

Duality symmetric actions with manifest space-time symmetries

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We consider a space-time-invariant duality symmetric action for a free Maxwell field and an $SL(2, \mathbf{R}) \times SO(6, 22)$ -invariant effective action describing a low-energy bosonic sector of the heterotic string compactified on a six-dimensional torus. The manifest Lorentz- and general coordinate-invariant formulation of the models is achieved by coupling dual gauge fields to an auxiliary vector field from an axionic sector of the theory.

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I. INTRODUCTION

The understanding of the important role played by duality in (super-)Yang-Mills theories [1–3], supergravity, and the theory of strings [4–8] has allowed one to gain new insight into the structure of these theories and to find a deep relationship between their different (dual) versions. If target space duality (T duality) and S duality (which is the generalization of the electric-magnetic duality) are the exact symmetries of string theory [4–6], it is natural to assume that there should be a version of the theory where these duality symmetries are manifest.

A T -duality symmetric string action was proposed by Tseytlin [9] and generalized to a case of the heterotic string by Schwarz and Sen [10]. This required modification of space-time transformations of fields. Note, however, that effective supersymmetric field actions which describe the low-energy behavior of superstrings do have a global noncompact symmetry related to T duality, while S duality is only a symmetry of the equations of motion [11–13].

In [13] Schwarz and Sen proposed models for describing antisymmetric gauge fields in D -dimensional space-time, where S -duality symmetry was lifted to the level of action, their results being the generalization of earlier work by Floreanini and Jackiw [14] and Henneaux and Teitelboim [15] who constructed actions for describing self-dual tensor fields in $(4p+2)$ -dimensional space-time ($p=0, 1, \dots$). The general feature of the models [9, 10, 13–15] is that, because of the explicit fixing of the time direction, they lose manifest Lorentz and general coordinate invariance, which, however, are replaced by some modified transformations. This is an example of how duality affects the symmetry structure of the theory.

In addition, one may try to look for a formulation of duality symmetric actions in which conventional space-time symmetries are restored. An attempt to do this for a duality symmetric version of Maxwell theory¹ [13] was undertaken

in [17]. Having been inspired by this paper we proposed in [18] a manifestly space-time and duality symmetric formulation of (supersymmetric) Maxwell theory by enlarging the Schwarz-Sen model [13] with an auxiliary vector field $u_m(x)$ and an antisymmetric tensor field $\Lambda_{mn}(x)$ ($m=0, 1, 2, 3$) in such a way that upon solving for equations of motion and gauge-fixing additional local symmetries associated with the auxiliary fields the model [18] could be reduced to that of Schwarz and Sen. The physical nature of $u_m(x)$ and $\Lambda_{mn}(x)$ was supposed to be a relic of a gravitational vielbein and an axionlike potential, respectively.

In the present paper we further develop the duality symmetric model of [18]. In particular, we will show that the model indeed has a local symmetry (anticipated in [18]) allowing one to choose $u_m(x)$ to be a constant unit-norm timelike vector and, thus, to demonstrate that the duality symmetric models [13], with their nonconventional space-time symmetries, correspond to a definite gauge choice for auxiliary fields in corresponding duality symmetric models with ordinary Lorentz and general coordinate invariance. Using the example of a model for two Abelian gauge fields in $D=1+3$, we shall show that duality between these two fields arises due to their specific coupling to a pseudoscalar sector of the theory through the field $u_m(x)$, and the latter can be gauge fixed to a constant timelike vector by use of a local counterpart of a Peccei-Quinn symmetry of axion models. Thus $u_m(x)$ can originate, in fact, from the pseudoscalar sector of the theory and, at the same time, carry some properties of a local Lorentz frame field. In Sec. III we will present a space-time and $SL(2, \mathbf{R}) \times O(6, 22)$ -invariant effective action which describes a low-energy limit of a toroidally compactified heterotic string.

II. DUALITY SYMMETRIC MAXWELL ACTION

Let us start with an action describing a free Maxwell field $A_m(x)$ and a pseudoscalar ("axion") field $a(x)$ in $D=1+3$ Minkowski space:

$$S = \int d^4x \left\{ -\frac{1}{4} F_{mn} F^{mn} - \frac{1}{2} [\partial_m a(x) - u_m(x)] \times [\partial^m a(x) - u^m(x)] - \epsilon^{pqmn} u_p \partial_q \Lambda_{mn} \right\}, \quad (1)$$

where the first term is the ordinary Maxwell Lagrangian with $F_{mn} = \partial_m A_n(x) - \partial_n A_m(x)$ and the last two terms form the

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¹Earlier an alternative version of duality invariant Maxwell action was considered in [16].

Lagrangian which, upon solving the equations of motion for $u_m(x)$ and using a local symmetry of (1) under transformations,

$$\delta a(x) = \varphi(x), \quad \delta u_m(x) = \partial_m \varphi(x), \quad (2)$$

produces (see, for the details [19–23] and references therein) the free Lagrangian for the scalar field $a(x)$

$$L = -\frac{1}{2} \partial_m a(x) \partial^m a(x), \quad (3)$$

or its dual

$$L = \frac{1}{3!} \partial_{[m} \Lambda_{np} \partial^{[m} \Lambda^{np]}, \quad (4)$$

with the duality relation between $a(x)$ and the antisymmetric field Λ_{np} to be

$$\partial^l a(x) = \epsilon^{lmnp} \partial_m \Lambda_{np}. \quad (5)$$

The role of $u_m(x)$ in (1) is to be the gauge field of the symmetry (2) and to ensure (5) on the mass shell.

In addition to (2) the action (1) is invariant under the Abelian gauge transformations of $A_m(x)$ and $\Lambda_{mn}(x)$:

$$\delta A_m(x) = \partial_m b(x), \quad \delta \Lambda_{mn} = \partial_{[m} b_{n]}(x). \quad (6)$$

Note that without violating local symmetry (2) one can couple fields from the pseudoscalar sector of (1) to the gauge field in an axionlike fashion:

$$S_{\text{int}} = - \int d^4x (\partial_m a - u_m) \epsilon^{mnpq} A_n F_{lp}. \quad (7)$$

The sum of (1) and (7) is explicitly invariant under (2). It is also invariant under the gauge transformations (6) if one requires that the variation of Λ_{mn} acquires the contribution $\delta_b \Lambda_{mn} = b(x) F_{mn}$. Eliminating either u_m and Λ_{mn} or u_m and $a(x)$ by use of equations of motions and local symmetries one gets the dual versions of the axion theory [20–23].

Now the question arises whether it is possible to replace (1) with an action which would be duality symmetric in the Maxwell field sector, still possess ordinary Lorentz invariance, and be equivalent (at least classically) to the action (1). The answer turns out to be positive [18].

From [13] we learn that for making the electric-magnetic duality manifest at the level of action one has to double the number of Abelian fields [introducing $A_m^\alpha(x)$ ($\alpha=1,2$)] and construct a duality symmetric action in such a way that equations of motion obtained from this action lead to the vanishing of the self-dual tensor

$$\mathcal{F}_{mn}^\alpha = \mathcal{L}^{\alpha\beta} F_{mn}^\beta - F_{mn}^{*\alpha} = \frac{1}{2} \epsilon_{mnpq} \mathcal{L}^{\alpha\beta} \mathcal{F}^{\beta pq}, \quad (8)$$

where $\mathcal{L}^{\alpha\beta} = -\mathcal{L}^{\beta\alpha}$ ($\mathcal{L}^{\alpha\alpha} = 1$) and $F_{mn}^{*\alpha} = \frac{1}{2} \epsilon_{mnpq} F^{lpq}$.

When

$$\mathcal{F}_{mn}^\alpha = 0 \quad (9)$$

one of the Abelian fields becomes completely determined through another one, and for the latter we get the free Maxwell equations by differentiating (9). Then the duality (9) between the two gauge fields reduces to the duality between the electric and magnetic strength of the gauge field which has been chosen to be independent.

The duality symmetric action proposed in [13], which gives (9), has the form

$$S = -\frac{1}{2} \int d^4x (B^{i\alpha} \mathcal{L}^{\alpha\beta} E_i^\beta + B^{i\alpha} B_i^\alpha), \quad (10)$$

where

$$E_i^\alpha = F_{0i}^\alpha = \partial_0 A_i^\alpha - \partial_i A_0^\alpha, \quad B^{i\alpha} = \frac{1}{2} \epsilon^{ijk} F_{jk}^\alpha = \epsilon^{ijk} \partial_j A_k^\alpha, \quad (11)$$

and $i, j, k = 1, 2, 3$ are spacial indices. The action (10) is invariant under modified space-time transformations of A_i^α (in the gauge $A_0^\alpha = 0$):

$$\delta A_i^\alpha = x^0 v^k \partial_k A_i^\alpha + v^k x^k \partial_0 A_i^\alpha + v^k x^k \mathcal{L}^{\alpha\beta} \mathcal{F}_{0i}^\beta, \quad (12)$$

where the first two terms describe the ordinary Lorentz boosts along a constant velocity v^i and the third term vanishes on the mass shell (9).

The model constructed this way [13] is classically and quantum mechanically [24] equivalent to the free Maxwell theory.

The covariantization of (10) is achieved by coupling the self-dual tensor (8) to the auxiliary field $u_m(x)$ from the pseudoscalar part of the Lagrangian (1) as follows:

$$S = \int d^4x \left(-\frac{1}{8} F_{mn}^\alpha F^{\alpha mn} + \frac{1}{4(-u_\mu u^\mu)} u^m \mathcal{F}_{mn}^\alpha \mathcal{F}^{\alpha np} u_p - \frac{1}{2} (\partial_m a - u_m) (\partial^m a - u^m) - \epsilon^{mnpq} u_m \partial_n \Lambda_{pq} \right). \quad (13)$$

Action (13) differs from that considered in [18] at the following point. In [18] $u_m(x)$ was required to have the negative unit norm

$$u^2 \equiv u_m u^m = -u_0 u_0 + u_i u_i = -1 \quad (14)$$

and played the role of a component of a local Lorentz frame. In (13) we weakened the normalization condition by introducing the norm of $u_m(x)$ only into the term containing \mathcal{F}_{mn}^α . We shall demonstrate the relationship between the two versions of the model later on.

The necessity to introduce the norm of $u_m(x)$ into the \mathcal{F}_{mn}^α term is dictated by the requirement to preserve the local symmetry (2). The action (13) is invariant under the transformations (2) provided $A_m^\alpha(x)$ and $\Lambda_{mn}(x)$ are transformed as follows:

$$\delta A_m^\alpha = \frac{\varphi(x)}{u^2} \mathcal{L}^{\alpha\beta} \mathcal{F}_{mn}^\beta u^n, \quad \delta \Lambda_{mn} = \frac{\varphi(x)}{(u^2)^2} \mathcal{F}_m^\alpha u_r \mathcal{F}_n^{\beta s} u_s \mathcal{L}^{\alpha\beta}. \quad (15)$$

Then, since the solution to the equation of motion of Λ_{mn} , obtained from (13), is

$$u_m(x) = \partial_m \hat{\varphi}(x), \quad (16)$$

where $\hat{\varphi}(x)$ is a scalar function, we can use the transformations (2) to put² $u_m = \delta_m^0$. In this gauge the action (13) reduces to (10), and the transformation of A_m^α in (15) (with $\varphi = x^i v^i$) is combined with the corresponding Lorentz transformation producing (12).

We see that $u_m(x)$ plays a double role. On the one hand it is the gauge field of local Peccei-Quinn symmetry and on the other hand it corresponds to a component of a local Lorentz frame.

The action (13) has another local symmetry [18] [which generalizes that of (10) [13]] under the following transformations of A_m^α and Λ_{mn} :

$$A_m^\alpha \rightarrow A_m^\alpha + u_m \varphi^\alpha(x), \quad (17)$$

$$\Lambda_{mn} \rightarrow \Lambda_{mn} - \frac{\varphi^\alpha}{u^2} \mathcal{F}_{mp}^\alpha u^p u_n + \frac{\varphi^\alpha}{u^2} \mathcal{F}_{np}^\alpha u^p u_m.$$

This symmetry allows one to reduce the general solution of the equations of motion of A_m^α

$$\epsilon^{lmnp} \partial_m (u_n \mathcal{F}_{pr}^\alpha u^r) = 0 \quad (18)$$

to Eq. (9) (see [13,18] for the details). In the gauge where $\mathcal{F}_{mn}^\alpha = 0$, the equations of motion of u_m lead to the same expressions for $a(x)$ and Λ_{mn} that follow from (1).

To transit from (13) to (1) we must solve Eqs. (18) for one of the gauge fields in terms of another one and substitute the solution back to (13) [13,18]. If we skip the kinetic term for $a(x)$ in (1) and (13), Λ_{mn} becomes a pure gauge as well, and the three actions (1), (10), and (13) become classically equivalent and describe dynamics of a single Maxwell field.

Now we shall demonstrate how the action (13) is related to the version considered in [18]. There the vector field u_m [in (13)] was subjected to the normalization condition (14) (we shall denote the normalized vector as \hat{u}_m). This caused a problem of establishing the explicit invariance of the model under the transformations (2), (15). To ensure this invariance one should couple (13) with the normalized \hat{u}_m [18] to scale-invariant gravity [25]. Then the action takes the form [we skip, for simplicity, the kinetic term of $a(x)$]

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{8} F_{mn}^\alpha F^{\alpha mn} + \frac{1}{4} \hat{u}^m \mathcal{F}_{mn}^\alpha \mathcal{F}^{\alpha np} \hat{u}_p - \frac{1}{\sqrt{-g}} \epsilon^{pqmn} \hat{u}_p \partial_q \Lambda_{mn} + R \Phi^2 + 6 \partial_m \Phi \partial^m \Phi \right), \quad (19)$$

where $g_{mn}(x)$ is a metric, $g = \det g_{mn}$, $R(x)$ is the scalar curvature, and $\Phi(x)$ is a conformal scalar field. The action (19) is invariant under (2), (15) provided $g_{mn}(x)$ and $\Phi(x)$ subject to the scale transformations

$$\delta g_{mn}(x) = (\hat{u}^m \partial_m \varphi)^2 g_{mn}(x), \quad \delta \Phi(x) = -(\hat{u}^m \partial_m \varphi) \Phi(x). \quad (20)$$

Making redefinition of the metric and \hat{u}_m as

$$g^{mn}(x) \rightarrow \Phi^2 g^{mn}, \quad \hat{u}_m(x) \rightarrow \frac{1}{\Phi} \hat{u}_m \quad (21)$$

[which preserves the condition³ (14)] we can rewrite (20) in the form which describes coupling the model to the Einstein gravity:

$$S = \int d^4x \sqrt{g} \left(-\frac{1}{8} F_{mn}^\alpha F^{\alpha mn} + \frac{1}{4} \hat{u}^m \mathcal{F}_{mn}^\alpha \mathcal{F}^{\alpha np} \hat{u}_p - \frac{1}{\sqrt{g}} \Phi \epsilon^{pqmn} \hat{u}_p \partial_q \Lambda_{mn} + R \right). \quad (22)$$

The only place where $\Phi(x)$ is present in (22) is the term $\Phi \epsilon^{pqmn} \hat{u}_p \partial_q \Lambda_{mn}$. Then, putting $\Phi(x) \hat{u}_m(x) = u_m(x)$, making use of (14) and, taking the flat limit we get the action (13).

This concludes the establishment of the links between different versions of the duality symmetric formulation of free Maxwell theory.

III. LOW-ENERGY EFFECTIVE ACTION IN STRING THEORY WITH MANIFEST $SL(2, \mathbf{R}) \times O(6, 22)$ AND SPACE-TIME SYMMETRY

In this section we present manifest space-time-invariant generalization of the $SL(2, \mathbf{R}) \times O(6, 22)$ -invariant low-energy effective action [13] describing heterotic string theory compactified on a six-dimensional torus [12,26]. To do this we should introduce 28 dual pairs [27] $A_m^{\alpha, a}$ ($a=1, \dots, 28$) of Abelian gauge fields and couple them to scalar fields in an $SL(2, \mathbf{R}) \times O(6, 22)$ -covariant way. This is achieved by modifying the self-dual tensor (8) as

$$\begin{aligned} \mathcal{F}_{mn}^{\alpha, a} &= \mathcal{L}^{\alpha\beta} \mathcal{L}^{ab} F_{mn}^{\beta, b} - (\mathcal{L}^T \mathcal{M} \mathcal{L})^{\alpha\beta} (\mathcal{L}^T \mathcal{M} \mathcal{L})^{ab} F_{mn}^{*\beta, b} \\ &\equiv \frac{\sqrt{-g}}{2} (-\mathcal{L}^T \mathcal{M})^{\alpha\beta} (\mathcal{L}^T \mathcal{M})^{ab} \epsilon_{mnpq} \mathcal{F}^{\beta, bpq}, \end{aligned} \quad (23)$$

where 2×2 matrix-valued scalar field

$$\mathcal{M} = \frac{1}{\lambda_2(x)} \begin{pmatrix} 1 & \lambda_1(x) \\ \lambda_1(x) & \lambda_1^2 + \lambda_2^2 \end{pmatrix}, \quad (24)$$

satisfies the conditions

$$\mathcal{M}^T = \mathcal{M}, \quad \mathcal{M} \mathcal{L} \mathcal{M}^T = \mathcal{L}. \quad (25)$$

\mathcal{M} , \mathcal{L} , and $A_m^{\alpha, a}$ transform under the global $SL(2, \mathbf{R})$ transformations ω as

$$\mathcal{M} \rightarrow \omega^T \mathcal{M} \omega, \quad \omega \mathcal{L} \omega^T = \mathcal{L}, \quad A_m = \omega^T A_m. \quad (26)$$

²To escape singularities we should require the norm of u_m to be nonzero.

³Note that one can introduce (14) into (19) with a Lagrange multiplier which transforms under the scale transformations in an appropriate way.

The 28×28 matrix-valued scalar field $M(x)$ satisfies the conditions

$$M^T = M, \quad M^T L M = L, \quad (27)$$

where

$$L = \begin{pmatrix} 0 & I_6 & 0 \\ I_6 & 0 & 0 \\ 0 & 0 & -I_{16} \end{pmatrix}. \quad (28)$$

Under a global $O(6,22)$ rotation M , L , and $A_m^{\alpha,a}$ transform as

$$M \rightarrow \Omega^T M \Omega, \quad \Omega^T L \Omega = L, \quad A_m \rightarrow \Omega^T A_m. \quad (29)$$

(See Ref. [13] for the details.) The transformation law of the self-dual tensor (23) under $SL(2, \mathbf{R}) \times O(6,22)$ is

$$\mathcal{F}_{mn} \rightarrow \omega^{-1} \Omega^{-1} \mathcal{F}_{mn}. \quad (30)$$

Using the properties of the fields described above one may convince oneself that the following general coordinate-invariant action has $SL(2, \mathbf{R}) \times O(6,22)$ symmetry:

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{8} F_{mn}^{\alpha,a} (\mathcal{L}^T \mathcal{M} \mathcal{L})^{\alpha\beta} (L^T M L)^{ab} F^{\beta,b}{}_{mn} + \frac{1}{4u^2} u^m \mathcal{F}_{mn}^{\alpha,a} \mathcal{M}^{\alpha\beta} M^{ab} \mathcal{F}^{\beta,b}{}_{np} u_p - \frac{1}{4} g^{mn} \text{tr}(\partial_m \mathcal{M} \mathcal{L} \partial_n \mathcal{M} \mathcal{L}) \right. \\ \left. + \frac{1}{8} g^{mn} \text{Tr}(\partial_m M L \partial_n M L) - \frac{1}{\sqrt{g}} \epsilon^{pqmn} u_p \partial_q \Lambda_{mn} + R \right). \quad (31)$$

It can be rewritten in a simpler form

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2u^2} u^m F_{mn}^{*\alpha,a} \mathcal{F}^{\alpha,a}{}_{np} u_p \right. \\ \left. - \frac{1}{\sqrt{-g}} \epsilon^{pqmn} u_p \partial_q \Lambda_{mn} + R - \frac{1}{4} g^{mn} \text{tr}(\partial_m \mathcal{M} \mathcal{L} \partial_n \mathcal{M} \mathcal{L}) \right. \\ \left. + \frac{1}{8} g^{mn} \text{Tr}(\partial_m M L \partial_n M L) \right) \quad (32)$$

which, upon fixing the gauge $u_m = (1/\sqrt{-g^{00}}) \delta_m^0$, $\Lambda_{mn} = 0$, directly reduces to the Schwarz-Sen action [13].

Note that we did not add the kinetic term of $a(x)$ (1), (13) to the actions (31) and (32). This is because we would like to identify $a(x)$ with $\lambda_1(x)$ which has already entered the actions (31) and (32) as part of the dilaton-axion matrix \mathcal{M} (24). The coupling of \mathcal{M} to the gauge fields ensures the manifest $SL(2, \mathbf{R})$ symmetry but breaks local transformations of $\lambda_1(x)$ [or $a(x)$ in (2)] down to the global Peccei-Quinn shifts which form a subgroup of the global⁴ $SL(2, \mathbf{R})$ (26). Such a coupling violates duality between the pseudoscalar field and Λ_{mn} in favor of the former and makes Λ_{mn} an auxiliary field, which can be eliminated from (31), (32) by solving for the equations of motion of u_m and Λ_{mn} , and substituting $u_m(x)$ back into (31), (32) in the form $u_m = \partial_m \hat{\varphi}(x)$ (16).

It is tempting to look for a version of the low-energy effective string action which would be manifestly duality symmetric not only in the gauge sector but in the axion sector (1), (7) as well. Might it imply a localization of the $SL(2, \mathbf{R})$?

IV. CONCLUSION AND DISCUSSION

We have constructed the space-time-invariant duality symmetric action for the free Maxwell theory and the $SL(2, \mathbf{R}) \times O(6,22)$ invariant effective action describing the low-energy bosonic sector of the heterotic string compactified on a six-dimensional torus. This has been achieved by coupling the self-dual stress tensor, constructed out of the dual gauge fields, to the auxiliary vector field from the axionic sector of the theory.

One can add to the bosonic action (13) the kinetic term for neutral fermions:

$$S_F = -i \int d^4x \bar{\psi} \gamma^m \partial_m \psi. \quad (33)$$

Then the full action becomes supersymmetric [13,18] under the following transformations with odd constant parameters $\epsilon^\alpha = i \gamma_5 \mathcal{L}^{\alpha\beta} \epsilon^\beta$:

$$\delta A_m^\alpha = i \bar{\psi} \gamma_m \epsilon^\alpha, \\ \delta \psi = \frac{1}{8} F^{\alpha mn} \gamma_m \gamma_n \epsilon^\alpha - \frac{1}{4u^2} \mathcal{L}^{\alpha\beta} u_p \mathcal{F}^{\alpha\beta pm} u^n \gamma_m \gamma_n \epsilon^\beta, \quad (34)$$

all other fields being inert under the supersymmetry transformations.

We see that the supersymmetric transformation law for the fermion $\psi(x)$ (34) is nonconventional and reduces to the ordinary one only on the mass shell (9). This reminds us of the problem with the Lorentz transformations (12) which we have just solved. Using the same reasoning that lead us to introducing $u_m(x)$ one may try to find a superpartner of $u_m(x)$ whose presence in the theory gives rise to a local fermionic symmetry [being a counterpart of (2), (15)] which involves $\psi(x)$ and leads to (34) upon gauge fixing the local

⁴Note that the action (31), (32) is still invariant under the local transformations of u_m (2), A_m , and Λ_{mn} (15).

fermionic symmetry. This construction may arise from coupling the models discussed above to supergravity, from which, in fact they originate.

The covariantization procedure for duality symmetric actions [14,15,9,13] proposed in [18] and developed herein is applicable to Abelian tensor fields in space-time dimensions other than $D=4$ and may turn out to be useful for finding new dual versions of $D=10$ supergravity [13].

Note added. After this paper was submitted for publication the authors became aware that earlier the nonmanifestly Lorentz-invariant duality symmetric action (10) was considered by Zwanziger [28] [see also [29] and references therein for detailed analysis of coupling the action (10) to electri-

cally and magnetically charged matter fields]. We are thankful to C. Preitschopf for pointing our attention to these references.

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