Infrared effects in a de Sitter background

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We have estimated higher order quantum gravity corrections to de Sitter spacetime. Our results suggest that, while the classical spacetime metric may be distorted by the graviton self-interactions, the corrections are relatively weaker than previously thought, possibly growing like a power rather than exponentially in time.

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I. INTRODUCTION

As is very well known Einstein field equations permit an addition of an extra term proportional to the metric tensor $\bar{g}_{\mu\nu}$:

$$\bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R} = 8\pi G T_{\mu\nu} + \Lambda \bar{g}_{\mu\nu}. \tag{1}$$

Here $T_{\mu\nu}$ is the energy-momentum tensor of matter, and Λ is the so-called cosmological constant. [We put a bar over all quantities here to distinguish them from the rescaled ones. See Eqs. (4), (5).] The last term in Eq. (1) may be interpreted as the vacuum energy-momentum tensor. Any natural estimate gives Λ or $\rho_{\text{vac}} = \Lambda/8\pi G$ much larger (by 50–100 orders of magnitude) than the observed upper bound in the present day Universe. This mysterious discrepancy is known as the cosmological constant problem and presents one of the most interesting challenges in modern physics (for a review see Refs. [1, 2]).

de Sitter spacetime is a solution of the Einstein equations with a dominant cosmological term. Written in the Robertson-Walker form for the special case of the spatially flat section the metric takes the form

$$ds^2 = dt^2 - a^2(t)d\vec{r}^2, \tag{2}$$

where $a(t) = \exp(Ht)$ with $H = \sqrt{\Lambda/3}$. The assumption of spatial flatness is not essential, and the results obtained below are true also for open and closed geometries. It is convenient to rewrite the metric in terms of conformal coordinates where, up to an overall scale factor, it has the Minkowski form

$$ds^2 = a^2(\tau)(d\tau^2 - d\vec{r}^2). \tag{3}$$

The conformal time τ is related to the physical one as $d\tau = -\exp(-Ht)dt$ and $a(\tau) = -1/H\tau$. Note that when t tends to future infinity, τ tends to $0-$. The de Sitter metric possesses the same degree of high symmetry as the Minkowski one and, for this reason, it is the sim-

plest nontrivial curved background for a quantum field theory. Despite its simplicity, it has several very interesting properties and in particular generates an infrared instability of a massless scalar field ϕ minimally coupled to gravity. It was shown [3, 4] that the vacuum expectation value (VEV) $\langle \phi^2 \rangle$ is singular at the zero mass limit, $\langle \phi^2 \rangle \sim H^4/m^2$. With the advent of the inflationary scenario, this phenomenon was rediscovered in a number of papers [5–7], where it was argued that $\langle \phi^2 \rangle$ rises linearly with time in the zero mass limit. This quantity however does not have a direct physical meaning and, in particular, the contribution of the field ϕ to the energy density does not rise with time. Nevertheless, this infrared instability afflicts the scalar field propagator and needs to be resolved to make a sensible quantum theory. Note in passing that this instability is a result of broken conformal invariance. To ensure the latter, one needs the coupling to the curvature scalar, $R\phi^2/6$, which gives rise to an infrared cutoff. In fact, any nonzero mass m_ϕ or nonminimal coupling $\xi R\phi^2$ with a non-negative coefficient is sufficient for infrared stability. In any case, this infrared divergence is a sign that the true vacuum state is not de Sitter invariant.

Conformal invariance is also broken for gravitons [8] and, for this reason, there should be infrared instability of the de Sitter vacuum due to quantum gravity effects. If this is indeed the case, it suggests that the solution of the long-standing cosmological constant problem might be found in this direction. One-loop graviton quantum corrections to the de Sitter metric were considered in this connection in Ref. [9] where it was found that they are time independent. However, it was suggested that higher loops may show evidence of this infrared instability. Indeed, it was claimed recently in several papers [10–12] that higher loop effects are much stronger, giving corrections that rise exponentially with physical time t or as a power of $1/\tau$ with conformal time. This is a very exciting result and, if confirmed, would mean that de Sitter space cannot exist indefinitely, opening a beautiful way for the solution of the cosmological constant problem in the framework of the normal quantum gravity without any drastic assumptions.

Before beginning our development, we note that there have been various suggestions that de Sitter space is infrared unstable. As early as in 1977, Gibbons and Hawk-

ing [14] pointed out that, in the presence of a scalar field, a comoving observer in de Sitter background would perceive a bath of thermal radiation. As in the Rindler problem in Minkowski space [15], this "particle production" is not to be interpreted as vacuum decay [16]. However, Mottola [17] has argued that, while this is true of the Hartle-Hawking (or "Euclidean") vacuum, a universe that evolves from realistic initial conditions would not be in equilibrium and would in fact decay. Similarly, Mottola [17] argues that general thermodynamic considerations in de Sitter space, as with black hole decay, would lead to vacuum decay.[1] Subsequently, Mazur and Mottola [18] and Antoniadis and Mottola [19] analyzed this instability dynamically, arguing that matter rapidly drives the conformal mode of the gravitation field away from its classical value. The extension of this, if possible, to pure gravity has not been made.

The present paper belongs to a long series of papers where the instability is sought by considering quantum gravity effects in de Sitter background. The starting observation is that the graviton propagator in de Sitter space grows for large spacelike separations [20–22]. This is closely related to the instability of the massless, minimally coupled scalar field in de Sitter background [3] for which the equation of motion is the same as for the physical (transverse-traceless) modes of the graviton field. Although one should be careful about gauge fixing[21] and, although there are still doubts on whether an analytical continuation exists from the Lorentz to Euclidean signature [23], the growth of the propagator for large separations in pure de Sitter space seems well established [20,21,24,26,27]. Whether that growth translates into an infrared divergence for observables remains a matter of some debate [24,25], but it seems to us quite likely. In any case, the origin of this divergence is the supposition that de Sitter space is eternal. Given a cosmology in which the vacuum energy dominates only after some initial time, this divergence disappears, and the graviton propagator does fall off for sufficiently large separations, as it must for a good quantum field theory [7,28].

In any case, the consideration of propagators is but a preliminary step, and eventually one concentrates on quantum corrections to physical quantities. There have been other interesting suggestions as to how quantum gravity might quench the cosmological constant [29]. However, these proposals do not work at the level of one-loop effective potential [30, 31]. As is claimed in Refs. [10–12], the two-loop corrections do result in variation of the cosmological constant which grows with time, although its origin appears to be somewhat different from infrared problems previously discussed. Our present paper is motivated by these claims.

Here we have reconsidered results of papers [10–12] using a different formalism and have found that such a

strong instability does not set in. The corrections at most behave as a power[2] of $\log|\tau| \sim t$.

II. EQUATIONS OF MOTION

For the conformally flat background metric we introduce the quantum graviton field $h_{\mu\nu}$ in the usual way:

$$\bar{g}_{\mu\nu} = a^2(\tau)g_{\mu\nu} \equiv a^2(\tau)(\eta_{\mu\nu} + h_{\mu\nu}). \tag{4}$$

The issue is whether, taking into account loop corrections, the VEV $\langle h_{\mu\nu}\rangle$ is nonzero and, in particular, whether it is time dependent. This would suggest that the background de Sitter metric is not self-consistent, although it must then be shown that this is a physical effect by calculating, for example, the curvature for the modified metric. Even then, it is important to ascertain whether the inconsistency involves strong coupling or whether it is an essentially negligible effect. The Einstein action (with nonzero cosmological constant) in terms of the new metric $g_{\mu\nu}$ can be rewritten as

$$A = \frac{1}{\kappa^2} \int d^4x \sqrt{\bar{g}}(\bar{R} + 2\Lambda)$$

$$= \frac{1}{\kappa^2} \int d^4x a^2 \sqrt{g} \left(R + 6\frac{a_{,\alpha}a^{,\beta}}{a^2} + 2\Lambda a^2\right), \tag{5}$$

with $\kappa^2 \equiv 16\pi/m_{\rm Pl}^2$. This implies the equations of motion

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda a^2 g_{\mu\nu} + \frac{4a_{,\mu}a_{,\nu}}{a^2} - \frac{2a_{;\mu\nu}}{a}$$

$$+ g_{\mu\nu}\left(\frac{2a_{;\alpha}^{;\alpha}}{a} - \frac{a_{,\alpha}a^{,\alpha}}{a^2}\right) = 0, \tag{6}$$

where tensor indices are raised and lowered with the new metric $g_{\mu\nu}$. To zeroth order in $h_{\mu\nu}$, we get the usual equation for the scale factor of the classical background metric, with solution $a(\tau) = -1/H\tau$ [see the notation after Eq. (3)].

It is of course well-known that this is a nonrenormalizable quantum field theory, but we regard the Einstein action as the first two terms in an infinite series of local operators of increasing dimension. This is an effective field theory, presumed valid on scales below $m_{\rm Pl}$, and is renormalizable in the sense that all divergences involving these vertices may be absorbed in a renormalization of one of the infinite number of coefficients of these local operators. While we have not written such terms explicitly, they should be understood to be present. In the following, we will deal exclusively with renormalized couplings and operators, with the tacit understanding that the counterterms are included in our interaction. To quantize the theory consistently, we must add a gauge-fixing term $\mathcal{L}_{\rm GF}$ and the corresponding Faddeev-Popov

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[1] The description of this mechanism in coordinates in which there is no Hawking radiation initially has not to our knowledge been developed.

[2] We understand that these authors, using an in-in formalism in contrast with the in-out formalism employed earlier, also do not get power law singularity but only powers of $\log \tau$ [R. Woodard, (private communication), and Ref. [13].]

(FP) ghosts $\mathcal{L}_{\rm FP}$. We choose $\mathcal{L}_{\rm GF} = -\frac{1}{2}F_\mu F_\nu \eta^{\mu\nu}$ as in Refs. [10, 11], with

$$F_\mu = a(\tau)\left(h_{\mu,\nu}^\nu - \frac{1}{2}h_{,\mu} + 2\delta_\mu^0 h_0^\nu \frac{a_{,\nu}}{a}\right). \qquad (7)$$

The ghost Lagrangian $\mathcal{L}_{\rm FP}$ may be found in Ref. [12]. Here and subsequently, when we consider perturbation theory in $h_{\mu\nu}$, the indices are raised with the Minkowski tensor $\eta_{\mu\nu}$. With these gauge conditions, the linear part of the equation of motion for $h_{\mu\nu}$ has a very simple form:

$$h_{\mu\nu,\alpha}^\alpha - \frac{2}{\tau}h_{\mu\nu,0} + \frac{2}{\tau^2}\left(\delta_\mu^0 h_{0\nu} + \delta_\nu^0 h_{0\mu}\right) - \frac{2}{\tau^2}\eta_{\mu\nu}h_{00} = 0. \qquad (8)$$

One can easily verify that time components $h_{0\mu}$ ($\mu = 0, 1, 2, 3$) are conformally invariant in the sense that the *rescaled* functions $\chi_{0\mu} = a(\tau)h_{0\mu}$ satisfy the free field equations of motion, $\partial^2 \chi_{0\mu} = 0$.

More interesting are the space-space components of metric h_{ij}. The equations of motion are not diagonal for them, but after a simple linear redefinition $f_{ij} = h_{ij} - \delta_{ij}h_{00}$ they are diagonalized and have the form

$$f_{ij,\alpha}^\alpha - \frac{2}{\tau}f_{ij,0} = 0. \qquad (9)$$

This is the same equation which is satisfied by a massless minimally coupled scalar field. The solutions with definite momenta can be written as

$$\phi_k(\tau, \vec{x}) = H\left(\tau - \frac{i}{k}\right)\exp(i\vec{k}\cdot\vec{x} - ik\tau). \qquad (10)$$

These nonconformal modes differ from the conformal modes by the presence of the i/k term.

The conventionally normalized Heisenberg field operator for quantum fluctuations of the gravitational field is defined as $\psi_{\mu\nu} \equiv h_{\mu\nu}/\kappa$. This may be expressed in the standard way in terms of creation-annihilation operators $a_{\mu\nu}(k)$ by the decomposition[3]

$$\psi_{\mu\nu} = \int \frac{d^3k}{(2\pi)^3\sqrt{2k}}\left[\psi_k(\tau, \vec{x})a_{\mu\nu}(\vec{k}) + \psi^*(\tau, \vec{x})a_{\mu\nu}^\dagger(\vec{k})\right] \qquad (11)$$

with a and $a^\dagger$ satisfying the commutation relations

$$\left[a_{\mu\nu}(\vec{k}), a_{\mu\nu}^\dagger(\vec{q})\right] = (2\pi)^3\delta^3(\vec{k} - \vec{q})$$

$$\text{(not summed on } \mu, \nu\text{)}. \qquad (12)$$

All other commutators are zero. For the nonconformal modes, the wave function $\psi_k(\tau, \vec{x})$ is given by Eq. (10), while for the conformal modes, they are plane waves $H\tau \exp(i\vec{k}\cdot\vec{x} - ik\tau)$. Respectively, the propagator for a conformal field (for example for the field $h_{0\alpha}$, see above) is proportional to that in flat space. It means that the

evolution of $h_{0\alpha}$ is trivial and in particular the corresponding Green's functions are obtained from the ones for the flat spacetime by the rescaling

$$G^{\rm dS}(x, x') = \frac{G^{\rm flat}(x, x')}{a(\tau)a(\tau')} = H^2\tau\tau' G^{\rm flat}(x, x'). \qquad (13)$$

Now, propagators (two-point functions) for nonconformal fields exhibit new features since the solutions (10) blow up at small k as compared to the solution for conformal fields. In particular, this implies that the anticommutator correlator G_1 is in fact not defined in the infrared. To be explicit, it is given by

$$G_1(\tau, \tau', \vec{k}) = \frac{H^2}{k}\left[\left(\tau\tau' + \frac{1}{k^2}\right)\cos(k\Delta\tau) - \frac{\Delta\tau}{k}\sin(k\Delta\tau)\right], \qquad (14)$$

where $\Delta\tau \equiv \tau - \tau'$. If we calculate the Fourier transform of (14) then it is logarithmically divergent at small k and undefined. However, derivatives of it are well defined.

It might also worth mentioning that in a de Sitter background the Green function for a nonconformal field can be also given in a closed form in four-dimensional notation. Namely we have

$$G(x, x') = H^2\tau\tau' \int \frac{d^4k}{(2\pi)^4}\frac{e^{ik\cdot(x-x')}}{k^2} - H^2 \int \frac{d^4k}{(2\pi)^4}\frac{e^{ik\cdot(x-x')}}{k^4}, \qquad (15)$$

where, as usual, depending on the $i\epsilon$ prescription we get either a Feynman, retarded, or advanced propagator. [The derivation of (15) follows the lines presented in the Appendix to Ref. [32] where the first terms in the WKB expansion are considered for arbitrary background field. In a de Sitter background, a similar argument fixes the exact propagator.] In particular the Feynman propagator takes the form [11]

$$G_F(x, x') = \frac{H^2}{4\pi^2}\left[\frac{\tau\tau'}{(x - x')^2} - \frac{1}{2}\ln(x - x')^2\right]. \qquad (16)$$

Note that the argument of the log is not defined, as a manifestation of the same infrared instability. What is specific about the de Sitter background (and $\xi = 1/6$ *or* 0) is that the expansion in k^{-2} in Eq. (15) terminates on the second term.

In fact, this infrared blowup of the Feynman propagator is the basic observation which gave rise to the hopes that quantum corrections to the de Sitter metric grow with time. Indeed, the quantum propagators are not vanishing in causally forbidden regions and may bring information on the overall expansion in the future, characteristic of the de Sitter solution, revealing in this way an instability of pure classical solution. We are going to scrutinize this suggestion.

The best way to approach the problem is to keep as close to classical consideration as possible. Indeed, the

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[3]See, e.g., Ref. [16] for further details.

classical solutions are known to be stable against perturbations [33]. The crucial difference between classical and quantum problems is that the development of classical fluctuations is governed by the retarded Green function G_R which is free of the infrared divergencies mentioned above. The Green function is obtained in the standard way:

$$G_R(x, x') = i\theta(\tau - \tau')\langle[\psi(\tau, \vec{x}), \psi(\tau', \vec{x}')]\rangle, \qquad (17)$$

where we have suppressed the various tensor indices. Although commonly expressed in terms of a VEV, it is important to recognize that G_R for the linearized theory is a purely classical construct that may be obtained directly from the classical equations of motion. Accordingly, it is completely independent of the definition of the vacuum state.

A simple calculation gives [12]

$$G_R(x, x') = H^2[\tau\tau' G_R^{\text{flat}}(x - x') + \theta(\Delta\tau)\theta(\Delta\tau - r)/4\pi], \qquad (18)$$

where $r = |\vec{x} - \vec{x}'|$, $\Delta\tau = (\tau - \tau')$, and

$$G_R^{\text{flat}} = \theta(\Delta\tau)\delta(r - \Delta\tau)/4\pi r \qquad (19)$$

is the retarded Green's function in the flat spacetime. We suppressed here the evident tensor indices. The second term in Eq. (18) is connected with the broken conformal invariance. For what follows it is essential that it vanishes for $\Delta\tau = 0$. In the mixed (τ, k) representation we find

$$G_R(\tau, \tau', \vec{k}) = \frac{H^2\theta(\Delta\tau)}{2k}\left[\left(\tau\tau' + \frac{1}{k^2}\right)\sin(k\Delta\tau) - \frac{\Delta\tau}{k}\cos(k\Delta\tau)\right], \qquad (20)$$

and in the limit if $k \to 0$ the retarded Green function for a nonconformal field is no more singular than for a conformal (or in flat space).

The Heisenberg picture of quantum field theory is closest to the classical one, and we expect that the infrared problem is most easily treated within this approach. With the initial condition $\psi_{\mu\nu}(\tau_0, \vec{x}) = \psi_{\mu\nu}^{\text{in}}(\tau_0, \vec{x})$, the operator equation (8) can be rewritten in the integral form

$$\psi_{\mu\nu}(\tau, \vec{x}) = \psi_{\mu\nu}^{\text{in}}(\tau, \vec{x}) + \frac{1}{m_{\text{Pl}}}\int_{\tau_0}^{\tau} d\tau'$$
$$\int d^3x' G_{R\ \mu\nu}^{\alpha\beta}(\tau, \vec{x}; \tau', \vec{x}')(H\tau')^{-2}V_{\alpha\beta}(\tau', \vec{x}'), \qquad (21)$$

where $V_{\alpha\beta}(\tau, \vec{x})$ is the interaction term. More specifically,

$$V_{\mu\nu} = \psi_{,\mu}^{\alpha\beta}\psi_{\alpha\beta,\nu} + \psi^{\alpha\beta}\psi_{\alpha\beta,\mu\nu} - \psi_{,\alpha}\Gamma_{\mu\nu}^{\alpha} + 2\Gamma_{\mu\nu}^{\alpha}\Gamma_{\alpha\beta}^{\beta}$$
$$- 2\Gamma_{\mu\beta}^{\alpha}\Gamma_{\nu\alpha}^{\beta} + 2\eta_{\mu\nu}\left(\frac{\psi^{\alpha\beta}\Gamma_{\alpha\beta}^{0}}{\tau} - \frac{2\psi_{0\alpha}\psi^{0\alpha}}{\tau^2}\right), \qquad (22)$$

where $\Gamma_{\mu\nu}^{\alpha} = (\psi_{\mu,\nu}^{\alpha} + \psi_{\nu,\mu}^{\alpha} - \psi_{,\mu\nu}^{\alpha})/2$ is the Christoffel symbol associated with the metric $g_{\mu\nu} = \eta_{\mu\nu} + \kappa\psi_{\mu\nu}$ to

first order in[4] $\psi_{\mu\nu}$. Note that the last two terms that are explicitly singular in τ are multiplied by combinations of fields whose wave functions, in lowest order, vanish when $\tau \to 0$. This result persists in higher order, as we shall argue below.

We will be investigating properties of the perturbative solution of (21). The use of (21) ensures that at least one propagator attached to each vertex is a retarded one and is smooth in the infrared limit. Another important advantage of the Heisenberg representation is that the state vectors in Hilbert space remain constant and the evolution of the system is contained in the time dependence of the field operators. Therefore one may study the time evolution of the physical quantities such as, e.g., energy density as the expectation values of the corresponding field operators over the initial quantum state |in⟩ which we take to be a no-particle, de Sitter invariant state. The time evolution of this state is quite a complicated problem by itself since the notion of particle production is ambiguous in general relatively (for discussion see, e.g., [16]). In the Heisenberg picture we do not confront this problem.

Although analytically quite complicated, it is straightforward in principle to develop the perturbative solution of the integral equation (21). [See Fig. 1(a).] Each three-point vertex carries a factor of $1/m_{\text{Pl}}$, so, for consistency, we should also include in $V_{\mu\nu}$ higher-point vertices obtained from the expansion of Eq. (6) as well as vertices of the same order from higher dimensional operators implicitly included in the action Eq. (5). We will discuss that in more detail in a subsequent paper where we will also consider in greater detail the renormalization of the theory. Here we concentrate only on a possible existence of singularities at $\tau = 0$. The contribution to ψ from the conformal Green's function reads

$$\Delta_{\text{conf}} = \frac{\tau}{m_{\text{Pl}}}\int_{\tau_0}^{\tau}\frac{d\tau'}{\tau'}(\tau - \tau')\int d\Omega V(\tau', \vec{x} + \vec{r}), \qquad (23)$$
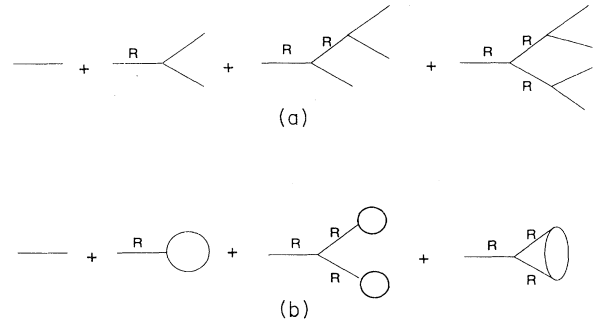

FIG. 1. (a) Perturbative solution of Eq. (21). The line denoted by R corresponds to the retarded propagator. (b) Loop corrections to metric fluctuations $\psi_{\mu\nu}$.

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[4]We remind the reader that counterterms are implicitly included in $V_{\mu\nu}$.

where $|\vec{r}| = \tau - \tau'$ and the angular integration is made over directions of $\vec{r}$. The nonconformal part coming from (18) contains two terms: the first of the form of Eq. (23) and the second given by

$$\Delta_{\text{nonconf}} = \frac{1}{4\pi m_{\text{Pl}}} \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau'^2} \int_0^{(\tau-\tau')} dr\, r^2 d\Omega V(\tau', \vec{x} + \vec{r}).$$

$$(24)$$

Our interest is in the VEV of this expression. Using translation invariance, one may simply set the spatial arguments to $\vec{0}$ to obtain

$$\langle \Delta_{\text{nonconf}} \rangle = \frac{1}{3m_{\text{Pl}}} \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau'^2} (\tau - \tau')^3 \langle V(\tau', \vec{0}) \rangle. \quad (25)$$

The generic form of the loop expansion is illustrated in Fig. 1(b). In calculating the VEV, one encounters quantum correlation functions of the type

$$G_1(\tau, \tau', \vec{x} - \vec{x}') \equiv \langle 0, \text{in} | \{\psi_{\text{in}}(\tau, \vec{x}), \psi_{\text{in}}(\tau', \vec{x}')\} | 0, \text{in} \rangle.$$

$$(26)$$

These correlators are not time-ordered and replace the Feynman propagators of the more familiar in-out formalism. Unlike the retarded propagator or commutator function, these are truly quantum-mechanical amplitudes that do not vanish for spacelike separated points. They depend upon the definition of the vacuum state $|0, \text{in}\rangle$, about which we shall have more to say shortly.

The dimension of $V_{\mu\nu}$ is mass^4. At one loop, it involves a single graviton propagator, which is proportional to H^2. It can be shown that each additional loop brings out another factor of H^2/m_{Pl}^2. Thus there are two powers of mass to be accounted for. Since the only remaining scale on which the VEV $\langle 0, \text{in} | V_{\mu\nu}(\tau, \vec{0}) | 0, \text{in} \rangle$ can depend is τ, one would naively expect, and will subsequently justify, a behavior as

$$\frac{H^2}{\tau^2} \left(\frac{H^2}{m_{\text{Pl}}^2} \right)^{L-1} \quad (27)$$

times possible powers of $\log |\tau|$. This behavior, when inserted into the integral equations, implies that the conformal modes are finite as $\tau \to 0$, while the nonconformal modes may diverge as a power of $\log |\tau|$. This would suggest that the de Sitter background would be unstable to quantum fluctuations, since the fluctuations would be growing with time. Although this is a much weaker singularity at $\tau = 0$ than previously suggested [10–12], this may nevertheless lead to a breakdown of perturbation theory and leave open the question of the ultimate future of the de Sitter metric.

It might be worth emphasizing that it is not only the use of G_R that softens the infrared behavior but the form of $V_{\mu\nu}$ as well. With one exception, the nonconformal fields ψ_{ij} enter the expression $V_{\mu\nu}$ in the form $\psi_{ij,\alpha}$, and the derivative renders their contributions infrared finite. The one exception is the second-to-last term in Eq. (22), which includes $\psi_{ij}\Gamma_{ij}^0/\tau$. The connection Γ_{ij}^0

involves space derivatives of conformal modes ψ_{0i}, as well as the time derivative of the nonconformal modes $\psi_{ij,0}$. Despite what one might think, $\psi_{ij,0}$ can be shown to satisfy a conformal field equation, so that all these terms in Γ_{ij}^0 involve an explicit factor of τ and satisfy the conformal integral equation. Thus, this term in $V_{\mu\nu}$ is a kind of cross product of a nonconformal field with a conformal field. It is only the expectation value of squares of conformal fields that manifest infrared problems, so one would not expect this to give trouble. In conclusion, there are no infrared divergences in quantities of interest to us here, and we may ignore for our purposes the fact that the true vacuum state is not de Sitter invariant.

Let us consider higher loop corrections to $\langle V(\tau, \vec{0}) \rangle$. The two-loop corrections take the generic form

$$F_2(\tau', \vec{0}) = \frac{1}{m_{\text{Pl}}^2} \int_{\tau_0}^{\tau'} \frac{d\tau_1}{\tau_1^2} \int_{\tau_0}^{\tau'} \frac{d\tau_2}{\tau_2^2} \int \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3}$$
$$\times \partial' G_R(\tau', \tau_1, \vec{k}) \partial' G_R(\tau', \tau_2, -\vec{k})$$
$$\times \partial^2 G_1(\tau_1, \tau_2, \vec{q}) \partial^2 G_1(\tau_1, \tau_2, \vec{k} - \vec{q}). \quad (28)$$

Here we have gone over to a mixed time-momentum representation, which is more convenient, since momentum is conserved, and power-counting is more familiar in momentum space. G_R is, as before, the retarded Green function, while G_1 is the anticommutator function[5] given in Eq. (26). Each partial derivative in Eq. (28) is to be interpreted either as a derivative with respect to time or multiplication by one power of momentum. Now the quantum correlators, such as G_1, depend in detail on the choice of vacuum state $|0, \text{in}\rangle$. This has generally been chosen to be the de Sitter invariant vacuum, but the infrared divergences discussed above and in Ref. [28] might lead one to worry about such a choice. The fact that these correlators only enter Eq. (28) with derivatives shows that those potential infrared singularities do not affect the quantity being calculated here. With regard to momentum integration, there will be various terms. In a mass-independent renormalization prescription, such as dimensional regularization with minimal subtraction, all divergences proportional to a power of the cutoff are completely removed by the counterterms. Thus, we need only concern ourselves with terms that are logarithmically divergent. Such contributions cannot change the power behavior of the time dependence, so that we simply have the result that one would guess on the basis of dimensional analysis. Thus, at worst, $F_2 \sim (\log |\tau'|)^p / \tau'^2$ for some power p. When inserted into the integral equation for the nonconformal ψ_{ij}, this yields a result that behaves at worst as $(\log |\tau|)^{p+1}$.

Thus it is safe to say that a quantum correction may bring only logarithmic dependence on τ. Terms proportional to powers of $\log |\tau|$ are not inherently problems for

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[5] There are also two-loop contributions from the commutator function.

perturbation theory. Consider, for example, the effect of a finite renormalization of the curvature, so that the scale factor $a(t)$ might be of the form $\exp[(H + \delta H)t]$. Expanding in powers of the perturbation gives

$$\exp[(H + \delta H)t] = \exp(Ht) \left[1 + \delta Ht + \frac{(\delta Ht)^2}{2} + \cdots\right].$$

(29)

In terms of the original conformal time $Ht \equiv -\log(H|\tau|)$, these corrections take the form of a power of $\log|\tau|$. Thus, it may be that logarithms arising from loop corrections, while suggesting a breakdown of perturbation theory, are merely finite renormalizations of the Hubble constant. To understand whether logarithmic corrections are a true instability rather than simply a finite renormalization, one would have to show that the breakdown is not simply due to large logs that can be summed up (as is often done using the renormalization group). Their physical interpretation may be simplified by computing their contribution to a gauge invariant quantity, such as the curvature.

III. CONCLUSION

To summarize, we have demonstrated that quantum corrections to the classical de Sitter solution generated by higher loops in quantum gravity can be at worst powers of logs of the conformal time τ. Moreover calculation of these logs is no longer a pure infrared problem. We presented then further arguments based on consideration of the one-loop effective action according to which one-loop effects in de Sitter background reduce to renormalization of the action for $\psi_{\mu\nu}$, with no logs involved. One needs also to include the effects of the FP ghosts as well as higher-point vertices of the same order in $1/m_{\rm Pl}$.

[1] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).
[2] A. D. Dolgov, in *The Quest for the Fundamental Constants in Cosmology*, Proceedings of the 24th Rencontre de Moriond, Les Arcs, France, 1989, edited by J. Audouze and J. Tran Thanh Van (Editions Frontieres, Gif-sur-Yvette, 1990), p. 227.
[3] L. H. Ford and L. Parker, Phys. Rev. D **16**, 245 (1977).
[4] T. S. Bunch and P. C. W. Davies, Proc. R. Soc. London **A360**, 117 (1978).
[5] A. Linde, Phys. Lett. **116B**, 335 (1982).
[6] A. A. Starobinsky, Phys. Lett. **117B**, 175 (1982).
[7] A. Vilenkin and L. Ford, Phys. Rev. D **26**, 1231 (1982).
[8] L. P. Grishchuk, Zh. Eksp. Teor. Fiz. **67**, 825 (1974) [Sov. Phys. JETP **40**, 1660 (1975)].
[9] L. H. Ford, Phys. Rev. D **31**, 710 (1985).
[10] N. C. Tsamis and R. P. Woodard, Phys. Lett. B **301**, 351 (1993).
[11] N. C. Tsamis and R. P. Woodard, Class. Quantum Grav. **11**, 2969 (1994).
[12] N. C. Tsamis and R. P. Woodard, Commun. Math. Phys. **162**, 217 (1994).
[13] N. C. Tsamis and R. P. Woodard, Ann. Phys. (N.Y.) **238**, 1 (1995).
[14] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2738 (1977).
[15] W. Rindler, Am. J. Phys. **34**, 1174 (1966).
[16] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982).
[17] E. Mottola, Phys. Rev. D **31**, 754 (1986); **33**, 1616 (1986); **33**, 2136 (1986).
[18] P. O. Mazur and E. Mottola, Nucl. Phys. **B278**, 694 (1986).
[19] I. Antoniadis and E. Mottola, Phys. Rev. D **45**, 2013 (1992).
[20] I. Antoniadis, J. Iliopoulos, and T. N. Tomaras, Nucl. Phys. **B261**, 157 (1985); Phys. Rev. Lett. **56**, 1319 (1986).
[21] B. Allen and M. Turyn, Nucl. Phys. **B292**, 813 (1987); M. Turyn, J. Math. Phys. **31**, 669 (1990).
[22] A. Higuchi, Class. Quantum Grav. **8**, 1961 (1991); **8**, 1983 (1991); **8**, 2005 (1991).
[23] A. Folacci, Phys. Rev. D **46**, 2553 (1992).
[24] B. Allen, Phys. Rev. D **32**, 3136 (1985).
[25] M. Turyn, Phys. Rev. D **39**, 2951 (1989).
[26] E. G. Floratos, J. Iliopoulos, and T. N. Tomaras, Phys. Lett. B **197**, 373 (1987).
[27] I. Antoniadis and E. Mottola, J. Math. Phys. **32**, 1037 (1991).
[28] A. D. Dolgov, M. B. Einhorn, and V. I. Zakharov, Acta Phys. Pol. (to be published).
[29] T. R. Taylor and G. Veneziano, Phys. Lett. B **228**, 311 (1989).
[30] E. Fradkin and A. A. Tseytlin, Nucl. Phys. **B234**, 472 (1984).
[31] T. R. Taylor and G. Veneziano, Nucl. Phys. **B345**, 210 (1990).
[32] T. S. Bunch and L. Parker, Phys. Rev. D **20**, 2499 (1979).
[33] E. M. Lifshitz, Zh. Eksp. Teor. Fiz. **16**, 587 (1946).