PHYSICAL REVIEW D VOLUME 52, NUMBER 2 15 JULY 1995

Cosmic no-hair conjecture and black-hole formation: An exact model with gravitational radiation

Jiří Bičák*
Department of Theoretical Physics, Faculty of Mathematics and Physics, Charles University, V Holešovičkách 2, 180 00 Prague 8, Czech Republic and Department of Physics, University of Utah, Salt Lake City, Utah 84112

Jiří Podolský†
Department of Theoretical Physics, Faculty of Mathematics and Physics, Charles University, V Holešovičkách 2, 180 00 Prague 8, Czech Republic

(Received 20 January 1995)

Robinson-Trautman radiative space-times with a positive cosmological constant are studied by analytical methods. They are shown to approach the Schwarzschild–de Sitter solution at large retarded times. Their global structure is analyzed, and it is demonstrated that they represent explicit models exhibiting the cosmic no-hair conjecture and black-hole formation under the presence of gravitational waves.

PACS number(s): 04.30.Db, 04.20.Jb, 04.70.Bw, 98.80.Hw

I. INTRODUCTION

Cosmological no-hair conjectures have been discussed in both relativistic and Newtonian contexts for more than a decade (see, e.g., [1, 2] for reviews and a number of references). The most common version claims that all expanding universes with a positive cosmological constant approach the de Sitter space-time locally. If an inflationary phase (described by the de Sitter universe) is really an "attractor" of cosmological space-times with positive Λ, the whole inflationary scenario becomes a more natural indication of the present isotropy and homogeneity of the Universe.

The cosmic no-hair conjecture has been studied under various additional assumptions. For example, it was rigorously demonstrated within the Bianchi cosmologies [3]. Counterexamples, however, were also found. A simple counterexample is given by the Schwarzschild–de Sitter space-time: the metric does not evolve into the de Sitter universe everywhere. Nevertheless, already Gibbons and Hawking [4] speculated that under the presence of a black hole, a space-time with positive cosmological constant would settle down to one of the Kerr–Newman–de Sitter solutions.

Just recently, the cosmic no-hair conjecture was also investigated under the presence of gravitational waves. Maeda, Nakamura, and their collaborators [5] have analyzed the dynamical evolution of axisymmetric gravitational waves in the asymptotically de Sitter space-time by using numerical computations. Their numerical work indicates that either the space-time evolves into the de Sitter space-time with small perturbations, or that a Schwarzschild–de Sitter–like space-time arises.

The purpose of the present work is to study the cosmic no-hair conjecture under the presence of gravitational waves by using analytical methods. Simultaneously, we shall discuss the formation of a Schwarzschild–de Sitter black hole in a vacuum radiative space-time with a positive cosmological constant. The model space-times used in our discussion are represented by the Robinson-Trautman metrics with $\Lambda > 0$. They will also be referred to as the "cosmological" Robinson-Trautman space-times. (Very recently, black-hole space-times with $\Lambda > 0$ have been discussed extensively in various contexts—see, e.g., [6] and references therein.)

It is well known that the Robinson-Trautman metrics are the general vacuum solutions which admit a geodesic, shear-free, and twist-free null congruence of diverging rays [7, 8]. The best candidates for describing gravitational radiation from isolated sources are the Robinson-Trautman metrics of the Petrov type II, which are asymptotically flat and, in contrast with "traditional" belief, contain no nodal singularities although, in general, a naked singularity is present at the center. These space-times have attracted an increased attention in the last decade, in particular in the works by Lukács *et al.* [9], Schmidt [10], Rendall [11], Singleton [12], and, most recently, by Chruściel [13, 14] and Chruściel and Singleton [15]. (We refer the reader to the last three papers for further references.) In these studies the Robinson-Trautman space-times were shown to exist globally for all positive "times," and to converge asymptotically to a Schwarzschild metric. Interestingly, the extension of these space-times across the "Schwarzschild-like" event horizon can only be made with a finite degree of smoothness. All these rigorous studies are based on the derivation and analysis of an asymptotic expansion describing the long-time behavior of the solutions of the nonlinear,

*Electronic address: bicak@aci.cvut.cz
†Electronic address: podolsky@aci.cvut.cz

parabolic Robinson-Trautman equation.

The Robinson-Trautman metrics can easily be generalized to solve the vacuum Einstein equations with a nonvanishing Λ [16]. The results proving the global existence and convergence of the solutions of the Robinson-Trautman equation can be taken over from the previous studies since Λ does not explicitly enter this equation.

In the next section we shall review the recent mathematical studies of the Robinson-Trautman equation. We try to give an intuitive ("physical") summary of the basic results obtained by using rigorous mathematical concepts and techniques. Section III is devoted to the analysis of the Schwarzschild–de Sitter solution. Its global structure is first described in terms of Kruskal-type coordinates. Then synchronous (Lemaître-type) coordinates based on free particles moving radially outwards towards infinity are introduced, and the properties of the Schwarzschild–de Sitter space-time are discussed in the context of the cosmic no-hair conjecture. The global structure of the Robinson-Trautman space-times of the Petrov type II with $\Lambda > 0$ is analyzed in Sec. IV. It is shown that these space-times approach the Schwarzschild–de Sitter space-time in a certain limit. They admit a smooth future (spacelike) infinity but, in general, a smooth past infinity does not exist (or, vice versa). The continuation of the metric across the "Schwarzschild–de Sitter–like" black-hole horizon can be made with a higher degree of smoothness than in the corresponding cases with $\Lambda = 0$.

Finally, in Sec. V the asymptotic properties of the Robinson-Trautman space-times with $\Lambda > 0$ are studied at future infinity and the validity of the cosmic no-hair conjecture is explicitly demonstrated. The results are briefly summarized and open problems indicated in Sec. VI.

Some of the following results were presented in a preliminary form in [17].

II. THE GLOBAL STRUCTURE OF THE ROBINSON-TRAUTMAN SPACE-TIMES WITH $\Lambda = 0$

In the standard coordinates, the Robinson-Trautman metrics have the form (see [7, 8, 16])

$$ds^2 = -\Phi du^2 - 2dudr + 2r^2P^{-2}d\zeta d\bar{\zeta} \ , \tag{1}$$

where $P = P(u,\zeta,\bar{\zeta})$, ζ is a complex spatial (stereographic) coordinate, $r \in [0,\infty)$ is the affine parameter along the rays $u =$ const, $\zeta =$ const, and

$$\Phi = \Delta \ln P - 2r(\ln P)_{,u} - \frac{2m}{r} \ . \tag{2}$$

Here $\Delta = 2P^2\partial^2/\partial\zeta\partial\bar{\zeta}$ and m is a constant related to the Bondi mass of the system ($m =$ const can always be achieved by a coordinate transformation [16]). The function P satisfies the fourth-order Robinson-Trautman equation

$$(\ln P)_{,u} = -\frac{1}{12m}\Delta\Delta(\ln P) \ . \tag{3}$$

The Robinson-Trautman equation can be formulated (see, e.g., [14, 13, 15]) by introducing a smooth metric $g^0_{ab}(x^c)$ on a two-dimensional, compact, orientable Riemannian manifold $\mathcal{M}^2$ and a u-dependent family of two-metrics,

$$g_{ab} = [f(u,x^c)]^{-2}\ g^0_{ab} \ , \tag{4}$$

which, with respect to the local stereographic coordinate ζ, take the form $2P^{-2}d\zeta d\bar{\zeta}$. In general, the last term in (1) can be written as $r^2f^{-2}g^0_{ab}dx^adx^b$, and the function f, as a consequence of the vacuum equations, satisfies

$$\frac{\partial f}{\partial u} = -\frac{f}{24m}\Delta_g R \ , \tag{5}$$

where R is the curvature scalar and Δ_g the Laplacian of the metric g_{ab}. Using R_0 and Δ_0 to denote the curvature scalar and the Laplacian of g^0_{ab}, one has

$$R = f^2(R_0 + 2\Delta_0 \ln f), \quad \Delta_g = f^2\Delta_0 \ . \tag{6}$$

In general, $\mathcal{M}^2$ can have the topology of a two-torus or be a higher genus surface; however, the corresponding space-times are then not asymptotically flat at null infinity [14]. Equation (5) is known in the mathematical literature as Calabi's equation [18] in dimension 2. It is defined on any manifold, but here we shall primarily concentrate on the physical case $\mathcal{M}^2 = S^2$. Choosing standard coordinates on the sphere,

$$\zeta = \sqrt{2}e^{i\varphi}\tan\frac{\theta}{2} \ , \tag{7}$$

we obtain

$$\begin{aligned} &P_0 = 1 + \frac{1}{2}\zeta\bar{\zeta} = \left(\cos^2\frac{\theta}{2}\right)^{-1} \ , \\ &2P_0^{-2}d\zeta d\bar{\zeta} = d\theta^2 + \sin^2\theta d^2\varphi \ , \\ &\Delta_0 \ln P_0 = 1 \ , \quad R_0 = +2 \ . \end{aligned} \tag{8}$$

The metric (1) with $P = P_0$ is just the Schwarzschild metric. Writing then [cf. (4)]

$$P = f\,P_0 \ , \tag{9}$$

we find the "original" Robinson-Trautman equation (3) to go over into Calabi's equation (5) with R, Δ_g given by (6) (and $R_0 = 2$).

As mentioned in the Introduction, the most general, rigorous, and detailed analysis of the existence and behavior of the solutions of the Robinson-Trautman equations was recently given by Chruściel [13, 14] and by Chruściel and Singleton [15] (cf. also [10, 19, 12]). We now very briefly summarize their main theorems and propositions; these are formulated and proved by employing various rigorous mathematical concepts and techniques, but we shall describe them in a simplified, intuitive way.

The main result of [13] (see Proposition 5.1 therein) shows that when $f_0 \equiv f(u = u_0, x^a)$ is an arbitrary, sufficiently smooth initial-value function for f, then f satisfying (5),(6) exists for all times $u \geq u_0$, and there always exists a constant C such that

$$|f(u,x^a) - f_{\rm Schw}| \leq C\,e^{-2u/m} \ , \tag{10}$$

where $f_{\rm Schw}$ corresponds to a, perhaps boosted, Schwarzschild solution. (The constant C depends on the initial data f_0.) Therefore, as $u \to +\infty$, *Robinson-Trautman metrics approach exponentially fast a boosted Schwarzschild metric.* Performing this boost, we can without loss of generality assume that

$$f_{\rm Schw} = 1 \ . \tag{11}$$

(How the boosts can be handled is described in Appendix B in [13].)

In a subsequent paper [14] Chruściel found an asymptotic expansion of $f(u, x^a)$ for large u to have the form

$$f = 1 + f_1 e^{-2u/m} + f_2 e^{-4u/m} + \cdots + f_{14} e^{-28u/m} + f_{\log} u e^{-30u/m} + f_{15} e^{-30u/m} + \cdots \ , \tag{12}$$

where $f_1, f_2, \ldots, f_{\log}, \ldots$ are smooth functions on S^2. Some of these functions may vanish, but Chruściel and Singleton [15] prove that there exist Robinson-Trautman space-times for which $f_{\log}$ is nonvanishing.

This implies a rather surprising fact that, in general, the Robinson-Trautman metrics cannot be smoothly extended through the null hypersurface $\mathcal{H}^+$ given by $u = +\infty$. There exist an infinite number of nonisometric Robinson-Trautman extensions through $\mathcal{H}^+$ which are obtained by gluing a given Robinson-Trautman space-time to any other Robinson-Trautman space-time with the same mass, as shown in Fig. 1. In particular, we may join the radiative metrics to the Schwarzschild metric so that the Robinson-Trautman space-time "settles down" to the Schwarzschild space-time including the interior of the Schwarzschild black hole. The extension through the horizon is C^5, in general. There exist extensions which are C^{117} through $\mathcal{H}^+$—an example can be obtained by gluing a copy of the Robinson-Trautman space-time to itself, as one does in the Kruskal diagram for two copies of the Schwarzschild space-time. (The extensions will even be smoother in special cases in which some of the coefficients of the terms $\sim u^j e^{-ku/m}$ vanish.)

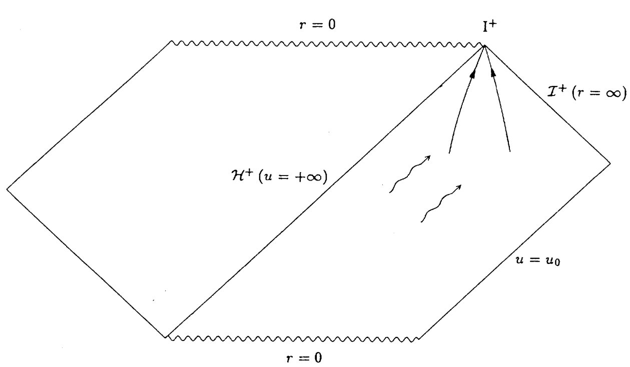

FIG. 1. Starting with arbitrary, smooth initial data for the Robinson-Trautman equation at $u = u_0$, the Robinson-Trautman metrics converge exponentially fast to a Schwarzschild metric as $u \to \infty$. However, the vacuum extension beyond the null hypersurface $\mathcal{H}^+ (u = +\infty)$ that includes, for example, the interior of the vacuum Schwarzschild black hole, can only be done with a finite degree of smoothness. (See the text for more details.)

In order to see the smoothness across $\mathcal{H}^+$, one introduces an advanced time coordinate v by $v = u + 2r + 4m \ln(r/2m - 1)$, and Kruskal-type coordinates $\hat{u}, \hat{v}$ by (see, e.g., [19])

$$\hat{u} = -\exp(-u/4m) \ , \quad \hat{v} = \exp(v/4m) \ . \tag{13}$$

[Such coordinates in the context of the Robinson-Trautman metrics were first used by Tod [19] and taken over by Chruściel [14]. Here we modified v used in [19, 14] by a constant additive factor so as to make the dimensions in v and in the final metric (14) correct.] The hypersurface $u = +\infty$ now becomes a boundary given by $\hat{u} = 0$. The original metric (1) becomes

$$ds^2 = -\frac{32m^3}{r} \exp(-r/2m) d\hat{u} d\hat{v} - 16m^2 \hat{\Phi} d\hat{u}^2 + 2r^2 P^{-2} d\zeta d\bar{\zeta} \ , \tag{14}$$

where

$$\hat{\Phi} = e^{u/2m} \left(\frac{1}{2} R - 1 + \frac{r}{12m} \Delta_g R \right) , \tag{15}$$

with R and Δ_g being given by (6). In terms of $\hat{u}$, the expansion (12) becomes

$$f = 1 + f_1 \hat{u}^8 + f_2 \hat{u}^{16} + \cdots + f_{14} \hat{u}^{112} - 4m f_{\log} (\ln |\hat{u}|)(\hat{u})^{120} + f_{15} \hat{u}^{120} + \cdots \ . \tag{16}$$

Because of the presence of the $\ln |\hat{u}|$ terms, the function f is not smooth at the $\hat{u} = 0$ — indeed it is C^{119} if $f_{\log} \neq 0$. The full metric (14) is C^{117}, since $\hat{\Phi}$ entering g_{00} contains the additional factor $e^{u/2m} \approx 1/\hat{u}^2$.

Although from a physical point of view a C^5 function may appear as good as a smooth function, the difference between the corresponding metrics is, in principle, observable. The resulting space-times, despite the fact that they have a "Schwarzschild-like" event horizon at $r = 2m$, should better be called Robinson-Trautman black holes.

Before we turn to the cosmological Robinson-Trautman metrics we shall discuss some properties of the Schwarzschild–de Sitter solution.

III. THE SCHWARZSCHILD–de SITTER METRIC AND SYNCHRONOUS COORDINATES

Although the solution generalizing the Schwarzschild metric to the case of a nonvanishing cosmological constant was found in 1918 [20], its global structure has been analyzed in detail only recently [21]. Gibbons and Hawking [4] and Lake and Roeder [22] give a brief discussion of its conformal diagram; however, only Bażański and Ferrari [21] performed its "Kruskalization" in detail. They considered the general case of a Schwarzschild black hole in a de Sitter universe, thus omitting "extreme" black holes and naked singularities. In the following, we shall also restrict ourselves to such general black-hole cases.

In the standard Schwarzschild-type coordinates the metric reads

$$ds^2 = -\left(1 - \frac{2m}{r} - \frac{\Lambda}{3}r^2\right) dt^2 + \left(1 - \frac{2m}{r} - \frac{\Lambda}{3}r^2\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \ , \quad (17)$$

where $m > 0$ is the mass of the central hole and $\Lambda > 0$ is the "repulsive" cosmological constant. If $\Lambda = 0$ we get the Schwarzschild metric; for $m = 0$ we get the de Sitter metric in static coordinates. Neither of these coordinate systems covers the whole manifold. The Killing vector $\partial/\partial t$ is timelike in a certain region of r provided that the condition

$$0 \le 9\Lambda m^2 < 1 \quad (18)$$

is satisfied. Hereafter we shall assume this, if not stated otherwise. As shown in [21], the metric (17) can be written in the form

$$ds^2 = -\Phi(r)dt^2 + \Phi^{-1}(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \ , \quad (19)$$

where

$$\Phi(r) = \frac{(r - r_+)(r_{++} - r)(r + r_+ + r_{++})}{r(r_+^2 + r_+r_{++} + r_{++}^2)} \ . \quad (20)$$

The parameters r_+, r_{++}, $0 < r_+ < r_{++}$, are two real positive roots of the equation $g_{00} = 0$; the third root r_- is negative. These roots can be parametrized as (see, e.g., [23])

$$\begin{aligned} r_+ &= \frac{2}{\sqrt{\Lambda}} \cos\left(\frac{\alpha}{3} + \frac{4\pi}{3}\right) \ , \\ r_{++} &= \frac{2}{\sqrt{\Lambda}} \cos\frac{\alpha}{3} \ , \\ r_- &= \frac{2}{\sqrt{\Lambda}} \cos\left(\frac{\alpha}{3} + \frac{2\pi}{3}\right) \ , \end{aligned} \quad (21)$$

where $\cos\alpha = -3m\sqrt{\Lambda}$. Here $r = r_+$ describes the black-hole horizon, and $r = r_{++}$ is the cosmological horizon. Only $r = 0$ is a real singularity. For $r_+ < r < r_{++}$, the solution (17) is static.

In [21], Kruskal-type coordinate systems are found in which the metric (19) is regular in the neighborhood of either the black-hole horizon or the cosmological horizon. Here we shall first convert the metric (17) into a Robinson-Trautman form.

Define the usual "tortoise-type" coordinate r^* by

$$r^* = \int \frac{dr}{\Phi(r)} = \delta_+ \ln\frac{|r - r_+|}{r + r_+ + r_{++}} - \delta_{++} \ln\frac{|r_{++} - r|}{r + r_+ + r_{++}} + \delta_+ \left[\ln\left(\frac{r_{++}}{r_+}\right) - \frac{1}{2}\right] \ , \quad (22)$$

where

$$\delta_+ = \frac{r_+}{1 - \Lambda r_+^2} \quad , \quad \delta_{++} = -\frac{r_{++}}{1 - \Lambda r_{++}^2} \ . \quad (23)$$

In contrast with [21], we have added a constant term to r^*; this makes the limit $\Lambda \to 0$ more transparent: $r_{++} \to \infty$, $\frac{\Lambda}{3}r_{++}^2 \to 1$, $\delta_{++} \to \frac{1}{2}r_{++}$, $\delta_+ \to r_+$. Now, by introducing the retarded time coordinate

$$u = t - r^* \ , \quad (24)$$

instead of t, we convert the metrics (17), and (19) into, respectively,

$$\begin{aligned} ds^2 &= -\left(1 - \frac{2m}{r} - \frac{\Lambda}{3}r^2\right) du^2 - 2dudr + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \\ &= -\Phi du^2 - 2dudr + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \end{aligned} \quad (25)$$

This is exactly the Robinson-Trautman form (1). (It is also the generalization of the Schwarzschild metric in the outgoing Eddington-Finkelstein coordinates.) It is not difficult to show that for fixed m but increasing Λ from $\Lambda = 0$ to $\Lambda = 1/9m^2$, the parameters r_+ and δ_+ monotonically increase within the intervals:

$$2m \le r_+ \le 3m \quad , \quad 2m \le \delta_+ \le +\infty \ , \quad (26)$$

whereas r_{++} decreases:

$$+\infty > r_{++} \ge 3m \ . \quad (27)$$

Further, it is easy to see that $\delta_{++} > 0$; it decreases from $+\infty$ for $\Lambda = 0$ to a minimum positive value, and then increases back to $+\infty$ for $\Lambda \to 1/9m^2$. At the black-hole horizon, we have $t \to +\infty$, $r^* \to -\infty$, and $u \to +\infty$; at the cosmological horizon, $t \to +\infty$, $r^* \to +\infty$, and u remains finite. On the other hand, the advanced time coordinate,

$$v = t + r^* \ , \quad (28)$$

remains finite at the black-hole horizon, while $v \to +\infty$ at the cosmological horizon.

In order to cover the black-hole horizon by regular coordinates, let us introduce Kruskal-type null coordinates

$$\begin{aligned} \hat{u} &= -\exp(-u/2\delta_+) \ , \\ \hat{v} &= \exp(v/2\delta_+) \ . \end{aligned} \quad (29)$$

The metric (25) now reads

$$\begin{aligned} ds^2 = &-\frac{4\Lambda\delta_+^2 r_+ e^{\frac{1}{2}}}{3r_{++}r}(r_{++} - r)^{1+\delta_{++}/\delta_+} \\ &\times(r + r_+ + r_{++})^{2-\delta_{++}/\delta_+} d\hat{u}d\hat{v} \\ &+ r^2(d\theta^2 + \sin^2\theta d\varphi^2) \ . \end{aligned} \quad (30)$$

Letting $\Lambda \to 0$, we obtain back the metric (14) with $\hat{\Phi} = 0$. Clearly, at the black-hole horizon with $r = r_+$, $\hat{u} = 0$, $\hat{v}$ finite, the metric (30) is regular. Analogously, we can find Kruskal-type null coordinates covering the cosmological horizon in which the metric is again regular; since, however, we shall not need them in the following, we refer to [21]. The conformal diagram of the Schwarzschild–de Sitter space-time is given in Fig. 2. Notice that both future and past infinities are spacelike as a consequence of $\Lambda > 0$. (In the following section, this point is discussed for a general cosmological Robinson-Trautman space-time.)

Next, we shall introduce the synchronous (Lemaître-type) coordinates in the Schwarzschild–de Sitter space-time. These coordinates have not been discussed so far in literature. We connect new coordinates with free par-

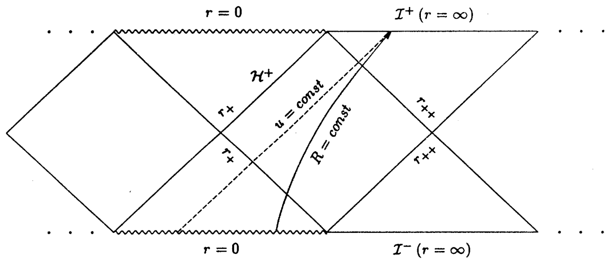

FIG. 2. The conformal diagram for the Schwarzschild–de Sitter space-time. The world line R = const (θ = const, φ = const) represents the history of a free particle moving radially outward across the past black-hole horizon and across the cosmological horizon towards future (spacelike) infinity. For an observer moving with such a particle, all "traces" of the hole disappear as he approaches $\mathcal{I}^+$; only the de Sitter geometry persists.

ticles moving radially *outward* across the past black-hole horizon and across the cosmological horizon towards infinity since such coordinates are suitable for the discussion of the cosmic no-hair conjecture. (The world line of such a particle is illustrated in Fig. 2.)

Introduce the function

$$h(r) = \left(\frac{2m}{r} + \frac{\Lambda}{3}r^2\right)^{\frac{1}{2}} , \tag{31}$$

and the coordinates τ, R by the relations

$$d\tau = dt - \frac{h}{1-h^2}dr , \tag{32}$$

$$dR = -dt + \frac{1}{h(1-h^2)}dr , \tag{33}$$

or, inversely,

$$dr = h(dR + d\tau) , \tag{34}$$

$$dt = \frac{1}{1-h^2}(d\tau + h^2 dR) . \tag{35}$$

Hence, we obtain $r(\tau, R)$ by integrating (34)

$$R + \tau = \int \left(\frac{2m}{r} + \frac{\Lambda}{3}r^2\right)^{-\frac{1}{2}} dr . \tag{36}$$

The integral can be calculated explicitly to yield

$$R + \tau = \frac{1}{\sqrt{3\Lambda}} \ln\left[2\sqrt{Y(Y+1)} + 2Y + 1\right] + \text{const} , \tag{37}$$

where

$$Y = \frac{\Lambda}{6m}r^3 . \tag{38}$$

Relation (37) can be inverted (we put the additive constant equal zero),

$$Y = \frac{(1-Z^3)^2}{4Z^3} , \tag{39}$$

where

$$Z = e^{-H(R+\tau)} , \tag{40}$$

and we introduced the "Hubble parameter"

$$H = \sqrt{\frac{\Lambda}{3}} . \tag{41}$$

In the coordinates τ, R the metric (17) reads

$$ds^2 = -d\tau^2 + \left(\frac{2m}{r} + H^2r^2\right) dR^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) , \tag{42}$$

with r being given in term of R and τ by (38)–(40). The metric is regular at the horizons r_+, r_{++}. The particles with R = const, θ = const, φ = const, satisfy the equations [see (32), (33)]

$$\begin{aligned} \frac{dr}{dt} &= \left(\frac{2m}{r} + H^2r^2\right)^{\frac{1}{2}} \left(1 - \frac{2m}{r} - H^2r^2\right) , \\ \frac{dr}{d\tau} &= \left(\frac{2m}{r} + H^2r^2\right)^{\frac{1}{2}} , \\ \frac{dt}{d\tau} &= \left(1 - \frac{2m}{r} - H^2r^2\right)^{-1} . \end{aligned} \tag{43}$$

Hence, these particles, with τ being their proper time, move from the singularity ($r = 0$, $R + \tau = 0$), across the past black-hole horizon ($r = r_+$, $t = -\infty$), through the static region ($r_+ < r < r_{++}$, $dt/d\tau > 0$), and then continue across the cosmological horizon ($r = r_{++}$, $t = +\infty$) towards infinity ($r = \infty$, $\tau \to +\infty$) (cf. Fig. 2). The equation for $dr/d\tau$ also shows immediately that the particles cross both horizons in a finite proper time and reach $r = \infty$ only with $\tau \to \infty$. Their local velocity, as measured by local static observers at r = const, θ = const, φ = const in the static region $r \in (r_+, r_{++})$, is given by $v_{\text{loc}} = dl_{\text{loc}}/d\tau_{\text{loc}} = (2m/r + H^2r^2)^{\frac{1}{2}}$. At the horizons $v_{\text{loc}} = 1$ since the local "static" observers themselves "move" there with the velocity of light.

In order to bring the metric (42) into the form in which the de Sitter metric in "standard" coordinates arises explicitly, we relabel the radial comoving coordinate R as a new coordinate χ given by

$$\chi = \left(\frac{m}{2H^2}\right)^{\frac{1}{3}} e^{HR} . \tag{44}$$

The metric (42) then reads

$$\begin{aligned} ds^2 = &-d\tau^2 + e^{2H\tau}(1-Z^3)^{\frac{4}{3}} \\ &\times \left[\left(\frac{1+Z^3}{1-Z^3}\right)^2 d\chi^2 + \chi^2(d\theta^2 + \sin^2\theta d\varphi^2)\right] , \end{aligned} \tag{45}$$

where Z is given by (40), or

$$Z = \left(\frac{m}{2H^2}\right)^{\frac{1}{3}} \chi^{-1} e^{-H\tau} \ . \tag{46}$$

This is the exact form of the Schwarzschild–de Sitter metric in the outgoing comoving coordinates. Hence it describes a white hole in the de Sitter universe (cf. Fig. 2). The past central singularity at $r = 0$ is given by $Z = 1$. As r increases, Z decreases; at $r \to \infty$, or $R + \tau \to \infty$, we find $Z \to 0$, and the metric (45) turns into

$$ds^2 = -d\tau^2 + e^{2H\tau}\left[d\chi^2 + \chi^2(d\theta^2 + \sin^2\theta d\varphi^2)\right] \ . \tag{47}$$

This is the de Sitter metric written in the standard synchronous Friedmann-Robertson-Walker form with the exponentially growing ("inflationary") expansion factor. Keeping the leading order terms in the expansion of the Schwarzschild–de Sitter metric (45) for $\tau \to \infty$, we obtain

$$\begin{aligned} ds^2 = &-d\tau^2 + e^{2H\tau}\left[d\chi^2 + \chi^2(d\theta^2 + \sin^2\theta d\varphi^2)\right] \\ &+ \frac{2m}{3H^2}\chi^{-3}e^{-H\tau}\left[2d\chi^2 - \chi^2(d\theta^2 + \sin^2\theta d\varphi^2)\right] \\ &+ O\left(e^{-4H\tau}\right) \ . \end{aligned} \tag{48}$$

Observe that the "traces" of the central black hole, characterized by the terms in which the mass m enters, completely disappear as $\tau \to \infty$ in full agreement with the cosmic no-hair conjecture. Of course, a completely different picture is seen by observers falling into the hole — those, however, are not described by our "outgoing" comoving coordinates.

Before concluding this section, let us give the asymptotic relation between the Robinson-Trautman-type coordinates $\{u, r, \theta, \varphi\}$ and the inflationary-type coordinates $\{\tau, \chi, \theta, \varphi\}$ for large r and large τ in the present case of the Schwarzschild–de Sitter space-time. It is easy to see that the exact relation (37) for large r and large τ implies

$$r \sim \left(\frac{m}{2H^2}\right)^{\frac{1}{3}} e^{H(R+\tau)} \ ; \tag{49}$$

i.e., regarding (44), we obtain asymptotically, at $r \to \infty$, $\tau \to \infty$,

$$r = \chi e^{H\tau} + O(1) + O\left(e^{-H\tau}\right) \ . \tag{50}$$

Next, for large r we can integrate (32)

$$H\tau = Ht + \ln Hr + \text{const} \ , \tag{51}$$

and by realizing that $r^* \to \text{const}$ at $r \to \infty$ [see (22)], we can choose a constant in the last relation so that [cf. (24)]

$$e^{Hu} \sim e^{H\tau}(Hr)^{-1} + \cdots = \frac{1}{H\chi} + O\left(e^{-H\tau}\right) \ . \tag{52}$$

Finally, let us note that the de Sitter metric (47) preserves its form at large τ up to terms $\sim e^{-2H\tau}$ under the transformation $(\tau, \chi) \to (\tau', \chi')$ such that

$$\begin{aligned} e^{H\tau} &= (H\chi')^2 e^{H\tau'} - e^{-H\tau'} + \cdots \ , \\ H\chi &= \frac{1}{H\chi'} + \frac{1}{(H\chi')^3}e^{-2H\tau'} + \cdots \ , \end{aligned} \tag{53}$$

which implies $\chi e^{H\tau} = \chi' e^{H\tau'} + O(e^{-3H\tau'})$. Therefore, omitting primes, we can write down the asymptotic relations at $\mathcal{I}^+$ ($r \to \infty$, $\tau \to \infty$) between the coordinate systems (u, r) and (τ, χ) in the form in which the leading terms are both linear in χ:

$$\begin{aligned} r &= \chi e^{H\tau} + O(1) \ , \\ e^{Hu} &= H\chi + O\left(e^{-H\tau}\right) \ . \end{aligned} \tag{54}$$

These relations obtained in the special case of the Schwarzschild–de Sitter space-time will motivate our ansatz for an asymptotic transformation for general Robinson-Trautman space-times in Sec. V.

IV. THE GLOBAL STRUCTURE OF THE ROBINSON-TRAUTMAN SPACE-TIMES WITH $\Lambda \neq 0$

When a Robinson-Trautman space-time with $\Lambda = 0$ is known in the standard form (1), it is straightforward to generalize it to the case of a nonvanishing Λ (cf. [16]). The metric still keeps the form (1) with P satisfying the Robinson-Trautman equation (3). The only place where Λ enters is the function Φ. The cosmological Robinson-Trautman metric reads

$$ds^2 = -\Phi_\Lambda du^2 - 2du dr + 2r^2P^{-2}d\zeta d\bar{\zeta} \ , \tag{55}$$

where

$$\Phi_\Lambda = \Delta \ln P - 2r(\ln P)_{,u} - \frac{2m}{r} - \frac{\Lambda}{3}r^2 \ . \tag{56}$$

We may still write $P = f\, P_0$, as in Eq. (9), where P_0 satisfies (8) and f satisfies (5), (6). Since Λ does not enter the

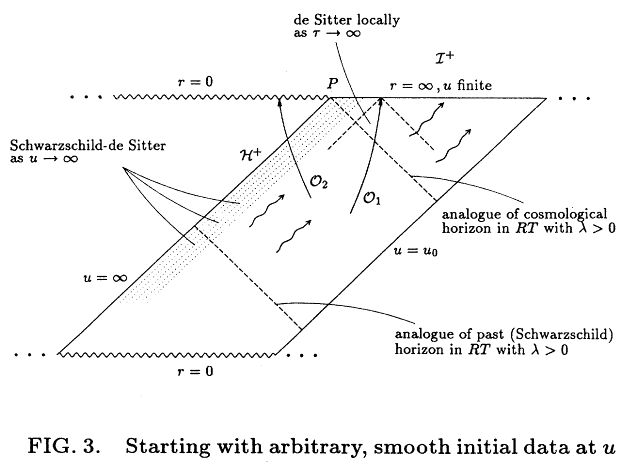

FIG. 3. Starting with arbitrary, smooth initial data at $u = u_0$, the Robinson-Trautman metrics with positive cosmological constant converge exponentially fast to a Schwarzschild–de Sitter metric as $u \to \infty$. In the neighborhood of P at future infinity $\mathcal{I}^+$, the metric is approaching the de Sitter metric exponentially fast. Although traces of gravitational waves will persist in other regions of future infinity, all geodesic observers (such as O_1) will observe the metric to approach the de Sitter metric exponentially fast within their past light cone. Observer O_2 falls into the black hole. The metric at the horizon $\mathcal{H}^+$ has only a finite degree of smoothness, however, although this can be higher than in the case with $\Lambda = 0$.

evolution equation for f, we may take over the results for $\Lambda = 0$ described in Sec. II. The Robinson-Trautman metric will now approach the Schwarzschild–de Sitter metric in the form (25) as $u \to \infty$ (see Fig. 3). The approach of f to its Schwarzschild–de Sitter form is again characterized by the inequality (10) and by the asymptotic expansion (12). We shall also assume an analogue of (11): i.e.,

$$f_{\text{Schw-de Si}} = 1 \; . \qquad (57)$$

The presence of a positive cosmological constant does not affect the smoothness of future infinity in these space-times; however, the future null infinity becomes spacelike in contrast to the cases with $\Lambda = 0$ (cf. Fig. 3). The smoothness of $\mathcal{I}^+$ is easily seen if one introduces an inverse radial coordinate $l = r^{-1}$ and uses $\Omega = l$ as a conformal factor. One finds

$$\Omega^2 ds^2 = 2du dl - l^2 \Phi_\Lambda du^2 + 2P^{-2} d\zeta d\bar{\zeta} \; , \qquad (58)$$

where

$$\Phi_\Lambda = \Delta \ln P - 2l^{-1} (\ln P)_{,u} - 2ml - \frac{\Lambda}{3} l^{-2} \; . \qquad (59)$$

It is easy to see that $l = 0$ is a regular hypersurface for smooth $P(u, \zeta, \bar{\zeta})$; and it is spacelike due to the presence of the last term in Φ_Λ. As in the Robinson-Trautman space-times with $\Lambda = 0$, *past* null infinity will not exist, in general, because the Robinson-Trautman (parabolic) equation does not admit smooth solutions both into the future and into the past (cf. e.g., [14]).

The presence of the cosmological constant, however, may have a considerable effect on the smoothness of the extensions through the null hypersurface $\mathcal{H}^+$ given by $u = +\infty$. The transformation to the Kruskal-type null coordinates is now given by (29). Hence, instead of (16), we find the expansion of f in terms of $\hat{u}$ to read

$$\begin{aligned} f = 1 &+ f_1 |\hat{u}|^{4\delta_+/m} + f_2 |\hat{u}|^{8\delta_+/m} + \cdots + f_{14} |\hat{u}|^{56\delta_+/m} \\ &-2\delta_+ f_{\log} (\ln|\hat{u}|) |\hat{u}|^{60\delta_+/m} + f_{15} |\hat{u}|^{60\delta_+/m} + \cdots \; . \end{aligned} \qquad (60)$$

The full metric now takes the form

$$\begin{aligned} ds^2 = &-\frac{4\Lambda\delta_+^2 r_+ e^{\frac{1}{2}}}{3r_{++} r} (r_{++} - r)^{1+\delta_{++}/\delta_+} \\ &\times (r + r_+ + r_{++})^{2-\delta_{++}/\delta_+} d\hat{u} d\hat{v} \\ &-4\delta_+^2 \hat{\Phi}_\Lambda \, d\hat{u}^2 + 2r^2 P^{-2} d\zeta d\bar{\zeta} \; , \end{aligned} \qquad (61)$$

where

$$\hat{\Phi}_\Lambda = e^{u/\delta_+} \left(\frac{1}{2} R - 1 + \frac{r}{12m} \Delta_g R \right) , \qquad (62)$$

and R, R_0, P, P_0, Δ_g are given by Eqs. (6)–(9), with f being of the form (60) above. The constant parameters r_+, r_{++}, δ_+, δ_{++} are the same as in Sec. III.

It is interesting to see how the presence of Λ influences the smoothness of the extension of the Robinson-Trautman metrics across the null hypersurface $\mathcal{H}^+$ ("horizon") given by $u = \infty$, or $\hat{u} = 0$. In particular, we may join the radiative Robinson-Trautman metrics to the Schwarzschild–de Sitter metric so that a Robinson-Trautman space-time with $\Lambda \neq 0$ "settles down" to the Schwarzschild–de Sitter space-time including the interior of the Schwarzschild–de Sitter black hole (see Fig. 3). As summarized in Sec. II, such an extension will, in general, be C^{117} in the case of vanishing Λ. With $\Lambda \neq 0$, much higher smoothness can be obtained. Indeed, since $\delta_+ = r_+/(1 - \Lambda r_+^2)$ [see (23)], the horizon $\mathcal{H}^+$ can be made "arbitrarily smooth" by letting Λ approach arbitrarily close its extremal value, $\Lambda \to 1/9m^2$ $(r_+ \to 3m)$, which corresponds to an almost extreme Schwarzschild–de Sitter black hole. Then δ_+ becomes arbitrarily large and the terms $\sim |\hat{u}|^{k(\delta_+/m)}$, $k = 4, 8, \ldots$, appearing in (60), will guarantee arbitrarily high smoothness of the function f at $\hat{u} = 0$; this in turn determines the smoothness of the functions R, P and thus of the full metric (61) at $\hat{u} = 0$. (Notice that Φ_Λ entering g_{00} contains, in addition, the factor $e^{u/\delta_+} \approx 1/\hat{u}^2$ which, however, decreases the smoothness of the metric by 2 only.)

On the other hand, there are cases when the presence of Λ can decrease the smoothness of the metric (61) across $\hat{u} = 0$. If $4\delta_+/m$ is not an integer, the second term in the expansion (60), $f_1 |\hat{u}|^{4\delta_+/m}$, is already not smooth. Since $4\delta_+/m \geq 8$ [see (26)], the function f is then at least C^8 and the full metric is at least C^6. For those values of Λ which imply $4\delta_+/m$ equals an integer, the smoothness is always better than for $\Lambda = 0$. In general, denoting by n the largest integer smaller than $4\delta_+/m$, the metric (61) is at least C^n at $\hat{u} = 0$. As mentioned above, $n \to \infty$ with $4\delta_+/m \to \infty$.

We also expect to find analogues of the past black-hole horizon and the cosmological horizon of the Schwarzschild–de Sitter metric in a Robinson-Trautman solution with $\Lambda > 0$ (see Fig. 3). Possible analogues can be considered in a way similar to Tod's treatment [19] for the case $\Lambda = 0$.

V. THE COSMIC NO-HAIR CONJECTURE

After having examined the behavior of the cosmological Robinson-Trautman metrics at the null hypersurface $u \to +\infty$, we shall now turn to their properties close to $\mathcal{I}^+$, i.e., near $r \to \infty$, u finite (cf. Fig. 3). If the cosmic no-hair conjecture is to be verified in these space-times, we should be able to demonstrate that they all approach the de Sitter space-time locally. For this purpose, it would be sufficient to convert the metrics (55), with Φ_Λ given by (56), into the Starobinsky asymptotic form [24]. As will be indicated below, such an asymptotic form automatically guarantees that the cosmic no-hair conjecture is satisfied.

Starobinsky's form represents the asymptotic behavior of an inhomogeneous cosmological model with $\Lambda > 0$ in synchronous coordinates at large τ (τ being the proper time measured by the observers at rest in these synchronous coordinates). In Sec. III, the Starobinsky form of the pure Schwarzschild–de Sitter metric was obtained in Eq. (48). We now wish to find a similar form under the presence of gravitational waves described by a general Robinson-Trautman metric (55).

In order to convert the metric (55) into the desired form, we first make an ansatz which is inspired by the asymptotic form of the transformation (54) between the Robinson-Trautman-type coordinates and the synchronous ("inflationary-type") coordinates for the Schwarzschild–de Sitter metric.

Assume that the required transformation is of the asymptotic form

$$r = \chi e^{H\tau} - H^{-2}(f_{\infty,u}/f_\infty) + \sum_{n=1}^{\infty} \psi_n e^{-nH\tau} ,$$
$$e^{Hu} = H\chi - e^{-H\tau} , \qquad (63)$$

where $H = \sqrt{\Lambda/3}$, ψ_n are functions of $\chi, \zeta, \bar\zeta$ to be chosen appropriately, and $f_\infty = f|_{\tau\to\infty} = f(u \to H^{-1}\ln|H\chi|, \zeta, \bar\zeta)$ is determined by the function $f(u,\zeta,\bar\zeta)$ entering the metric (55). {Whenever the gravitational waves are present, $f \neq 1$; only $f_{\mathrm{Schw-de\ Si}} = 1$ [cf. (57)].} Under transformation (63), the metric (55) becomes

$$ds^2 = -d\tau^2 + e^{2H\tau}\left[d\chi^2 + (\chi/f_\infty P_0)^2\, 2d\zeta d\bar\zeta\right] + h_{ij}^{(0)}\, dx^i dx^j + e^{-H\tau} h_{\mu j}^{(1)} dx^\mu dx^j + \sum_{m=2}^{\infty} e^{-mH\tau} h_{\mu\nu}^{(m)}\, dx^\mu dx^\nu . \qquad (64)$$

Here f_∞ depends only on the spatial coordinates $\{x^i, i = 1,2,3\} = \{\chi,\zeta,\bar\zeta\}$; all $h_{\mu\nu}$'s appearing in (64) also depend only on x^i, but $\{x^\mu\} = \{\tau, x^i\}$ so that the last two terms in (64) involve $d\tau$ as well. Nevertheless, the nondiagonal components of the metric $h_{0j}^{(1)}, h_{0j}^{(2)}, \ldots$ can be transformed away by transformations of the form

$$\chi^{(m-1)} = \chi^{(m)} + a_m\, e^{-(m+2)H\tau} ,$$
$$\zeta^{(m-1)} = \zeta^{(m)} + b_m\, e^{-(m+2)H\tau} , \qquad (65)$$

where $m = 1,2,3,\ldots$, $\{\chi^{(0)},\zeta^{(0)},\bar\zeta^{(0)}\} = \{\chi,\zeta,\bar\zeta\}$, and the coefficients a_m, b_m depend on $\{\chi^{(m)},\zeta^{(m)},\bar\zeta^{(m)}\}$. After choosing a's and b's so that $h_{0j}^{(m)} = 0$, the metric (64) reads

$$ds^2 = -d\tau^2 + e^{2H\tau}\left[d\tilde\chi^2 + (\tilde\chi/f_\infty)^2 (1 + \tfrac{1}{2}\tilde\zeta\bar{\tilde\zeta})^{-2}\, 2d\tilde\zeta d\bar{\tilde\zeta}\right] + \sum_{m=0}^{\infty} e^{-mH\tau} \tilde h_{ij}^{(m)}\, d\tilde x^i d\tilde x^j + \sum_{m=2}^{\infty} e^{-mH\tau} \tilde h_{00}^{(m)}\, d\tau^2 . \qquad (66)$$

Furthermore, the metric coefficients $\tilde h_{00}^{(m)}(\tilde x^{(i)})$ can be transformed away by performing a transformation of the form (63) in which ψ_n are chosen appropriately.

To summarize, a transformation of the form

$$r = \chi e^{H\tau} - H^{-2}(f_{\infty,u}/f_\infty) + \sum_{n=1}^{\infty} A_n\, e^{-nH\tau} ,$$
$$e^{Hu} = H\chi - e^{-H\tau} + \sum_{n=3}^{\infty} B_n\, e^{-nH\tau}, \qquad (67)$$
$$\zeta = \eta + \sum_{n=3}^{\infty} C_n\, e^{-nH\tau} ,$$

in which A_n, B_n, C_n are suitable functions of $\chi, \eta, \bar\eta$ brings the metric (55) into the asymptotic form

$$ds^2 = -d\tau^2 + e^{2H\tau}\left[d\chi^2 + f_\infty^{-2}\chi^2\,(d\theta^2 + \sin^2\theta d\varphi^2)\right] + \sum_{m=0}^{\infty} e^{-mH\tau} h_{ij}^{(m)}\, dx^i dx^j , \qquad (68)$$

where we have reintroduced the standard angular coordinates θ, φ [$\eta = \sqrt{2}e^{i\varphi}\tan(\theta/2)$—cf. Eq. (7)], $f_\infty = f|_{\tau\to\infty} = f(u = H^{-1}\ln|H\chi|, \theta, \varphi)$, and $h_{ij}^{(m)}$ depend on $\{x^i\} = \{\chi,\theta,\varphi\}$ only. This asymptotic form of the metric shows explicitly that for $\tau \to \infty$ the space-time metric does *not* approach the de Sitter metric globally. In contrast with the case of the pure Schwarzschild–de Sitter black hole where outside of the hole all its "traces" disappear as $\tau \to \infty$ [cf. Eq. (47)], the gravitational waves now leave "an imprint" on the $\mathcal{I}^+$ which is demonstrated by the presence of the function f_∞ in the "angular" part of the metric (68). Only as $u \to \infty$, $\chi \to \infty$, $r \to \infty$, so that $f_\infty \to 1$ (see the point P in Fig. 3), do the Robinson-Trautman metrics approach the de Sitter metric since the terms in (68) with the factor $e^{2H\tau}$ dominate those proportional to $e^{-mH\tau}$.

In fact, the cosmic no-hair conjecture claims that a universe with a positive cosmological constant approaches exponentially fast the de Sitter space-time only locally in general. That this is so in our case can easily be demonstrated as follows (cf. [25]).

Write the metric (68) in the form

$$ds^2 = -d\tau^2 + e^{2H\tau} a_{ij}(x^s)\, dx^i dx^j + O(1) . \qquad (69)$$

Consider a world line $x^i = x^i_{(0)} = \text{const}$. It is a geodesic (in Fig. 3 indicated by O_1). Make a linear transformation $x^i \to \bar x^i$, $x^i - x^i_{(0)} = A^i_k \bar x^k$ such that $\bar a_{ij}(0) = \delta_{ij}$. Now introduce new coordinates y^i, T by

$$\bar x^i = y^i\, e^{-HT}/(1 - H^2R^2)^{\frac{1}{2}} ,$$
$$e^{H\tau} = e^{HT}(1 - H^2R^2)^{\frac{1}{2}} , \qquad (70)$$

where $R^2 = (y^1)^2 + (y^2)^2 + (y^3)^2$. Putting $y^1 = R\sin\tilde\theta\cos\tilde\varphi$, $y^2 = R\sin\tilde\theta\sin\tilde\varphi$, $y^3 = R\cos\tilde\theta$, we find that (69) takes the form

$$ds^2 = -\left(1 - \frac{\Lambda}{3}R^2\right) dT^2 + \left(1 - \frac{\Lambda}{3}R^2\right)^{-1} dR^2 + R^2\,(d\tilde\theta^2 + \sin^2\tilde\theta d\tilde\varphi^2) + \sum_{m=1}^{\infty} e^{-mHT} \tilde a_{\mu\nu}^{(m)}\, dy^\mu dy^\nu , \qquad (71)$$

where $\tilde a_{\mu\nu}^{(m)}$ are functions of $\{y^i\} = \{R,\tilde\theta,\tilde\varphi\}$, and $y^0 = T$. The transformation (70) cannot be applied globally — it is meaningful only for $1 - H^2R^2 > 0$. A geodesic observer (say O_1 in Fig. 3) moving with $y^i = 0$ will see, inside his past light cone, the space-time approach the de Sitter space-time exponentially fast as $T \to \infty$. Thus for a freely falling observer the observable universe becomes quite bald. This is what the cosmic no-hair conjecture claims.

VI. CONCLUDING REMARKS

Starting from the *exact* model radiative space-times with a positive cosmological constant, we demonstrated the validity of the cosmic no-hair conjecture by purely analytic means. We have also shown that these cosmological Robinson-Trautman solutions settle down to the Schwarzschild–de Sitter solution at large retarded times. The interior of a Schwarzschild–de Sitter black hole can be joined to an "external" cosmological Robinson-Trautman space-time across the horizon with a higher degree of smoothness than in the corresponding case with $\Lambda = 0$. As far as we are aware, these models represent the only exact analytic demonstration of the cosmic no-hair conjecture under the presence of gravitational waves. They also appear to be the only exact examples of black-hole formation in nonspherical space-times which are not asymptotically flat. Hopefully, these models may serve as tests of various approximation methods, and as test beds in numerical studies of more realistic situations.

Finally, let us note that the ideas discussed here can also be applied to the Robinson-Trautman space-times containing an incoherent homogeneous radiation field (null fluid). For $\Lambda = 0$, such space-times were studied by Bičák and Perjés [26] who showed that they approach the Vaidya metric asymptotically. With $\Lambda \neq 0$ the radiating Vaidya metric in the de Sitter universe should arise.

ACKNOWLEDGMENTS

We want to thank Piotr Chruściel for several very enlightening discussions of his work on the Robinson-Trautman space-times, and Joe Romano for reading the manuscript. J.B. is grateful to Karel Kuchař for the warm hospitality in Salt Lake City where this work was finished. We also acknowledge the support of the U.S.-Czech Science and Technology Grant No. 92067, and Grants Nos. GACR-0503 and GAUK-318 of the Czech Republic and Charles University.

[1] K. Maeda, in *Fifth Marcel Grossman Meeting on General Relativity*, Proceedings, Perth, Australia, 1988, edited by D.G. Blair, M.J. Buckingham, and R. Ruffini (World Scientific, Singapore, 1989), p. 145.

[2] J.D. Barrow and G. Götz, Class. Quantum Grav. **6**, 1253 (1989).

[3] R.M. Wald, Phys. Rev. D **28**, 2118 (1983).

[4] G.W. Gibbons and S.W. Hawking, Phys. Rev. D **15**, 2738 (1977).

[5] K. Nakao, T. Nakamura, K. Oohara, and K. Maeda, Phys. Rev. D **43**, 1788 (1991); K. Nakao, K. Maeda, T. Nakamura, and K. Oohara, *ibid.* **47**, 3194 (1993); M. Shibata, K. Nakao, T. Nakamura, and K. Maeda, *ibid.* **50**, 708 (1994).

[6] D.R. Brill and S.A. Hayward, Class. Quantum Grav. **11**, 359 (1994).

[7] I. Robinson and A. Trautman, Phys. Rev. Lett. **4**, 431 (1960).

[8] I. Robinson and A. Trautman, Proc. R. Soc. London Ser. A **265**, 463 (1962).

[9] B. Lukács, Z. Perjés, J. Porter, and Á. Sebestyén, Gen. Relativ. Gravit. **16**, 691 (1984).

[10] B.G. Schmidt, Gen. Relativ. Gravit. **20**, 65 (1988).

[11] A.D. Rendall, Class. Quantum Grav. **5**, 1339 (1988).

[12] D.B. Singleton, Class. Quantum Grav. **7**, 1333 (1990).

[13] P.T. Chruściel, Commun. Math. Phys. **137**, 289 (1991).

[14] P.T. Chruściel, Proc. R. Soc. London Ser. A **436**, 299 (1992).

[15] P.T. Chruściel and D.B. Singleton, Commun. Math. Phys. **47**, 133 (1992).

[16] D. Kramer, H. Stephani, M.A.H. MacCallum, and H. Herlt, *Exact Solutions of the Einstein's Field Equations* (VEB Deutscher Verlag der Wissenschaften, Berlin/Cambridge University Press, Cambridge, 1980).

[17] J. Bičák and J. Podolský, in *Relativistic Astrophysics and Cosmology*, edited by S. Gottlöber, J.P. Mücket, and V. Müller (World Scientific, Singapore, 1992), p. 167.

[18] J.E. Calabi, in *Seminar on Differential Geometry*, edited by S.T. Yau (Princeton University Press, Princeton, 1982).

[19] K.P. Tod, Class. Quantum Grav. **6**, 1159 (1989).

[20] F. Kottler, Ann. Phys. (Leipzig) **56**, 410 (1918).

[21] S.L. Bażański and V. Ferrari, Nuovo Cimento **91B**, 126 (1986).

[22] K. Lake and R.C. Roeder, Phys. Rev. D **15**, 3513 (1977).

[23] M.J. Jaklitsch, C. Hellaby, and D.R. Matravers, Gen. Relativ. Gravit. **21**, 941 (1989).

[24] A.A. Starobinsky, Pis'ma Zh. Eksp. Teor. Fiz. **37**, 55 (1983) [JETP Lett. **37**, 66 (1983)].

[25] W. Boucher and G.W. Gibbons, in *The Very Early Universe*, edited by G.W. Gibbons, S.W. Hawking, and S.T.C. Siklos (Cambridge University Press, Cambridge, 1983), p. 273.

[26] J. Bičák and Z. Perjés, Class. Quantum Grav. **4**, 595 (1987).