Parton-hadron duality: Resonances and higher twists

Xiangdong Ji and Peter Unrau
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 10 February 1995)

We explore the physics of the parton-hadron duality in the nucleon structure functions measured in lepton-nucleon scattering. We stress that duality allows one to extract the higher-twist matrix elements from data in the resonance region, or conversely to learn about the properties of resonances if the matrix elements are known. As an example, we construct the moments of $F_2(x, Q^2)$ for the low and medium Q^2 region, and from which we study the interplay between higher twists and the resonance contributions.

PACS number(s): 13.60.Hb, 12.38.Aw, 12.40.Nn, 14.20.Dh

I. INTRODUCTION

In electron-nucleon scattering, one probes the substructure of the nucleon with virtual photons of mass Q^2 and energy ν. The measured total response is characterized by the nucleon structure functions $W_{1,2}$ and $G_{1,2}$, etc. Before the advent of quantum chromodynamics (QCD), Bloom and Gilman [1] discovered an interesting feature in the structure function $W_2(\nu, Q^2)$, measured at SLAC. Simply speaking, when plotted in terms of the *improved scaling variable* $\omega' = 1 + W^2/Q^2$, where W is the final-state hadron mass, the scaling function $F_2(Q^2, \omega') = \nu W_2/m_N$ in the resonance region ($W < 2$ GeV) roughly averages to (or duals) that in the deep-inelastic region ($W > 2$ GeV). Referring to a similar phenomenon observed in hadron-hadron scattering, they called it *parton-hadron duality.* The occurrence of duality appears to be local, in the sense that it exists for each interval of ω' corresponding to prominent nucleon resonances. In fact, the assumption of an exact local duality allows a rough estimate of the nucleon's elastic form factor from the deep-inelastic scaling function.

An explanation of the Bloom-Gilman duality in QCD was first offered by De Rújula, Georgi, and Politzer in 1977 [2]. Following the operator product expansion, they studied the moments of the scaling function in the Nachtmann scaling variable $\xi = 2x/(1 + \sqrt{1 + 4x^2m_N^2/Q^2})$, where $x = Q^2/2m_N\nu$. They argued that the nth moment $M_n(Q^2)$ of F_2 has the twist expansion

$$M_n(Q^2) = \sum_{k=1}^{\infty} \left(\frac{nM_0^2}{Q^2}\right)^{k-1} B_{n,k}(Q^2)\,, \qquad (1)$$

where M_0^2 is a mass scale $\sim$(400–500 MeV)^2 and $B_{n,k}(Q^2)$ depends logarithmically on Q^2 and is roughly on the order of $B_{n,0}$. According to Eq. (1), there exists a region of n and Q^2 ($n \le Q^2/M_0^2$) where the higher-twist contribution is neither large nor negligible, and where the dominant contribution to the moments comes from the low-lying resonances. The phenomenon of local duality reflects the very existence of this region. A recent work on duality can be found in Ref. [3].

While these original studies of the parton-hadron duality were largely qualitative, enormous progress has been made in understanding QCD in the past twenty years. The radiative corrections have been evaluated to the next-to-leading order for the twist-two part of the scaling function [4]; the structure of the higher-twist expansion has been clarified to the order of $1/Q^2$ and some at the order of $1/Q^4$ [5]. The physics of the parton-hadron duality has been exploited ingeniously in the vacuum correlation functions, from which a useful technique for calculating hadron properties, the QCD sum rule method, has emerged [6]. Experimentally, a large body of lepton-nucleon scattering data has been collected in the past 25 years [7]. With the Continuous Electron Beam Accelerator Facility (CEBAF) becoming available for making systematic, high-precision measurements in the resonance region, it is timely to reexamine duality in its original context, and further explore the physics content of this important concept.

In this paper we seek to sharpen the explanation of the duality offered by the authors in Ref. [2], with a few crucial differences. First, we choose to work with the moments of Cornwall and Norton, instead of those of Nachtmann, thereby avoiding the unphysical region of $\xi > \xi(x = 1)$. Second, we look for a way to describe more clearly the contribution of the resonances to the moments. Finally, we emphasize a thorough exploitation of the consequences of duality. We furnish our discussions with the example of F_2, for which the abundant data allow an accurate construction of its moments in the low and medium Q^2 region. These moments offer a unique opportunity for studying the effects of higher-twists and the resonance contributions.

II. PARTON-HADRON DUALITY REVISITED

The Cornwall-Norton moments of a scaling function $F(x, Q^2)$ are defined as

0556-2821/95/52(1)/72(6)/$06.00 ©1995 The American Physical Society

$$M_n(Q^2) = \int_0^1 dx x^{n-2} F(x, Q^2) , \quad (2)$$

where the upper limit includes the elastic contribution. According to the operator product expansion [8], the moments can be expanded in powers of $1/Q^2$,

$$M_n(Q^2) = \sum_{k=0}^{\infty} E_{nk}(Q^2/\mu^2) M_{nk}(\mu^2) \left(\frac{1}{Q^2}\right)^k , \quad (3)$$

where E_{nk} are dimensionless coefficient functions, which can be calculated perturbatively as a power series in the strong coupling constant $\alpha_s(\mu^2)$,

$$E_{nk}(Q^2/\mu^2) = \sum_{i=0}^{\infty} \alpha_s^i(\mu^2) e_{nk}^i(Q^2/\mu^2) , \quad (4)$$

and $M_{nk}(\mu^2)$ are the nucleon matrix elements of higher-twist operators composed of quark and gluon fields. The renormalization scale (μ^2) dependence cancels in the product of E_{nk} and M_{nk}; however, when we talk about them separately, μ^2 is implicitly chosen at the hadron mass scale. The terms beyond the first in Eq. (3) are called the higher-twist corrections, which include both the target mass corrections and the true higher-twist effects, which physically represent initial and final state interactions between the struck quark and the remnants of the target.

The double expansions in Eq. (3) are asymptotic at best. Nonperturbative effects can invalidate both expansions at higher orders, and can mix the two, rendering the separation of radiative and power corrections ambiguous [9]. In the following discussion, however, we assume that in the Q^2 region of our interest the size of the twist-four term ($1/Q^2$) is significantly larger than the smallest term in the asymptotic expansion for E_{n0}, beyond which the evaluation of E_{n0} cannot be improved by including higher-order terms, and so the ambiguity in defining the higher-twist corrections can be neglected. We shall henceforth focus only on the structure of the twist expansion.

Following Ref. [2], we assume the ratio of the twist-four to the leading twist term in the nth moment is roughly nM_0^2, where M_0 is on the order of quark transverse momentum in the nucleon. We further assume that the twist expansion is an asymptotic series in parameter nM_0^2/Q^2. According to these assumptions, we can classify the higher-twist contributions to the moments. Consider the n-Q^2 plane as shown in Fig. 1(a), which is separated into three regions by two solid lines. Region A is defined by $nM_0^2 \ll Q^2$ where the higher-twist effects are negligible. Region B is where the higher-twist corrections become important but stay perturbative, and thus only the first few terms in the twist expansion are of practical importance. Physically, this means the scattering in this region can be described by few-parton processes. Region C is where the higher-twist effects become nonperturbative and the power-expansion diverges. In this region of many-parton coherent scattering, the dominating mechanism for resonance production requires infinitely-many power terms kept in the expansion.

Now let us consider the resonance contribution to the Cornwall-Norton moments by examining the x-Q^2 plane shown in Fig. 1(b), on which the resonance region lies roughly above the curve $W = 2$ GeV. For a large, fixed Q^2 (say 15 GeV^2), the resonance contribution to the lowest few moments is essentially negligible. When n increases, the resonance contribution weights more and more and finally becomes significant. We draw a dashed line, as Q^2 varies, in the n-Q^2 plane to indicate the transition between the two regions. The dashed line certainly cannot be in region A, because the nonresonance experimental data have already shown the higher-twist effects [12]. If the dashed line is in the region C, then the perturbative higher-twist effects, i.e., the few-parton process, have nothing to do with resonance physics. The more interesting possibility is when the dashed line lies in the region B, in which case one can study the interplay between resonances and higher twists.

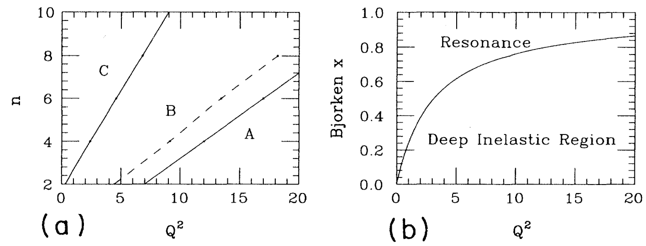

FIG. 1. (a) Three regions of differing importance to higher-twists: Region A, negligible higher-twists; Region B, perturbative higher twists; and region C, the twist-expansion breaks down. (b) Kinematic regions corresponding to the resonance and deep-inelastic scattering.

When the dashed line is located in region B, then to the left of the line in the region the following statements are true: (1) the higher-twist corrections are perturbative, so the moments are not too different from those at larger Q^2, and (2) the resonance contributions to the moments are significant. Thus in this region the resonances must organize themselves to follow the deep-inelastic contribution apart from a perturbative higher-twist correction, or conversely the structure of the higher-twist expansion constrains the behavior of the resonance contribution. It appears that the physics in this region can be described in terms of either resonance production or scattering of a few partons. Both languages dual each other. The degree of duality is determined by the size of the region: the larger the region, the more the moments are constrained, and the more local the duality will become.

Why should duality occur at all in QCD? On one hand, the quark transverse momentum in the nucleon, which governs the magnitude of the higher twists, is about 400 MeV. This makes the higher-twist corrections perturbative down to very small Q^2. On the other hand, the resonance contribution to the moments are already significant at $Q^2 \sim 5$ GeV^2 for low n. Thus the occurrence of the duality seems unavoidable, unless QCD had two widely different scales.

The consequences of duality, like duality itself, are twofold. If one knows data in the resonance region, one can extract the matrix elements of the higher-twist operators.

The extraction, of course, is limited by our ability to calculate higher-order radiative corrections. On the other hand, if one knows the higher-twist matrix elements from other sources, such as lattice QCD calculations, one can utilize them to extract the properties of the resonances. This second use of duality has been pursued vigorously in the QCD sum rule calculations, from which a large number of interesting results have been obtained [6]. In the present case, however, the number of higher-twist matrix elements is large, and they are difficult to estimate in general. This severely limits our ability to check, for example, the internal consistency of duality predictions.

III. TWIST-FOUR MATRIX ELEMENTS FROM $F_2(x, Q^2)$

We make the above discussion more concrete and quantitative by using the example of the F_2 scaling function, for which rich data exist in a large kinematic region. Most of the low Q^2 data were taken in the late 1960s and early 1970s at SLAC and DESY, and they nearly cover the whole resonance region at large x. The data were fitted by Brasse *et al.* [10] to a function with three parameters for each fixed W. In Ref. [11], Bodek *et al.* have made a more extensive but different fit, covering higher Q^2 resonance data. The deep-inelastic data were systematically taken by SLAC, BCDMS, the European Muon Collaboration (EMC), and other collaborations during the 1970s and 1980s, and they have recently been shown to be consistent with each other [12]. New measurements from the New Gluon Collaboration (NMC) at CERN have extended these data to lower Q^2 and x [14]. In Fig. 2, we have shown the F_2 data as a function of Bjorken x at Q^2 = 0.5, 1.0, 2.0, 4.0, 8.0, and 16.0 GeV^2 from the two fits [11,14] made in complementary kinematic regions.

The salient features of the data can be summarized as follows. At high Q^2, the data are almost entirely deep inelastic except for a small resonance contribution at large x. The scaling function near $x = 0$ shows a rise due to perturbative QCD effects. As Q^2 decreases, small bumps become visible and slide toward low x. These prominent excitations are believed to be the $\Delta(1232)$, $S_{11}(1535)$ or $D_{13}(1520)$, and $F_{15}(1680)$ resonances. The resonance excitations become strong near $Q^2 = 2$ GeV^2 and clearly dominate F_2 in the large x region below $Q^2 = 1$ GeV^2. As $Q^2 \to 0$ the data are compressed toward $x = 0$ due to simple kinematic reason. At $Q^2 = 0$, the whole photoproduction physics is shrunk to $x = 0$. Of course, one should not forget about the elastic contribution, which contributes a δ function at $x = 1$ and is not shown in the figure.

We construct the experimental moments by integrating the fitted $F_2(x, Q^2)$ according to Eq. (2). As usual, assumptions are needed to extrapolate the data beyond the experimentally measured region. However, for the moments we are considering, the contribution from the small x region is quite small due to the phase space sup-

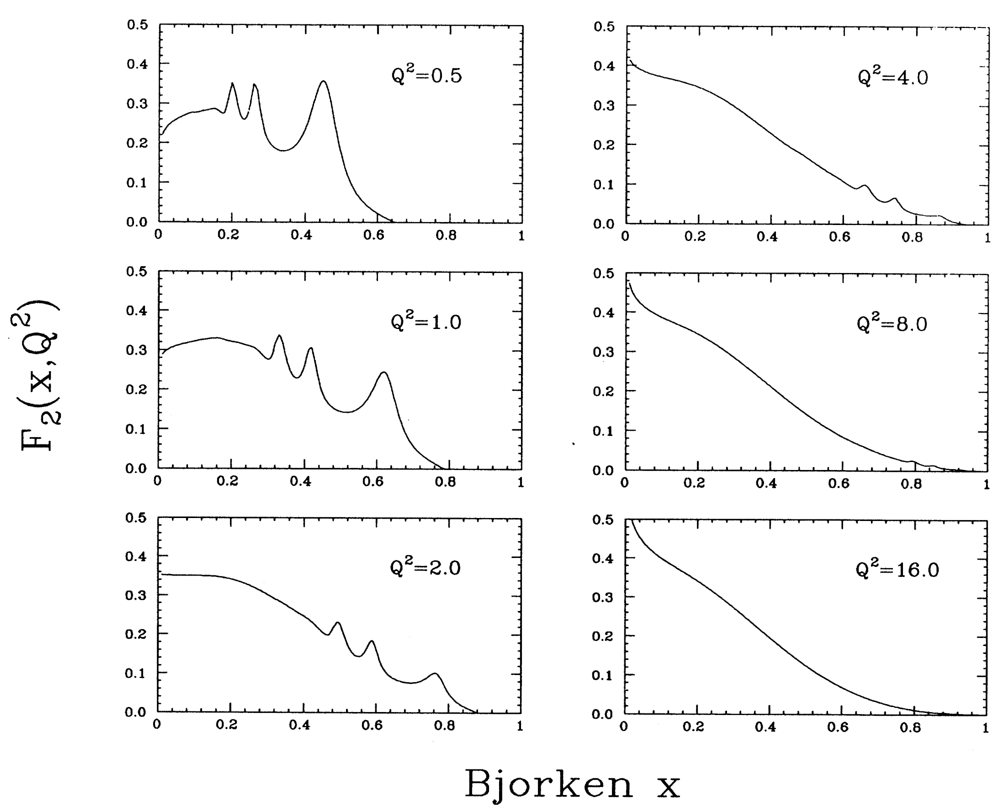

FIG. 2. Scaling function obtained from the fits to experimental data in Refs. [11,14].

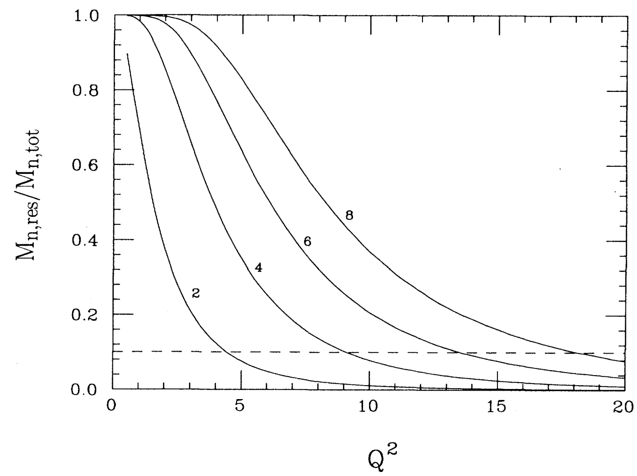

FIG. 3. Ratio of the moments from the resonance region, including the elastic contribution, to that of the total.

pression, in contrast with the first moment of $g_1(x, Q^2)$ [13]. We have not included the experimental errors for all data points and studied their effects on the moments although it is a quite desirable thing to do. Rather we have taken the errors on moments to be 3% uniformly, independent of kinematic variables. Though very crude, the number is a representative average of errors from a typical experiment.

To understand the role of the resonance-region contribution to the Cornwall-Norton moments, we plot in Fig. 3 the ratio of this contribution to the total, where the resonance region is defined by a cut on $W < 2$ GeV. If one uses 10% as a yardstick to measure the importance of the resonance-region contribution, this threshold is reached for the lowest moment ($n = 2$) at $Q^2 \sim 4$ GeV^2. For higher moments, the transition occurs approximately at $2n$ GeV^2. This is a bit surprising because it says that the resonance effects are appreciable for the eighth moment even at $Q^2 = 16$ GeV^2. At $Q^2 = 8$ GeV^2, the same moment receives 50% of the contribution from the resonance region. The dashed line in Fig. 1(a) roughly corresponds to the 10% line shown in Fig. 3.

The experimental F_2 moments can be used to extract the matrix elements of higher-twist operators. To do this, we first need to subtract the twist-two contribution in the moments. We use a twist-two parton distribution (CTEQ2 from [15]) fitted to a large number of data from hard processes, and calculate the moments for each quark flavor and gluon distribution at some large Q^2 (=20 GeV^2 in our case). Then we evolve these twist-two moments to lower Q^2 using the perturbative QCD formula accurate to next-to-leading order. Theoretical errors in evolution are mainly from the uncertainty in

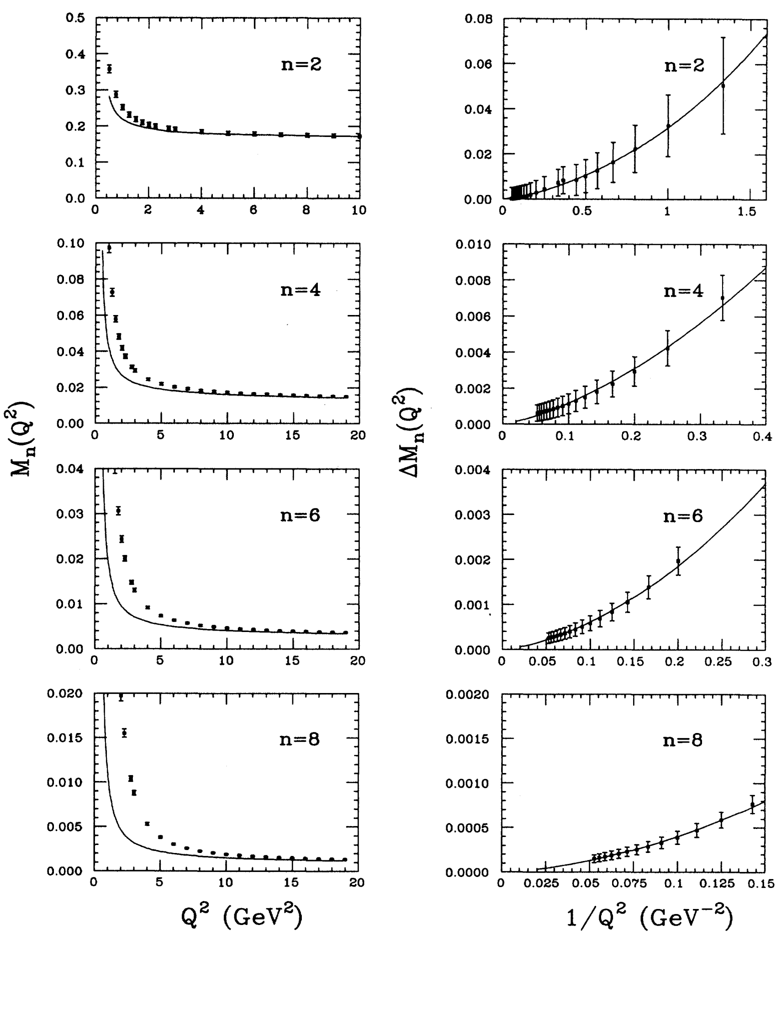

FIG. 4. (a) Moments as functions of Q^2, extracted from the scaling function in Fig. 2. The solid lines refer to the contribution from the leading twist and target mass corrections. (b) Residue moments from the higher-twist contribution. The solid lines are the fits described in the text.

$\Lambda_{\rm QCD}$ and unknown higher-order terms in the coefficient functions. In our work, we take $\Lambda^{(4)}_{\rm QCD} = 260 \pm 50$ GeV from a global fit for α_s [16]. The target mass corrections are further subtracted from the moments according to the formula in Ref. [17]. In Fig. 4(a), we show the experimental moments as a function of Q^2 (data points) and also the twist-two part plus the target mass corrections (solid lines). The residual moments, which are our definition of higher twist contributions, are shown in Fig. 4(b) as functions of $1/Q^2$.

We choose to fit the Q^2 evolution of the residual moments with a pure twist-four contribution,

$$\Delta M_n(Q^2) = a_n(1)\left(\frac{\alpha_s(Q^2)}{\alpha_s(1)}\right)^{\gamma_n}\frac{1}{Q^2}\,, \tag{5}$$

where we have included phenomenologically the leading-log effects with an adjustable exponent. The fitted γ_n represents an average of the anomalous dimensions of the spin-n, twist-four operators, weighted by the size of individual matrix elements. The coefficient a_n is the sum of the twist-four matrix elements at the scale $\mu^2 = 1$ GeV^2. Inclusion of a twist-six term creates strong correlations among the parameters and renders the fits indeterminate. Thus we have neglected such a term by restricting the fit to the $Q^2 > n$ GeV^2 region, where according to Eq. (1) the twist-six contribution is presumably suppressed by a factor of $(0.4\text{–}0.5)^2$.

The result of our fit is shown in Table I. Let us discuss in some detail the correction to the $n = 2$ moment, which is the well-known momentum sum rule. With the twist-four and target mass corrections, the sum rule reads

$$\int_0^1 F_2(x,Q^2)dx = \sum_f e_f^2 \Delta P_f(Q^2) + \frac{A_4(Q^2)}{2}\frac{M^2}{Q^2} + \frac{a_2(1)}{Q^2}\left(\frac{\alpha_s(Q^2)}{\alpha_s(1)}\right)^{\gamma_2} + \cdots\,, \tag{6}$$

where the summation extends over three light flavors and $\Delta P_f(Q^2)$ is the momentum fraction of the nucleon carried by the quark flavor f. The matrix element A_4 arises from the target mass correction and is defined by

$$\langle P|\bar\psi\gamma^{(\mu_1}iD^{\mu_2}iD^{\mu_3}iD^{\mu_4)}\mathcal{Q}^2\psi|P\rangle = 2A_4(\mu^2)P^{(\mu_1}P^{\mu_2}P^{\mu_3}P^{\mu_4)}\,. \tag{7}$$

The twist-four correction is summarized by the matrix element $a_2(\mu^2)$ [5]:

$$\langle P|\tfrac{1}{2}g^2\bar\psi\gamma_{(\mu}\gamma_5 t^a\mathcal{Q}\psi\bar\psi\gamma_{\nu)}\gamma_5 t^a\mathcal{Q}\psi + \tfrac{5}{16}g^2\bar\psi\gamma_{(\mu}t^a\mathcal{Q}^2\psi\bar\psi\gamma_{\nu)}t^a\psi + \tfrac{1}{16}g\bar\psi iD_{(\mu}\tilde F_{\nu)\alpha}\gamma^\alpha\gamma_5\mathcal{Q}^2\psi|P\rangle = 2a_2(\mu^2)(P_\mu P_\nu - M^2 g_{\mu\nu}/4), \tag{8}$$

where $(\mu\nu)$ means symmetrization of the indices and subtraction of the trace and $\mathcal{Q}$ is the quark charge matrix. For $Q^2 = 2$ GeV^2, the leading-twist contribution in Eq. (6) is 0.187. The target mass correction contributes 0.007. From the fitted parameter in Table I, we find that the twist-four contribution is 0.015, about 10% of the leading twist. This level of twist-four effect is also expected in the Bjorken sum rule for the spin-dependent structure function $g_1(x,Q^2)$ measured at SLAC [13].

Table I shows that the exponent γ_n of the leading-log corrections increases gradually with n, in accord with general expectations. The uniform errors on γ_n are the result of our fits, not from any constraint. The constant magnitude of the twist-four matrix elements a_n as n increases is in sharp contrast to the diminishing leading-twist contribution. It reflects, though, the fact that the higher-twist contribution becomes more important for higher moments at a fixed Q^2, and is a precursor for onset of the resonance region. In QCD, this can be explained by an increasing number of twist-four operators compensated by a decrease in strength of individual matrix element. The pattern of a_n indicates a twist-four distribution negative at small x, positive at large x and peaked near $x = 1$, qualitatively consistent with the fits in Ref. [12], where the resonance data were entirely ignored.

Finally, we test the assumption for the higher-twist matrix elements shown in Eq. (1). We have shown in the fourth column of Table I the ratios of the higher-twist matrix elements to the leading-twist ones. From this, we extract an effective M_0, the characteristic scale for higher twists, by dividing the ratios by n and taking a square root. The result is shown in the fifth column and is approximately n independent, though there is a slight hint of M_0 getting larger for larger n. However, this should not be taken too seriously because of the errors and limited number of moments. The twist-expansion shall become nonperturbative when 50% of the moments come from the twist-four corrections. According to this prescription, we can find a characteristic Q^2 for each moment where this transition takes place. For $n = 2$, this hap-

TABLE I. Extracted twist-four matrix elements a_n, effective anomalous dimension γ_n, ratio to the leading twist contribution, and the effective mass scale M_0.

n	a_n (GeV^2)	γ_n	$a_n/(E_{n0}M_{n0})$	M_0 (GeV)
2	0.030 ± 0.003	1.0 ± 0.5	0.14	0.26 ± 0.02
4	0.042 ± 0.013	1.5 ± 0.5	1.00	0.50 ± 0.08
6	0.047 ± 0.021	2.5 ± 0.5	2.47	0.64 ± 0.17
8	0.038 ± 0.018	2.5 ± 0.5	3.45	0.66 ± 0.19
10	0.052 ± 0.025	3.5 ± 0.5	4.73	0.69 ± 0.19

pens at ~ 0.3 GeV^2, and for higher moments at roughly $n - 1$ GeV^2. The solid line which separates regions B and C in Fig. 1 marks this transition.

To illustrate the converse use of duality, one could, for instance, use the higher-twist contribution extracted from the pure deep-inelastic region (as done in [12]), or from some theoretical calculations, to determine the nucleon's elastic form factor. However, we feel that the higher-twist matrix elements have not been determined in these methods to a sufficient accuracy to allow a quantitative extraction of the resonance properties. Qualitatively, however, knowing the higher-twist contribution will surely improve the nucleon form factor extracted in Ref. [2], which showed a systematic deviation from the directly measured G_M, a clear indication of higher-twist effects.

To sum up, we explored in this work the physics of the parton-hadron duality. We emphasized that the existence of duality allows one to determine the higher-twist matrix elements from data in the resonance region, or alternatively knowing the matrix elements enables one to determine the properties of the resonances. We studied the duality picture offered by the F_2 scaling function, and extracted the matrix elements of the lowest few spin, twist-four operators. Clearly, this study can be applied straightforwardly to the spin-dependent structure function G_1 once more data become available.

ACKNOWLEDGMENTS

We thank A. Bodek, J. Morfin, and P. Stoler for information on the experimental data and the CETQ2 parton distributions. P.U. acknowledges the support of NSERC Canada. This work was supported in part by funds provided by the U.S. Department of Energy (DOE) under Cooperative Agreement No. DF-FC02-94ER40818.

---

[1] E. D. Bloom and F. J. Gilman, Phys. Rev. D **4**, 2901 (1971).
[2] A. De Rújula, H. Georgi, and H. D. Politzer, Ann. Phys. (N.Y.) **103**, 315 (1977).
[3] C. E. Carlson and N. C. Mukhopadhyay, Phys. Rev. D **47**, R1737 (1993).
[4] A. J. Buras, Rev. Mod. Phys. **52**, 199 (1980).
[5] E. V. Shuryak and A. I. Vainshtein, Nucl. Phys. B **199**, 451 (1982); R. L. Jaffe and M. Soldate, Phys. Rev. D **26**, 49 (1982); R. R. Ellis, W. Furmanski, and R. Petronzio, Nucl. Phys. **B212**, 29 (1983).
[6] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979); **B147**, 447 (1979).
[7] M. Virchaux, in *QCD—20 years later*, Proceedings of the Workshop, Aachen, Germany, 1992, edited by P. M. Zerwas and H. A. Kastrup (World Scientific, Singapore, 1993).
[8] F. J. Yndurain *Quantum Chromodynamics* (Springer-Verlag, New York, 1983).
[9] A. Mueller, Phys. Lett. B **308**, 355 (1993).
[10] F. W. Brasse *et al.*, Nucl. Phys. **B110**, 413 (1976).
[11] A. Bodek *et al.*, Phys. Rev. D **20**, 1471 (1979).
[12] M. Virchaux and A. Milsztajn, Phys. Lett. B **274**, 221 (1992).
[13] See, for instance, E143 Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **74**, 346 (1995).
[14] The New Muon Collaboration, P. Amaudruz *et al.*, Phys. Lett. B **295**, 159 (1992).
[15] H. L. Lai, J. Botts, J. Huston, J. G. Morfin, J. F. Owens, J. W. Qiu, W. K. Tung, and H. Weerts, "Global QCD Analysis and the CTEQ Parton Distributions," hep-ph/9410404, 1994 (unpublished).
[16] Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992); G. Altarelli, in *QCD—20 years later* [7].
[17] H. Georgi and H. D. Politzer, Phys. Rev. D **14**, 1829 (1976).