# Gravity and global symmetries

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There exists a widely held notion that gravitational effects can strongly violate global symmetries. If this is correct, it may lead to many important consequences. We argue, in particular, that nonperturbative gravitational effects in the axion theory lead to a strong violation of $CP$ invariance unless they are suppressed by an extremely small factor $g \lesssim 10^{-82}$. One could hope that this problem disappears if one represents the global symmetry of a pseudoscalar axion field as a gauge symmetry of the Ogievetsky-Polubarinov-Kalb-Ramond antisymmetric tensor field. We show, however, that this gauge symmetry does not protect the axion mass from quantum corrections. The amplitude of gravitational effects violating global symmetries could be strongly suppressed by $e^{-S}$, where $S$ is the action of a wormhole which may absorb the global charge. Unfortunately, in a wide variety of theories based on the Einstein theory of gravity the action appears to be fairly small, $S \sim 10$. However, we find that the existence of wormholes and the value of their action are extremely sensitive to the structure of space on the nearly Planckian scale. We consider several examples (Kaluza-Klein theory, conformal anomaly, $R^2$ terms) which show that modifications of the Einstein theory on the length scale $l \lesssim 10 M_P^{-1}$ may strongly suppress violation of global symmetries. We find also that in string theory there exists an additional suppression of topology change by the factor $e^{-\frac{8\pi^2}{g^2}}$. This effect is strong enough to save the axion theory for the natural values of the stringy gauge coupling constant.

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## I. INTRODUCTION

The most elegant way to solve the strong $CP$ violation problem is given by the Peccei-Quinn (PQ) mechanism [1]. This mechanism is based on the assumption that there exists a complex scalar field $\Phi(x) \equiv \frac{f(x)}{\sqrt{2}} e^{i\theta(x)}$, which after spontaneous symmetry breaking can be represented as $\frac{\phi(x)+f_0}{\sqrt{2}} \exp\left(\frac{ia(x)}{f_0}\right)$. The Goldstone field $a(x)$ (axion) has the coupling $\frac{a}{32\pi^2 f_0} F_{\mu\nu}\tilde{F}^{\mu\nu}$, similar to the famous $\theta$ term $\frac{\bar{\theta}}{32\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$. Nonperturbative effects in QCD lead to the appearance of the condensate $\langle F_{\mu\nu}\tilde{F}_{\mu\nu}\rangle$ and to the effective potential of the axion field proportional to $\Lambda^4_{\rm QCD}[1-\cos(\bar{\theta}+\frac{a}{f_0})]$. This potential has a minimum at $\frac{a}{f_0} = -\bar{\theta}$. In this minimum the terms $\frac{a}{32\pi^2 f_0} F_{\mu\nu}\tilde{F}^{\mu\nu}$ and $\frac{\bar{\theta}}{32\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$ cancel each other, and strong $CP$ violation disappears. This effect gives the axion a small mass $m_a \sim \frac{\Lambda^2_{\rm QCD}}{f_0}$.

In addition to providing a possible solution to the strong CP violation problem, the invisible axion field [2] is one of the best dark matter candidates [3]. It naturally appears in all phenomenological models based on superstring theory [4]. The axion field possesses many interesting properties near black holes [5]. Finally, axions may be responsible for the possible existence of wormholes in the baby universe theory [6]. Therefore there exists an extensive literature on axions. This literature includes at least two different formulations of the axion theory, which are not completely equivalent, and several modifications of these formulations (for a review see [7]).

The axion theory has many problems. First of all, it is not easy to make this theory compatible with cosmology. If the spontaneous symmetry breaking toward a state with $f_0 \neq 0$ occurs after the end of inflation, then the standard axion model is compatible with cosmological and astrophysical constraints only if $10^{10}$ GeV $\lesssim f_0 \lesssim 10^{12}$ GeV [8]. A recent investigation with an account of cosmological effects of the axion strings suggests that the upper bound may be even more tight, so that the "axion window" becomes almost closed, $10^{10}$ GeV $\lesssim f_0 \lesssim 10^{11}$ GeV [9]. On the other hand, if the spontaneous symmetry breaking occurs during inflation, then the constraint $f_0 \lesssim 10^{12}$ disappears [10], but typically it implies that the Hubble constant at the end of inflation should be sufficiently small, $H \lesssim 10^9$ GeV [11, 12]. Inflationary models of this type can be easily suggested [12, 13], but one should keep in mind that not every inflationary model satisfies this condition.

Three years ago it was pointed out [14] that the axion theory faces another difficult problem, which we are going to discuss in this paper.

The standard potential in the axion theory (ignoring small QCD corrections) is given by

$$V_0(\Phi) = \lambda(|\Phi|^2 - f_0^2/2)^2 . \tag{1}$$

In this approximation the axion is massless due to the global symmetry $\Phi \to \Phi\, e^{i\theta}$. However, there are some reasons to expect that nonperturbative quantum gravity effects do not respect global symmetries. The simplest way to understand it is to remember that global charges can be absorbed by black holes, which subsequently may evaporate. One may expect that a similar effect can occur because of the nonperturbative formation and evaporation of "virtual black holes" in the presence of a global charge. A somewhat more developed (even though still very speculative) approach is based on investigation of wormholes, which may take a global charge from our Universe to some other one. Indeed, it was claimed in [6, 15, 16] that such effects do actually take place, and can be described by additional terms (vertex operators) in the effective Lagrangian which break the global symmetry.

As an example, one may consider the terms of the type [14]

$$V_g(\Phi) = g_n \frac{|\Phi|^{2m}\,\Phi^{4-2m+n}}{M_P^n} + \text{H.c.}\,, \qquad (2)$$

where $g_n$ is some dimensionless constant. Naively, one could expect that these operators should be at least of the fifth order in $\Phi$, so that they should be suppressed by $M_P^n$ in the denominator, with $n > 0$. The authors of [14] concentrated on the simplest (and the most dangerous) term $g_5 \frac{|\Phi|^4(\Phi+\Phi^*)}{M_P}$. They have shown that, for $f_0 \sim 10^{12}$ GeV, this term destroys the standard solution of the strong $CP$ problem. Indeed, this term changes the shape of the effective potential and moves its minimum away from $\frac{a}{f_0} = -\bar{\theta}$. If $\frac{a}{f_0}$ changes by more than $10^{-9}$, the corresponding effects of $CP$ violation become too strong. In order to avoid such effects, one should have an extremely small coupling constant $g$ of the symmetry breaking operator $g_5 \frac{|\Phi|^4(\Phi+\Phi^*)}{M_P}$: $g_5 < 10^{-54}$ for $f_0 \sim 10^{12}$ GeV [14]. Thus, instead of the problem of explaining why the angle $\bar{\theta}$ in the theory of strong interactions is smaller than $10^{-9}$, we must explain now why some other parameter is smaller than $10^{-54}$. This does not look like a fair trade.

In fact, the situation is even more complicated. The idea to consider only the terms containing $M_P$ in the denominator was based on the assumption that the quantum gravity effects should be suppressed in the limit $M_P \to \infty$. Indeed, the $n$-loop quantum gravity corrections contain factors $M_P^{-n}$. However, the effects we are interested in are *nonperturbative.* These effects may give rise to vertex operators of the type of $g_1 M_P^3(\Phi + \Phi^*)$, or other operators which do not contain $M_P$ in the denominator [15, 16]. The way to see it is, e.g., to consider the averages of the type $M_P^3\langle(\Phi+\Phi^*)\rangle$. One can show that in the presence of wormholes such terms do not vanish, but they are suppressed by the same exponential factor $e^{-S}$ as the terms $\langle\frac{|\Phi|^4(\Phi+\Phi^*)}{M_P}\rangle$ [15]. Here $S$ is the action of a wormhole which can absorb a unit of a global charge associated with the field $\Phi$. This implies that if the effective vertex operators $g_5 \frac{|\Phi|^4(\Phi+\Phi^*)}{M_P}$ appear in the theory due to nonperturbative gravitational effects, one may expect that the operators $g_1 M_P^3(\Phi + \Phi^*)$ should appear as well, with a comparable coupling constant, $g_1 \sim g_5 \sim e^{-S}$.

The vertex operator $g_1 M_P^3(\Phi + \Phi^*)$ is most dangerous for the axion physics. One can easily show, by analogy with [14], that this term leads to a strong $CP$ violation unless $g_1 \lesssim 10^{-82} \frac{f_0}{10^{12}\,\text{GeV}}$. This constraint is almost 30 orders of magnitude stronger that the constraint following from the investigation of the operators $g_5 \frac{|\Phi|^4(\Phi+\Phi^*)}{M_P}$. For comparison of our constraint with the results of our future calculations of the wormhole action it is convenient to express this constraint (for $f_0 \sim 10^{12}$ GeV) in the form $g_1 \lesssim e^{-189}$.

Note, that this constraint depends on the value of $f_0$, but not too strongly: it is proportional to $f_0$. Thus, for $f_0 \sim 10^{10}$ GeV one should have $g_1 \lesssim e^{-193}$, whereas for $f_0 \sim 10^{19}$ GeV our constraint is $g_1 \lesssim e^{-172}$. In what follows we will suppose, for definiteness, that $f_0 = 10^{12}$ GeV, even though, as we have already emphasized, $f_0$ in the axion theory may be either 2 orders of magnitude smaller, or much greater than $10^{12}$ GeV.

Similar arguments are valid for other theories possessing global symmetries. For example, it was shown in [17] that the theory of cosmic textures may work only if the constant $g$ in the term $g_5 \frac{|\Phi|^4(\Phi+\Phi^*)}{M_P}$ is extremely small: $g_5 < 10^{-91}$. One can easily show that this constraint becomes even much stronger if one takes into account the above-mentioned terms linear in $\Phi$: $g_1 < 10^{-103} \sim e^{-237}$.

The same situation appears in the so-called "natural inflation" model [18]. In this model it is assumed that the effective potential has the form (2) with $f_0 \gtrsim M_P$, and then the Goldstone field, just like the axion field, acquires mass $\Lambda^2/f_0 \sim 10^{13}$ GeV. This can be achieved in a natural way, e.g., for $\Lambda \sim 10^{16}$ GeV and $f_0 \sim M_P$. However, the term $g_1 M_P^3(\Phi + \Phi^*)$ will destroy this nice picture unless the coupling $g_1$ is extremely small, $g_1 \lesssim 10^{-6}$. As compared with the constraints on the models of axions and textures, this condition looks relatively mild. Still, the existence of this additional constraint is rather disappointing. [On the other hand, it would be quite encouraging to find a natural mechanism which would lead to the gravity-induced terms $\sim g_1 M_P^3(\Phi + \Phi^*)$ with $g_1 \lesssim 10^{-6}$, since some terms breaking the global symmetry are necessary in this scenario.]

As we already mentioned, the main reason why quantum gravity may break global symmetries is associated with the possibility that the global charge can be absorbed by wormholes (or virtual black holes) and taken away from our Universe. It is commonly believed, however, that local charges, such as an electric or magnetic one, cannot disappear, and therefore quantum gravity does not break local symmetries. The reason can be easily understood if one thinks about electric (or magnetic) charges falling into black holes. Because of the Gauss law, the flux of electric field cannot disappear when the charge falls into a black hole. Charged black holes cannot evaporate entirely and take the electric charge away from our Universe. Instead of that, they eventually form charged extreme black holes which do not evaporate any further.

It would be very tempting to use a similar mechanism to save the axion theory. Indeed, it is well known that the theory of a massless pseudoscalar axion field in a topologically trivial space is equivalent to the theory of an antisymmetric tensor gauge field $b_{\mu\nu}$. This field, which was introduced by Ogievetsky and Polubarinov and later also by Kalb and Ramond [19], naturally appears in string theory. It is related to the field $a$ by the duality transformation. This suggests an idea that if one formulates the axion theory in terms of the antisymmetric *gauge* field, then the low mass of the axion will be protected not by global but by local (i.e., gauge) invariance, and it will not be destroyed by quantum gravity.

Of course, one may immediately argue that this cannot work. Indeed, if the axion charge can disappear in the standard formulation of the theory, its disappearance may have an adequate description in the theory of an antisymmetric tensor field. However, this argument has some caveats, since in fact these two theories are not completely equivalent at the quantum level. For example, conformal anomaly associated with the pseudoscalar field differs from the conformal anomaly in the theory of the antisymmetric tensor field [20].

There is another problem which appears to be much more important in the context of our discussion. Whereas a massless antisymmetric tensor field in a topologically trivial space can be converted into a pseudoscalar, not all pseudoscalars can be replaced by antisymmetric tensors. The necessary condition is that the Lagrangian of a pseudoscalar field should depend only on its derivatives.

Indeed, if one starts with the theory of the antisymmetric tensor gauge field, duality transformation between the field strength of the antisymmetric tensor field $\epsilon^{\mu\nu\lambda\delta}\partial_\nu b_{\lambda\delta}$ and derivative of the pseudoscalar field $\partial^\mu a$ exists and can be used to prove the equivalence of these two theories [21]. Duality transformation is always possible from the $b_{\mu\nu}$ side to the $a$ side and one ends up with the theory of the massless pseudoscalar field with derivative coupling only. Vice versa, if one starts with the theory of the massless pseudoscalar field with derivative coupling only, one can use the duality transformation and have an equivalent $b_{\mu\nu}$ version of the theory.

If the effective action depends on the pseudoscalar field $a$ without derivatives (and this is the case if the axion field has a small mass), no clear information about the relation of this theory to the $b_{\mu\nu}$ theory was available. To clarify the relation between $b_{\mu\nu}$ versus $a$ theories one should understand how one can describe appearance of the small mass $m_a \sim \frac{\Lambda^2_{\rm QCD}}{f_0}$ in terms of the gauge theory of the field $b_{\mu\nu}$. Is this effect possible at all, or is it associated with some kind of gauge symmetry breaking?

We have analyzed this question and found that the axion mass generation can be consistently described in terms of the antisymmetric tensor field, and that this effect does not involve any gauge symmetry breaking. This effect is quite interesting in its own terms, independently of the initial goal of our investigation. It provides a generalization of the phenomenon studied by Polyakov in three-dimensional compact QED, where the massive scalar excitation appears in the presence of monopoles [22]. In our case the mass of the antisymmetric tensor field appears because of its interaction with the usual QCD instantons. The deep physical reason why the antisymmetric tensor field can acquire mass without breaking gauge invariance is that both the electromagnetic field in $d = 3$ space-time and the antisymmetric tensor field in $d = 4$ space-time have only one degree of freedom. Thus, the condition of transversality for these theories, which typically protects excitations from becoming massive, does not apply to *physical* degrees of freedom in these theories. We will describe this effect in Sec. II of this paper.

Even though there is no reason to expect that the Peccei-Quinn symmetry is protected by the gauge invariance of the antisymmetric tensor field, one may still hope that the symmetry violation should be very small for some other reason. For example, it is not so easy for a quantized axion charge to be squeezed into a wormhole or a black hole: They should be large enough to absorb a unit of the axion charge. In the language of Euclidean quantum gravity this translates into the question of what is the Euclidean action of the wormhole which could absorb a unit of the quantized axion charge. If this action $S$ is large enough, then one could expect that the violation of the Peccei-Quinn symmetry is strongly suppressed by a factor $e^{-S}$. Our estimates of the effects related to the term $g_1 M_P^3(\Phi + \Phi^*)$ indicate that in order to suppress dangerous effects of global symmetry violation in the axion theory one should have the wormhole action $S \gtrsim 190$.

Wormholes which could absorb the global charge have been first discovered by Giddings and Strominger in the formulation of the axion theory in terms of the antisymmetric tensor field [6]. The simplest of their solutions corresponds to the fixed value of the radial component of the scalar field. The pseudoscalar representation of the Giddings-Strominger wormhole was obtained by Kimyeong Lee [23]. In Sec. III of this paper we will rederive their expression for the wormhole action. Our result for the value of the action of the wormhole configuration without the boundary at the wormhole neck coincides with the result obtained by Giddings, Strominger, and Lee. However, we point out that if one takes into account boundary terms including the contribution of the boundary at the wormhole throat, the action becomes about three times smaller. In any case, the action is proportional to $\frac{M_P}{f_0}$, which is as large as $10^7$ for the axion theory with $f_0 \sim 10^{12}$ GeV. This could suggest that for the axion theory one does not have any problem whatsoever since the symmetry violating effects will be suppressed by the factor $\sim 10^{-10^7}$, which is more than enough to explain why $g_1 \lesssim 10^{-82}$.

Unfortunately, this attitude proves to be too optimistic. As was first pointed out in [15], in realistic models of the axion field the radial component $f(x)$ of the axion field on the wormhole solutions does not remain equal to $f_0$. Near the wormhole throat this field typically acquires some value of the order of $M_P \gg f_0$. A detailed investigation of solutions with an account taken of the spatial dependence of $f(x)$ was performed by Abbott and Wise [15] and by Coleman and Lee [16] for the case without spontaneous symmetry breaking. The corresponding Euclidean action which was found in these papers linearly diverged on extremely large length scales. It was argued that despite the action is infinite, wormholes do lead to charge nonconservation and global symmetry breaking since the corresponding effects appear on a relatively small scale, where the large scale behavior of the wormhole solutions is irrelevant.

Unfortunately, the most interesting case of the theories with spontaneous symmetry breaking was only briefly mentioned in [15, 16]. It required some additional work to obtain results in a form in which one could compare them with the expectations expressed in [14]. Perhaps this was the reason why the authors of Ref. [14] did not make any attempt to use the results obtained in [6, 23, 15, 16].

In Sec. IV of this paper we describe wormhole solutions in several different theories with spontaneous symmetry breaking. Whereas in some cases we could obtain important information about these solutions by the methods of Ref. [15], in general it was necessary to use numerical calculations. These calculations were extremely tedious, especially for the axion theories with $f_0 \sim 10^{12}$ GeV. The results which we obtained are in a qualitative agreement with the expectations of [15, 16]. We have shown that for a very wide class of potentials the action is finite and to a good accuracy is given by a simple expression $S \sim \ln \frac{M_P}{f_0}$. This means that if the global symmetry breaking is suppressed by $e^{-S}$, this suppression is approximately given by the factor $\frac{f_0}{M_P}$. This factor is of the order of $10^{-7}$ for the axion theory with $f_0 \sim 10^{12}$ GeV, and it is of the order of $10^{-3}$ for the texture theory with $f_t \sim 10^{16}$ GeV. This is clearly insufficient to save the axion and the texture theory.

In fact, the situation becomes even more complicated if one takes into account that, according to [15, 16], the symmetry breaking vertex operators $g_n \frac{|\Phi|^{2m}\Phi^{4-2m+n}}{M_P^n}$ + H.c. are suppressed only by the part of the Euclidean action $S$ corresponding to integration in a small vicinity of the throat of the wormhole. In this case suppression of the global symmetry breaking in the theory $-m^2|\Phi|^2 + \lambda|\Phi^2|^2$ practically disappears.

One could expect that this result should be strongly model dependent. Indeed, we have found that near the wormhole throat the field $f(x)$ typically becomes as big as $M_P$. In certain theories, such as the theory of superstrings or supergravity, the effective potential may acquire large additional terms, which exponentially grow at large $f$. In such theories the behavior of the wormhole solutions near the throat becomes quite different from the one envisaged in [15, 16]. Here our use of numerical methods was absolutely crucial. The results, however, have not been very encouraging: Even if one considers the effective potentials growing at large $f$ as fast as $\exp\left(\frac{500f}{M_P}\right)$, the resulting Euclidean action remains quite small. Thus, one cannot make the violation of the global symmetries small by changing the effective potential of the scalar field in any reasonable way.

Fortunately, during our investigation we found several ways to fix this problem. First of all, when we make the effective potential more and more steep (keeping its minimum at $f_0$), the corresponding action tends to increase toward the very large action of the Giddings-Strominger-Lee wormhole. We found that the main reason why it happens is an increase of the size of the wormhole: The action is (approximately) proportional to the square of the radius of the wormhole throat $R(0)$, $S \sim M_P^2 R^2(0)$. According to our calculations, increasing of the minimal radius of the throat just a few times as compared with $M_P^{-1}$ can make the "natural inflation" scenario viable. The situation with axions and textures is more complicated, but still these two theories can be saved by an increase of the radius of the throat up to about $10M_P^{-1}$ or $15M_P^{-1}$.

It is very difficult to increase this radius by making the effective potential steep. However, there may be some other reasons why the wormhole throat cannot be small. For example, in string theory the effective "minimal length" may be somewhat greater than the Planck length $M_P^{-1}$. Our investigation contained in Sec. V indicates that the size of the wormhole throat can be very large in Kaluza-Klein theories with a sufficiently large radius of compactification. This suggests that the axion theory can be quite viable in the context of a theory in which the gravitational effects on the length scale $l \lesssim 10\, M_P^{-1}$ cannot be described by the standard Einstein theory of gravity in four-dimensional space-time. Another example pointing in the same direction is related to $\mathcal{R}^2$ corrections and effects of conformal anomaly. We show that an account taken of conformal anomaly (even if the corresponding terms are relatively small) may lead to the disappearance of the wormhole solutions. We also discuss the observation made in [6, 24] which suggests that there are no wormhole solutions in certain versions of the string theory.

In Sec. VI we discuss the possibility of an additional strong suppression of wormhole effects because of the Gauss-Bonnet term $\frac{\gamma}{32\pi^2}{}^*\mathcal{R}^*\mathcal{R}$. This term does not change any observational consequences of the Einstein theory, but it tends to suppress transitions with the change of topology. Similar terms appear in the heterotic string theory with the coefficient proportional to $\alpha'$. We show that these terms may suppress the wormhole effects by the factor $e^{-\frac{8\pi^2}{g^2}} = e^{-\pi\frac{M_P^2}{M_{\rm str}^2}}$, where $g$ is the gauge coupling constant, $M_{\rm str}^2$ is the stringy mass scale. This result is very similar to the standard result $e^{-\frac{8\pi^2}{g^2}}$ for the suppression of the instanton effects in QCD. The possibility to have this suppression factor smaller than $e^{-190}$ is quite consistent with the present picture of stringy phenomenology. This suppression becomes even stronger if one adds the usual part of the action to the topological contribution discussed above.

Our results and conclusions are summarized in Sec. VII. A considerable part of our results is based on numerical investigation of differential equations for wormholes. In many cases it was impossible to solve these equations using standard numerical recipes [25]. In the Appendix we describe an improved method which we have used in our work.

## II. AXION THEORY. PSEUDOSCALAR VERSUS ANTISYMMETRIC TENSOR

### A. Pseudoscalar formulation of the axion theory

The simplest version of the pseudoscalar axion theory [1] adds to the standard model Lagrangian $\mathcal{L}_{\rm SM}$ the terms

$$\mathcal{L} = \frac{1}{2}(\partial_\mu a)^2 + \left(\bar\theta + \frac{a}{f_0}\right)\frac{g^2}{32\pi^2}F^a_{\mu\nu}\,\tilde F^{a\mu\nu}\ . \qquad (3)$$

For a review of various models see [7]. Peccei-Quinn global U(1) symmetry

$$a(x) \to a(x) + C \tag{4}$$

is broken spontaneously when the operator $F^a_{\mu\nu}\tilde{F}^{a\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\delta}F^{\mu\nu}F^{\lambda\delta}$ has a nonvanishing vacuum expectation value $\langle F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\rangle$. The term $F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$ can be represented as a total derivative, $F^a_{\mu\nu}\tilde{F}^{a\mu\nu} = \partial_\mu K^\mu$, where

$$K_\mu(x) = \epsilon_{\mu\alpha\beta\gamma}A^\alpha_a\left(F^{\beta\gamma}_a - \frac{g}{3}f_{abc}A^\beta_b A^\gamma_c\right) . \tag{5}$$

The fact that $F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$ is a total derivative is not sufficient for providing the symmetry since the action is not invariant in presence of instantons. The variation of the action with an account taken of the surface terms is

$$\delta S_{\text{eff}} = \frac{C}{f_0}\frac{g^2}{32\pi^2}\int d^4x\, F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$$
$$= \frac{C}{f_0}\frac{g^2}{32\pi^2}\int d^4x\, \partial_\mu K^\mu \neq 0 . \tag{6}$$

Spontaneous breaking of the global U(1) Peccei-Quinn symmetry allows us to generate a potential for the axion field. This gives the axion a small mass $m_a^2 \sim \frac{m_\pi f_\pi}{f_0^2} \sim \frac{\Lambda^4_{\text{QCD}}}{f_0^2}$. The lightness of the axion is provided by the fact that $f_0 \gg \Lambda_{\text{QCD}}$.

Note, that the Peccei-Quinn symmetry is global. If the axion would not interact with non-Abelian fields, this symmetry could be promoted to a gauge one. Indeed, one can introduce an Abelian vector field [5] and replace everywhere $\partial_\mu a$ by $\partial_\mu a + eA_\mu$. Then the global PQ symmetry becomes the local one,

$$a \to a + C(x) \ , \quad A_\mu \to A_\mu - \partial_\mu C(x) \ . \tag{7}$$

However, in the presence of a non-Abelian gauge coupling, which is an essential part of all realistic axion models, one cannot promote the global PQ symmetry to the local one. Indeed, the non-Abelian coupling can be represented in the form $\partial_\mu a\, K^\mu$. After the promotion of the global PQ symmetry to the local one we would obtain the term $eA_\mu K^\mu$ in the action. This term would violate the non-Abelian gauge symmetry since the variation of the Chern-Simons term $K_\mu$ vanishes only when it is coupled to the longitudinal part of $A_\mu$ but not to the transverse part of it.

Still it is possible to represent the global PQ symmetry as a local one if one goes to a dual formulation of the axion theory, in which the axion pseudoscalar field $a$ is represented by the antisymmetric tensor field $b_{\mu\nu}$. This version of the axion theory naturally appears in the context of string theory [4].

### B. Dual version of the axion theory with the non-Abelian coupling in the broken symmetry phase

The noninteracting two-index antisymmetric tensor gauge field $b_{\rho\sigma}$ with the gauge symmetric action $\frac{1}{2}(\partial_{[\nu}b_{\rho\sigma]})^2$ is equivalent to a noninteracting massless pseudoscalar field with the action $\frac{1}{2}(\partial_\mu a)^2$. The simplest way to see this is to perform a duality transformation of the classical action. This leads to a replacement of $\partial^\mu a$ by a pseudovector $\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\partial_{[\nu}b_{\rho\sigma]}$ which is dual to the tree tensor field strength $\partial_{[\nu}b_{\rho\sigma]}$ of the two-form field $b_{\rho\sigma}$. Another way to see this equivalence is to perform gauge fixing of the gauge symmetry $\delta b_{\rho\sigma} = \partial_{[\rho}\Lambda_{\sigma]}$, $\delta\Lambda_\sigma = \partial_\sigma\Lambda$, which requires two generations of ghosts. In addition to 6 $b_{\mu\nu}$ fields one gets 8 anticommuting ghosts in first generation and 3 commuting ones in the second generation. This provides a net number of propagating degrees of freedom equal to one commuting field.

The equivalence of the massless pseudoscalar and massless antisymmetric tensor field coupled to the non-Abelian gauge field has been shown in [21] starting with the first-order-type action using duality transformation. In what follows we will perform this duality transformation for the theory which describes the complex scalar theory with the axion $\theta$ coupled to the non-Abelian vector fields:

$$\mathcal{L} = |\partial_\mu\Phi|^2 - V(|\Phi|) - \partial_\mu\theta\,\Omega^\mu \ . \tag{8}$$

Here

$$\Omega_\mu(x) = \frac{g^2}{32\pi^2}\epsilon_{\mu\alpha\beta\gamma}A^\alpha_a\left(F^{\beta\gamma}_a - \frac{g}{3}f_{abc}A^\beta_b A^\gamma_c\right)$$
$$\equiv \frac{g^2}{32\pi^2}K_\mu \ . \tag{9}$$

One can also write this Lagrangian as

$$\mathcal{L}_\theta = \tfrac{1}{2}f^2(\partial_\mu\theta)^2 - \partial_\mu\theta\,\Omega^\mu + \mathcal{L}(f) \ , \tag{10}$$

where

$$\mathcal{L}(f) \equiv \tfrac{1}{2}(\partial_\mu f)^2 - V(f) \tag{11}$$

and

$$\Phi = \frac{\phi(x) + f_0}{\sqrt{2}}\exp\left(\frac{ia(x)}{f_0}\right)$$
$$\equiv \frac{f(x)}{\sqrt{2}}\exp[i\theta(x)] \ . \tag{12}$$

However, it is useful to start with a more general Lagrangian which depends both on the pseudoscalar $\theta$ and on some pseudovector field $H_\mu$:

$$\mathcal{L}_{\theta,H} = i\partial_\mu\theta(H^\mu + i\Omega^\mu) + \tfrac{1}{2}f^{-2}H_\mu H^\mu + \mathcal{L}(f) \ . \tag{13}$$

One can solve the field equations for $H_\mu$,

$$H_\mu = -if^2\partial_\mu\theta \ \ , \tag{14}$$

and on shell for $H_\mu$ the Lagrangian acquires the form (10), which describes the pseudoscalar axion field $\theta$.

On the other hand, one can vary $\mathcal{L}_{\theta,H}$ (13) over $\theta(x)$ and obtain the constraint on $H_\mu$:

$$\partial_\mu H^\mu + i\frac{g^2}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{a\mu\nu} = 0 \ . \tag{15}$$

The solution to this constraint is

$$H^{\mu} = -\frac{i}{2}\epsilon^{\mu\nu\rho\sigma}H_{\nu\rho\sigma},$$

$$H_{\nu\rho\sigma} = \partial_{[\nu}b_{\rho\sigma]} + \frac{g^2}{32\pi^2}A^a_{[\nu}\left(F_{a\rho\sigma]} - \frac{g}{3}f_{abc}A^b_\rho A^c_{\sigma]}\right), \tag{16}$$

where $\partial_{[\nu}b_{\rho\sigma]}$ is the field strength of the two-form field.

The Lagrangian which follows from (13) describes the antisymmetric massless field $b_{\mu\nu}$ interacting with the non-Abelian vector fields as well as with the radial component of the scalar field,

$$\mathcal{L}_{b_{\mu\nu}} = \tfrac{1}{2}f^{-2}H^2_{\mu\nu\lambda} + \mathcal{L}(f) . \tag{17}$$

*There are no global symmetries in the dual version of the axion theory, but there are two types of local symmetries*, the Maxwell-type gauge symmetry of the two-form field,

$$\delta b_{\mu\nu} = \partial_{[\mu}\Lambda_{\nu]} , \qquad \delta\Lambda_\nu = \partial_\nu\Lambda , \tag{18}$$

and the Nicolai-Townsend-type non-Abelian gauge symmetry [21]

$$\delta A^a_\mu = \nabla^{ab}_\mu \Lambda_b , \qquad \delta b_{\mu\nu} = 2\frac{g^2}{32\pi^2}\Lambda^a\partial_{[\mu}A_{a\nu]} . \tag{19}$$

Will these local symmetries of the dual version allow us to avoid the problem which destroys the nice features of the axion theory in the standard version? To address this issue we will first study the origin of the light axion mass generated by QCD instantons in the dual version of the theory.[1]

### C. How does the axion become massive in the dual theory?

The possibility that a scalar or pseudoscalar particle can acquire a nonvanishing mass due to nonperturbative effects does not look very surprising. However, in the dual formulation of the axion theory the axion is massless because of a gauge symmetry (18). It is commonly believed that gauge symmetry protects massless particles from becoming massive. This would solve all our problems. However, if it were true, then the axion field in its dual formulation would not get its mass $m_a \sim \frac{\Lambda^2_{\rm QCD}}{f_0}$. Therefore before going any further we must first resolve this puzzle.

With an account of QCD instantons the effective action of the pseudoscalar field acquires a potential

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2\frac{f^2}{f_0^2} + \Lambda^4_{\rm QCD}\cos\frac{a}{f_0} . \tag{20}$$

[1]We are grateful to M. Dine for the suggestion to investigate this problem.

Starting with this action one cannot make a transition to the dual version of the theory with the $b_{\mu\nu}$ field using any of the methods discussed above. For example, one cannot perform duality transformation of the action starting with the action depending both on $\partial_\mu a$ and $H_\mu$ as we did before. Indeed, there is now a term in the pseudoscalar version of the theory depending on $a$ rather than on $\partial_\mu a$ and the procedure does not work. If we will try to go from the side of the gauge invariant $b_{\mu\nu}$ theory and perform duality transformation, we will get no terms depending on $a$, but only terms depending on $\partial_\mu a$.

Does the impossibility to perform duality transformation mean that the dual version of the axion theory is incapable of explaining the mechanism of generating a small axion mass? Will the same mechanism protect axion from getting very heavy?

To understand the effect at the level which is more subtle than just performing duality transformation over the effective action we will first go back to the Polyakov model of compact QED in $d = 3$ [22]. In this theory the massless scalar is dual to the massless Abelian vector $A_\mu$ and not to the $b_{\mu\nu}$ field.

Polyakov's main idea is that the existence of magnetic monopoles, which are instantons in $d = 3$, changes the behavior of the correlators: due to instantons there are no massless transverse excitations in the system. Instead there is an excitation corresponding to a longitudinal component of a gauge invariant operator, which may be interpreted as a massive scalar. The effect has a nonperturbative nature and does not violate gauge symmetry.

We will show that something very similar happens with the antisymmetric tensor field in our $d = 4$ theory. Let us first describe the situation in $d = 3$ [22]. One starts with the dual to the vector field strength

$$H_\alpha = \tfrac{1}{2}\epsilon_{\alpha\beta\gamma}F_{\beta\gamma} . \tag{21}$$

If one allows certain magnetic charge density in the system of the form $\rho(\vec{x}) = \sum q_a\delta(\vec{x} - \vec{x}_a)$, in the quasiclassical approximation this would correspond to

$$H_\mu = \frac{2\pi i k_\mu}{k^2}\rho(k) . \tag{22}$$

The main steps in the calculation of the correlator of two gauge invariant operators $H_\mu$ are the following. The bare part without monopoles is

$$\langle H_\mu(k)H_\nu(-k)\rangle^{(0)} = e^2\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) . \tag{23}$$

This expression corresponds to the free photon propagator coming from the action $\frac{1}{e^2}F^2_{\mu\nu}$. In addition, there is a contribution from the instantons such that the full correlator is

$$\langle H_\mu(k)H_\nu(-k)\rangle = \langle H_\mu(k)H_\nu(-k)\rangle^{(0)} + (2\pi)^2\frac{k_\mu k_\nu}{k^4}\ \langle\rho(k)\rho(-k)\rangle . \tag{24}$$

The correlator of charge densities is calculated from the generating functional for the charge density of the plasma and is given in [22]:

$$Z(\eta) = \left\langle \exp\left(i\int d^3x\,\eta(\vec{x})\rho(\vec{x})\right)\right\rangle = \frac{1}{Z(0)}\int \mathcal{D}\chi\,\exp\left[-\left(\frac{e}{2\pi}\right)^2\int\{[\nabla(\chi-\eta)]^2 - M^2\cos\chi\}\right] , \tag{25}$$

where $M^2 = \left(\frac{2\pi}{e}\right)^2 e^{-\frac{C}{e^2}}$, $C \sim 1$ is some constant. The second variational derivative over $\eta$ gives the expression for the correlator of $\rho$,

$$\langle\rho(k)\rho(-k)\rangle = \left(\frac{e}{2\pi}\right)^2\left(k^2 - \frac{k^4}{M^2+k^2}\right) = \left(\frac{e}{2\pi}\right)^2\frac{M^2k^2}{M^2+k^2} . \tag{26}$$

The final answer is

$$\frac{1}{e^2}\langle H_\mu(k)H_\nu(-k)\rangle = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{M^2+k^2} . \tag{27}$$

Thus the correlation function of two gauge invariant operators $\frac{1}{2}\epsilon_{\alpha\beta\gamma}F_{\beta\gamma}$ has only one longitudinal excitation as if one would calculate the two-point correlator of the derivatives of a massive scalar field.

For the antisymmetric tensor field in $d = 4$ things work not exactly the same way but very close to it. The only gauge invariant operator in our theory where we keep the antisymmetric tensor coupled to the non-Abelian vector field is

$$H^\mu = -\frac{i}{2}\epsilon^{\mu\nu\rho\sigma}\left[\partial_\nu b_{\rho\sigma} + \frac{g^2}{32\pi^2}A^a_\nu\left(F_{a\rho\sigma} - \frac{g}{3}f_{abc}A^b_\rho A^c_\sigma\right)\right] \equiv h^\mu - i\Omega^\mu . \tag{28}$$

This is the only operator which is gauge invariant under both Maxwell and Nicolai-Townsend non-Abelian symmetry. Separately, the operator $h^\mu$ which is dual to a field strength of the antisymmetric tensor field is invariant under the Abelian gauge symmetry, but it is not invariant under the Yang-Mills symmetry without the Chern-Simons term.

To examine our problem we may ignore small quantum fluctuations of the radial component of the scalar field. These fluctuations are irrelevant when one investigates the possibility that gauge symmetries protect axion theory from getting a mass. Thus, the action we consider is the action (17) with the fixed radial component $f(x) = f_0$,

$$\mathcal{L}_{b_{\mu\nu}} = \tfrac{1}{2}f_0^{-2}H^2_{\mu\nu\lambda} . \tag{29}$$

We may start by treating the coupling of the $b_{\mu\nu}$ field to the non-Abelian field perturbatively, i.e., we may study the correlator of two gauge invariant operators $H_\mu$ by treating the coupling as a correction to the value of the correlator of two operators $h_\mu$ without coupling. The bare correlation function of two operators $h_\mu$ must be transverse since

$$\partial_\mu h^\mu = -\frac{i}{2}\partial_\mu\epsilon^{\mu\nu\rho\sigma}\partial_\nu b_{\rho\sigma} = 0 . \tag{30}$$

Thus, as in the Polyakov case, we may expect

$$f_0^{-2}\langle H_\mu(k)H_\nu(-k)\rangle = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + \frac{k_\mu k_\nu}{k^4}\langle\rho(k)\rho(-k)\rangle + \cdots , \tag{31}$$

where the ellipsis corresponds to the corrections to a transverse part of the correlator. We have parametrized the longitudinal excitation by some two-point correlator of a "charge density" $\rho(x)$. The divergence of our gauge invariant operator $H_\mu$, which we may associate with $\rho$, is given by

$$f_0^{-1}\partial_\mu H^\mu(x) = -if_0^{-1}\partial_\mu\Omega^\mu = -i\frac{g^2}{32\pi^2 f_0}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}(x) \equiv -i\rho(x) . \tag{32}$$

The correlator of two divergences of the Yang-Mills Chern-Simons currents was calculated by Shifman, Vainstein, and Zakharov in [2]. They have shown that in a certain approximation it is proportional to the axion mass

$$\left\langle\left(\frac{g^2}{32\pi^2 f_0}F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\right)(k)\;\left(\frac{g^2}{32\pi^2 f_0}F^b_{\lambda\delta}\tilde{F}^{b\lambda\delta}\right)(-k)\right\rangle \sim m_a^2 . \tag{33}$$

This serves as an indication that one may obtain in the $b_{\mu\nu}$ theory the longitudinal excitation of the gauge invariant operator corresponding to a scalar massive particle instead of a massless $b_{\mu\nu}$ field.

One can confirm these expectations by calculating the correlator $\langle H_\mu(k)H_\nu(-k)\rangle$ directly, without separating our operator $H_\mu$ into its free part $h_\mu$ and the interaction part $\Omega_\mu$. For this purpose we will perform the generalized duality transformation in the functional integral describing the theory. Consider the "first-order" functional integral for the action we have considered above with the additional term including the source $\eta^\mu$ to the current $f_0^{-1}H_\mu$:

$$Z(\eta) = \int\mathcal{D}a\mathcal{D}H_\mu\mathcal{D}A^a_\alpha\,\exp\left(i\int\left[if_0^{-1}\partial_\mu a\,(H^\mu + i\Omega^\mu) + \tfrac{1}{2}f_0^{-2}H_\mu H^\mu + \eta^\mu f_0^{-1}H_\mu + \mathcal{L}_{\rm YM}\right]\right) . \tag{34}$$

The functional integration is performed over the pseudoscalar $a$ as well as over the pseudovector $H_\mu$ and over the Yang-Mills fields. For the sake of simplicity we do not write down explicitly the integration over the ghosts related to the gauge fixing of the non-Abelian symmetry. We may evaluate this functional integral by integrating over the pseudoscalar $a$ first. This will produce a constraint given in Eq. (16). In this way we will get rid of the $a$ integration and the remaining integral will reduce to the integration over the constrained $H_\mu$. Equivalently the integration over the constrained $H_\mu$ may be replaced by the integration over the unconstrained $b_{\mu\nu}$ together with the proper gauge-fixing procedure in the path integral. As before, $H_\mu(b,A)$ is defined in terms of $b_{\mu\nu}$ and Yang-Mills fields in Eq. (16). Thus we get

$$Z(\eta) = \int \mathcal{D}b_{\mu\nu} DC_{\rm gh} \mathcal{D}A^a_\alpha \, \exp\left[ i \int \tfrac{1}{2} f_0^{-2} H_\mu H^\mu(b,A) + \eta_\mu f_0^{-1} H^\mu(b,A) + \mathcal{L}_{\rm YM} + \mathcal{L}_{\rm gfix} + \mathcal{L}_{\rm gh} \right] \,, \tag{35}$$

where we have written down the integration over the complete set of ghost fields $C_{\rm gh}$ related to both types of gauge symmetries. $\mathcal{L}_{\rm gfix}$ is the gauge-fixing part of the action and $\mathcal{L}_{\rm gh}$ is the action of the ghost fields. By varying this functional twice over $\eta^\mu$ we will get the correlation function of two gauge invariant operators $f_0^{-1} H_\mu(b,A)$ defined in Eq. (16) and calculated in the dual version of the axion theory.

On the other hand we can perform the integration over $H_\mu$ first. This does not change the fact that the second derivative of $Z(\eta)$ gives the correlator of two $f_0^{-1}H$'s. It is just an alternative method of calculations. After integration over $H_\mu$ we get

$$Z(\eta) = \int \mathcal{D}a \mathcal{D}A^a_\alpha \, \exp\left[ i \int -\tfrac{1}{2}(i\partial_\mu a + \eta_\mu)^2 + \frac{g^2 a}{32\pi^2 f_0} F^a_{\mu\nu} \tilde{F}^{a\mu\nu} + \mathcal{L}_{\rm YM} \right] \,. \tag{36}$$

The integration over the instanton configurations in the dilute gas approximation produces the effective action for the field $a$

$$Z(\eta) \approx \int \mathcal{D}a \mathcal{D}A^a_\alpha \, \exp\left[ i \int \tfrac{1}{2}(\partial_\mu a - i\eta_\mu)^2 + \Lambda^4_{\rm QCD} \cos\frac{a}{f_0} + \mathcal{L}_{\rm YM} \right] \,. \tag{37}$$

Double variation of this generating functional gives us an expression for the correlation function of two operators $f_0^{-1} H_\mu$ in the form

$$f_0^{-2} \langle H_\mu(k) H_\nu(-k) \rangle = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{m_a^2 + k^2} \,. \tag{38}$$

This result is in a complete correspondence with the Polyakov result for $d = 3$. The first term on the right-hand side (RHS) of Eq. (38) comes from the $\eta_\mu^2$ term in the generating functional, and the second term comes effectively from the correlator of two derivatives of the massive field $a$.

Our conclusion is the following. The antisymmetric gauge field is coupled to the Yang-Mills fields. With an account taken of instantons the behavior of the system becomes that of the massive scalar field theory. Note that this is *not* a mass of the field $b_{\mu\nu}$. Rather it is the mass of the only gauge-invariant degree of freedom associated with $b_{\mu\nu}$.

Thus we have found a mechanism of getting small QCD mass for the axion in the dual version of the theory, which does not violate any of the gauge symmetries. This result has an important implication that there is no reason to expect that the gauge symmetry which exists in the dual formulation of the axion theory can protect the axion mass from becoming very large because of the gravitational effects.

## III. AXIONIC INSTANTONS OR WORMHOLE SOLUTIONS WITH FIXED RADIAL COMPONENT OF THE SCALAR FIELD

The axionic wormholes which may provide mass to the axion via gravitational effects have been found originally in the dual version of the axion theory with antisymmetric tensor field [6]. The theory of the complex scalar field does not have such wormhole solutions unless the functional integral is supplemented by the proper boundary conditions. The corresponding investigation has been performed in [23, 16]. The conclusion was that there exists a consistent procedure to obtain the same wormhole solutions in both versions of the axion theory.

In this section we discuss wormholes with a frozen radial component of the field, following Giddings and Strominger [6] and Lee [23]. We will mainly reproduce their results. However, in addition we will discuss the subtlety related to the boundary terms and the value of the Euclidean action. We will find out that if one considers the configuration with the boundary at the wormhole throat, see Fig. 1(a), and calculates the contribution to the action from the boundary terms in a standard way, one ends up with the action which is only $(1 - \frac{2}{\pi}) \sim 0.36$ of the original action, which was calculated in [6, 23] without an account taken of the boundary terms on the wormhole throat.[2]

Giddings and Strominger have found their wormhole solution in the string-inspired theory with the action

$$S_H = \int d^4x \sqrt{g} \left( -\frac{M_P^2}{16\pi} \mathcal{R} + f_0^{-2} H^2_{\mu\nu\lambda} \right) - \frac{M_P^2}{16\pi} \int_{\partial V} d^3S (K - K_0) \,. \tag{39}$$

Here $H_{\mu\nu\lambda} = \partial_{[\mu} b_{\nu\lambda]}$ is the field strength of the antisymmetric tensor field. The last term in the action is the Gibbons-Hawking surface term.

[2]We are grateful to A. Strominger for the discussion of this subtlety.

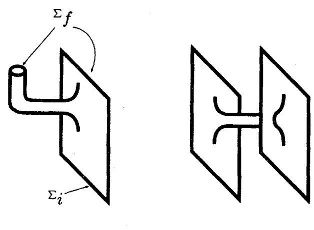


FIG. 1. (a) The geometry of the Giddings-Strominger-Lee wormhole with initial surface $\Sigma_i$ which is $R^3$ and final surface $\Sigma_f$ which is $R^3 \times S^3$. This wormhole may describe the tunneling from $R^3$ to $R^3 \times S^3$. (b) Extended solution which connects two asymptotically Euclidean regions $r \to \pm\infty$.

Meanwhile, Lee obtained wormhole solutions in the theory of a pseudoscalar axion field $\theta \equiv \frac{a}{f_0}$ with the action

$$S = \int d^4x\sqrt{g}\left(-\frac{M_P^2}{16\pi}\mathcal{R} + \frac{1}{2}f_0^2\,(\partial_\mu\theta)^2\right) - \frac{M_P^2}{16\pi}\int_{\partial V} d^3S(K - K_0)\ . \quad (40)$$

The wormhole geometry has the form

$$ds^2 = dr^2 + R(r)^2 d^2\Omega_3\ . \quad (41)$$

As we already mentioned, there are no true solutions of the Lagrange equations following from (40). However, it was pointed out in [23, 16] that these solutions appear if one takes into account the charge conservation condition in space (41). The global charge $n$, defined as an integral over the three-space from the zero component of the Noether current,

$$n = \int_{S_3} R^3 f^2\theta' = 2\pi^2 R^3 f_0^2\theta' = \text{const}\ , \quad (42)$$

is an integer upon quantization.[3]

The conclusion of Refs. [23, 16] was that if one makes the variation of the action under the condition $n = \text{const}$, one obtains equations of motion which are equivalent to the equations in the theory (39). In particular, the gravitational equation of motion in both theories looks like

$$R'(r)^2 = 1 - \left(\frac{R(0)}{R(r)}\right)^4\ . \quad (43)$$

Here $R(0)$ is the size of the throat of the wormhole defined by the condition $R'(0) = 0$. It is given by

$$R(0) = \left(\frac{n^2}{3M_P^2\pi^3 f_0^2}\right)^{\frac{1}{4}}\ . \quad (44)$$

[3]Note that in the situation with spontaneous symmetry the global charge is carried not by charged particles, but by the vortices of the classical scalar field with the time-dependent phase $\theta$.

The wormhole geometry $R(r)$ can be expressed analytically in terms of elliptic integrals [6].

The wormhole action with an account taken of the boundary terms both at the outer boundary at $r = r_0$, $r_0 \to \infty$, and at the inner boundary at $r = 0$ is

$$S_{\text{total}} = \frac{3M_P^2}{4\pi}\int d\Omega \int_0^\infty dr R(r)R'(r)[1 - R'(r)]\ . \quad (45)$$

The integration can be performed as

$$S_{\text{total}} = \frac{3M_P^2}{4\pi}\,2\pi^2\,\frac{1}{2}\int_{R^2(0)}^\infty dR^2\left(1 - \sqrt{1 - \frac{R^4(0)}{R^4}}\right) = \frac{3\pi^2}{8}M_P^2 R^2(0)\left(1 - \frac{2}{\pi}\right)\ . \quad (46)$$

Giddings-Strominger-Lee (GSL) on shell action (without the inner boundary terms) is [6]

$$S_{\text{no bound}} = \frac{3\pi M_P^2}{2}\int_0^\infty dr R(r)\{1 - [R'(r)]^2\} = \frac{3\pi^2}{8}M_P^2 R^2(0)\ . \quad (47)$$

The boundary term which we added,

$$S_{\text{bound}} = -\frac{3\pi M_P^2}{2}\int_0^\infty dr R(r)[1 - R'(r)] = \frac{3\pi^2 M_P^2 R^2(0)}{8}\left(-\frac{2}{\pi}\right)\ , \quad (48)$$

is $-0.637$ of the action without the boundary term. To understand better the boundary term contribution consider it in the form

$$S_{\text{bound}} = \frac{3M_P^2}{8\pi}\int d\Omega\int_0^\infty dr\frac{\partial}{\partial r}\Big[R^2(r)[1 - R'(r)]\Big]\ . \quad (49)$$

The contribution comes only from the throat where $R(r) = R(0)$, $R'(r) = 0$. It equals

$$S_{\text{bound}} = -\frac{3M_P^2}{8\pi}\,2\pi^2\,R^2(0) = -\frac{3\pi}{4}M_P^2 R^2(0)\ , \quad (50)$$

which is in complete agreement with (48). Note that the extrinsic curvature term $K$ at the $S^3$ boundary at $r = 0$ [the second term in Eq. (49)] vanishes, in agreement with the discussion in [26], but an additional nonvanishing contribution comes from the term $K_0$, which was added to remove the divergence of the action at the outer boundary.

The total action for the configuration with the boundary at the throat is thus only 0.363 of the value of the action without the surface term. This gives an additional support to the idea that in gravitational problems one has to be very careful with the boundary terms. The outer boundary surface term $K$ has to be corrected by $K_0$; otherwise, the action is infinite. Does it mean that the surface term has to have the same functional form $K - K_0$ on both boundaries? If the answer is yes we have to subtract $\frac{2}{\pi}$ part of the action and make it almost three times smaller than the action obtained in [6, 23].

One should note, that the problem of obtaining a proper contribution from the inner boundary is very complicated. For example, recently it was argued that an adequate account taken of the inner boundary of an extreme charged black hole changes its Euler number [27], which makes its entropy zero [28, 29, 27]. However, even for the black hole case this issue is rather nontrivial. The situation with the wormholes is even more complicated and ambiguous. If one considers the wormholes without the inner boundary contribution as in [6, 23, 15, 16], one has to add to our result for the action the universal term $\frac{3\pi}{4}M_P^2R^2(0)$, which depends only on the size of the wormhole throat. One may also want to calculate the action on a symmetric configuration $-\infty < r < +\infty$, Fig. 1(b). In this case one will not have any contribution from the inner boundary, and the action will be two times greater than the GSL action (47).

In what follows we will work with the action defined with both boundaries and with the surface term $K - K_0$ on both boundaries since this prescription seems to be more internally consistent. Another advantage of this prescription is that it gives us the smallest action as compared with other prescriptions mentioned above. Therefore if we find a way to avoid the strong violation of global symmetries in our approach, we will simultaneously solve the corresponding problem in other approaches as well.

The radius of the throat of the Giddings-Strominger-Lee wormhole depends on the parameter $f_0$ and on the value of the charge $n$. We may therefore express the action in terms of these parameters as

$$S_{\text{total}} = \frac{\sqrt{3}\pi}{8}\left(1-\frac{2}{\pi}\right)\frac{nM_P}{f_0} \;. \tag{51}$$

For completeness, we will give here also the value which has been obtained in [6, 23] without an account taken of the inner boundary:

$$S_{\text{no bound}} = \frac{\sqrt{3}\pi}{8}\left(\frac{nM_P}{f_0}\right) \;. \tag{52}$$

If one takes the smallest action (51) with $n = 1$ and $f_0 = 10^{12}$ GeV, one obtains an enormously strong suppression $\sim \exp(-10^6)$. This would immediately solve the problem of the global symmetry violation. Unfortunately, however, things are much more complicated. As it was pointed out in [15], it is almost impossible to keep the field $f(x)$ close to $f_0$ on the wormhole solution. In what follows we will show that if one allows the field $f$ to depend on $r$, this field typically grows to $f \sim M_P \gg f_0$ near the wormhole throat. Therefore one needs to make a separate investigation to calculate the wormhole action in realistic theories with spontaneous symmetry breaking. This investigation will be contained in the next section.

## IV. WORMHOLE SOLUTIONS WITH DYNAMICAL COMPLEX SCALAR FIELD AND SPONTANEOUS SYMMETRY BREAKING

### A. Equations for the scalar field in the wormhole geometry

In this section we will discuss wormhole solutions for several different theories with spontaneous symmetry breaking. In particular, we will investigate the dependence of the action on the value of the vacuum expectation value $f_0$ of the radial part of the scalar field far away from the wormhole.

We will study interaction of gravity with the complex scalar field $\Phi(x) = \frac{f(x)}{\sqrt{2}}\,e^{i\theta(x)}$. The corresponding action is

$$S = \int d^4x\sqrt{g}\left(-\frac{M_P^2}{16\pi}\mathcal{R} + \frac{1}{2}(\partial_\mu f)^2 + \frac{1}{2}f^2(\partial_\mu\theta)^2 + V(f)\right) - \frac{M_P^2}{16\pi}\int_{\partial V} d^3S^a(K_a - K_{0a}) \;. \tag{53}$$

We will assume that the potential $V(f)$ has a minimum at some value $f = f_0$. The vacuum energy vanishes in this minimum,

$$V'(f)|_{f=f_0} = V(f)|_{f=f_0} = 0 \;. \tag{54}$$

Equations of motion corresponding to the analytic continuation of the Euclidean theory of the two-form version of the axion theory [23, 15, 16] are

$$f'' - \frac{3R'f'}{R} - \frac{dV(f)}{df} - \frac{n^2}{4\pi^4 f^3R^6} = 0 \;, \tag{55}$$

$$R'^2 - 1 + \frac{8\pi}{3M_P^2}R^2\left(V(f) + \frac{n^2}{8\pi^4f^2R^6} - \frac{f'^2}{2}\right) = 0 \;, \tag{56}$$

and the value of $\theta' = \frac{n}{2\pi^2f^2R^3}$ has been already substituted in the equations.

Using these equations one can derive the following expression for the wormhole action:

$$S_{\text{total}} = 2\pi^2\int_0^\infty dr\left(R^3f'^2 + \frac{3M_P^2}{4\pi}RR'(1-R')\right) \;. \tag{57}$$

We are looking for the wormhole solutions with the geometry given in Eq. (41) and with the fields $R(r)$ and $f(r)$ solving the system of Eqs. (55) and (56). Our boundary conditions are

$$R'(0) = 0 \;, \tag{58}$$

$$f'(0) = 0 \;, \tag{59}$$

$$f'(r) \to 0 \;, \quad r \to \infty \;, \tag{60}$$

$$f(r) \to f_0 \;, \quad r \to \infty \;. \tag{61}$$

We will examine the potentials

$$V_1(f) = \frac{\lambda}{4}\left(f^2 - f_0^2\right)^2 \;, \tag{62}$$

$$V_2(f) = \frac{\lambda}{6M_P^2}\left(f^6 - 3f_0^4f^2 + 2f_0^6\right) \;, \tag{63}$$

$$V_3(f) = \frac{\lambda}{4}e^{\beta f M_P^{-1}}\left(f^2 - f_0^2\right)^2 \;. \tag{64}$$

The first potential is a standard potential of a theory with spontaneous symmetry breaking, the second one is inspired by some phenomenological models based on supergravity with $f_0^2 \sim \frac{M_P m_{3/2}}{\sqrt{\lambda}}$, the third one is inspired by string theory. We will find the wormhole solutions numerically using the fact that the field $f$ has a definite value $f_0$ at infinity. In particular, for axions we will be interested in $f_0 \sim 10^{12}$ GeV and for the textures in $f_0 \sim 10^{16}$ GeV.[4] Typically the radial component of the axion field $f$ will grow many orders of magnitude from the value $f_0$ to $f \sim M_P$ when approaching the wormhole throat.

Note that our expression for the total action (57) does not have any *explicit* dependence on the effective potential $V(f)$ and on the charge $n$. It is important also that $0 \leq R'(r) < 1$ for all wormhole solutions which we are going to study. As a result, the integrand is always positive, despite the fact that the gravitational contribution to the action may be negative. One of the consequences of this result is that if we make a cutoff at some radius $r_c$ and integrate from $r = 0$ to $r = r_c$, the result will be always smaller than the total action (57).

### B. Wormholes in the theories without symmetry breaking

In this paper we study the theories with spontaneous symmetry breaking. However, on a small scale $R \lesssim m^{-1}(f) \sim (\sqrt{\lambda} f)^{-1}$ the wormholes in the theories with the effective potential $\frac{\lambda}{4} f^4$ described in [15, 16] are very similar to the wormholes which we have found in many theories with spontaneous symmetry breaking. Therefore we will briefly discuss here the wormhole solutions in the theory $\frac{\lambda}{4}\phi^4$, following [15].

Near the wormhole throat the solution was obtained in [15] numerically. This solution should be matched to the solution obtained analytically at large $r$. According to Eq. (56), far away from the throat one has $R(r) = r$. Therefore Eq. (55) in this case (for $n = 1$) reads

$$f'' - \frac{3f'}{r} - \lambda f^3 - \frac{1}{4\pi^4 f^3 r^6} = 0 \ . \tag{65}$$

This equation has an exact solution

$$f = \frac{\delta}{r} \ , \tag{66}$$

where $\delta$ is determined from the equation

$$\frac{1}{4\pi^4 \delta^4} - \lambda \delta^2 = 1 \ . \tag{67}$$

If one adds to the effective potential the term $\frac{m^2}{2} f^2$ with $m^2 > 0$ (no spontaneous symmetry breaking), the asymptotic behavior at large $r$ changes to $f \sim (2\pi^2 m r^3)^{-1/2}$.

It was assumed in [15] that the contribution to the action from the region near the wormhole throat [until the solution approaches its regime (66)] is very small. On the other hand, integration far away from the throat gives the contribution

$$S = 2\pi^2 \int r^3 dr \left( \frac{1}{2} f'^2 + \frac{1}{8\pi^4 f^2 r^6} + V(f) \right) \ . \tag{68}$$

The total action integrated up to some $r_{\max}$ has the general form

$$S_{\text{total}}(r_{\max}) = -\left(1 - \frac{3\lambda}{8\pi^2}\right) \ln(m r_n) + m r_{\max} + \Delta S \ . \tag{69}$$

Here $r_n$ corresponds to the place where the numerical solution near the wormhole throat matches the solution (66), $\Delta S$ stands for several other terms which have not been determined in [15]. One can calculate $\Delta S$ using our expression for the action (57); typically this term is fairly small, $\Delta S \sim 1$.

Thus the two most interesting terms in (69) are the logarithmic term and the term which linearly diverges at large $r_{\max}$. As we will see, a similar logarithmic contribution appears in the theories with spontaneous symmetry breaking as well. However, the linear divergence at $r_{\max} \to \infty$ is a particular property of the theory without symmetry breaking, which is a consequence of the asymptotic behavior $f \sim (2\pi^2 m r^3)^{-1/2}$ at large $r$. One could conclude that the wormhole action is infinite in the theories without spontaneous symmetry breaking, and therefore these theories cannot lead to the global symmetry violation.

However, it was argued in [15, 16] that this is not the case, and in fact the effects of global symmetry violation are quite significant. It was suggested that these effects are suppressed only by some small part of action $S_w$ coming from the region close to the throat of the wormhole. The value of $S_w$ was not calculated in [15, 16], but it was estimated to be of the order of 1. We will return to the discussion of this issue when we will consider wormholes in the theories with spontaneous symmetry breaking.

### C. Wormholes in the theory with the simplest potential $\frac{\lambda}{4}(f^2 - f_0^2)^2$

The equation for the scalar field $f$ in the theory $\frac{\lambda}{4}(f^2 - f_0^2)^2$ on the wormhole configuration looks very similar to Eq. (65):

$$f'' - \frac{3f'}{r} - \lambda f (f^2 - f_0^2) - \frac{1}{4\pi^4 f^3 r^6} = 0 \ . \tag{70}$$

Therefore the wormhole solutions at $f \gg f_0$ behave just as their counterparts in the theory without spontaneous symmetry breaking. In particular, far away from the wormhole throat $f = \frac{\delta}{r}$, where $\delta$ is determined by $\frac{1}{4\pi^4\delta^4} - \lambda\delta^2 = 1$.

The main difference between the wormhole solutions with and without symmetry breaking appears on the scale $r \gtrsim (4\sqrt{2}\pi^3\lambda)^{-1/5} f_0^{-1}$, where the field $f$ approaches $f_0$ very rapidly (though not exponentially, as anticipated in [15]). We have found that, at large $r$,

[4]We should emphasize again that the value of the parameter $f_0$ may be quite different from $10^{12}$ GeV. However, as we will see, our results depend on $f_0$ only logarithmically. Also, for textures one should take a theory with another group of symmetries, which, however, should not considerably change our results.

$$f(r) - f_0 = \frac{1}{8\pi^4 \lambda f_0^5 r^6} , \tag{71}$$

which leads to a finite (and very small) contribution to action from the region $r \gtrsim (4\sqrt{2}\pi^3\lambda)^{-1/5} f_0^{-1}$. Thus, in our theory we do not have any problems with infinite wormhole action.

To study the behavior of our solutions in all regions from $r = 0$ to $r \to \infty$, and to avoid having numerical uncertainties associated with the value of action coming from each of the regions we performed a numerical investigation of the wormhole solutions in our theory. This investigation was rather complicated since the solutions happen to be extremely sensitive to the boundary conditions. Whereas it was possible to obtain some results for $f_0 \sim 10^{18}$ GeV using standard numerical recipes [25], it was necessary to substantially improve these methods in order to study the physically interesting regime with $f_0 \sim 10^{12}$ GeV. Since this improvement may be useful not only for finding the wormhole solutions, we will describe our method in the Appendix.

We have found it useful to make our numerical calculations in dimensionless variables $\rho$, $A(\rho)$, $F$, and $U(F)$, where

$$\rho = rM_P \sqrt{\frac{3\lambda}{8\pi}} , \quad A = RM_P \sqrt{\frac{3\lambda}{8\pi}} , \quad F = \frac{f}{M_P}\sqrt{\frac{8\pi}{3}} ,$$

$$U(F) \equiv \lambda^{-1} V(\phi) . \tag{72}$$

We also introduce the combination $Q \equiv \frac{n^2\lambda^2}{8\pi^4}$ [15].

Equations of motion in these variables are given by

$$F''(\rho) + \frac{3A'(\rho)F'(\rho)}{A(\rho)} - \frac{dU(F)}{dF} - \frac{2Q^2}{F^3(\rho)A^6(\rho)} = 0 , \tag{73}$$

$$A'^2(\rho) - 1 + A(\rho)^2 \left( U(F) + \frac{Q^2}{F^2(\rho)A^6(\rho)} - \frac{1}{2}F'^2(\rho) \right) = 0 . \tag{74}$$

The on shell action is

$$S_{\text{total}} = \frac{n}{\sqrt{2Q}} \int_0^\infty d\rho \left[ A^3(\rho)F'^2(\rho) + 2A(\rho)A'(\rho)[1 - A'(\rho)] \right] . \tag{75}$$

Our numerical solutions of the system of differential equations (73) for the theory $\frac{\lambda}{4}(f^2 - f_0^2)^2$ depend on the value of the coupling constant $\lambda$ and on the asymptotic value of the field $f_0$. In all cases we are interested in the strongest possible violation of global symmetries by gravity and therefore we will consider the smallest value of the charge $n = 1$. We will present most of our results for $\lambda = 0.1$ and $\lambda = 1$, but in all figures we will show only the case $\lambda = 0.1$. Different asymptotic values of the field $f_0$ are considered, from $f_0 = 10^{12}$ GeV to $f_0 = 10^{18}$ GeV. The corresponding values of the dimensionless field $F_0$ are

$$F_0 = f_0 \sqrt{\frac{8\pi}{3M_P^2}} = f_0 \times 2.4 \times 10^{-19}\,\text{GeV}^{-1} . \tag{76}$$

For example, for $f_0 = 10^{12}$ GeV one has $F_0 = 2.4 \times 10^{-7}$ and for $f_0 = 10^{16}$ GeV one has $F_0 = 2.4 \times 10^{-3}$. The solution for the dimensionless radial component of the scalar field $F$ is represented in Fig. 2 where the value of $\log_{10} F$ is plotted as a function of $\log_{10}\rho$ for seven different values of $F_0$, corresponding to $f_0 = (10^{18}, 10^{17}, 10^{16}, 10^{15}, 10^{14}, 10^{13}, 10^{12})$ GeV. One can see from Fig. 2 that all solutions with different asymptotic values of $f_0$ behave in the same way near to the wormhole throat. This means that the axion-type field $F$ which far away from the wormhole was $\sim 10^{-7}$, increases 7 orders of magnitude near the throat to become $\sim 1$, and the texture-type field $F$ which far away from the wormhole was $\sim 10^{-3}$, increases 3 orders of magnitude near the throat to reach the same value $\sim 1$ corresponding to $f \sim M_P$. Thus, the solutions which we obtained differ very much from the Giddings-Strominger-Lee solutions with a fixed value $f(r) = f_0$. On the other hand, the fact that the solutions at small $r$ do not depend on $f_0$ confirm our expectations that the behavior of these solutions near the wormhole throat does not depend on spontaneous symmetry breaking.

This conclusion becomes even more obvious if one considers Fig. 3, which gives the value of the function $A(\rho)$ [i.e., $R(r)$]. This function, describing the geometry is completely insensitive to the asymptotic value of the field $F_0$ (i.e., of $f_0$). Therefore we have only one curve for all seven cases above.

One can also express our results in the usual dimensional variables. One can show, in particular, that if the coupling constant $\lambda$ is very small, then, just as in the theory without symmetry breaking [15], the value of the field $f$ at the wormhole throat and the radius of the throat are given by the simple expressions independent on $\lambda$ and $f_0$:

$$f(0) \approx M_P \sqrt{\frac{3}{8\pi}} , \quad R(0) \approx \frac{2}{\sqrt{3\sqrt{2}\pi}} M_P^{-1} . \tag{77}$$

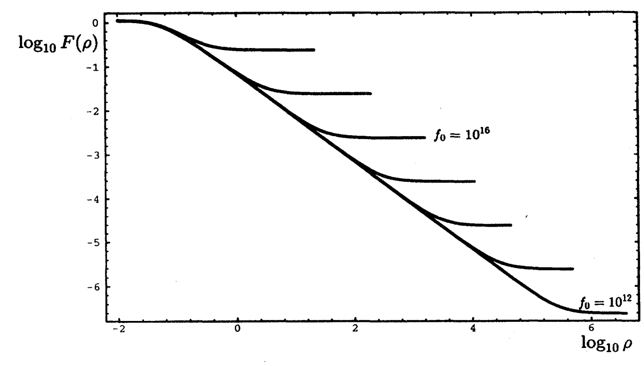


FIG. 2. The distribution of $\log_{10} F$ as a function of $\log_{10}\rho$ for the wormhole solution in the theory with an effective potential $\frac{\lambda}{4}(f^2 - f_0^2)^2$. Here $\rho = rM_P\sqrt{\frac{3\lambda}{8\pi}}$, $F = \frac{f}{M_P}\sqrt{\frac{8\pi}{3}}$.

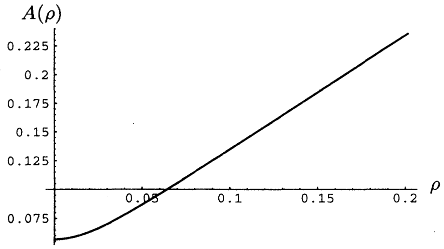


FIG. 3. Wormhole geometry for the potential $\frac{\lambda}{4}(f^2 - f_0^2)^2$. Here $\rho = rM_P\sqrt{\frac{3\lambda}{8\pi}}$, $A = RM_P\sqrt{\frac{3\lambda}{8\pi}}$.

Meanwhile the total action does depend on $f_0$, and this dependence is pretty simple. Figure 4 represents the value of the action as the function of $-\ln F_0$ for $\lambda = 0.1$. The black line and dots give the total action with the boundary term, the grey one shows the action without the contribution of the boundary term at the inner boundary (see the discussion of this possibility in the previous section). All data fit the following simple equation for the action:

$$S_{\text{total}} = a - b\ln F_0 \ . \tag{78}$$

The values of $a$, $b$ for two values of $\lambda$ are

$$a = 0.186 \pm 0.005 \ , \quad b = 1.001 \pm 0.005 \ , \quad \lambda = 0.1, \tag{79}$$

$$a = 0.188 \pm 0.01 \ , \quad b = 1.010 \pm 0.002 \ , \quad \lambda = 1 \ . \tag{80}$$

Thus the dependence on $\lambda$ is not strong and the total action is small.

If we consider the configuration without the boundary term, the action increases slightly. The values of $a$ and $b$ for two values of $\lambda$ are

$$a = 0.850 \pm 0.005, \quad b = 1.001 \pm 0.005, \quad \lambda = 0.1 \ , \tag{81}$$

$$a = 1.33 \pm 0.01, \quad b = 1.010 \pm 0.002, \quad \lambda = 1 \ . \tag{82}$$

However, even in this case the total action remains of the order of 15 for $f_0 \sim 10^{12}$ GeV. This is much smaller than what we need.

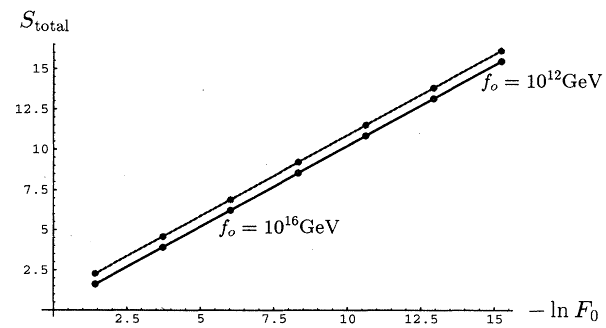


FIG. 4. Total action as a function of $-\ln F_0 \equiv \ln \frac{M_P}{f} + \frac{1}{2}\ln\frac{3}{8\pi}$ for $\lambda = 0.1$. The black dots and the line show the total action with the boundary term at the throat, the grey ones give the corresponding values for the action without the boundary term. Note that $S_{\text{total}} \sim 15$ for $f_0 \sim 10^{12}$ GeV.

This conclusion may seem somewhat unexpected. Indeed, the only natural length scale in the theory of gravity is $M_P^{-1}$. How could it be possible for a wormhole with a throat of a radius $\sim M_P^{-1}$ to absorb a vortex with $f^2\dot{\theta} \neq 0$ or a particle of a typical size $m^{-1} \gg M_P^{-1}$? Indeed, if wormholes do not change the value of the scalar field $f$, such processes are extremely strongly suppressed, as we have seen for the case of the Giddings-Strominger-Lee wormhole. However, in our case the total action is rather small, and it depends on $f_0$ only logarithmically. A possible interpretation of our results is the following. It does not cost the wormhole almost any action to squeeze the vortex to the size $r \sim \lambda^{-1/5} f_0^{-1}$. Later (in the Euclidean time $r$) by increasing the scalar field $f$ the wormhole squeezes the vortex to the Planckian size and easily swallows it. One may say that our wormholes have a small throat but a very big mouth; they compactify the charge before absorbing it.

Note that the total action provides the maximum value of suppression of the violation of global symmetries. However, as we have already mentioned, this suppression in fact may be even much weaker, if it is controlled not by the total action but only by the contribution to the action from the small vicinity of the wormhole throat [15, 16].

Indeed, nonperturbative effects are controlled by the total action only if one can use the dilute gas approximation and consider contribution of each wormhole separately. If the wormholes are very compact and their action is very large, then it is indeed the case. Otherwise one may consider a possibility that the wormholes carrying away opposite charges can screen the large-scale "tails" of each other, and their effective action then will be determined by the integration from $r = 0$ to $r_c$, where $2r_c$ is a typical distance between the throats of different wormholes. If $r_c$ is not much different from the radius of the wormhole throat $R(0)$, the corresponding action should be small, and there will be no suppression of the wormhole-induced global symmetry violation.

To make an estimate of the cutoff radius $r_c$ (assuming that we are already in the regime $R \approx r$) one may write an approximate condition implying that there are no wormholes within the distance $2r_c$ from each other:

$$2\pi^2 \left(\frac{r_c}{R(0)}\right)^4 e^{-S_{\text{total}}(r_c)} \sim 1 \ . \tag{83}$$

Here $e^{-S_{\text{total}}(r_c)}$ appears due to the exponential suppression of the wormhole-like fluctuations on the scale $r_c$, and $2\pi^2\left(\frac{r_c}{R(0)}\right)^4$ is our estimate for the subexponential factor.

Using these results and Eq. (83) one can obtain the value of $e^{-S_{\text{total}}(r_c)}$ which should be associated with the effective coupling constant of the operators violating global symmetries. In the case we are considering right now it is a pretty easy problem to solve. Indeed, at $r \gg R(0)$ our wormhole solution enters the regime $R = r$, $f = \frac{\beta}{r}$, and its action at this stage with a very good accuracy is equal to $\ln\frac{r}{R(0)}$ [compare with (78)]. Therefore our condition (83) in this case reads

$$2\pi^2\left(\frac{r_c}{R(0)}\right)^4 e^{-\ln\frac{r}{R(0)}} = 2\pi^2\left(\frac{r_c}{R(0)}\right)^3 \sim 1 . \quad (84)$$

It is clear that this condition cannot be satisfied for $r > R(0)$. Thus, $r_c \lesssim R(0)$ for the wormholes in the theory $\frac{\lambda}{4}(f^2 - f_0^2)^2$. This gives $S_{\rm total}(r_c) \sim 1$, which implies existence of the vertex operators $gM_P^3(\Phi + \Phi^*)$ violating global U(1) symmetry with the unacceptably large effective coupling constant $g \sim e^{-S_{\rm total}(r_c)} \sim 1$.

Thus, spontaneous symmetry breaking *per se* does not imply any suppression of the wormhole-induced violation of the global symmetries. One should be warned, however, that this conclusion was based on a rather crude estimate of $e^{-S_{\rm total}(r_c)}$ using Eq. (83). This equation is based on the assumption that if one has many wormholes at a distance $2r_c$ from each other, the action of each wormhole is (approximately) equal to the action of a single wormhole solution integrated from $r = 0$ up to $r_c$. Note, however, that the scalar field $f$ at $r = r_c$ remains extremely large, $f(r_c) \sim M_P \gg f_0$. Thus one could argue that if space were populated by many wormholes displaced at a very small distance from each other, this would not describe our original situation where the average amplitude of the radial component is equal to $f_0 \ll M_P$. This argument does not really invalidate the multiwormhole scenario. The radial part of the field $\Phi$ may be quite large in the vicinity of each wormhole, but the presence of the charge implies that the phase $\theta$ depends on $r$ and can be different for each of the wormholes. Therefore, even though the value of the field $\Phi$ near each of the wormholes is of the order of $M_P$, the average field $\Phi$ in the whole space (after taking the average over fluctuations of $\theta$) can have a very small radial component $f_0 \ll M_P$.

To return to a more solid ground, one should note that even if one does not want to consider this multiwormhole picture and decrease the wormhole action by introducing a cutoff at $r = r_c$, one still has a problem. Indeed, in the theory with spontaneous symmetry breaking, unlike in the theory without symmetry breaking considered in [15, 16], the total wormhole action is finite *and small.* The largest action which we have obtained for the axion theory with $f_0 \sim 10^{12}$ GeV is only about 15. This is more than ten times smaller than the action $S \sim 190$ which is necessary to save the axion theory. Therefore we will study now other, more complicated models, where one may hope to obtain larger values of $S$.

Note that if we find the wormholes which have a very large action $S \gtrsim 190$ given by the integration in a small vicinity of the wormhole throat (and we will find such solutions), then our equation (83) will suggest that the cutoff radius $r_c$ should be many orders of magnitude greater than $R(0)$. Therefore in all situations where we will find a solution to the problem of strong violation of the global symmetries, the effective coupling constants in our vertex operators of the type of $g_1 M_P^3(\Phi+\Phi^*)$ will be determined by the total wormhole action $S_{\rm total}$ rather than by its small part originated by the integration near the wormhole throat. This will eliminate all uncertainties with the interpretation of the vertex operators and multiwormhole solutions described above.

### D. A more complicated polynomial potential

The next theory on our list has an effective potential

$$V_2(f) = \frac{\lambda}{6M_P^2}\left(f^6 - 3f_0^4 f^2 + 2f_0^6\right) , \quad (85)$$

where $f_0^2 \sim \frac{M_P\, m_{3/2}}{\sqrt{\lambda}}$. This potential was suggested to us by Dine as a useful phenomenological potential for the axion theory which might follow from supergravity. We have found the wormhole solution in this theory and performed the calculation of the action. There was practically no difference in the value of the action as compared with the action in the theory $\frac{\lambda}{4}(f^2 - f_0^2)^2$. Again we have recovered the logarithmic dependence on the value of $f_0$. All figures practically coincide with the ones obtained in the previous case. The conclusion is that the mild change of the potential responsible for spontaneous symmetry breaking does not change the value of the wormhole action and therefore in such theories we have to face a strong violation of global symmetries.

The reason can be easily understood. In the theory $\frac{\lambda}{4}(f^2 - f_0^2)^2$ the field $f$ far away from the wormhole throat behaved as $f = \frac{\delta}{r}$, with $\delta$ being determined by $\frac{1}{4\pi^4\delta^4} - \lambda\delta^2 = 1$. When the field $f$ decreases as $f = \frac{\delta}{r}$, the contribution of the term $\lambda f^3$ to the equation of motion (70) decreases as fast as the contribution of all other terms (for $f \gg f_0$). If now one has an effective potential which depends on $f$ as $f^6$, then its contribution to the equations of motion in the regime $f \sim \frac{\delta}{r}$ decreases even faster than other terms, and the field behaves as in the theory $\frac{\lambda}{4}(f^2 - f_0^2)^2$ in the small $\lambda$ limit.

Thus the existence of the regime $f \sim \frac{\delta}{r}$ is a very general property of our wormhole solutions. This leads to the familiar logarithmic dependence of the action on $f_0$. Therefore it is very difficult to increase the action $S$ by changing $f_0$. However, as we will see now, one can considerably increase the action if one succeeds to increase the radius of the wormhole throat $R(0)$.

### E. Exponential potential

The third class of potentials includes an exponential dependence on the radial component of the field which forces the field to remain close to its minimum value and not grow so fast as in the previous cases, $V_3(f) = \frac{\lambda}{4}e^{\beta f M_P^{-1}}\left(f^2 - f_0^2\right)^2$. In terms of our dimensionless variables the potential is

$$U_3(F) = \tfrac{1}{4}e^{\beta F}\left(F^2 - F_0^2\right)^2 . \quad (86)$$

Our previous calculations have shown that the wormhole action is small when the theory allows the radial component of the field to grow near the throat. By introducing the exponent to the potential we were hoping to achieve several different purposes.

(i) We were trying to keep interactions far away from the wormhole the same as in the usual theory of spontaneously broken symmetry. Indeed at small $F$ this potential coincides with the standard potential $\frac{1}{4}(F^2 - F_0^2)^2$.

(ii) When approaching small distances the value of the exponent will change strongly; the growth of the field $F$ will not be able to proceed. This should increase the size of the throat of the wormhole.

(iii) The exponent in the potential should imitate a gravitational theory which does not allow distances smaller than some values. In this way we could have a model where the size of the wormhole throat cannot be small. As we will see, this will give us large wormholes with the action approaching the limiting case of the Giddings-Strominger-Lee wormhole with the radial component of the scalar field frozen to its value at infinity, $F = F_0$.

Our expectations were indeed confirmed by the calculations. We present the wormhole solutions for five different values of the exponent $\beta = (0, 10, 100, 300, 500)$. We work with $\lambda = 10^{-1}$ and $f_0 = 10^{16}$ GeV (i.e., $F_0 = 2.4 \times 10^{-3}$). Figure 5 shows the plot of $\log_{10} F$ as the function of $\log_{10} \rho$. At infinity all solutions are given by the same line, corresponding to $F_0 = 2.4 \times 10^{-3}$. However they behave differently when approaching the throat. The solutions with the largest values of the exponent do not grow as fast as the solution with smaller exponents, the grows increases when the exponent decreases. The behavior of the radial component of the field shows the tendency to approach the GSL solution where the field $F$ is fixed. In our case this is a property of the solution of a system of equations (73) and (74), whereas the GSL solution is a solution of the gravitational equation (74) only, in which one assumes that $F' = 0$. It is gratifying that the dynamical system with the standard mechanism of spontaneous symmetry breaking can be brought to the regime of the almost frozen radial component of the field which results in the increase of the wormhole action and suppression of the violation of global symmetries. The geometric properties of the wormhole are represented in Fig. 6, which shows the $A(r)$ and in Fig. 7, which shows $A'(\rho) \equiv R'(r)$. One can see a dramatic change of the geometry with an increase of the exponent. With zero exponent we have the same situation as in the theory $\frac{1}{4}(F^2 - F_0^2)^2$. The wormhole throat is very small, $A'$ becomes close to the flat space value $A' = 1$ very fast. With the growth of $\beta$ the picture changes: $A'$ remains smaller than 1 even far away from $\rho = 0$, and the size of the throat $A(0)$ becomes much larger.

The total action was calculated for several different

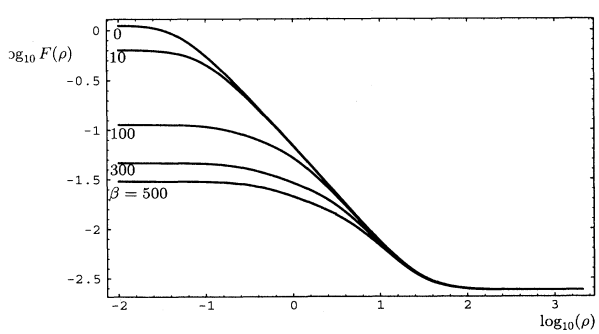


FIG. 5. Plot of $\log_{10} F$ for different values of the exponent and $f_0 = 10^{16}$ GeV .

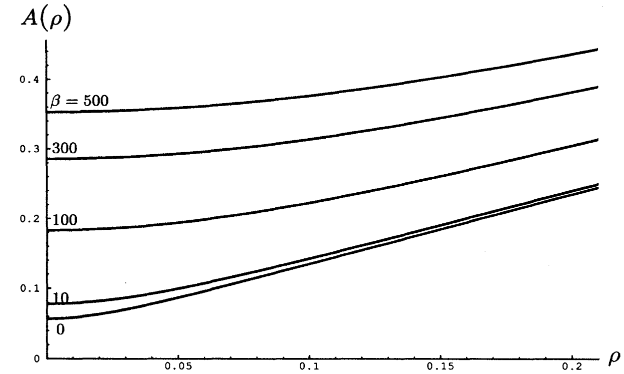


FIG. 6. Scale factor $A(\rho)$ for different values of the exponent and $f_0 = 10^{16}$ GeV.

values of the exponent $\beta$. The results are plotted in Fig. 8. We have found that in a wide range of values of $\beta$ from 0 to 500 the action is given by a very simple expression

$$S_{\text{total}} \approx a + b\beta \ . \tag{87}$$

The values of $a, b$ for the action with the surface terms are $a = 5.7$, $b = 0.034$. The corresponding dots and the line in Fig. 9 are black. For completeness we will also give the results for the action without the boundary term at the throat (see discussion in Sec. III). In this case $a = 6.8$, $b = 0.083$, see the grey line in Fig. 8.

Note that the value of the action without the boundary term for the largest value of our exponent is twice as large as the one with the boundary term, although with vanishing exponent the difference is very small. The reason is that at a higher value of the exponent the value of the wormhole neck is getting much larger and therefore the term $\frac{3\pi}{4} M_P^2 R^2(0)$ is also getting larger.

We have also calculated the part of the action coming only from the area close to the wormhole throat, where the geometry is much different from the geometry of a flat space. This part does not take into account the log-

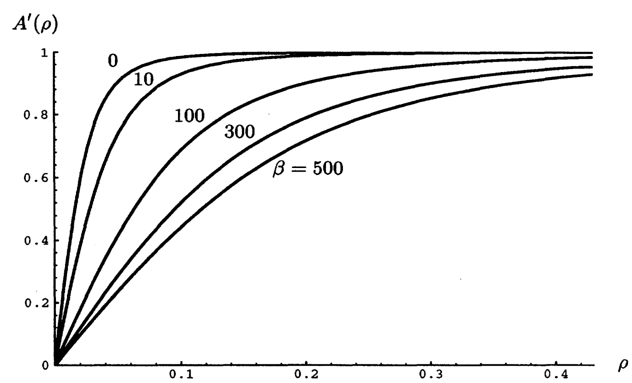


FIG. 7. Derivative of the scale factor $A(\rho)$ for different values of the exponent and $f_0 = 10^{16}$ GeV. Wormhole geometry approaches geometry of a flat Euclidean space when $A'(\rho)$ approaches 1.

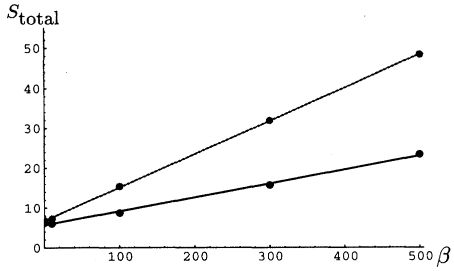


FIG. 8. The total action in the theory with the exponential potential $V_3(f) = \frac{\lambda}{4} e^{\beta f M_P^{-1}} (f^2 - f_0^2)^2$ as a function of the exponent $\beta$. Black dots give the action with the boundary term, grey ones give the action without the boundary term at the wormhole throat.

arithmic contribution which comes from the region with $R' \approx 1$. We have called $S_w$ the part of the action which comes from the region from $r = 0$ to the radius $r$ at which $R'(r)$ grows up to 0.9. On dimensional grounds one could expect that $S_w$ should be a quadratic function of the size of the throat. This is indeed the case. The results can be described rather well by the quadratic expression

$$S_w = 0.16 + 85A^2(0) \approx 0.16 + M_P^2 R^2(0) \ . \tag{88}$$

(Note that $85A^2 \approx M_P^2 R^2$ for $\lambda = 0.1$.) Five dots correspond to the five values of the exponent $\beta$: the largest action comes from the theory with the largest exponent, see Fig. 9.

The total wormhole action (which is the main interest for us) also grows as $M_P^2 R^2(0)$, but with a greater coefficient:

$$S_{\text{total}} \approx 5 + 1.7 M_P^2 R^2(0) \ . \tag{89}$$

Because of the computational difficulties we have not performed the calculation for the values of exponent greater than 500. At $\beta < 500$ the logarithmic terms which are taken into account in $S_{\text{total}}$ (but not in $S_w$) also give a considerable contribution. Therefore the quadratic fit for $S_{\text{total}}$ is much less accurate than the fit for $S_w$, and the coefficient 1.7 in (89) is obtained with a rather limited

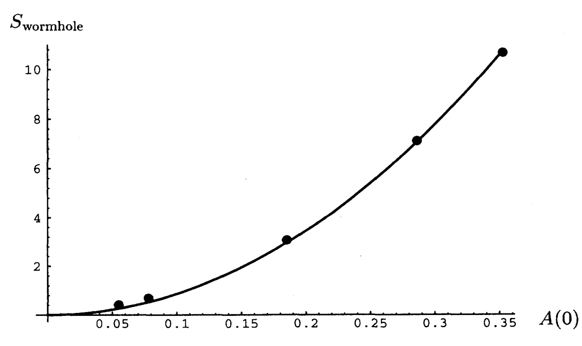


FIG. 9. The action obtained by integration in the region near the wormhole throat [where $A'(\rho) = R'(r) < 0.9$] for different values of the exponent $\beta$.

accuracy. However, the behavior of the solutions allows us to make a plausible assumption. We expect that with further growth of the exponential factor the field $f(r)$ will become frozen near $f_0$, and the total wormhole action will approach our expression for the action (46) and (51) for the Giddings-Strominger-Lee wormhole:

$$\begin{aligned} S_{\text{total}} &= \frac{\sqrt{3\pi}}{8} \left(1 - \frac{2}{\pi}\right) \frac{M_P}{f_0} \\ &= \frac{3\pi^2}{8} \left(1 - \frac{2}{\pi}\right) M_P^2 R^2(0) \\ &= 1.34 M_P^2 R^2(0) \ . \end{aligned} \tag{90}$$

This expression is consistent with our approximate quadratic fit (89).

## V. GLOBAL SYMMETRIES AND PLANCK SCALE PHYSICS

### A. Kaluza-Klein wormholes

As we have seen, it is extremely difficult to increase the wormhole action by changing the effective potential of the scalar field. However, as a result of our investigation we have learned that in those cases when we were able to make the action large, the value of the action could be estimated by a simple expression $S_{\text{total}} \sim 1.34 R^2(0) M_P^2$. Remember also that the action becomes almost three times larger if one does not include the inner boundary contribution. This suggests that one can obtain a very large action if there exists some reason why the wormhole throat should be large. Indeed, as we mentioned in the Introduction, we may not have any problems with axions if the effective coupling constants of the operators violating the global symmetry are smaller than $e^{-190}$. Equation (88) suggests that this happens if the radius of the wormhole throat becomes greater than $15 M_P^{-1}$. We were unable to make the throat that large by changing the effective potential of the scalar field, but there exist other possibilities to do so.

Indeed, we have assumed that our space remains four dimensional and that gravitational interactions are described by the standard Einstein theory at all length scales. Meanwhile, each of these assumptions may be wrong.

First of all, according to Kaluza-Klein theories, the number of dimensions of space-time is much greater than four, but space-time becomes effectively four dimensional at $R \gtrsim R_c$, where $R_c$ is the radius of compactification. What if $R_c \gg M_P^{-1}$? Then our equations should be considerably modified at $R \lesssim R_c$, which may lead to the wormhole throat of a large size $R(0) \sim R_c$.

It is not easy to test our hypothesis in any realistic theory, but we may play with a toy model. First of all, at $R \gg R_c$ our equations should coincide with our original equations (55) and (56). Meanwhile we will assume that at $R \ll R_c$ space-time becomes ten dimensional, as in string theory. In this case at $R \ll R_c$ the charge conservation equation instead of $n = 2\pi^2 f^2 \theta' R^3$ gives $n = \frac{\pi^2 f^2 \theta' R^9}{12 R_c^6}$. (We assumed that six dimensions form a sphere $S_6$ of radius $R_c$ and area $\pi^3 R_c^3$, and took into account that the area of a sphere $S_9$ of radius $R$ is $\frac{\pi^5 R^9}{12}$.) Modified equations (55) and (56) look as follows:

$$f'' - \frac{9R'f'}{R} - \frac{dV(f)}{df} - \frac{144n^2R_c^{12}}{\pi^4 f^3 R^{18}} = 0 \ , \quad (91)$$

$$R'^2 - 1 + \frac{8\pi}{3M_P^2}R^2\left(V(f) + \frac{72n^2R_c^{12}}{\pi^4 f^2 R^{18}} - \frac{f'^2}{2}\right) = 0 \ . \quad (92)$$

Note also that the factor 3 in front of $\frac{R'f'}{R}$ in the first of these equations was replaced by 9, which corresponds to space time with $d = 10$.

The idea of a phenomenological description of possible wormhole solutions in this situation is to solve equations which at $R \gg R_c$ look like (55) and (56), but at $R \ll R_c$ look like (91) and (92). It can be achieved, e.g., by introducing an interpolating factor $(1 + \frac{24^{1/6}R_c}{R})^{-6}$, which changes the nine-dimensional volume, and, correspondingly, the conservation law, $n = 2\pi^2 f^2\theta' R^3(1 + \frac{24^{1/6}R_c}{R})^{-6}$. This equation gives $n = 2\pi^2 f^2 \theta' R^3$ at $R \gg R_c$, and $n = \frac{\pi^2 f^2\theta' R^9}{12R_c^6}$ at $R \ll R_c$. After this and some other obvious modifications the interpolating equations can be represented in the following way:

$$f'' - \frac{3R'f'}{R}\left(1 + 2\frac{24^{1/6}R_c}{R_c + R}\right) - \frac{dV(f)}{df} - \frac{n^2}{4\pi^4 f^3 R^6}\left(1 + \frac{24^{1/6}R_c}{R}\right)^{12} = 0 \ , \quad (93)$$

$$R'^2 - 1 + \frac{8\pi}{3M_P^2}R^2\left(V(f) + \frac{n^2}{8\pi^4 f^2 R^6}\left(1 + \frac{24^{1/6}R_c}{R}\right)^{12} - \frac{f'^2}{2}\right) = 0 \ . \quad (94)$$

We have solved these equations numerically for various values of $R_c$, see Fig. 10. As expected, the radius of the wormhole throat $R(0)$ was found to be approximately equal to the compactification radius $R_c$. Of course, our investigation of Kaluza-Klein wormholes cannot be considered conclusive. Still it indicates that the global symmetry-breaking problem may disappear in the theories where the radius of compactification $R_c$ is sufficiently large.

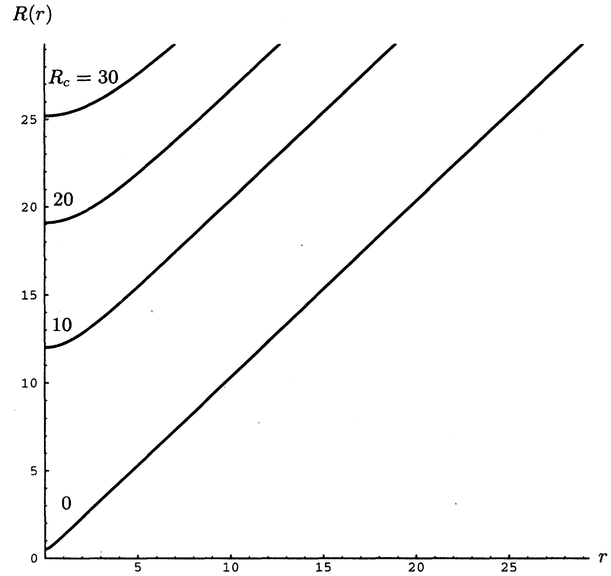


FIG. 10. Behavior of the radius $R$ of the wormhole solution at small $r$ in our model of a 10-dimensional Kaluza-Klein theory with different values of the compactification radius $R_c$. All quantities are given in the Planck units $M_P^{-1} = 1$.

### B. One-loop effects in quantum gravity

Another possibility is related to the $\mathcal{R}^2$ corrections which may appear in the effective Lagrangian or in equations of motion of the gravitational field. The simplest example is the conformal anomaly, which gives the contribution $\frac{1}{6M_0^2}\,{}^{(1)}H_{\mu\nu} + \frac{1}{H_0^2}\,{}^{(3)}H_{\mu\nu}$ to the gravitational equations. Here

$${}^{(1)}H_{\mu\nu} = 2\left(\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla^2\right)\mathcal{R} + 2\mathcal{R}\mathcal{R}_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}\mathcal{R}^2 \ , \quad (95)$$

$${}^{(3)}H_{\mu\nu} = \mathcal{R}_\mu^\lambda\mathcal{R}_{\lambda\nu} - \tfrac{2}{3}\mathcal{R}\mathcal{R}_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}\mathcal{R}^{\rho\sigma}\mathcal{R}_{\rho\sigma} + \tfrac{1}{4}g_{\mu\nu}\mathcal{R}^2 \ .$$

The parameters $H_0$ and $M_0$ are of the same order as $M_P$, but they can be much smaller than $M_P$ if there are many matter fields (of spin 0, 1/2, and 1) contributing to the conformal anomaly. The equation for the scalar field (55) remains unchanged with an account of conformal anomaly, but the gravitation field equation (56) acquires some new terms. To get an idea of the possible influence of quantum corrections on the structure of wormholes, let us assume for simplicity that $H_0 \ll M_0$, so that the second term in (95) can be neglected. In this case the gravitational equation looks as follows:

$$R'^2 - 1 + \frac{8\pi R^2}{3M_P^2}\left(V(f) + \frac{n^2}{8\pi^4 f^2 R^6} - \frac{f'^2}{2}\right) + \frac{1}{R^2H_0^2}(R'^2 - 1)^2 = 0 \ . \quad (96)$$

This equation on the throat yields

$$R^2(0) = H_0^{-2} - \frac{8\pi R^4(0)}{3M_P^2}\left(V(f) + \frac{n^2}{8\pi^4 f^2 R^6(0)}\right) \ . \quad (97)$$

In the limit $H_0 \gg M_P$ an additional term $\frac{1}{R^2H_0^2}(R'^2 - 1)^2$ does not alter our wormhole solutions. However, for smaller values of $H_0$ the character of our solution changes dramatically, see Fig. 11. The throat of the wormhole becomes considerably wider, and the interval of $r$ where $R' \ll 1$ becomes very small. Finally, this interval disappears altogether, and for $H_0 \lesssim 2M_P$ regular wormhole solutions with $R'(0) = 0$ cease to exist. In other words, even small quantum gravity corrections can lead to absence of wormhole solutions.

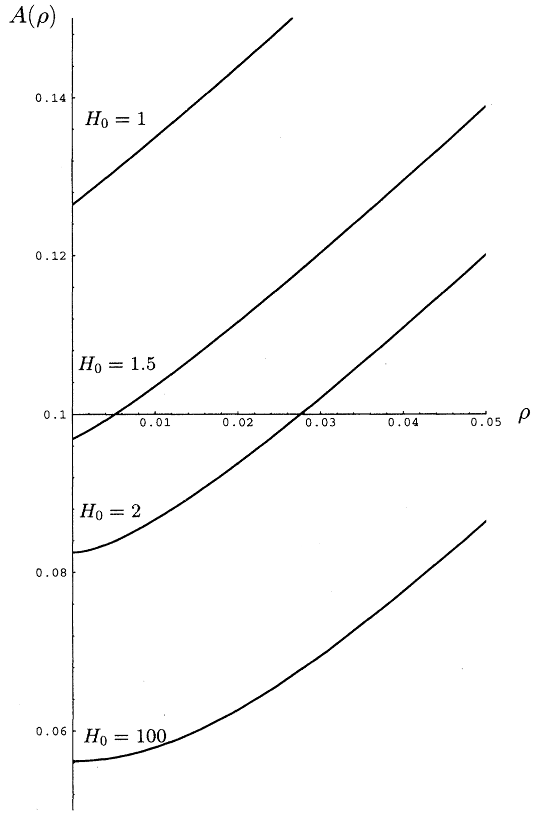


FIG. 11. Behavior of the radius $A(\rho)$ of the wormhole solution at small $\rho$ in the model taking into account conformal anomaly. Note that regular wormhole solutions with $A'(0) = 0$ [i.e., with $R'(0) = 0$] disappear for $H_0 \lesssim 2M_P$, even though in this regime the coefficient $\frac{1}{H_0^2}$ in front of the conformal anomaly term is still very small.

### C. String-inspired models

We mentioned in Sec. III that the first wormhole solution was in fact obtained in the version of the axion theory where instead of the pseudoscalar axion field one has the field $H_{\mu\nu\lambda}$ [6]. In the same paper [6] Giddings and Strominger have obtained a family of wormhole solutions in the string-inspired version of the theory of the field $H_{\mu\nu\lambda}$ with the effective action

$$S_{\rm dual} = \int d^4x\sqrt{g}[-\mathcal{R} + \tfrac{1}{2}(\partial_\mu\phi)^2 + e^{\beta\phi}\, H^2_{\mu\nu\lambda}] - \int_{\partial V} d^3S(K - K_0) \ . \qquad (98)$$

Here $\phi$ is the dilaton field, $\beta$ is a phenomenological parameter; $\beta = 2$ in string theory. For simplicity we used here units in which $\frac{M_P^2}{16\pi} = 1$. (Wormhole solutions in a more general class of theories have been obtained later by Coule and Maeda [24].) The solution for $R(r)$ in this theory does not depend on $\beta$. It coincides with the corresponding solution in the theory (39). However, the situation with the dilaton field is more complicated. The solution can be written in the form

$$e^{-\frac{\beta}{2}\phi(r)} = e^{-\frac{\beta}{2}\phi(0)}\ \cos\left(\frac{\sqrt{3}\beta}{2}\ \arccos\frac{R^2(0)}{R^2(r)}\right) \ . \qquad (99)$$

For $\beta < \frac{2}{\sqrt{3}}$ this equation describes the wormhole solution with $\phi(r)$ which gradually increases at large $r$ from its maximal value $\phi(0)$. However, for $\beta > \frac{2}{\sqrt{3}}$ this regime becomes impossible. Indeed, in this case $\phi(r)$ becomes infinitely large at the point where $\frac{\sqrt{3}\beta}{2}\arccos\frac{R^2(0)}{R^2(r)}$ becomes equal to $\frac{\pi}{2}$. At this point derivatives of the scalar field diverge, and the action becomes infinitely large. The conclusion of Ref. [6] was that in such a situation there are no regular wormhole solutions in this theory.

Does it mean that the gravitational effects cannot make the field $H_{\mu\nu\lambda}$ massive in the theory (98)? We have already discussed a similar situation in the theory of a scalar field without symmetry breaking, where the total action was infinite, and the conclusion was that the effects responsible for the global symmetry violation are suppressed only by the part of the action coming from a small vicinity of the wormhole throat. Thus one could argue that the global symmetry violation could occur in the theory (98) as well, even despite the absence of the wormhole solutions, if one considers a small part of the configuration (99) near the wormhole throat.

However, the main reason why this argument could work for the theory of a scalar field was the existence of two vastly different length scales. The wormhole throat had a nearly Planckian size $\sim M_P^{-1}$, whereas the typical scale on which the scalar field significantly changed was much greater, of the order of $m^{-1}$. Therefore it was possible to pack many wormhole throats inside the region of size $m^{-1}$. This is not the case for configurations (99). In the most interesting case of the string theory with $\beta = 2$ the field $\phi(r)$ becomes infinitely large at the point where

$$R(r) = \frac{R(0)}{\cos\sqrt{\frac{\pi}{2\sqrt{3}}}} \approx 1.27\, R(0) \ . \qquad (100)$$

Thus the total size of our field configuration almost coincides with the size of its throat. Since each of such configurations has infinite action and is not of a wormhole type, we do not expect them to lead to global symmetry breaking.

This does not necessarily imply that there are no wormhole solutions in string theory. The model considered above does not contain any potential $V(\phi)$. Also, it is very hard to associate the value of the dilaton field $\phi$, which is typically assumed to be of the order of $M_P$, with the parameter $f_0 \sim 10^{12}$ GeV. Nevertheless, this model clearly shows that the existence of wormhole solutions in the axion theory is by no means automatic. In this model the wormhole solutions disappear as soon as one considers effects associated with the dilaton field.

## VI. TOPOLOGICAL SUPPRESSION OF WORMHOLE EFFECTS IN GRAVITY AND STRING THEORY

All our previous results have been obtained by an investigation of particular solutions which may or may not appear in different theories. However, there is one general reason which may lead to suppression of wormhole effects. These effects lead to the change of topology of space by creating a universe capable of carrying a global charge away from our space. On the classical level such processes simply cannot occur. Our use of Euclidean methods to describe such processes still needs to be fully justified. But even if these methods are valid, there is an easy way to suppress the probability of the wormhole formation in the Einstein theory.

In order to explain it we remind the reader that the standard Lagrangian of the Yang-Mills field in QCD originally was written as $\frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu}$, but later it was recognized that one can add to this Lagrangian the term $\frac{\theta}{32\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}$. This term does not modify the Yang-Mills equations, but it gives a contribution to the nonperturbative processes involving change of topology of the Yang-Mills field.

A similar situation occurs in the Einstein theory of gravity, where the standard Einstein Lagrangian $-\frac{M_P^2}{16\pi}\mathcal{R}$ can be supplemented by two different topological terms:

$$S = -\int d^4x\sqrt{g}\left(\frac{M_P^2}{16\pi}\mathcal{R} + \frac{\theta_g}{32\pi^2}\mathcal{R}_{\mu\nu\lambda\delta}{}^*\mathcal{R}^{\mu\nu\lambda\delta} + \frac{\gamma}{32\pi^2}{}^*\mathcal{R}_{\mu\nu\lambda\delta}{}^*\mathcal{R}^{\mu\nu\lambda\delta}\right) . \tag{101}$$

Here ${}^*\mathcal{R}_{\mu\nu\lambda\delta} \equiv \frac{1}{2}\epsilon_{\mu\nu\mu'\nu'}\mathcal{R}^{\mu'\nu'}{}_{\lambda\delta}$.

The last two terms do not give any contribution to equations of motion, and therefore do not change the theory at the classical level. Therefore, just as the term $\frac{\theta}{32\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}$, these two terms can be considered as an integral part of general relativity. The first of these terms is very similar in its nature to the term $\frac{\theta}{32\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}$. It contributes to the effective potential determining the value of the axion field. Fortunately, it is not expected to lead to any problems with strong $CP$ violation. The effects induced by this term are related not to the wormhole physics, but to the Abelian Eguchi-Hanson instantons, and typically they are exponentially small, being suppressed by $e^{-\frac{4\pi^2}{e^2}}$ where $e$ is the electromagnetic coupling constant [30].

Meanwhile, the Gauss-Bonnet term $-\frac{\gamma}{32\pi^2}{}^*\mathcal{R}_{\mu\nu\lambda\delta}{}^*\mathcal{R}^{\mu\nu\lambda\delta}$ gives a nonvanishing topological contribution to the wormhole action:

$$S_{\rm topol} = -\frac{\gamma}{32\pi^2}\int d^4x\sqrt{g}\;{}^*\mathcal{R}_{\mu\nu\lambda\delta}{}^*\mathcal{R}^{\mu\nu\lambda\delta} = -\frac{\gamma}{32\pi^2}\int d^4x\sqrt{g}\left(\mathcal{R}_{\mu\nu\lambda\delta}\mathcal{R}^{\mu\nu\lambda\delta} - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}^2\right) . \tag{102}$$

Therefore it may control the strength of the global symmetry breaking.

This term was considered in the early works on wormholes [6]. The constant $\gamma$ was called there a topological coupling constant. Just as the $\theta$ parameter, the value of this constant in gravitational theory is not determined *a priori.*

It is useful to remind the reader that the reason why the effects of the Yang-Mills instantons are suppressed by the factor $e^{-\frac{8\pi^2}{g^2}}$. The semiclassical Yang-Mills action $\frac{1}{4g^2}F_{ab}F^{ab}$ is not topological. Therefore the variation of this action produces equations of motion which have instanton solutions. The semiclassical action calculated on these solutions is known to suppress the instantons $F_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\delta}F_{\lambda\delta}$ as

$$e^{-S} = \exp\left(-\frac{1}{4g^2}\int d^4x F_{ab}F^{ab}\right) = e^{-\frac{8\pi^2}{g^2}\nu}, \tag{103}$$

where $\nu$ is the winding number of the gauge configuration.

In gravity things seem to work differently, but the results are very similar. It is the Einstein action with matter which gives us the equations of motions. Those equations have wormhole solutions. For the Euclidean wormhole geometry (41) with the wormhole radius $R(r)$ the topological (Gauss-Bonnet) contribution to the action is given by the integral

$$S_{\rm topol} = -\frac{\gamma}{32\pi^2}\int d^4x\sqrt{g}\left(\mathcal{R}_{\mu\nu\lambda\delta}\mathcal{R}^{\mu\nu\lambda\delta} - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}^2\right) = \frac{3\gamma}{4\pi^2}\int d^4x\frac{R''(r)[1-R'^2(r)]}{R^3(r)} . \tag{104}$$

After the angular integration the integral becomes

$$S_{\rm topol} = \frac{3\gamma}{2}\int_0^\infty dr R''(r)[1-R'^2(r)] . \tag{105}$$

Keeping in mind that the Gauss-Bonnet part can be brought to a form where it is a total derivative, we can rewrite this integral as

$$S_{\text{topol}} = \frac{3\gamma}{2}\int_0^\infty dr\frac{\partial}{\partial r}[R'(r) - \tfrac{1}{3}R'^3(r)]$$
$$= \frac{3\gamma}{2}[R'(r) - \tfrac{1}{3}R'^3(r)]_{r\to\infty} = \gamma\ . \qquad (106)$$

In performing this calculation we did not use any particular form of the wormhole solution; we have used only the fact that $R'(0) = 0$ and $R'(\infty) = 1$. Thus, independently of any suppression calculated in the previous sections of this paper, there exists an additional exponential suppression of the wormhole-related effects in quantum gravity by the factor $e^{-\gamma}$. This agrees with the result obtained in [6].

It is important that this additional suppression is equally related to the total probability of the wormhole formation and to the values of the vertex operators. Indeed, for all wormhole solutions which are known to us the derivative $R'(r)$ approaches its asymptotic regime $R'(r) = 1$ at $r$ comparable with the radius of the wormhole throat. Thus, the integral in Eq. (106) rapidly converges to $\gamma$ in the vicinity of the wormhole throat. It means that all vertex operators become suppressed by the factor $e^{-\gamma}$.

Note that the value of parameter $\gamma$ is arbitrary; it does not change any experimentally tested predictions of general relativity. If one takes $\gamma > 190$, all our problems with axions in quantum gravity immediately disappear. One may or may not like having a large parameter $\gamma$ in gravitational theory, but it is certainly not forbidden, and it solves the problem of the global symmetry breaking.

Still it would be nice to find some reasons why this parameter should be large. One of the ideas is to consider string theory and to study an analogous topological term there.

In string theory it is considered plausible that the Gauss-Bonnet term appears at the level of $\alpha'$ corrections in a specific form since it is related to the Green-Schwarz mechanism of cancellation of anomalies. One expects that this part of stringy corrections has the form [4]

$$\mathcal{L}_{\text{stringy}} = \frac{\alpha'}{16\kappa^2}\Big[F_{ab}F^{ab} - (\mathcal{R}_{\mu\nu\lambda\delta}\mathcal{R}^{\mu\nu\lambda\delta}$$
$$-4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}^2)\Big]\ . \qquad (107)$$

Here $\alpha'$ is a function of the fundamental dilaton field $\varphi$

$$\alpha' = \frac{4\kappa^2}{g_0^2}e^{-\kappa\varphi}\ , \qquad \kappa^2 = \frac{8\pi}{M_P^2}\ . \qquad (108)$$

Our normalization corresponds to the standard normalization of the Yang-Mills Lagrangian,

$$\mathcal{L}_{\text{YM}} = \frac{\alpha'}{16\kappa^2}F_{ab}F^{ab} = \frac{1}{4g^2}F_{ab}F^{ab}\ , \qquad (109)$$

and we consider $\frac{e^{-\kappa\varphi(x)}}{g_0^2}$ as the "running" gauge coupling constant $\frac{1}{g^2(x)}$.

What follows is a direct generalization of our purely gravitational calculation. The only difference is the presence of the function $\alpha'(x)$. Unfortunately, we do not know much about the dependence of $\alpha'$ on $x$, which makes the results which we are going to obtain less rigorous but perhaps still rather plausible. First of all, since we study spherically symmetric configurations, we suppose that $\alpha'$ depends only on $r$ and does not depend on the angular variables. The integral becomes

$$S_{\text{topol}} = 12\pi^2\int_0^\infty dr\frac{1}{g^2(r)}R''(r)[1 - R'^2(r)]. \qquad (110)$$

As we have seen in the previous sections, on a sufficiently large distance $r_w$ from the wormhole throat the wormhole geometry becomes undistinguishable from the geometry of a flat Euclidean space because its scale factor $R(r)$ becomes almost exactly equal to $r$, and its derivative $R'(r)$ rapidly approaches 1. Typically it happens at the distance of the same order of magnitude as $R(0)$; precise value of $r_w$ will not be particularly important for us. It is important, however, that at a sufficiently large $r > r_w$ the term $[1 - R'^2(r)]$ in the integral (110) becomes very small, which implies that the total value of the integral is dominated by integration in a region $r < r_w$. Note also, that on all our solutions we had $[1 - R'^2(r)] > 0$ and $R'' > 0$. Therefore one can represent the integral (110) in the following way:

$$S_{\text{topol}} \approx 12\pi^2\int_0^{r_w} dr\frac{1}{g^2(r)}R''(r)\{1 - [R'^2(r)]\}$$
$$\equiv \frac{12\pi^2}{g_w^2}\int_0^{r_w} drR''(r)[1 - R'^2(r)]\ , \qquad (111)$$

where $g_w$ is some average value of the gauge coupling constant in the region $0 < r < r_w$ defined by Eq. (111). Since we expect $r_w$ to be of the same order of magnitude as $R(0)$, and $R(0)$ should be determined by typical stringy length scale [just as the natural scale for $R(0)$ in the Einstein theory was given by $M_P^{-1}$], we will identify $g_w$ with the typical value of the gauge coupling constant $g_{\text{str}}$ on the stringy scale. The subsequent evaluation of the integral in (111) goes exactly as in (106), because $R' \approx 1$ on the boundary $r = r_w$. This gives

$$S_{\text{topol}} = \frac{8\pi^2}{g_{\text{str}}^2}\ . \qquad (112)$$

Note that this is a topological contribution to the action, which practically does not depend on the detailed form of the wormhole solutions (this dependence is concentrated in our definition of $g_w \approx g_{\text{str}}$).

This part of the action is a precise analogue of the one-instanton action $\frac{8\pi^2}{g^2}$ in the Yang-Mills case. This leads to an additional suppression of the wormhole-induced effects by the factor

$$e^{-S_{\text{topol}}} = e^{-\frac{8\pi^2}{g_{\text{str}}^2}}\ . \qquad (113)$$

We would like to stress that whereas in the Yang-Mills theory the suppression of the instanton effects comes from the semiclassical action, the suppression of the wormhole effects described above did not come from the action of Einstein gravity with matter but from the topological term in the action which in string theory appears at the level of $\alpha'$ corrections. Thus, the term $S_{\text{topol}} = \frac{8\pi^2}{g_{\text{str}}^2}$ appears here *in addition* to the usual wormhole action.

Let us estimate the numerical value of $S_{\rm topol}$ in the realistic theories. We do not really know the value of the effective coupling constant $g^2_{\rm str}$ on the wormhole throat (i.e., approximately on the stringy scale). The simplest idea would be to identify $g^2_{\rm str}$ with the gauge coupling constant related to the grand unification in supersymmetric grand unified theories (GUT's), $\alpha_{\rm GUT} = \frac{g^2_{\rm GUT}}{4\pi} \sim \frac{1}{26}$ [31]. This would give $S_{\rm topol} \sim 163$. Thus, the topological suppression alone can be strong enough to eliminate the effects of the rank-five operators $g_5 \frac{|\Phi|^4(\Phi+\Phi^*)}{M_P}$. (Remember that the coupling constant $g_5$ for $f_0 \sim 10^{12}$ GeV should be smaller than $10^{-54} \sim e^{-124}$ [14].) To get the factor $e^{-190}$ required to suppress the most dangerous term $g_1 M_P^3(\Phi+\Phi^*)$ we need a slightly smaller gauge coupling constant:

$$\frac{g^2_{\rm str}}{4\pi} \lesssim \frac{1}{30} \ . \tag{114}$$

Note that the factor $\frac{8\pi^2}{g^2_{\rm str}}$ is completely analogous to the topological coupling constant $\gamma$ which we discussed in the case of pure gravity. In that case $\gamma$ could take any possible value, and it was not quite clear whether it is natural or not to take it as large as 190. In the case of the string theory the condition $\gamma > 190$ corresponds to a very natural constraint $\frac{g^2_{\rm str}}{4\pi} \lesssim \frac{1}{30}$. This requirement seems quite reasonable since the effective gauge coupling constant $\frac{g^2_{\rm GUT}}{4\pi} \sim \frac{1}{26}$ can slightly decrease on the way from the GUT scale $2\times 10^{16}$ GeV to the stringy scale, which is much higher. Moreover, one can solve all problems even with $\frac{g^2_{\rm str}}{4\pi} \sim \frac{1}{26}$ if in addition to the topological action (110) one takes into account the standard contribution to the action $S$ which we studied in previous sections.

It is very hard to discuss these issues in the absence of a well-established string-inspired phenomenological theory. Nevertheless we will try to make some simple estimates. With this purpose we should remember the relation between the stringy gauge coupling constant $g_{\rm str}$, the stringy mass scale $M_{\rm str}$ (i.e., the mass of the first massive stringy excitation), the stringy length scale $l_{\rm str}$ (compactification radius), and the parameter $\alpha'$ in the heterotic string theory [31, 32]:

$$M_{\rm str} = \frac{2}{\sqrt{\alpha'}} = \frac{2}{l_{\rm str}} = \frac{g_{\rm str}\, M_P}{\sqrt{8\pi}} \ . \tag{115}$$

Here $\alpha'$ is the effective value of the parameter $\alpha'$ on the scale of the wormhole throat. Therefore the topological action $\frac{8\pi^2}{g^2_{\rm str}}$ can also be written as

$$S_{\rm topol} = \frac{8\pi^2}{g^2_{\rm str}} = \frac{\pi}{4}\alpha' M_P^2 = \frac{\pi}{4} M_P^2 l^2_{\rm str} = \pi\left(\frac{M_P}{M_{\rm str}}\right)^2 \ . \tag{116}$$

Let us first concentrate on the expression $S_{\rm topol} = \pi\left(\frac{M_P}{M_{\rm str}}\right)^2$. Uncertainty in the value of $g^2_{\rm str}$ translates into the uncertainty of $M_{\rm str}$. If one simply takes $M_{\rm str} \sim \frac{g_{\rm GUT}\, M_P}{\sqrt{8\pi}}$ one obtains $M_{\rm str} \sim 1.66\times 10^{18}$ GeV, which then leads to the estimate for the stringy unification scale $E_{\rm str} \sim 4\times 10^{17}$ GeV. However, this implies the existence of a large gap between the stringy unification scale and the supersymmetric (SUSY) GUT unification scale $2\times 10^{16}$ GeV. Therefore there is a tendency to assume that for some reason the stringy scale is in fact considerably smaller than $M_{\rm str} \sim 1.66\times 10^{18}$ GeV [31]. In order to obtain a sufficiently strong suppression due to topological effects it would be enough to have $M_{\rm str} \lesssim 1.5\times 10^{18}$ GeV, which is quite consistent with the present ideas about the value of $M_{\rm str}$.

Moreover, this constraint on $M_{\rm str}$ can be somewhat relaxed and reduced to $M_{\rm str} \lesssim 2\times 10^{18}$ GeV if one takes into account the standard (nontopological) contribution to the wormhole action. Indeed, as we have seen, the usual contribution to the GSL wormhole action can be represented as $1.34 M_P^2 R(0)^2$ (90). This result was quite consistent with our results for the theory with the exponential potential and with simple dimensional estimates suggesting that the action of the wormhole with the radius of the wormhole throat $R(0)$ should be of the order $M_P^2 R(0)^2$. Note that the topological contribution also has the same structure (although with a slightly smaller coefficient), $S_{\rm topol} = \frac{\pi}{4} M_P^2 l^2_{\rm str}$. If the wormhole solutions exist at all in the string theory, one may expect that the wormhole throat $R(0)$ should be greater than the "elementary length scale" $l_{\rm str}$. This suggests that the nontopological part of the action $\sim M_P^2 R(0)^2$ should be greater than $M_P^2 l^2_{\rm str}$. If this is correct, the total action of the wormholes, including the topological part, should be about two times greater than the topological contribution. In fact, if one simply adds to the topological action $\frac{\pi}{4} M_P^2 l^2_{\rm str}$ the GSL action $1.34 M_P^2 R(0)^2$ for $R(0) \sim l_{\rm str}$, the value of the topological action almost triples. In such a situation one can expect that the total action should become greater than 400 even if one takes $\frac{g^2_{\rm str}}{4\pi} \sim \frac{g^2_{\rm GUT}}{4\pi} \sim \frac{1}{26}$. This is more than sufficient to solve the problem of the global symmetry violation. One would get the total action $S \gtrsim 190$ for $\frac{g^2_{\rm str}}{4\pi} \lesssim \frac{1}{15}$, which looks like a very safe bet.

Thus, instead of the absolutely incredible fine-tuning of the values of possible coupling constants of the interaction terms breaking the global symmetry in the axion theory, our estimates gave us rather mild constraints on the gauge coupling constant and on the stringy mass scale $M_{\rm str}$. This provides a natural possibility to make stringy gravity compatible with the existence of the light axion in nature.

## VII. DISCUSSION

One of the main obstacles in the way of development of quantum gravity is the problem of experimental verification of its predictions. Indeed, gravitational interactions between elementary particles become strong only at energies comparable to the Planck mass $M_P \sim 1.2\times 10^{19}$ GeV. Such energies are far beyond our reach.

Fortunately, there exist some indirect ways to test quantum gravity experimentally. For example, it is quite possible that a consistent theory of gravity requires supersymmetry. Then one may study different versions of the theory by investigation of the properties of light superpartners of the graviton. Another possibility is related to cosmology, which provides us with experimental data originated at the very early stages of the evolution of the Universe.

In addition to that, there may exist some nonperturbative gravitational effects which may have important experimentally testable consequences even at very low energies. Some of these effects have been investigated in this paper. We have found, under the assumptions specified in this paper, that gravitational effects strongly violate global symmetries in a wide class of theories, including the theory of axions. Surprisingly enough, this strong violation occurs even if one drastically modifies the effective potential of the theory, for example, if one multiplies it by $\exp\left(\frac{C\Phi}{M_P}\right)$, where the factor $C$ can be as big as $10^3$.

Of course, it is quite possible that our methods based upon the Euclidean approach to quantum gravity are inadequate. However, we must admit that when we began our investigation we expected that it would be very easy to fix this problem either by using a formulation where the global symmetries become local, or by finding a simple modification of the theory which leads to wormholes with a very large action. We found that it is almost impossible to do so in the context of the standard Einstein theory in four-dimensional space.

However, as soon as we allowed ourselves to modify the theory of gravity or the properties of space at the scale about $10M_P^{-1}$ we found many different possibilities to improve the situation. One of them is the possibility that our space is compactified, with a compactification radius $r_c \gtrsim 10M_P^{-1}$. We have seen also that wormholes may simply disappear if one takes into account conformal anomaly, or if one considers certain string-inspired axion models. In addition to all these effects we found that in string theory there exists a specific strong suppression of topology change by the factor $e^{-\frac{8\pi^2}{g^2_{\rm str}}} = e^{\pi\left(\frac{M_P}{M_{\rm str}}\right)^2}$. This is a topological effect which does not depend on many particular details of wormhole configurations. Our estimates show that with an account taken of topological terms, the effects related to the global symmetry breaking become strongly suppressed if the string mass scale is sufficiently small, $M_{\rm str} \lesssim 2\times 10^{18}$ GeV. Equivalently, the problem of the wormhole-induced global symmetry breaking disappears if the gauge coupling constant is sufficiently small on the stringy scale of energies, $\frac{g^2_{\rm str}}{4\pi} \lesssim \frac{1}{15}$. These values of parameters are quite consistent with the existing picture of stringy phenomenology. Therefore at present we do not see any reason to reject the theories with global symmetries.

On the other hand, we have found that the existence of (approximate) global symmetries and the possibility to solve the strong $CP$ violation problem by the Peccei-Quinn mechanism are very sensitive to the choice of the theory of quantum gravity. If the axions in the mass range of $m_a \sim 10^{-5}$ eV will be discovered experimentally [33], it may give us important information about the structure of space and properties of particle interactions at the Planckian scale.

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## APPENDIX: NUMERICAL METHODS OF FINDING WORMHOLE SOLUTIONS

Finding the wormhole solutions numerically was a rather complicated problem. We found that the behavior of the solutions was extremely sensitive to the choice of initial conditions. Whereas it was relatively easy to find solutions with small action $S$, in the most interesting cases where the action was large the standard numerical methods failed. Therefore it was necessary to develop a more advanced method of calculation. There is a chance that our method can be useful for solving other problems as well. Therefore in this appendix we will first describe the standard numerical methods of obtaining solutions to the differential equations (73) and (74) and then we will describe our method.

### Standard shooting method

The first method we try is the standard shooting method [25]. We fix some value for $F(0)$. Then we find $A(0)$ using (58). This involves solving a cubic equation for $A^2(0)$:

$$A^4(0) - A^6(0)\,V[F(0)] - \frac{Q}{F^2(0)} = 0\ . \qquad \text{(A1)}$$

If there are no roots, then Eqs. (73) and (74) do not have a solution for the fixed value of $F(0)$. Equation (A1) has at most two roots. We find that the bigger root does not lead to a solution satisfying boundary conditions at infinity. Hence we pick the smallest root.

Once we have the initial conditions we use any numerical ordinary differential equation solver (for example, Runge-Kutta method). According to the behavior of the solution at infinity we modify our guess of $F(0)$. Using a simple iterative procedure we can determine the correct value for $F(0)$.

First we have used the Runge-Kutta method to obtain the numerical solution once the initial conditions (boundary conditions at $x = 0$) are set. However, for solutions with large action, the solution has to be computed to a very high accuracy. We have found that the Runge-Kutta method does not work for actions above 6. The Bulirsch-Stoer method, which we used next, works for actions below 10. As a result, simple shooting method is inadequate for our purposes. The reason it does not work is that $F(0)$ has to be determined to a very high accuracy, otherwise the numerical solution will not come close to the boundary conditions at infinity (at $x \approx \frac{10}{F_0}$). On a computer, $F(0)$ can be determined only to machine precision. The difference between $F(0)$ rounded to machine precision and the correct $F(0)$ causes an exponentially large deviation between the numerical solution and the correct solution for large $x$, hence making the standard shooting method fail.

### Improved shooting method

With the standard shooting method we cannot get a numerical solution to closely approximate the correct solution for the wormholes with large action. The idea of an alternative method is to sacrifice the correctness of the numerical solution at some $x$, and use the gained freedom to make it closer to the correct solution. The disadvantage is that the resulting numerical solution does not satisfy the differential equation at all points. The advantage is that it is close to the correct solution.

We solve the differential equation in several stages. The first stage uses the standard shooting method with the Bulirsch-Stoer method as the ordinary differential equation solver. After $F(0)$ has been determined within machine precision, the first stage is completed. Every time a solution is generated with the Bulirsch-Stoer method, the program remembers at which $x$ the solution rapidly veered up or down so that it is clear it will not satisfy the boundary conditions at infinity. Once the stage is completed, the program recalls solutions $F_{\rm up}(x)$ and $F_{\rm down}(x)$ which are the curves which have passed next to the correct solution for the longest time. $F_{\rm up}(x)$ has passed above the correct solution (so it veered up) and $F_{\rm down}(x)$ passed below the correct solution.

The program finds the maximum $x$ for which the two solutions are still very close to each other: $|F_{\rm up}(x) - F_{\rm down}(x)| < \epsilon$. Let this point be $x_m$. Then we have found a good approximation to the correct solution up to $x = x_m$. The second stage is to try to find an approximation to the solution for $x > x_m$. For the second stage, and all further stages, the initial conditions are different from the ones at the first stage. As in the case of the first stage, we are free to choose $F(x_m)$ with the restriction $F_{\rm up}(x_m) \geq F(x_m) \geq F_{\rm down}(x_m)$. Notice that by choosing $F(x_m)$ we introduce a point in the numerical solution which does not satisfy the differential equation. The error introduced is very small, though, since $F_{\rm up}(x_m)$ is very close to $F_{\rm down}(x_m)$.

We also need to know $A(x_m)$. We find it by linear interpolation between $A_{\rm up}(x_m)$ and $A_{\rm down}(x_m)$ once $F(x_m)$ is set. In the same way we determine $F'(x_m)$. As already mentioned, the jump in $F(x), A(x), F'(x)$ at $x = x_m$ is small and can be made as small as we like by reducing the parameter $\epsilon$. During the second stage $F(x_m)$ is determined to machine precision by the same iterative technique used in the first stage to determine $F(0)$. Then we proceed to the third stage, and so on, until we reach $x \approx \frac{10}{F_0}$ after which the solution is nearly constant: $F(x) \sim F_0,\ x \to \infty$.

The method described above can be interpreted in a way similar to the interpretation of the standard shooting method. We aim and shoot for the boundary conditions at infinity. Having missed several times, we follow the trajectory the best arrows followed and approach the target a little closer. Then we shoot from the new position. After finally hitting the target we present the set of trajectories of different arrows as one nearly correct trajectory which a single arrow could have followed all the way from the initial position to the target. Being a bad shot, we would not have been able to shoot the arrow to follow the correct trajectory all the way from the boundary at $x = 0$ to the boundary at infinity. However, using the method described above we can find a good approximation to the correct arrow trajectory.


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