Exact supersymmetric massive and massless white holes

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(Received 13 July 1995)

We study special points in the moduli space of vacua at which the supersymmetric electric solutions of the heterotic string theory become massless. We concentrate on configurations for which the supersymmetric nonrenormalization theorem is valid. These are ten-dimensional supersymmetric string waves and generalized fundamental strings with SO(8) holonomy group. From these we find the four-dimensional spherically symmetric configurations which saturate the BPS bound, in particular, near the points of the vanishing ADM mass. The nontrivial massless supersymmetric states in this class exist only in the presence of non-Abelian vector fields. We also find a new class of supersymmetric massive solutions, closely related to the massless ones. A distinctive property of all these objects, either massless or massive, is the existence of gravitational repulsion. They reflect all particles with nonvanishing mass and/or angular momentum, and therefore they can be called white holes (repulsons), in contrast with black holes which tend to absorb particles of all kinds. If such objects can exist we will have the first realization of the universal gravitational force which repels all particles with the strength proportional to their mass and therefore can be associated with antigravity.

PACS number(s): 11.25.−w, 04.65.+e, 04.70.Dy, 11.30.Pb

I. INTRODUCTION

The purpose of this paper is to exhibit some peculiar features of exact supersymmetric solutions of the heterotic string theory. We will consider the most unusual properties of these solutions which all saturate the supersymmetric positivity bound in the limit when the mass of such configurations tends to zero:

$$M^2 = Z^A \mathcal{R}_{AB}(\Phi_0) Z^B \to 0 \ . \qquad (1)$$

Here $\mathcal{R}$ is a continuous function of the asymptotic values of the scalar fields Φ_0 and Z are electric and magnetic charges. It has been pointed out recently by Hull and Townsend [1], Witten [2], and Strominger [3] that since the matrix $\mathcal{R}$ is a continuous function of Φ_0, the masses of the Bogomolny states are also continuous functions of Φ_0. In particular, for some values of Φ_0 massless states may exist which saturate the bound.

The first explicit example of such a state was given by Behrndt [4] and it was interpreted as a massless black hole. This was a significant progress. A wider class of similar solutions was obtained in [5]. The solutions were interpreted as $N_L = 0$ states of the toroidally compactified heterotic string. However, all these solutions did not include non-Abelian fields which are necessary to cancel anomalies of supersymmetry. Therefore it was not quite clear whether these solutions survive and remain massless with an account taken of α' corrections. The importance of quantum corrections to supersymmetry transformations increases in the situation when it is known that the massless states may present the points of enhanced gauge symmetry. Anomalies of supersymmetry result from the Lorentz anomaly in the effective heterotic string action and may spoil all conjectures about the exactness of the Bogomolny-Prasad-Sommerfield (BPS) bound. The anomalies can be cured when Yang-Mills fields are included according to the Green-Schwarz mechanism of cancellation of anomalies.

In this paper we will consider a general class of supersymmetric solutions including non-Abelian fields. We will find massless states which may correspond either to black holes or to waves, and which remain exact solutions of equations of motion due to the presence of non-Abelian fields even with an account taken of α' corrections. In other words, we will find massless BPS states which saturate the supersymmetric positivity bound and which are free of anomalies of supersymmetry.

We will find also a class of anomaly free massive configurations closely related to the massless black holes. A rather unusual property of all these configurations (either massive or massless) is that instead of the usual black hole horizon, which *absorbs* all particles falling into the black hole, they have a repulsive (i.e., antigravitating) naked singularity which *reflects* all test particles. Since the totally reflecting surface is not black but white, it is more proper to call these new singular configurations supersymmetric white holes, or repulsons.[1] The solutions

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[1]The name "white holes" referring to the complete reflection (as opposite to the complete absorbtion by black holes) seems to be most adequate. Unfortunately, many years ago this name was used for hypothetical objects associated with the time reversal of the gravitational collapse. Physical relevance of such objects is rather doubtful [6]. Therefore we believe that the name "white holes" is essentially vacant and can be used for the repulsive singular configurations discussed in our paper. However, in order to completely avoid collisions with old terminology, we will often call our massive and massless white hole configurations "repulsons."

can be obtained either directly in four-dimensional space or by dimensional reduction of ten-dimensional gravitational waves. The repulsive singularity discussed above is not present in the ten-dimensional noncompactified version of the solution. It appears after the compactification in those places of the four-dimensional space where the volume of the six-dimensional compactified space shrinks to zero.

Our basic strategy in looking for massless states is the following. If a configuration has one-half of unbroken $N = 4$ supersymmetry and describes a solution of $N = 4$ supergravity interacting with some Abelian and non-Abelian vector multiplets, the vanishing ADM mass simultaneously means the vanishing dilaton charge[2]:

$$M \to 0 \qquad \Longleftrightarrow \qquad \Sigma \to 0 \; . \tag{2}$$

If we know any ten-dimensional solution with one-half of unbroken $N = 1$ supersymmetry, we may use the fact that the corresponding four-dimensional dilaton $e^{2\phi}$ is related to the ten-dimensional dilaton $e^{2\hat{\phi}}$ as follows:

$$e^{-2\phi} = e^{-2\hat{\phi}}\sqrt{\det G} \; , \tag{3}$$

where the matrix G describes the geometry of the internal six-dimensional space. Knowledge of the ten-dimensional solutions means that both $e^{-2\hat{\phi}}$ and $\det G$ are known. The massless four-dimensional configurations saturating the BPS bound are the ones in which the ten-dimensional dilaton charge $\hat{\Sigma}$ is compensated by the modulus field charge σ, where we define

$$\begin{aligned} e^{-2\phi} &= e^{-2\phi_0} + \frac{\Sigma}{r} + \cdots, \\ e^{-2\hat{\phi}} &= e^{-2\hat{\phi}_0} + \frac{\hat{\Sigma}}{r} + \cdots, \\ \sqrt{\det G} &= (\sqrt{\det G})_0 + \frac{\sigma}{r} + \cdots \; . \end{aligned} \tag{4}$$

The BPS state is massless when

$$(\sqrt{\det G})_0 \, \hat{\Sigma} + e^{-2\hat{\phi}_0} \, \sigma = 0 \; . \tag{5}$$

It is clear that for the flat six-dimensional solution with $\sqrt{\det G} = \sqrt{\det G_0}$ the massless supersymmetric state of pure $N = 4$ supergravity is a trivial flat space. The nontrivial solutions exist only when there are matter multiplets. We will find that for our class of solutions not only $N = 4$ nongravitational gauge multiplets must be present but also some of them have to be non-Abelian to keep supersymmetry preserved with an account taken of quantum corrections.

The class of massive supersymmetric solutions which we are going to study will be closely related to the massless configurations. They will also have the property that the mass of the configuration is proportional to the charge of the four-dimensional dilaton. Therefore the new mass formulas for the white holes which will be obtained in this paper will simultaneously give the dilaton charge formulas.

II. SUPERSYMMETRIC STRING WAVES AND GENERALIZED FUNDAMENTAL STRINGS

We have found various examples of configurations with vanishing four-dimensional dilaton charge by using some known solutions of the equations of motions of effective action of the heterotic string. Some of them belong to the class of exact supersymmetric heterotic string backgrounds and require the non-Abelian gauge fields to be part of the solution, some others do not seem to preserve the unbroken supersymmetry when α' corrections are taken into account. The first class of exact solutions with SO(8) special holonomy is given by supersymmetric string waves (SSW's) [7] and their T-dual partners, generalized fundamental strings (GFS's) [8]. Both the *pp* waves and fundamental strings admit a null Killing vector and belong to the class of supersymmetric gravitational waves. The Killing spinor for these solutions satisfies a null constraint, and the dimensionally reduced form of these solutions always gives electrically charged configurations.

We will consider here the u, v-independent part of these solutions, which is described in terms of *nine harmonic functions*, satisfying the flat space eight-dimensional equation $\sum_{i=1}^{8} \partial_i \partial_i \, h(x^i) = 0$. These configurations solve the cohomology constraint $\mathrm{tr} R^2 - \mathrm{tr} F^2 = 0$.

The second class of supersymmetric solutions, associated with the chiral null model [9], is described by *ten harmonic functions*. When one of the harmonic functions is taken to be a constant, the chiral null model is reduced either to SSW solutions or to GFS solutions. Therefore only for these solutions the embedding of the spin connection into the gauge group is possible. This leads to the preservation of unbroken space-time supersymmetry with an account taken of α' corrections, as well as to the left-right world-sheet supersymmetry. However, when all ten functions are present in the solutions the status of unbroken supersymmetry in presence of α' corrections is not clear. The holonomy of torsionful spin connections of this theory, related to the properties of α' corrections, is a subgroup of the noncompact SO(1.9) Lorentz group. This was established in [10] for uplifted electrically charged $a = 1$ dilaton black holes (which form a particular case of the chiral null model) and for the complete chiral null model in [9]. The spin embedding into the gauge group is the only known way to preserve unbroken supersymmetry. It does not work for generic chiral null model since the gauge group of the heterotic string is compact.

All solutions described above admit the null Killing vector and therefore may be called gravitational waves. Dimensional reduction of these solutions was performed in [11].

Here we would like to describe first the solutions which solve the cohomology constraint $dH = 0$ and which remain supersymmetric even with an account taken of α' corrections. After this is done we will look for the massive four-dimensional supersymmetric black holes and study

[2]For solutions without the fundamental axion charge this can be shown using the supersymmetry rules.

how they approach the massless states. For this purpose we start with anomaly free ten-dimensional solutions of $N = 1$ supergravity coupled to supersymmetric Yang-Mills theory. The exact SSW as well as exact GFS solutions admit a null Killing vector l^μ with $l^2 = 0$. This Killing vector generates an isometry in the v direction where we use light-cone coordinates $x^\mu = (u, v, x^i)$, $i = 1, \ldots, 8$. The solution consists of the ten-dimensional metric, dilaton, two-form, and the non-Abelian gauge fields. The metric and the two-form field are both described in terms of the dilaton $e^{-2\hat{\phi}(x^i)}$ and one vector function $A_\mu(x^i)$ of the transverse coordinates x^i:

$$A_\mu(x^i) = \left\{A_u(x^i) \equiv -\frac{K(x^i)}{2},\ A_v = 0,\ A_i(x^i)\right\}. \qquad (6)$$

For SSW the dilaton has to be taken constant, $e^{-2\hat{\phi}(x^i)} = e^{-2\hat{\phi}_o}$, for GFS the function $K(x^i)$ has to be a constant, $K(x^i) = K_0$. Under these conditions both solutions can be described as [3]

$$ds^2 = 2e^{2\hat{\phi}} du(dv + A_\mu dx^\mu) - \sum_1^8 dx^i dx^i, \quad A_v = 0\ , \qquad (7)$$

$$B = 2e^{2\hat{\phi}} du \wedge (dv + A_\mu dx^\mu)\ . \qquad (8)$$

The non-Abelian gauge field $V_\mu{}^{IJ}$ is obtained by embedding of the torsionful spin connection into the gauge group of the heterotic string:

$$V_\mu{}^{IJ} = l_\mu V^{IJ} \equiv \Omega_{\mu-}{}^{ab} = e^{2\hat{\phi}} l_\mu \mathcal{A}^{ab},$$

$$a, b, I, J = 1, \ldots, 8. \qquad (9)$$

The Yang-Mills indices are in the adjoint representation of SO(8). The equations that the dilaton $e^{-2\hat{\phi}(x^i)}$, $K(x^i)$, and $A_i(x^i)$ have to satisfy for the configuration to be supersymmetric and solve equations of motion are

$$\triangle e^{-2\hat{\phi}} = \triangle K = 0, \quad \triangle \partial^{[i} A^{j]} = 0, \quad \partial^{[i} A^{j]} \equiv \mathcal{A}^{ij}, \qquad (10)$$

where the Laplacian is taken over the transverse directions only ($\triangle \equiv \sum_1^8 \partial_i \partial_i$). This solution has SO(8) symmetry. Obviously, SSW with constant dilaton and GFS with constant K are special solutions of this system of equations.[4] However, for any other solutions when both the dilaton and the function K are nonconstant, the proof of unbroken supersymmetry is not available. The holonomy of the generalized connections in this case includes the noncompact subgroup of the Lorentz group, since $\Omega_{\mu-}{}^{0i}$ is not vanishing.[5] This spin connection cannot be embedded into SO(32) or $E_8 \times E_8$ gauge group of the heterotic string. However, without such spin embedding the preservation of supersymmetry at the quantum level is questionable. Therefore in what follows we will consider only SSW and GFS with non-Abelian fields included as exact supersymmetric solutions in our class.

We will limit ourselves to the solutions which in four dimensions correspond only to static configurations and not to the stationary ones. For this purpose we take $A_1 = A_2 = A_3 = 0$. Our next specification will be to consider the solutions which depend only on x^1, x^2, x^3. Thus the SO(8) symmetry is broken. The only nonvanishing components of the non-Abelian vector field $V_\mu{}^{IJ}$ in this case are $V_\mu{}^{im}$ with $i = 1, 2, 3$ and $m = 4, 5, 6, 7, 8$.

III. ZERO MASS CONFIGURATIONS WITH ASYMPTOTICALLY FLAT INTERNAL GEOMETRY

The effective action describing the dynamics of the massless fields of toroidally compactified heterotic string is described by the bosonic part of the action of $N = 4$ supergravity interacting with 22 Abelian $N = 4$ vector multiplets. Ten-dimensional supergravity, dimensionally reduced to four dimensions provides 6 of them. The additional 16 are coming from the ten-dimensional vector multiplets, or from the gauge sector of the heterotic string. The action in the form given by Maharana-Schwarz [12] and Sen [13] is

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-\det g}\, \Big[R - 2g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{8} g^{\mu\nu} \mathrm{Tr}(\partial_\mu \mathcal{M} L \partial_\nu \mathcal{M} L) - \frac{1}{12} e^{-2\phi} (H_{\mu\nu\rho})^2 - e^{-2\phi} g^{\mu\mu'} g^{\nu\nu'} F^a_{\mu\nu}\, (L\mathcal{M}L)_{ab}\, F^b_{\mu'\nu'} \Big]\ , \qquad (11)$$

where ϕ is the four-dimensional dilaton, and $\mathcal{M}$ is the 28×28 matrix valued scalar field, describing the moduli. Vector fields include graviphoton as well as vectors from the gauge multiplets. The 28×28 symmetric matrix L with 22 eigenvalues -1 and 6 eigenvalues $+1$ defines the metric in the O(6, 22) space. We use the units where $G_N = 1$, which should be taken into account when comparing with $G_N = 2$ units often used for identification of string states.

Dimensional reduction of the chiral null model (with-

[3]In the chiral null model [9] $e^{2\hat{\phi}} = F$.

[4]This solution with constant $K = K_0$ is equivalent to the one presented in [8] after the shift in the isometry direction $v' = v - K_0 u/2$.

[5]There is an apparent discrepancy between the statement about the holonomy of generalized fundamental string solutions in [8] and [9]. Using T-duality rotation from the waves, we have found that the holonomy of the SSW as well as of GFS is SO(8). Meanwhile in [9] only the holonomy of SSW is qualified as that of SO(8) and the case of GFS is considered as not special. The analysis shows that there is no discrepancy, however. Spin connections are frame dependent: in the frame used in [9] one can still find that the curvatures are such that the spin embedding into the SO(8) gauge group solves the problem with α' corrections for GFS configurations.

out Yang-Mills fields) was performed in [11]. The supersymmetric four-dimensional solutions of the field equations following from the action (11) have the metric (in the canonical frame)

$$ds^2_{\rm can} = e^{2\phi}dt^2 - e^{-2\phi}d\vec{x}^2 \ , \tag{12}$$

and the four-dimensional dilaton is

$$e^{-2\phi} = \left(e^{-2\hat{\phi}} K - \sum_{n=4}^{8}(A_n)^2\right)^{\frac{1}{2}} . \tag{13}$$

Other fields can be also deduced from the ten-dimensional solution by dimensional reduction.

The massless solution was found by Behrndt [4] in the framework of toroidally compactified heterotic string theory, and it was further generalized in [5]. It has vanishing dilaton charge and is obtained when the functions defining the solutions are taken in the form [11]

$$e^{-2\hat{\phi}} = 1 + \frac{2\tilde{m}}{r} \ , \quad K = 1 + \sum_{1}^{s} \frac{2\hat{m}}{r} \ , \quad A_n = \frac{2q_n}{r} \ , \quad n = 4, \ldots, 8, \quad r^2 \equiv \vec{x}^2. \tag{14}$$

This choice corresponds to the asymptotically flat internal space. The four-dimensional dilaton is given by

$$e^{-2\phi} = \left(1 + \frac{2(\tilde{m} + \hat{m})}{r} - \frac{4(q^2 - \tilde{m}\hat{m})}{r^2}\right)^{\frac{1}{2}}, \tag{15}$$

where $q^2 \equiv \sum_{n=4}^{8}(q_n)^2$.

In terms of the right- and left-handed charges the dilaton is given by

$$e^{-2\phi} = \left(1 + \frac{2\sqrt{2}|Q_R|}{r} - \frac{2(|Q_L|^2 - |Q_R|^2)}{r^2}\right)^{\frac{1}{2}} . \tag{16}$$

Right-handed charge is the charge corresponding to the gravi-photon, and left-handed charge is the charge corresponding to the vector fields of the matter multiplets. There is only one possibility to make these solutions massless: to take $\sqrt{2}|Q_R| = \hat{m} + \tilde{m} = 0$. The dilaton becomes

$$e^{-2\phi} = \left(1 - \frac{4(\tilde{m}^2 + q^2)}{r^2}\right)^{\frac{1}{2}} = \left(1 - \frac{2|Q_L|^2}{r^2}\right)^{\frac{1}{2}} . \tag{17}$$

Using the fact that for toroidally compactified string supersymmetric configurations $|Q_L|^2 - |Q_R|^2 = -2(N_L - 1)$, one can see that for vanishing Q_R the state is characterized by $N_L = 0$. Here N_L is a non-negative integer, describing the total oscillator contribution to the squared mass of a state from the left moving oscillators of the string. Rescaling this solution for arbitrary value of the dilaton at infinity, $e^{2\phi_0} \equiv g^2$, we get finally the canonical four-dimensional metric in the form [5]

$$ds^2_{\rm can}({\rm el}) = \left(1 - \frac{4g^2}{r^2}\right)^{-\frac{1}{2}} dt^2 - \left(1 - \frac{4g^2}{r^2}\right)^{\frac{1}{2}} d\vec{x}^2 \ . \tag{18}$$

The singularity of this configuration at $r = 2g$ was found in [5] to be a true singularity since the scalar curvature is given by

$$R^{\rm el}_{\rm can} = \frac{4g^2(2g^2 + r^2)}{r(r^2 - 4g^2)^{5/2}} \ . \tag{19}$$

The origin of this singularity can be traced back to the fact that the volume of the compactified six-dimensional space shrinks to zero for $r = 2g$ since

$$\det G = e^{-4\phi}e^{4\hat{\phi}} = \frac{r^2 - 4g^2}{(r + 2\tilde{m})^2} \ . \tag{20}$$

In the magnetic case $g \to \frac{1}{g}$, and the volume of the compactified space is

$$\det G = e^{-4\phi}e^{4\hat{\phi}} = \frac{r^2 g^2 - 4}{g^2(r + 2\tilde{m})^2} \ . \tag{21}$$

It shrinks to zero at the point $r = \frac{2}{g}$ where the four-dimensional magnetic solution is singular:

$$ds^2_{\rm can}(m) = \left(1 - \frac{4}{g^2r^2}\right)^{-\frac{1}{2}} dt^2 - \left(1 - \frac{4}{g^2r^2}\right)^{\frac{1}{2}} d\vec{x}^2 \ . \tag{22}$$

All this analysis is valid in the framework of the toroidally compactified heterotic string with only Abelian vector fields in the solutions, and when one ignores the issue of α' corrections to supersymmetry. From now on we will consider only exactly supersymmetric SSW and GFS supplemented by the non-Abelian fields and described in Sec. II.

This means that when both $\tilde{m}$ and $\hat{m}$ are nonvanishing, supersymmetry is anomalous. If only one of these two numbers vanishes, which is acceptable from the point of view of the exactness of the solution, we do not approach the massless state. Only for $\hat{m} = \tilde{m} = 0$ we can have a massless solution without anomalies. This special configuration has the dilaton field given by

$$e^{-2\phi} = \left(1 - \frac{4q^2}{r^2}\right)^{\frac{1}{2}} . \tag{23}$$

The presence of the Yang-Mills vector fields in the ten-dimensional solution will add some non-Abelian vectors as well as scalars to the four-dimensional solutions. It is quite remarkable that for this solution to be nontrivial the presence of the non-Abelian vector field is necessary. In fact, using Eq. (9) we will find that for the configuration given in (14) the non-Abelian part of the solution in ten dimensions is given by [8]

$$V_\mu{}^{[\rm in]} = l_\mu \left(1 + \frac{2\tilde{m}}{r}\right)^{-1} \frac{x^i q^n}{|x|^3}, \quad i = 1, 2, 3, \quad n = 4, 5, 6, 7, 8. \tag{24}$$

For the massless configuration presented above with $\hat{m} = \tilde{m} = 0$ the Yang-Mills field is

$$V_\mu{}^{[\rm in]} = l_\mu \frac{x^i q^n}{|x|^3}, \quad i = 1, 2, 3, \quad n = 4, 5, 6, 7, 8. \tag{25}$$

The ten-dimensional manifold in the process of compactification is split into the four-dimensional manifold M^4 with coordinates $v = t, x^1, x^2, x^3$ and a six-dimensional manifold M^6 with coordinates $x^4, \ldots, x^8$, $u = x^9$ [11]. The moduli space of our configuration is defined by the six-dimensional metric G_{rs}, $r = 4, \ldots, 9$, and by the six-dimensional matrix B_{rs}:

$$G_{rs} = \begin{pmatrix} -\delta_{mn} & e^{2\hat\phi} A_n \\ e^{2\hat\phi} A_m & -e^{2\hat\phi} K \end{pmatrix},$$
$$B_{rs} = \begin{pmatrix} 0 & e^{2\hat\phi} A_n \\ -e^{2\hat\phi} A_m & 0 \end{pmatrix}. \qquad (26)$$

In addition, there are non-Abelian vectors and scalars which will come from the ten-dimensional Yang-Mills field (24). The two matrices G and B together form the $O(6,6)$ matrix $\mathcal{M}$ [12] which appears in the Bogomolny bound [13]. The ansatz (14) used in Eq. (26) has the following properties.

(i) The $O(6,6)$ matrix $\mathcal{M}$ built out of G and B at infinity ($r \to \infty$) is

$$\mathcal{M}_0 = \begin{pmatrix} -I & 0 \\ 0 & -I \end{pmatrix}. \qquad (27)$$

(ii) Given this asymptotic value of the matrix $\mathcal{M}$, there is only one solution for the massless state which is not changed by quantum corrections:

$$G_{rs} = \begin{pmatrix} -\delta_{mn} & \frac{2q_n}{r} \\ \frac{2q_m}{r} & -1 \end{pmatrix},$$
$$B_{rs} = \begin{pmatrix} 0 & \frac{2q_n}{r} \\ -\frac{2q_n}{r} & 0 \end{pmatrix}, \quad e^{-2\phi} = \left(1 - \frac{4q^2}{r^2}\right)^{\frac{1}{2}}. \qquad (28)$$

The metric of this configuration coincides with the metric (18). Besides, there are Abelian vector fields and non-Abelian vectors and scalars. The most unusual property of this solution is that the massless state is described by a static configuration. We will return to this issue later after we find more general exact massless and massive solutions.

IV. SPECIAL POINTS IN THE MODULI SPACE

One can find a solution describing a more general family of BPS states with a vanishing ADM mass. For this purpose we may use the fact that in gravitational wave solutions in $d = 10$ one can use more general harmonic functions. For the one-black-hole case one can take

$$e^{-2\hat\phi} = e^{-2\hat\phi_0} + \frac{2\hat m}{r}, \quad K = K_0 + \frac{2\tilde m}{r},$$
$$A_n = (A_n)_0 + \frac{2q_n}{r}, \quad n = 4, \ldots, 9. \qquad (29)$$

However, for the solution to be exact we have one constraint:

$$\hat m \tilde m = 0 . \qquad (30)$$

Indeed, this means that either $\tilde m = 0$ and the dilaton is constant or $\hat m = 0$ and K is constant, which are the conditions for exactness. The four-dimensional dilaton is now given by

$$e^{-2\phi} = \left(\left(e^{-2\hat\phi_0} + \frac{2\hat m}{r}\right)\left(K_0 + \frac{2\tilde m}{r}\right) - \left[(A_n)_0 + \frac{2q_n}{r}\right]^2 \right)^{\frac{1}{2}} . \qquad (31)$$

This expression can be reorganized as follows:

$$e^{-2\phi} = e^{-2\phi_0}\left(1 + \frac{4M}{r} - \frac{4g^2q^2}{r^2}\right)^{\frac{1}{2}}, \qquad (32)$$

where

$$e^{-2\phi_0} = \left[e^{-2\hat\phi_0} K_0 - (A_n)_0^2\right]^{1/2} \equiv \frac{1}{g^2}. \qquad (33)$$

The mass formula for the exact SSW case ($\tilde m = 0$) is

$$M = \frac{g^2}{2}\left[e^{-2\hat\phi_0}\,\hat m - 2(A_n)_0\, q_n\right] \geq 0 , \qquad (34)$$

whereas the mass formula for the case of the exact GFS ($\hat m = 0$) is

$$M = \frac{g^2}{2}\left[K_0\,\tilde m - 2(A_n)_0\, q_n\right] \geq 0 . \qquad (35)$$

The mass has to be non-negative due to supersymmetric positivity bound, but the vanishing value of the mass M is not forbidden by supersymmetry.

Thus the metric of the exact non-Abelian electrically charged black hole is given by

$$ds^2_{\text{can}} = \left(1 + \frac{4M}{r} - \frac{4g^2q^2}{r^2}\right)^{-\frac{1}{2}} \times dt^2 - \left(1 + \frac{4M}{r} - \frac{4g^2q^2}{r^2}\right)^{\frac{1}{2}} d\vec x^2 . \qquad (36)$$

The moduli space is presented by the matrix $\mathcal{M}$ which asymptotically (in the limit $r \to \infty$) is described in terms of asymptotic values of the matrices G and B:

$$(G_{rs})_0 = \begin{pmatrix} -\delta_{mn} & (A_n)_0 \\ (A_n)_0 & -e^{2\hat\phi_0} K_0 \end{pmatrix},$$
$$(B_{rs})_0 = \begin{pmatrix} 0 & (A_n)_0 \\ -(A_n)_0 & 0 \end{pmatrix}. \qquad (37)$$

Thus the asymptotic value of the matrix $\mathcal{M}$ is very different from the simple diagonal form in Eq. (27). A complete expression for these matrices is

$$G_{rs} = \begin{pmatrix} -\delta_{mn} & (A_n)_0 + \frac{2q_n}{r} \\ (A_n)_0 + \frac{2q_m}{r} & -e^{2\hat\phi} K \end{pmatrix},$$
$$B_{rs} = \begin{pmatrix} 0 & (A_n)_0 + \frac{2q_n}{r} \\ -(A_n)_0 + \frac{2q_n}{r} & 0 \end{pmatrix}, \qquad (38)$$

where for SSW and for GFS we have, respectively,

$$e^{2\hat{\phi}}K = e^{2\hat{\phi}_0}\left(K_0 + \frac{2\hat{m}}{r}\right) , \quad \tilde{m} = 0,$$

$$e^{-2\hat{\phi}}K = \left(e^{-2\hat{\phi}_0} + \frac{2\tilde{m}}{r}\right)^{-1} K_0 , \quad \hat{m} = 0. \tag{39}$$

The non-Abelian fields for both configurations in the four-dimensional form can be deduced from the ten-dimensional form (24).

The moduli space is rather involved and allows to approach the critical points of the massless configuration continuously when the right-hand side in Eqs. (34) and (35) tends to zero.

In all cases considered the four-dimensional configuration has a new singularity which was not present in the ten-dimensional case. This singularity is present in the compactified solution when the volume of the compactified space shrinks to zero. For the solutions described above we have

$$\det G = e^{-4\phi}e^{4\hat{\phi}} = g^4\left(1 + \frac{4M}{r} - \frac{4g^2q^2}{r^2}\right)\left(e^{-2\hat{\phi}_0} + \frac{2\tilde{m}}{r}\right)^{-2} . \tag{40}$$

At the singularity point

$$r_0 = 2(\sqrt{M^2 + g^2q^2} - M) \tag{41}$$

the determinant of the metric of the compactified six-dimensional space vanishes:

$$\det G(r_0) = 0 . \tag{42}$$

In stringy frame the geometry of the electric configuration (36) is given by

$$ds^2_{\rm str} = \left(1 + \frac{4M}{r} - \frac{4g^2q^2}{r^2}\right)^{-1} dt^2 - d\vec{x}^2 . \tag{43}$$

Note that metric in stringy frame is well defined even in the region $r < r_0$, where $1 + \frac{4M}{r} - \frac{4g^2q^2}{r^2} < 0$. It may seem meaningless to continue metric to the region $r < r_0$, since the singularity at $r = r_0$ is a real curvature singularity. However, it may be important to have such a continuation in order to investigate the possibility of tunneling through the singularity, see Sec. V. One may suggest the continuation of the canonical metric (36),

$$g^{\rm can}_{\mu\nu} = g^{\rm str}_{\mu\nu}\sqrt{\left|1 + \frac{4M}{r} - \frac{4g^2q^2}{r^2}\right|} , \tag{44}$$

which gives the following generalization of the canonical metric (36):

$$ds^2_{\rm can} = \left(1 + \frac{4M}{r} - \frac{4g^2q^2}{r^2}\right)^{-1} \times\left(\left|1 + \frac{4M}{r} - \frac{4g^2q^2}{r^2}\right|\right)^{\frac{1}{2}} \times dt^2 - \left(\left|1 + \frac{4M}{r} - \frac{4g^2q^2}{r^2}\right|\right)^{\frac{1}{2}} d\vec{x}^2. \tag{45}$$

This continuation preserves an important property of metric (43): the determinant of metric changes its sign at $r = r_0$.

V. EXACT SUPERSYMMETRIC BLACK HOLES ARE WHITE

It is very tempting to associate singular spherically symmetric configurations (36), (45) with black holes. However, it would not be quite correct. Black holes got their name for the reason that they strongly attract all particles, so that even light cannot escape from a black hole. Gravitational attraction can be described by the Newtonian potential $\Phi = \frac{1}{2}(g_{00} - 1)$. This yields the usual Newtonian attractive potential $\Phi = -\frac{M}{r}$ at a large distance from a massive Schwarzschild black hole. Meanwhile, the potential corresponding to the metric $g_{00} = g_{rr}^{-1} = \left(1 + \frac{4M}{r} - \frac{4q^2}{r^2}\right)^{-1/2}$ at large r is given by $\Phi = -\frac{M}{r} + \frac{q^2}{r^2}$, and the strength of the gravitational field is proportional to $\Phi' = \frac{M}{r^2} - \frac{2q^2}{r^3}$. (For notational simplicity we take here the coupling constant $g^2 = 1$.) Thus, in the limit $r \to \infty$ we still have gravitational attraction, but only for the configurations with a positive Arnowitt-Deser-Misner (ADM) mass M. However, there is a stable equilibrium for test particles at $r_c = 2q^2/M$, and there is a gravitational *repulsion* (antigravity) for $r < r_c$. (For massless states the gravitational force is repulsive at all r.) This repulsion becomes infinitely strong near the singularity, which appears at $r_0 = 2(\sqrt{M^2 + q^2} - M)$. Infalling particles cannot touch the singularity at $r = r_0$ and become totally reflected.

Indeed, one can write an equation of motion for a test particle of a small mass m in an external spherically symmetric background (36), see [14]:

$$t = E\int dr \frac{g_{rr}}{\sqrt{g_{00}}}\left(g_{rr}\, E^2 - g_{00}\,\frac{L^2}{r^2} - g_{00}g_{rr}\, m^2\right)^{-1/2} . \tag{46}$$

Here E is the test particle energy at $r \to \infty$, and L is its angular momentum with respect to the center of our configuration. For $g_{00} = g_{rr}^{-1} = \left(1 + \frac{4M}{r} - \frac{4q^2}{r^2}\right)^{-1/2}$ Eq. (46) reads

$$t = E\int dr\left(1 + \frac{4M}{r} - \frac{4q^2}{r^2}\right)\left(E^2\left(1 + \frac{4M}{r} - \frac{4q^2}{r^2}\right) - \frac{L^2}{r^2} - m^2\sqrt{1 + \frac{4M}{r} - \frac{4q^2}{r^2}}\right)^{-1/2} . \tag{47}$$

It is clear from this equation that test particles with any initial energy E cannot reach the singularity at $r = r_0$. One can easily show that each test particle within a finite time reaches some minimal radius $r_{\rm min} > r_0$, and becomes reflected. For example, in the case $M = 0$, the singularity is at $r_0 = 2|q|$, and all massless test particles with $L \neq 0$ become reflected at $r_{\rm min} = 2|q|\sqrt{1 + \frac{L^2}{4q^2E^2}} > r_0$. (For comparison, all massless particles with $L < 2ME$

are swallowed by the usual Schwarzschild black hole.) In the case $M = 0$, $L = 0$ massive test particles are reflected at $r_{\min} = 2|q|\left(1 - \frac{m^4}{E^4}\right)^{-1/2} > r_0$. The only possible exception is the behavior of massless test particles in the S state ($m = L = 0$). In this case one should use quantum mechanical treatment similar to the one developed in [15, 16]. An investigation of this question indicates that even in this special case particles are totally reflected. In this sense our solutions describe white holes rather than the black ones.

To verify the last statement and to get an additional insight into the nature of the repulsive singularity at $r = r_0$ we will study the wave equation for a massive scalar field, taking the metric of an electric white hole (repulson) in the form which allows continuation to $r < r_0$, see Eq. (45). For the S wave, the scalar field equation $\partial_\mu(\sqrt{|g|}g^{\mu\nu}\partial_\nu\phi) = -m^2\phi$ in this metric reads

$$\left(1 + \frac{4M}{r} - \frac{4q^2}{r^2}\right)\ddot{\phi} - \phi'' - \frac{2\phi'}{r} = -m^2\phi \ . \tag{48}$$

The solution of this equation in the WKB approximation for $m \neq 0$ reproduces our previous results about total reflection, being strongly suppressed at $r < r_{\min}$. However, exact solutions of this equation both for $m \neq 0$ and for $m = 0$ do not vanish and do not exhibit any kind of singular behavior at the point $r = r_0$. To give a particular example, one may consider this equation for $M = 0$, $m = 0$. In this case Eq. (48) has a stationary solution $\phi = e^{-iEt}\chi$ in terms of Bessel functions:

$$\chi = r^{-1/2}\, J_\nu(Er), \quad \nu_\pm = \pm\frac{1}{2}\sqrt{1 + 4q^2E^2} \ . \tag{49}$$

These solutions behave in a regular way at $r = r_0$. This suggests that the singularity at $r = r_0$ at the quantum level is transparent for massless particles in the S wave. Only one of these two functions, the Bessel function with $\nu_+ = +\frac{1}{2}\sqrt{1 + 4q^2E^2}$, is normalizable. It decreases near the singularity at $r = 0$ as $r^{\nu_+ - 1/2}$. In such a situation the probability flux near the singularity $r = 0$ vanishes, which shows that even the massless particles in the S wave ($m = 0$, $L = 0$) are totally reflected, though not by the singularity at $r = r_0$ but by the singularity at $r = 0$.

Here one should make a cautionary note. In general, test particles may influence the background. For $M \neq 0$ this does not lead to any problems: one may consider test particles with energy $E \ll M$, in which case their influence on the white hole background can be neglected. Therefore massive states described in our paper do exhibit the antigravity regime and can be called white holes, or repulsons.

On the other hand, in the limit $M = 0$ our semiclassical considerations may become somewhat misleading. Indeed, gravitational repulsion changes momentum of a test particle. This may happen only if the white hole itself acquires the same momentum with an opposite sign, which would imply that the massless white hole should start moving with the speed of light. This may suggest that the state corresponding to a massless white hole at rest is unstable with respect to infinitesimally small external fluctuations, and therefore generically such states should be described as particles (waves) moving with the speed of light [1, 17].

However, it might be impossible to give any boost to a massless white hole without either forming a bound state with it or making it massive. Indeed, these states can be considered massless only at an infinitely large distance from them, but in this case they do not interact at all. Repulsive force $-\frac{2q^2}{r^3}$, which appears at a finite distance from the center of a massless white hole, may be interpreted as a gravitational interaction with its massive core. Thus, gravitational interaction occurs only with an internal part of the massless white hole, which leads to its deformation. Such a deformation may change energy and the effective mass of the white hole, and then it will be able to carry finite momentum without being accelerated to the speed of light.

Another problem appears when one tries to understand the nature of the repulsive gravitational field. The formal reason of the repulsion is the existence of the nondiagonal terms in the metric of six-dimensional compactified space, see (28). However, it would be nice to have a simple intuitive four-dimensional picture describing the repulsive force from the phenomenological point of view. One way of thinking about it is that the singularity acts on test particles as a body with a negative mass. This mass becomes "screened" by positive energy density of physical fields. Therefore the absolute value of the effective gravitating mass of a sphere of a radius r decreases at large r as $-\frac{M^2}{r}$, and finally the total mass vanishes in the limit $r \to \infty$.

This intuitive picture is, in fact, rather counterintuitive. The states with negative energy do exist in general relativity. For example, the total energy of a closed universe is equal to zero as a result of exact cancellation of positive energy of matter and negative energy of gravitational field. Still it is hard to imagine how massive or massless white holes with a repulsive core could be created in the process of gravitational collapse of normal matter with positive energy density. This could make such solutions very suspicious. One should note, however, that the same is true for the usual charged stringy black holes as well. Typically such black holes cannot be formed in the process of gravitational collapse of charged elementary particles. Indeed, in most cases there are no such charged particles in the underlying Lagrangian. The description of charged stringy black holes is somewhat unconventional as compared with the ordinary Reissner-Nordstrom black holes containing charged elementary particles. One may consider a sourceless flux of electric or magnetic field, and then imagine a situation where the gravitational force squeezes the flux into a singularity. Then the singularity will look like an electrically or magnetically charged particle. To describe such a situation at a more formal level, one should find the flux of electric, magnetic, and gravitational fields at infinity, and then find an extremum of action with these boundary conditions, but without imposing any boundary conditions and solving Lagrange equations at the singularity. In particular, there is no requirement that the effective charge of the singularity is carried by an elementary particle, or that the singularity looks like a normal particle with a positive mass. Such requirements appear only if one imposes an additional condition that the black hole is formed as a result of gravitational collapse of elementary particles. For the reason discussed above, this condition does not necessarily apply to charged stringy black holes. The best constraint which one can obtain on the black hole mass is the supersymmetric Bogomolny positivity bound. This constraint applies not to the "effective mass" of the singularity, but to the total ADM mass, and it is satisfied by the massless and massive white holes considered in this paper.

VI. DISCUSSION

In the previous papers [4] and [5] the massless $N_L = 0$ states of the toroidally compactified heterotic string have been found. Those states saturate the supersymmetric positivity bound. In this paper we have found exact supersymmetric electrically charged four-dimensional configurations whose ADM mass can vanish without the solutions being trivial. One of the features of these solutions is the necessary presence of non-Abelian vectors and scalars besides the metric, Abelian vectors and scalars. These solutions have been obtained as classical solutions of the effective ten-dimensional action of the heterotic string theory.

The configurations which we have discussed here cannot be associated with the toroidally compactified string with O(6,22) duality symmetry. The presence of Yang-Mills fields required by preservation of supersymmetry at the quantum level means that we have only O(6,6) symmetry with 6 Abelian gravi-photons and 6 Abelian vector multiplets. But instead of the 16 additional Abelian vector multiplets, which are extending the symmetry of the toroidally compactified string from 6 to 22, we have non-Abelian vector multiplets.

Therefore the interpretation of these new configurations as the states of the properly quantized string still has to be investigated. The quantization conditions used for toroidally compactified string should be generalized for the presence of non-Abelian vector multiplets.

The metric of the exact supersymmetric configurations which we have studied has the general form

$$ds^2_{\text{can}} = \left(1 + \frac{2\sqrt{2}|Q_R|}{r} - \frac{2(|Q_L|^2 - |Q_R|^2)}{r^2}\right)^{-\frac{1}{2}} dt^2 - \left(1 + \frac{2\sqrt{2}|Q_R|}{r} - \frac{2(|Q_L|^2 - |Q_R|^2)}{r^2}\right)^{\frac{1}{2}} dx^2 \,. \tag{50}$$

Here $2(|Q_L|^2 - |Q_R|^2) = q_n^2$, and the Yang-Mills field is a necessary part of the solutions when $q_n \neq 0$. The massive $a = \sqrt{3}$ electrically charged black hole of Gibbons and Perry [18] is included into this class. In fact it is the only case with $|Q_L|^2 - |Q_R|^2 = 2q_n^2 = q_n = 0$, $V^{YM} = 0$ for which the configuration does not need the presence of non-Abelian vector fields to be exact: quantum corrections vanish due to null properties of the curvature of the pp waves with $K = 1 + \frac{M}{r}$ and $e^{-2\hat{\phi}} = 1, A_n = 0$ [7]. This is a one-parameter extreme black hole solution:

$$ds^2 = \left(1 + \frac{4M}{r}\right)^{-\frac{1}{2}} dt^2 - \left(1 + \frac{4M}{r}\right)^{\frac{1}{2}} dx^2 \,. \tag{51}$$

When the mass of this solution tends to zero, it becomes trivial, and one half of unbroken supersymmetry gets restored to the completely unbroken supersymmetry of the flat space. Apart from this massive black hole solution, every other one has

$$|Q_L|^2 - |Q_R|^2 = 2q_n^2 > 0 \,. \tag{52}$$

This means that any solution in this group can become massless ($Q_R = 0$) and still the metric and the right-handed Abelian vectors (from the gravitational supermultiplet) will have some nonvanishing $1/r^2$ terms. The left-handed Abelian vectors (from the nongravitational vector supermultiplet) as well as the Yang-Mills fields are also present since $|Q_L|^2 = 2q_n^2 > 0$.

Thus the nontrivial supersymmetric massless configurations described in this paper do not exist without the non-Abelian multiplets. Even in the limit of the vanishing mass one half of the supersymmetry is unbroken and the other half is broken and serves to form the supercharge of the ultrashort multiplet.

A very unusual property of the new set of exact supersymmetric solutions described in this paper is the presence of a repulsive singularity when the condition $|Q_L|^2 - |Q_R|^2 = 2q_n^2 > 0$ is valid. This singularity appears both for massless ($Q_R = 0$) and for massive ($Q_R \neq 0$) solutions. As a result, these solutions can be better classified as white holes rather than the black ones.

Note that the *gravitational* repulsion which we are discussing here is quite different from the repulsive component of interaction between extreme black holes, which appears due to the *nongravitational* interaction of their electric, magnetic, and dilaton charges [19]. White holes (repulsons) considered in this paper repel all particles, either charged or not, with the strength proportional to their mass. This repulsion, unlike the nongravitational repulsion considered in [19], does not violate the equivalence principle. If existence of repulsons is confirmed, we will have the first realization of the universal gravitational force which repels all particles and therefore can be associated with antigravity.

One should note, however, that the interpretation of our solutions as white holes (repulsons) is rather straightforward for massive states, but, as we already mentioned, interaction of particles with massless states requires a more detailed investigation, and the classical concept of gravitational repulsion in this case may become inapplicable. Formally white holes with vanishing ADM mass are still described by a static four-dimensional geometry. The limit to the massless state has to be considered with special care since normally one would expect that a massless state has to be described by a wave configuration which admits a null Killing vector. However, to have a link to extreme white hole solutions it is natural to consider those white holes which do not become trivial when the mass equals zero. One may try to boost this solution to get the wave-type configuration.

Alternatively, after we have found the special points in the moduli space where the four-dimensional white holes become massless, we may return to the original form of the ten-dimensional configuration, which from the beginning admitted a null Killing vector. The simplest one, whose four-dimensional metric is given in Eq. (18), is indeed a supersymmetric *pp* wave [7] described by the metric, the constant dilaton, the two-form, and the Yang-Mills field:

$$ds^2 = 2dudv - du^2 + \sum_{n=4}^{8} \frac{2q_n}{r} dy^n du - \sum_{i=1}^{3} dx^i dx^i - \sum_{n=4}^{8} dy^n dy^n, \qquad e^{2\hat{\phi}} = 1,$$
$$B = 2du \wedge \left(dv + \frac{2q_n}{r} dx^n \right), \qquad V_u{}^{in} = \frac{x^i q^n}{r^3}, \qquad i = 1,2,3, \quad n = 4, \ldots, 8. \tag{53}$$

The remarkable feature of this solution is the fact that only if the Yang-Mills field V does not vanish, i.e., $q_n \neq 0$, the geometry and the two-form are not trivial. At $q_n = 0$ the metric becomes that of the flat space, $ds^2 = dt^2 - \sum_{i=1}^{9} dx^i dx^i$, and the three-form H vanishes. The supersymmetry is not broken at all, it is that of the flat space. However, as long as $q_n \neq 0$, one half of the supersymmetries is broken, the condition which the Killing spinor satisfy is $\gamma^u \epsilon = 0$ [7].

This solution, as well as the more general ones presented in Eqs. (7) and (9) and described in Sec. IV, are exact solutions with one half of unbroken supersymmetry with an account taken of perturbative quantum corrections in α'. The generalization consists in allowing more general asymptotic values of the ten-dimensional geometry, which is equivalent to allowing the four-dimensional scalars to have more general vacuum expectation values. The new mass formulas for exact supersymmetric non-Abelian white holes are presented in Eqs. (34) and (35).

Thus we have described the exact supersymmetric non-Abelian configurations either as ten-dimensional gravitational waves or as electrically charged four-dimensional white holes, which may be also called repulsons. Our exact supersymmetric massless configurations do not exist without the Yang-Mills fields which form a part of the white hole configuration. One may expect various nonperturbative effects including confinement/condensation of electric/magnetic white holes near the special points of the moduli space where these solutions become massless. It would be most appropriate to study these effects, but it is outside of the scope of the present paper. Our main purpose here was to demonstrate the possibility of the existence of a new class of supersymmetric configurations with very unusual properties, and to prepare a framework for their subsequent investigation.

ACKNOWLEDGMENTS

We are grateful to K. Behrndt, G. Horowitz, A. Sen, L. Susskind, S. Theisen, and A. Tseytlin for extremely useful discussions. This work was supported by NSF Grant No. PHY-8612280.

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