Superspace formulation of Yang-Mills theory. II. Inclusion of gauge-invariant operators and scalars

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In a superspace formulation of Yang-Mills theory previously proposed, we show how gauge-invariant operators and scalars can be incorporated keeping intact the (broken) OSP$(3,1|2)$ symmetry of the superspace action. We show, in both cases, that the WT identities can be cast in a simple form $\partial\bar{W}/\partial\theta = 0$.

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I. INTRODUCTION

The theory of Yang-Mills fields forms the backbone of all the successful high energy physics of today, viz. the standard model. Yang-Mills fields are characterized by a non-Abelian local gauge invariance. The consequences of gauge invariance are formulated as the Ward-Takahashi (WT) identities of gauge theories. A study of these is highly important in any dealing with the gauge theories. In particular, the establishment of renormalizability requires use of these. The derivation and study of WT identities and of the renormalization program in gauge theories is facilitated greatly by the introduction of the alternate global symmetry of the effective action, viz. the Becchi-Rouet-Stora (BRS) symmetry [1–3]. Hence any program that will shed light on BRS symmetry, Ward identities, renormalization of gauge theories, etc., is of value.

BRS transformations contain an anticommuting parameter. This has naturally lead to construction of a superfield or superspace formulation of gauge theories in order to make the underlying BRS structure evident in a simple way [4,5]. An excellent review of earlier works is found in the last of reference [5].

With a view to simplify the WT identities and renormalization program in gauge theories (especially for gauge-invariant operators), an improved superspace formulation was proposed [6]. This formulation has the advantage that the superfields were completely unrestricted (and not constructed by hand as in earlier formulations [4,5]). It further had an OSp$(3,1|2)$ broken invariance, which, unlike earlier formulations, was not a very formal device [5], but one where superspace rotation could be actually carried out. Further, the source terms for composite BRS variation operators, so crucial to a simple formulation of WT identities, were generated from within. The sources for BRS variations and fields came from supermultiplets of (super)sources. The sources for BRS variations and fields came from supermultiplets of (super)sources. These led to a simple formulation of WT identities [7], in the elegant form $\partial\bar{W}/\partial\theta = 0$. These identities were further understood as arising from a broken OSp$(3,1|2)$ symmetry of the superspace action [8]. The formulation of Ref. [6] has also lead to an understanding of the interrelation between renormalization transformations in gauge theories [9]. The formulation of Ref. [6] has also been generalized to incorporate the most general BRS and/or anti-BRS symmetry in linear gauges [10].

A difficult problem in the renormalization program of gauge theories is the problem of renormalization of gauge-invariant operators. This problem is relevant in phenomenological applications of QCD through the use of operator product expansion [11]. Though solved long ago [12], the long treatment could well be simplified. We expect the present superspace formulation to do this, because it simplifies the WT identities in form, and hence hopefully their solution too. The first step towards a solution to this problem in this formulation is to show how to incorporate the gauge-invariant operators in the formulation without spoiling OSp$(3,1|2)$ symmetry (broken). The next step is to show that the WT identities retain their simple form in presence of gauge-invariant sources. We propose to do these two steps in the present work. The final step of solving the WT identity will be reported elsewhere.

In passing, we also show that the scalars can be incorporated in a straightforward manner in this formulation. This will have application in the superspace formulation of spontaneously broken gauge theories.

We now summarize the plan of the paper. In Sec. II, we introduce the superspace formulation of Ref. [6] and propose its extension to include gauge-invariant sources without spoiling broken OSp$(3,1|2)$ symmetry of the Lagrange density. In Sec. III, we show that the proposed generating functional does in fact contain all the information about the Green's functions with one insertion of a gauge-invariant operator. In Sec. IV, we exhibit how scalars in an arbitrary representation of the gauge group can be included straightforwardly. In Sec. V, we show that in both cases the WT identities retain their

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simple form $\partial\bar{W}/\partial\theta = 0$. Appendixes A and B deal with the general structure of gauge-invariant operators in Minkowski space and superspace. In Appendix C we have listed the Jacobi identities and commutation or anticommutation relations satisfied by the superspace covariant derivatives.

II. SUPERSPACE ACTION TO INCLUDE GAUGE-INVARIANT OPERATOR INSERTION

A. Superspace notations and superspace action

In this section we shall briefly introduced the notation (for more details, the reader is referred to Ref. [6]). We shall then introduce the superspace action for ordinary gauge theories and show how it is to be generalized to include an insertion of a gauge-invariant operator of arbitrary dimensions and Lorentz structure.

We shall work in a six-dimensional superspace $\bar{x}^i = (x^\mu, \lambda, \theta)$ with two anticommutating coordinates λ and θ. The superspace formulation of Ref. [6] utilizes an anticommutating antighost superfield $\zeta^\alpha(\bar{x})$ and a commutating superfield $\bar{A}_i^\alpha(\bar{x})$. The former is a scalar under rotations [8] characterized by the group OSp(3,1|2) and the latter is a covariant vector. The sources for fields and composite operators for the BRS variations also come in the form of a (commutating) scalar superfields $t^\alpha(\bar{x})$ and an (anticommutating) vector superfield $\bar{K}^i(\bar{x})$. We will not go into further details here; the reader is referred to Refs. [6] and [8]. We only state the superspace action

$$\begin{aligned}\bar{S} &= \int d^4x\left[-\frac{1}{4}g^{ik}g^{jl}\bar{F}_{ij}\bar{F}_{kl}\right] + \int d^4x\frac{\partial}{\partial\theta}\left\{\bar{K}^i(\bar{x})\bar{A}_i(\bar{x}) + \zeta^\alpha(\bar{x})\left[\partial^\mu A_\mu^\alpha(\bar{x}) + \frac{1}{2\eta_0}\zeta^\alpha_{,\theta} + t^\alpha\right]\right\}\\ &\equiv \bar{S}_0 + \bar{S}_1\\ &\equiv \int d^4x\bar{\mathcal{L}}_0 + \int d^4x\bar{\mathcal{L}}_1\ ,\end{aligned} \tag{2.1}$$

where $\bar{F}_{ij}$ is the superspace generalization of the field strength [6]. [The index i runs over 0 to 5 while the index μ runs over 0 to 3.)

The superspace generating functional for the superspace action is given by

$$\bar{W}[\bar{K}(\bar{x}), t(\bar{x})] = \int\{dA\}\{d\zeta\}\exp(i\bar{S}[\bar{A},\zeta,K,t])\ , \tag{2.2}$$

where the measure has been defined in Ref. [6].

$\bar{\mathcal{L}}_0$ is invariant under OSp(3,1|2) superspace rotations. $\bar{W}$ is related to the generating functional of ordinary Yang-Mills Green's functions by [6]

$$\int[dk^4][dk^4_{,\theta}]\bar{W}[\bar{K},t] = W[K^\mu_{,\theta}(\bar{x}), K^5_{,\theta}(\bar{x}), -t_{,\theta}(\bar{x}), K^\mu(\bar{x}), K^5(\bar{x}), t(\bar{x})]\ . \tag{2.3}$$

B. Inclusion of gauge-invariant operators

We want to generalize $\bar{\mathcal{L}}_0$ to include the source term for a gauge-invariant operator so that it still has OSp(3,1|2) invariance.

Now consider a typical gauge-invariant operator $O = O_{\mu_1\cdots\mu_n}[A]$. First O belongs to a representation of the Lorentz group O(3,1) and not to a representation of OSp(3,1|2). Secondly, it only contain the x-dependent part of the gauge fields $A_\mu(\bar{x})$, viz. $A_\mu(x)$ (and its space-time derivatives). Now in order to construct a superspace analog of O, viz. $\bar{O}$, we have to (i) alter the external index structure, (ii) change $A_\mu(x)$ to superfield $\bar{A}_i(\bar{x})$, and (iii) change space-time derivatives to superspace derivatives. This will be done as follows.

As shown in Appendix A, a gauge-invariant operator can be written entirely in terms of $D_\mu^{\alpha\beta}$ and $F^\alpha_{\nu\lambda}$'s. We replace each $D_\mu^{\alpha\beta}$ by the superspace covariant derivative operator $D_i^{\alpha\beta} = -\delta^{\alpha\beta}\vec{\partial}_i + gf^{\alpha\beta\gamma}\bar{A}_i^\gamma(\bar{x})$ and each $F^\alpha_{\nu\lambda}$ by $\bar{F}^\alpha_{jk}(\bar{x})$ defined in [6]. The contracted indices continue to be contracted but now range over 0 to 5 instead of 0 to 3. One thus obtains an operator $\bar{O}_{i_1\cdots i_n}[\bar{A}(\bar{x})]$ which belongs to a representation of OSp(3,1|2). We now introduce a superspace source $\bar{N}^{i_1\cdots i_n}(\bar{x})$, assumed also to belong to an appropriate representation of OSp(3,1|2) such that $\bar{N}^{i_1\cdots i_n}(\bar{x})O_{i_1\cdots i_n}(\bar{A})$ is an OSp(3,1|2) scalar. We then modify $\bar{\mathcal{L}}_0$ to a

$$\bar{\mathcal{L}}_0' = \bar{\mathcal{L}}_0 + \bar{N}^{i_1\cdots i_n}(\bar{x})\bar{O}_{i_1\cdots i_n}[\bar{A}] \tag{2.4}$$

and define the generating functional $\bar{W}'$ accordingly so that

$$\bar{W}'[\bar{K},t,\bar{N}] = \int\{dA\}\{d\zeta\}\exp\left(i\int d^4x[\bar{\mathcal{L}}_0' + \mathcal{L}_1]\right)\ . \tag{2.5}$$

Then $\frac{\delta\bar{W}'}{\delta\bar{N}^{i_1\cdots i_n}}|_{\bar{N}=0}$ generates the Green's functions of the superspace theory with one insertion of $\bar{O}_{i_1\cdots i_n}$. Among these we are interested in those with one insertion of $\bar{O}_{\mu_1\cdots\mu_n}$. We shall show how we can recover the Green's functions of the Yang-Mills theory with one insertion of $O_{\mu_1\cdots\mu_n}$ in a later section [see Eq. (3.9)].

III. EVALUATION OF $\bar{W}'[\bar{K}, t, \bar{N}]$

We shall show, in this section, how the generating functional $\bar{W}'$ can be evaluated partially, by performing integrations over $A_{i,\theta}$ and $A_{i,\lambda}$. The procedure is very much the same as that in Ref. [6]. Hence we shall be brief and indicate only the main steps successively.

(1) In

$$\bar{W}' = \int \{dA\}\{d\zeta\} \exp\left[i\bar{S} + i\int d^4x\, \bar{N}\bar{O}\right] , \qquad (3.1)$$

dependence of $\bar{S} + \int \bar{N}\bar{O}\, d^4x$ on $A_{i,\theta}$ and $A_{i,\lambda}$ arising *solely* out of dependence of $\bar{A}(\bar{x})$ on these via

$$\bar{A}_i = A_i(x) + \theta\tilde{A}_{i,\theta} + \lambda\tilde{A}_{i,\lambda} + \lambda\theta\tilde{A}_{i,\lambda\theta} \qquad (3.2)$$

is immaterial as it can be removed by the equation of motion of $\bar{A}$ (this is explained in detail in Ref. [6].) Hence only the bona fide dependence of $\bar{S} + \int \bar{N}\bar{O}\, d^4x$ on $A_{i,\theta}$ and $A_{i,\lambda}$ needs to be retained.

(2) As the original $\bar{W}$ does not depend on $A_{i,\lambda\theta}$ and as these are not dynamical variables, we shall omit (put to zero) terms in $\bar{N}\bar{O}$ depending on $A_{i,\lambda\theta}$ in evaluating $\bar{W}'$.

(3) In $\bar{W}'$ we are only interested in the terms to the first order in $\bar{N}$. Hence we consider

$$\bar{W}' = \int \{dA\}\{d\zeta\} \exp(i\bar{S}) \left[1 + i\int d^4x\, \bar{N}\bar{O}\right]$$
$$\equiv \bar{W} + i\left\langle\!\!\left\langle \int d^4x\, \bar{N}\bar{O} \right\rangle\!\!\right\rangle \qquad (3.3)$$

and evaluate it in this form. Then later we put it back in the form of exponential.

(4) In evaluating the term $\langle\langle \bar{N}\bar{O}\rangle\rangle$ in (3.3), we can use the equation of motion for $\bar{S}$ (i.e., in absence of $\bar{N}\bar{O}$) in it as this term is already of order $\bar{N}$.

(5) In performing the integrals over $A_{\mu,\lambda}$, $A_{\mu,\theta}, c_{4,\lambda}, c_{5,\theta}, c_{5,\lambda}$, we can choose instead as integration variables $F_{\mu 5}, F_{\mu 4}$, F_{44}, F_{55}, F_{45} as the Jacobian of the relevant transformations is one [see relation of Eq. (3.3) of Ref. [6] for elaboration].

(6) The equation of motion for $F_{\mu 5}, F_{\mu 4}$, F_{44}, and F_{55} in absence of the $\bar{N}\bar{O}$ term reads

$$\begin{aligned} F_{4\mu} + K_\mu - \partial_\mu\zeta &= 0 , \\ F_{5\mu} &= 0 , \\ F_{55} &= 0 , \\ F_{44} - 2K^5 &= 0 , \end{aligned} \qquad (3.4)$$

(7) In relating $\bar{W}'$ to the Yang-Mills (YM) generating functional with one insertion of O, we shall be needing

$$W' = \int [dK^4][dK^4_\theta]\bar{W}' \qquad (3.5)$$

just the same way as Eq. (2.3) involves $\int [dK^4][dK^4_\theta]\bar{W}$. Hence we shall begin with (3.5) which in effect puts to zero c_4 and $c_{4,\theta}$. Then the modified equation of motion for $c_{5,\lambda}$ or equivalently $\tilde{F}_{45} = F_{45}|_{c_4=0,c_{4,\theta}=0}$ reads .

$$F_{45} = 0 . \qquad (3.4a)$$

(8) Now, it is easy to show that the effect of doing integration over $A_{\mu,\theta}, A_{\mu,\lambda}$, $c_{4,\lambda}, c_{5,\theta}, c_{5,\lambda}$ in (3.3) is simply to use equation of motion (3.4) and (3.4a) in the $\bar{N}\bar{O}$ term. [Hence we assume dimensional regularization which puts to zero $\delta^4(0)$ and derivatives of $\delta^4(x)$ at $x = 0$.] Thus we obtain

$$\int [dK^4][dK^4_{,\theta}][dK^4_{,\theta}]\bar{W}'[\bar{K}, t, \bar{N}]$$
$$\simeq \int [dK^4][dK^4_{,\theta}] \left[\bar{W} + i\left\langle\!\!\left\langle \int d^4x\, \bar{N}\bar{O}' \right\rangle\!\!\right\rangle\right] , \qquad (3.6)$$

where $\bar{O}'$ is obtained from $\bar{O}$ by the following operations.

(a) Put c_4 and $c_{4,\theta}$ equal to zero.

(b) Remove $A_{i,\theta}$ and $A_{i,\lambda}$ dependence of $\bar{O}$ arising solely out of $\bar{A}(\bar{x})$.

(c) Put $A_{i,\lambda\theta}$ to zero.

(d) Express $\bar{O}$ in terms of $F_{\mu 5}, F_{\mu 4}, F_{44}$, F_{55}, F_{45} instead of $A_{\mu,\theta}, A_{\mu,\lambda},\ldots,c_{5,\lambda}$, and use (3.4) and (3.4a) in them. In particular note that it replaces $F_{5\mu}$, F_{55}, F_{45} to zero and $F_{4\mu}$ by $-(K_\mu - \delta_\mu\zeta)$ and F_{44} by $2K^5$.

(9) We shall show in Appendix B that $\frac{\delta\bar{O}_{\mu_1\cdots\mu_n}}{\delta F_{4\mu}}$ and $\frac{\delta\bar{O}_{\mu_1\cdots\mu_n}}{\delta F_{44}}$ contain terms of which contains $F_{5\mu}, F_{55}$, or F_{45} as a factor, the latter being zero by Eq. (3.4). Hence the statement made in 8(d) above *all* dependence of O on $A_{\mu,\theta}, A_{\mu,\lambda}, \cdots, c_{5,\lambda}$ and of course c_4 and $c_{4,\theta}$ drops out. This leaves a possible c_5 dependence alone while it is impossible as c_5 carries a ghost number one while $\bar{O}$ does not and there is no field left in $\bar{O}'$ to compensate the ghost number of c_5. Hence

$$\bar{O}'_{\mu_1\cdots\mu_n} = O_{\mu_1\cdots\mu_n} . \qquad (3.7)$$

Note that the above argument does not generally apply to $\bar{O}_{i_1\cdots i_n}$ with $i_k = 4$ or 5 for some k. This should not bother us as we are *not* interested in insertion of such spurious operators. They have been added to recover formal OSp(3, 1|2) invariance of $\bar{\mathcal{L}}'_0$ which is expected to be useful in formal manipulation of $\bar{W}'$. Hence we note that

$$W' = \int [dK^4][dK^4_{,\theta}]\bar{W}'[\bar{K}, t, \bar{N}]$$
$$= \int [dK^4][dK^4_{,\theta}] \exp\left(i\bar{S} + i\int d^4x\, \bar{N}_{\mu_i\cdots\mu_n} O_{\mu_i\cdots\mu_n}[A]\right) + O(\hat{N}_{i_1\cdots i_k\cdots i_n}) + O(N^2) , \qquad (3.8)$$

where $\hat{N}_{i_1 \cdots i_k \cdots i_n}$ denotes a source with at least one $i_k = 4$ or 5.

(10) Now the integral on the right-hand side of (3.8) can be done as in Ref. [6]. The term $\bar{N}O$ does not interfere in any way with $A_{\mu,\theta}, A_{\mu,\lambda}, \cdots c_{5,\lambda}$ integration as it does not depend on anything but $A_\mu(x)$. The result is, self-evidently,

$$W' = \int [dA][dc_5][d\zeta] \exp\Bigg(i \Bigg[\int \mathcal{L}_0 d^4x + \int \bar{N}_{\mu_1 \cdots \mu_n} O_{\mu_1 \cdots \mu_n} d^4x$$
$$+\text{source terms for fields and BRS composite operators} \Bigg] \Bigg)$$
$$= W[K^{\alpha\mu}_{,\theta}, K^{\alpha 5}_{,\theta}, -t^\alpha_{,\theta}, K^{\alpha\mu}(\bar{x}), K^{\alpha 5}(\bar{x}), t^\alpha(\bar{x}), \bar{N}] \; . \tag{3.9}$$

where W on the right-hand side of Eq. (3.9) is W Eq. (2.3) modified to include the gauge-invariant source $\int \bar{N} O d^4x$. Equation (3.8) together with Eq. (3.9) shows the equivalence of $W' = \int [dK^4][dK^4_{,\theta}] \bar{W}'[\bar{K}, t, \bar{N}]$ and the Yang-Mills generating functional with one insertion of $O_{\mu_1 \cdots \mu_n}$, viz. W of (3.9) up to terms irrelevant for our purpose.

We have thus in Eq. (2.5) constructed a generating functional $\bar{W}'$, containing $\bar{\mathcal{L}}_0'$ [see Eq. (2.4)] that possesses symmetry, which yields us correctly the Green's function of Yang-Mills theory with one insertion of an arbitrary gauge-invariant operator $O_{\mu_1 \cdots \mu_n}$. We shall consider the WT identity satisfied by $\bar{W}'$ in Sec. V.

IV. INCLUSION OF SCALARS

If one is to apply this superspace formulation to the discussion of Higgs mechanism say in the Weinberg-Salam model, one must show how the scalar fields can be incorporated in superspace formulation. In this section we shall show that the scalars can be incorporated trivially in the superspace formulation and show in Sec. V that the corresponding WT identities continue to retain the simple form $\partial \bar{W}/\partial\theta = 0$.

Consider a set of scalars in some representation of gauge group G. Let $\{T^\alpha\}$ be the representation of generators of G corresponding to the representation to which the scalars belong. Let the scalars be represented by a real column vector $\Phi(x)$. The covariant derivative of Φ is

$$D_\mu \Phi = (\partial_\mu - igT^\alpha A^\alpha_\mu)\Phi \; . \tag{4.1}$$

We now introduce a scalar superfield (a group multiplet) $\bar{\Phi}(\bar{x})$ which transforms as a scalar under OSp(3, 1|2) and has as its first component the column vector $\Phi(\bar{x})$: i.e.,

$$\Phi(\bar{x}) = \Phi(x) + \theta\tilde{\Phi}_{,\theta} + \lambda\tilde{\Phi}_{,\lambda} + \lambda\theta\tilde{\Phi}_{,\lambda\theta} \; . \tag{4.2}$$

Let the usual Yang-Mills action including scalars be

$$\mathcal{L}_{0\phi} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2}(D_\mu\Phi)^T (D^\mu\Phi) + V(\Phi) \; , \tag{4.3}$$

where $V(\Phi)$ contains the mass terms and interactions (independent of $\partial_\mu\Phi$). We then define the generating functional for the new superspace action by

$$\bar{W}_\phi[\bar{K}, t, J(\bar{x})] = \int \{dA\}\{d\zeta\}\{d\Phi\} \exp(i\bar{S}_\phi[A, \zeta, \Phi, K, t, J]) \; , \tag{4.4}$$

where

$$\bar{S}_\phi[A, \zeta, \Phi, K, t, J] = \bar{S} + \int d^4x \frac{\partial}{\partial\theta}[J(\bar{x})^T \Phi(\bar{x})] + \int d^4x \left[\frac{1}{2}[D_i\Phi(\bar{x})]^T [D_j\Phi(\bar{x})] g^{ij} + V(\Phi(\bar{x})) \right] \tag{4.5}$$

and

$$\{d\Phi(\bar{x})\} \equiv \prod_i \{d\Phi_i(\bar{x})\} = \prod_i \prod_x d\Phi(x) d\Phi_{i,\lambda}(x) d\Phi_{i,\theta}(x) \; . \tag{4.6}$$

The $\bar{W}_\phi$ of Eq. (4.4) is nothing but a straightforward generalization of $\bar{W}$ of Eq. (2.2). We shall now show that $\bar{W}_\phi$ contains all information about the Green's function S of the Lagrange density of Eq. (4.3). Incidentally note that the $\bar{\mathcal{L}}_{0\phi}$ [with obvious interpretation] has the full symmetry under superspace rotations of OSp(3, 1|2).

To do this, we need only to concentrate on two new integrals over $\Phi_{i,\lambda}(x)$ and $\Phi_{i,\theta}(x)$. This is done in a few steps.

(i) First, dependence of $\bar{S}$ on $\Phi_{i,\lambda}$ and $\Phi_{i,\theta}$ arising solely from that of Φ_i on these variables via Eq. (4.2) can be ignored completely on account of equations of motion of Φ_i [6].

(ii) Secondly, $\Phi_{i,\lambda\theta}$ do not appear in $\bar{W}_\phi$ as dynamical variables.

(iii) Consider now the terms in $\bar{S}$ dependent on $\Phi_{i,\lambda}$ and $\Phi_{i,\theta}$. They are (with corresponding integrals)

$$\int [d\Phi_{,\lambda}[d\Phi_{,\theta}] \exp\left(i \int d^4x \{ J^T_{,\theta}(\bar{x})\Phi(x) - J^T(x)\Phi_{,\theta}(x) + (\Phi_{,\lambda} - igT^\alpha c^\alpha_4 \Phi)^T (\Phi_{,\theta} - igT^\alpha c^\alpha_5 \Phi)\} \right) \; . \tag{4.7}$$

We can now change the variables of integration to $(D_4\Phi)_i$ and $(D_5\Phi)_j$ themselves. The Jacobian of the transformation is one. Integration over $(D_4\Phi)_i$ yields an anticommuting δ-function $\prod_{i,x}(D_5\Phi)_i(x)$. This can be used to replace $\Phi_{,\theta}$ in the source term by $(-igT^\alpha c_5^\alpha\Phi)$. Then the $(D_5\Phi)_i$ integration is done trivially. Thus the net scalar dependent piece in the expression for $\bar{W}_\phi$ reads

$$\int[d\Phi(x)]\exp\left(i\int d^4x[\mathcal{L}_{0\phi}+J^T_{,\theta}(\bar{x})\Phi(x) - J^T(\bar{x})igT^\alpha c_5^\alpha\Phi(x)]\right) . \quad (4.8)$$

The last term in the exponent of (4.8) gives the term for the BRS variation of Φ:

$$\delta\Phi_{\text{BRS}} = -igT^\alpha c_5^\alpha\Phi\delta\Lambda . \quad (4.9)$$

The rest of the integrals can be carried out straightforwardly as in Ref. [6]. The result is

$$\bar{W}_\phi[\bar{K},t,J(\bar{x})] = \prod_{\alpha,x}[\delta(K^{\alpha 4}(x))\delta(K^{\alpha 4}_{,\theta}(x))] \times W[K^\mu_{,\theta}K^5_{,\theta},-t_{,\theta},K^\mu,K^5,t,J,J_{,\theta}] , \quad (4.10)$$

where W is the generating functional for the action $\mathcal{L}_{0\phi}$ of Eq. (4.3) with sources for BRS variations of $A_\mu, c, \bar{c}$ and Φ included. This concludes the discussion of how scalars can be included in the present superspace action.

V. WT IDENTITIES

In this section, we shall give a brief derivation of the WT identities of the gauge theories with (i) one insertion of the gauge-invariant operator and (ii) scalars. The derivation is much the same way as in Ref. [7]. Hence we only explain the steps qualitatively.

The steps are (i) evaluate $\delta\bar{W}/\delta\theta$ from the integrated out form; (ii) Recognize that these are just the terms in the BRS variation of the integrated out action under the standard BRS transformations of the fields [7].

We only mention new relevant points.

A. Gauge-invariant operator insertion

We call

$$W' = \int[dK^4][dK^4_\theta]\bar{W}'[\bar{K},t,\bar{N}] . \quad (5.1)$$

In W', the exponent now contains a new term $\bar{N}_{\mu_1\cdots\mu_n}(\bar{x})O_{\mu_1\cdots\mu_n}[A]$. It will contribute to $\partial W'/\partial\theta$ only through $\partial\bar{N}/\partial\theta$. But we can always set $\partial\bar{N}/\partial\theta = 0$ in the end being an independent field of x^μ. In the BRS variation, the new term does not contribute as $O[A]$ is a gauge-invariant operator. Hence the proof of WT identities goes through as before, and we have

$$\frac{\partial W'}{\partial\theta} = 0 \quad (5.2)$$

[omitting $O(\hat{N}_{i_1\cdots i_k\cdots i_n}, O(N^2), O(dN/d\theta)$ terms)].

B. Scalars

In this case there are the following new terms in the integrated out action:

$$\int d^4x\left\{J^T_{,\theta}(\bar{x})\Phi(x) - J^T(\bar{x})igT^\alpha A^\alpha_\mu\Phi + \frac{1}{2}(D_\mu\Phi)^T(D_\mu\Phi) - V[\Phi]\right\} . \quad (5.3)$$

In taking $\partial\bar{W}/\partial\theta$, $-J^T(\bar{x})igT^\alpha A^\alpha_\mu\Phi$ terms contribute. There is no further contribution. But this new contribution is recognized as exactly being the BRS variation of $J^T_{,\theta}\Phi(x)$, with other new terms being BRS invariant. Hence the derivation of [7] goes through leading to the result

$$\frac{\partial\bar{W}}{\partial\theta} = 0 . \quad (5.4)$$

Thus the WT identities continue to retain their elegant form.

APPENDIX A: GENERAL STRUCTURE OF A LOCAL GAUGE-INVARIANT OPERATOR

We shall derive a very simple result in this Appendix. It is about the general structure of a gauge-invariant operator constructed out of Yang-Mills fields only. The result, while it is rather trivial, is very much needed in the discussion of superspace generalization.

We shall show that for a simple gauge group G, the most general local gauge-invariant operator constructed out of Yang-Mills fields has the structure

$$O = \sum t^{\alpha_1\cdots\alpha_n}X^{\alpha_1}_{(1)}\cdots X^{\alpha_n}_{(n)} , \quad (A1)$$

where each of the $X^{\alpha_k}_{(k)}$ can be written as a string of covariant derivatives, $D^{\alpha\beta}_\mu$ acting on one field strength, $F^\gamma_{\nu\lambda}$. For example,

$$X^{\alpha_1}_{(1)} = D^{\alpha_1\beta}_\mu D^{\beta\gamma}_\nu F^\gamma_{\lambda\delta} . \quad (A2)$$

The $X^{\alpha_k}_{(k)}$'s can be distinct for each k. Further $t^{\alpha_1\cdots\alpha_n}$ is a group tensor. Σ denotes formally a sum of terms of this form. That O of (A1) is a gauge-invariant operator is obvious. This is so since each $X^{\alpha_k}_{(k)}$ is a group vector and $t^{\alpha_1\cdots\alpha_n}$ is a group tensor.

To prove (A1), imagine expanding an arbitrary local gauge-invariant operator O of dimension D as

$$O = \sum_{p=0}^{D} g^p O_{(p)}[A] . \quad (A3)$$

As O is invariant under

$$\delta A_\mu^\alpha = -\partial_\mu \theta^\alpha(x) + g f^{\alpha\beta\gamma} A_\mu^\beta \theta^\gamma(x) \ , \tag{A4}$$

$O_{(0)}[A]$ is invariant under the Abelian part of (A4): viz.,

$$\delta A_\mu^{\alpha(0)} = -\partial_\mu \theta^\alpha \ . \tag{A5}$$

Hence $O_{(0)}[A]$ can be expressed in terms of "Abelian" field strengths $F_{\mu\nu}^{(0)} = \partial_\mu A_\nu^\alpha - \partial_\nu A_\mu^\alpha$ and their derivatives,

$$O_{(0)}[A] = t^{\alpha_1 \cdots \alpha_n} Y_{0(1)}^{\alpha_1} \cdots Y_{0(n)}^{\alpha_n} \ , \tag{A6}$$

where $t^{\alpha_1\cdots\alpha_n}$ are constant and each $Y_{0(k)}^{\alpha_k}$ contains an arbitrary order of the derivative of $F^{(0)\alpha_k}$:

$$Y_{0(k)}^{\alpha_k} = \partial_{\mu_1} \cdots \partial_{\mu_q} F_{\nu\lambda}^{(0)\alpha_k} \ . \tag{A7}$$

Further each $O_{(p)}[A]$ must be separately invariant under global transformations of the group

$$\delta A_\mu^\alpha = g f^{\alpha\beta\gamma} A_\mu^\beta \theta^\gamma; \quad \theta^\gamma = \text{const} \ . \tag{A8}$$

Nothing that each of $Y_{0(k)}^{\alpha_k}$ transforms as a *global vector* under G, it follows that the invariance of $O_{(0)}[A]$ under global transformations of G require that $t^{\alpha_1\cdots\alpha_n}$ is a group tensor.

Now we construct

$$Y_{(k)}^{\alpha_k} = D_{\mu_1}^{\alpha_k\beta_1} D_{\mu 2}^{\beta_1\beta_2} \cdots D_{\mu_q}^{\beta_{q-1}\beta_q} F_{\nu\lambda}^{\beta_q} \ . \tag{A9}$$

Note that $Y_{0(k)}^{\alpha_k} = Y_k^{\alpha_k}|_{g=0}$ and

$$\tilde{O}[A] = t^{\alpha_1\cdots\alpha_n} Y_{(1)}^{\alpha_1} \cdots Y_{(n)}^{\alpha_n} . \tag{A10}$$

Evidently $\tilde{O}[A]$ is a gauge-invariant operator. Now consider $O - \tilde{O}$. It is gauge-invariant operator of the same dimension D and no term in it of $O(g^0)$. It has the expansion

$$O - \tilde{O} = \sum_{p=1}^{D} g^p O'_{(p)}[A] \ . \tag{A11}$$

Now, we can repeat the discussion for the coefficient of lowest power of g, viz. $O'_{(1)}[A]$. We then find a gauge-invariant operator $\tilde{\tilde{O}}$ of the form of (A10) such that

$$O - \tilde{O} - \tilde{\tilde{O}} = \sum_{p=2}^{D} g^p O''_{(p)}[A] \ . \tag{A12}$$

This process can be continued. It is guaranteed to end since the highest power of g is finite (D). Then

$$O = \tilde{O} + \tilde{\tilde{O}} + \cdots \tag{A13}$$

is of the form (A10). Hence (A1) is verified.

Hence we can choose the basis for gauge-invariant operators in which every term is of the form

$$O = t^{\alpha_1\cdots\alpha_n} X_{(1)}^{\alpha_1} \cdots X_{(n)}^{\alpha_n} \ . \tag{A14}$$

APPENDIX B

We have noted in Appendix A that a typical gauge-invariant operator generalized to a six-dimensional superspace contains a chain of the form

$$X^\alpha = D_i^{\alpha\beta} \cdots D_j^{\gamma\delta} \cdots F_{kl}^\eta \ , \tag{B1}$$

where the indices $i\cdots j\cdots kl$ are free to go from 0 to 5 when the group indices have been successively contracted. Now for a given chain all the indices $i\cdots j\cdots kl$ may be free (they may be contracted elsewhere in the expression for O) or some of them may be contracted. Thus the chain X^α may contain terms in which all of $i\cdots j\cdots kl$ take values between 0 to 3 and it may contain terms in which at least one of these is 4 and/or 5. In the latter group there are terms in which 4 and/or 5 appear *only* in F but not on any D and finally there are terms in which at least one D caries an index 4 or 5. It is this last type of term that we focus our attention on. All the rest of the terms in X are such that 4 and/or 5 appears only in F. We shall now show that the last type of term can, by use of Jacobi identities, be cast as a sum over terms each of which has D_i's with i going only from 0 to 3; and 4 and/or 5 appears possibly only on F's (or c_5). This would then show that the entire X has this property.

For simplicity, first consider a simple example. Let X contain only one D with index (say) 4 and other D's have only some spacetime index ($\mu = 0,1,2,3$):

$$X^\alpha = D_\mu^{\alpha\beta} \cdots D_4^{\gamma\delta} D_\nu^{\delta\eta} \cdots F_{\sigma\lambda}^\xi \ . \tag{B2}$$

We can now compute D_4 across D_γ and all the rest of further D's by using (C2b) of Appendix C. Finally when D_4 hits $F_{\sigma\lambda}$ we use (C1b) of Appendix C to express

$$D_4 F_{\sigma\lambda} = -D_\lambda F_{4\sigma} - D_\sigma F_{\lambda 4} \ .$$

In each term in X^α, now, the index 4 always appears on an F and all D's are with Lorentz indices.

Now consider the case when the indices on the last F contains a 4 and/or 5. All discussion is the same as before except, when D_4 hits F, we use one of the following (see Appendix C):

$$\begin{aligned}
D_4 F_{4\mu} &= -\frac{1}{2} D_\mu F_{44} \ , \\
D_4 F_{5\mu} &= -D_5 F_{4\mu} - D_\mu F_{45} = -g f c_5 F_{4\mu} \\
&\quad -D_\mu F_{45} (\text{at } c_4 = 0 = c_{4,\theta}) \ , \\
D_4 F_{44} &= 0 \ , \\
D_4 F_{45} &= -\frac{1}{2} f c_5 F_{44} (\text{at } c_4 = 0 = c_{4,\theta}) \ , \\
D_4 F_{55} &= -2 D_5 F_{45} = 2 f c_5 F_{45} (\text{at } c_4 = 0 = c_{4,\theta})
\end{aligned} \tag{B3}$$

[we note that in the evaluation in Sec. III, we need to consider $\bar{O}$ only at $c_4 = 0 = c_{4,\theta}$] proving the necessary result.

Now consider a somewhat more complicated case. Let there be two D_4 in X^α. We pass through the first D_4 until it hits the second; then use $D_4 D_4 = \frac{g}{2} f F_{44}$. Next consider the case with three D_4's:

$$X^\alpha = D_\mu^{\alpha\beta} \cdots D_4^{\gamma\delta} \cdots D_4^{\gamma\delta} \cdots D_4^{\eta\xi} D_\nu^{\xi\kappa} \cdots D_4^{\lambda\sigma} \cdots F_{ij}^\tau \ .$$

Here we need only exhibit how to pass the third $D_4^{\gamma\delta}$ after the second $D_4^{\eta\xi}$ has been passed through:

$$D_4^{\eta\xi} D_\nu^{\xi\kappa} = D_\nu^{\eta\xi} D_4^{\xi\kappa} + g f^{\eta\kappa\tau} F_{4\nu}^\tau \ . \tag{B4}$$

This generates at term like $F^{\eta\kappa\tau} F_{4\nu}^\tau$ through which the third D_4 must be passed. This is done using the identity

$$\begin{aligned} D_4^{\zeta\eta} f^{\eta\kappa\tau} F_{4\nu}^\tau &= f^{\zeta\kappa\tau} (D_4^{\tau\eta} F_{4\nu}^\eta) - f^{\zeta\eta\tau} F_{4\nu}^\tau D_4^{\eta\kappa} \\ &= f^{\zeta\kappa\tau} \left(-\frac{1}{2}\right) (D_\mu F_{44}) - f^{\zeta\eta\tau} F_{4\nu}^\tau D_4^{\eta\kappa} \ . \end{aligned} \tag{B5}$$

In the final term when all D's hit each other we use $D_4 D_4 D_4 = 0$ Eq. (C1i). This process can be evidently generalized to more than three D_4's.

An exactly identical procedure follows when there are only D_5's (any number of them) in X^α. Finally, we must deal with the mixed case when there are both D_4's and D_5's. First consider the simplest version of these:

$$X^\alpha = D_\mu^{\alpha\beta} \cdots D_5^{\eta\xi} \cdots D_4^{\lambda\sigma} \cdots F_{ij}^\zeta \ .$$

The general procedure is similar to the previous cases and uses in particular modification of (B4). Ultimately, one ends up with

$$D_5 D_4 F_{ij} = [D_5 D_4 - D_4 D_5] - g f F_{45} F_{ij} \ .$$

Now, it is easily shown using (C1) of Appendix C that the term $(D_5 D_4 - D_4 D_5) F_{ij}$ case by case contains only F's and D_μ's but no D_4 or D_5 in the final result. ($c_4 = 0 = c_{4,\theta}$ has been employed.)

A similar procedure can be given for more D_5's and/or more D_4's. Now, what holds for each X^α holds for the entire operator O of (B1). Thus O can be written entirely without the use of D_4 and D_5 anywhere (i.e., in the term of F_{ij}, D_μ, c_5 only).

Now we consider the dependence of $\bar{O}_{\mu_1 \cdots \mu_n}$ on $A_{\nu,\lambda}$. First, $A_{\nu,\lambda}$ is present only in some $F_{4,\nu}$ in $\bar{O}$ (and not, say, D_4 acting on A_ν) because O can be written without D_4. The subscript 4 must be contracted somewhere to 5 as $\bar{O}_{\mu_1 \cdots \mu_n}$ has no free subscript 4. Now consider the chain X which has this index 5. From the above discussion (with $4 \leftrightarrow 5$ everywhere) it is clear that each such term contains either $F_{5\lambda}$ or F_{54} or F_{55}, all of which vanish by equation of motion. The same applies to F_{44}. This fact has been used in Sec. III to simplify integration over $A_{\mu,\lambda}, c_{4,\lambda}$, etc. in the presence of an operator. Thus we have the result.

All terms in $\bar{O}_{\mu_1 \cdots \mu_n}$ depending on $F_{4\mu}$ and F_{44} vanish by equation of motion. Those depending on $F_{5\mu}$, F_{45}, F_{55} also vanish because these quantities themselves vanish by equation of motion.

APPENDIX C

1. Jacobi identities

The Jacobi identities are

$$D_\mu F_{\nu\lambda} + D_\nu F_{\lambda\mu} + D_\lambda F_{\mu\nu} = 0 \ , \tag{C1a}$$

$$D_4 F_{\mu\nu} + D_\mu F_{\nu 4} + D_\nu F_{4\mu} = 0 \ , \tag{C1b}$$

$$D_5 F_{\mu\nu} + D_\mu D_{\nu 5} + D_\nu F_{5\mu} = 0 \ , \tag{C1c}$$

$$D_4 F_{5\mu} - D_5 F_{\mu 4} + D_\mu F_{45} = 0 \ , \tag{C1d}$$

$$2 D_4 F_{4\mu} + D_\mu F_{44} = 0 \ , \tag{C1e}$$

$$2 D_5 F_{5\mu} + D_\mu F_{55} = 0 \ , \tag{C1f}$$

$$D_5 F_{44} + 2 D_4 F_{45} = 0 \ , \tag{C1g}$$

$$D_4 F_{55} + 2 D_5 F_{45} = 0 \ , \tag{C1h}$$

$$D_4 F_{44} = 0 \ , \tag{C1i}$$

$$D_5 F_{55} = 0 \ . \tag{C1j}$$

2. Commutation or anticommutation relation of the covariant derivatives

The commutation or anticommutation relations are

$$[D_\mu, D_\nu]^{\alpha\beta} = -g f^{\alpha\beta\gamma} F_{\mu\nu}^\gamma \ , \tag{C2a}$$

$$[D_4, D_\mu]^{\alpha\beta} = -g f^{\alpha\beta\gamma} F_{4\mu}^\gamma \ , \tag{C2b}$$

$$[D_5, D_\mu]^{\alpha\beta} = -g f^{\alpha\beta\gamma} F_{5\mu}^\gamma \ , \tag{C2c}$$

$$\{D_4, D_5\}^{\alpha\beta} = -g f^{\alpha\beta\gamma} F_{45}^\gamma \ , \tag{C2d}$$

$$\{D_4, D_4\}^{\alpha\beta} = -g f^{\alpha\beta\gamma} F_{44}^\gamma ; \text{i.e.,} D_4^{\alpha\gamma} D_4^{\gamma\beta} = -\frac{g}{2} f^{\alpha\beta\gamma} F_{44}^\gamma \ , \tag{C2e}$$

$$\{D_5, D_5\}^{\alpha\beta} = -g f^{\alpha\beta\gamma} F_{55}^\gamma ; \text{i.e.,} D_5^{\alpha\gamma} D_5^{\gamma\beta} = -\frac{g}{2} f^{\alpha\beta\gamma} F_{55}^\gamma \ . \tag{C2f}$$

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