only [22]. This wave function is annihilated by the supercharges $Q\Psi=\bar{Q}\Psi=0$ and it is these constraints that represent the Dirac-type square root of the Wheeler-DeWitt equation. They are given by a set of coupled, linear differential equations:

$$\left[\frac{\partial}{\partial w}+\frac{\partial I}{\partial w}\right]A_+=0, \quad \left[\frac{\partial}{\partial z}+\frac{\partial I}{\partial z}\right]A_+=0,$$

$$\left[\frac{\partial}{\partial w}+\frac{\partial I}{\partial w}\right]B_1-\left[\frac{\partial}{\partial z}+\frac{\partial I}{\partial z}\right]B_0=0,$$

$$\left[\frac{\partial}{\partial w}-\frac{\partial I}{\partial w}\right]B_0-\left[\frac{\partial}{\partial z}-\frac{\partial I}{\partial z}\right]B_1=0,$$

$$\left[\frac{\partial}{\partial w}-\frac{\partial I}{\partial w}\right]A_-=0, \quad \left[\frac{\partial}{\partial z}-\frac{\partial I}{\partial z}\right]A_-=0, \tag{25}$$

where I is given by Eq. (20).

These equations may be solved exactly and the supersymmetric wave function has the form

$$\Psi=e^{-I}+2n\lambda^{1/4}[H_{n-1}(\lambda^{1/4}w)H_n(\lambda^{1/4}z)\theta^0 + H_n(\lambda^{1/4}w)H_{n-1}(\lambda^{1/4}z)\theta^1]e^{-I}+e^I\theta^0\theta^1, \tag{26}$$

where $n\geq 0$. The solutions $A_\pm$ represent the empty and filled fermion sectors of the wave function, respectively. As can be seen directly from Eq. (26), they are manifestly self-dual. Moreover, the empty fermion sector of this wave function is identical to the ground states $\Psi_{n=0}$ and $\Psi_{b=1}$ of the purely bosonic spectra discussed above. Hence, this supersymmetric approach to quantum cosmology naturally selects the bosonic state of lowest energy.

In summary, we have found that a class of scalar-tensor cosmologies, including the vacuum Brans-Dicke model, exhibit a scale factor duality invariance. By mapping the model onto the zero-energy oscillator-ghost-oscillator pair, we identified a continuous spectrum of quantum states whose ground state is the self-dual wave function. We further showed that this duality symmetry of the classical action is associated with a hidden supersymmetry that exists at the quantum level in a wide region of parameter space. Included in this regime is the bosonic string effective action. This is important because supersymmetric quantum cosmology may be able to resolve the problems encountered in the standard approach when one attempts to construct a non-negative norm from the wave function [22]. Furthermore, Dereli, Önder, and Tucker [23] have developed an alternative spinor model of quantum cosmology that is also based on the constrained oscillator-ghost-oscillator. The direct correspondence between Eqs. (3) and (13) suggests that their results will also apply for the model discussed here and it would be of interest to compare and contrast these two approaches in more detail.

It is also of interest to investigate whether the scale factor duality invariance of action (1) can be extended to more general space-times. The simplest extensions are to include spatial curvature and anisotropy. However, spatial curvature introduces a term proportional to $e^{-2\alpha}$ into the Lagrangian of action (3) and this explicitly breaks the symmetry of the model. Thus, within the context of spatially isotropic Universes, the duality symmetry and associated hidden supersymmetry of action (1) are only respected by the spatially flat model.

If the Universe is anisotropic, however, with a line element given by $ds^2=-dt^2+e^{2\alpha_i(t)}dx_i^2$, Eq. (1) takes the form

$$S=\int dt\,\exp\left(\sum_i \alpha_i-\Phi\right)\left[2\dot{\Phi}\sum_i \dot{\alpha}_i-\sum_{i,j}\dot{\alpha}_i\dot{\alpha}_j + \sum_i \dot{\alpha}_i^2+\omega\dot{\Phi}^2-2\Lambda\right], \tag{27}$$

where the summations are over $i,j=1,\ldots,(D-1)$. We may now consider the generalization of transformation (6) given by

$$\alpha_i=c_1\tilde{\alpha}_i+c_2\tilde{\Phi}, \quad \Phi=c_3\sum_{i=1}^{D-1}\tilde{\alpha}_i+c_4\tilde{\Phi}, \tag{28}$$

where the c_l's are arbitrary constants. Substitution of Eq. (28) into Eq. (27) leads to six constraints that must be simultaneously satisfied if the action is to remain invariant. However, the only nontrivial solution to these constraints is given by $c_1=-1$, $c_2=0$, $c_3=-2$, $c_4=1$, and $\omega=-1$. Thus, the scale factor duality invariance (28) is unique to the string effective action if an anisotropic Universe is considered. If $\omega\neq -1$, the symmetry only holds for spatially flat, isotropic models.

Although these conclusions apply to cosmologies derived from the scalar-tensor action (1), they are also relevant to the higher-dimensional, vacuum Einstein action $S=\int d^Tx\sqrt{-g}[R-2\Lambda]$, where $T>D$. In general, scale factor duality maps a constant gravitational coupling onto a time-dependent one. Consequently, it is not respected by Einstein gravity when the Universe is spatially isotropic. However, it can be if a suitable dimensional reduction of the theory is considered. If the higher-dimensional space-time M is viewed as the product $M=R\times J\times K$, where J is the $(D-1)$-dimensional "external" space and K is a Ricci-flat, isotropic d-dimensional space, the dimensionally reduced action has the form of Eq. (1), where $\omega=-1+1/d$ and Φ is related to the radius of K. Since $\omega>-1$ in this model for all physical values of d, it follows from the above discussion that the dimensionally reduced action will not be invariant under Eq. (28) if the external space is anisotropic. On the other hand, the action is invariant under the classical scale factor duality (6) if J is isotropic and spatially flat. In this case, one may then verify that there exists a hidden $N=2$ supersymmetry for all values of D and d if $\Lambda<0$, but not if $\Lambda>0$.

In conclusion, therefore, we have identified a dimensionally reduced version of the higher-dimensional, vacuum Einstein action that is invariant under a scale factor duality transformation. This symmetry is respected if the higher-dimensional Universe is anisotropic with a spatial geometry given by the product of two spaces that are both flat and isotropic. The symmetry discussed in this work is a natural generalization of the scale factor duality exhibited by the genus-zero string effective action. If the cosmological constant is negative, it may be further associated with a hidden $N=2$ supersymmetry.

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