

Black hole complementarity versus locality

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The evaporation of a large mass black hole can be described throughout most of its lifetime by a low-energy effective theory defined on a suitably chosen set of smooth spacelike hypersurfaces. The conventional argument for information loss rests on the assumption that the effective theory is a *local* quantum field theory. We present evidence that this assumption fails in the context of string theory. The commutator of operators in light-front string theory, corresponding to certain low-energy observers on opposite sides of the event horizon, remains large even when these observers are spacelike separated by a macroscopic distance. This suggests that degrees of freedom inside a black hole should not be viewed as independent from those outside the event horizon. These nonlocal effects are only significant under extreme kinematic circumstances, such as in the high-redshift geometry of a black hole. Commutators of spacelike separated operators corresponding to ordinary low-energy observers in Minkowski space are strongly suppressed in string theory.

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I. INTRODUCTION

According to the principle of black hole complementarity [1], an observer who remains outside the horizon of a black hole can describe the black hole as a very hot membrane, the stretched horizon, which lies just above the mathematical event horizon, and absorbs any matter, energy, and information which fall onto it. The information which is absorbed by the stretched horizon is eventually reemitted in the Hawking radiation, albeit in a very scrambled form. An observer who falls freely into the black hole sees things very differently: no membrane, no high temperature, no irregularities of any kind as the observer crosses the event horizon. The principle of black hole complementarity asserts the consistency of these apparently contradictory descriptions. According to this principle, the matter which has fallen past the event horizon and the Hawking radiation are not different objects. They are complementary descriptions of a single system, viewed from very different reference frames which are related by an enormous Lorentz boost. A similar viewpoint has long been advocated by 't Hooft [2] and more recently by Kiem, Verlinde, and Verlinde [3]. At present, such strange behavior cannot be ruled out because it involves physics at energy scales far beyond anything with which we have any experience [4].

Although no logical contradiction is known to follow from the principle of black hole complementarity, it nevertheless seems to contradict our ordinary ideas about locality. In actuality, black hole complementarity does not require any observer to detect nonlocal effects. In the membrane picture the observation of information in the Hawking radiation by a distant observer involves nothing acausal, since the Hawking radiation is in causal contact with the stretched horizon at all times. As for infalling observers, they see the ordinary low-energy laws of nature

until the singularity is approached. It is only in certain correlations between events on either side of the horizon that nonlocality is required. Such correlations are unobservable in the sense that they cannot be established by any single observer without violating known laws of physics [4]. On the other hand, if physics is described in terms of a local effective field theory, then correlations across the event horizon will in fact be in contradiction with black hole complementarity, as we will discuss below. If black hole complementarity is correct, the usual principles of local quantum field theory must break down, not only at small scales, but at all scales. And yet these violations of locality must be undetectable in ordinary low-energy experiments. Their only role should be to reconcile the two complementary descriptions. Evidently, the nonlocality must be of an extraordinarily special and subtle kind.

The nature of the required nonlocality can be illustrated by examining an argument, which we will call the *nice-slice argument*, which is often put forward in support of the idea of information loss. The essence of this argument is very simple. Since the process of gravitational collapse and subsequent evaporation of a very large black hole begins and ends with very low-energy particles, and since the evolution of the black hole is very slow on microscopic time scales, the adiabatic theorem should ensure that high-energy degrees of freedom decouple. In other words, the process can be described using only a local low-energy effective field theory defined on a (slowly varying) background geometry, and fluctuations of the gravitational field can be neglected. Of course, the final burst of energy involves a few high-energy particles, but this is irrelevant for our considerations, since such a small number of particles cannot carry off an appreciable amount of the information that originally fell into the black hole. It is then straightforward to argue that the known behavior of local field theories prohibits informa-

tion retrieval.

The nice-slice argument is formalized in Sec. II of this paper. In Sec. III, we present evidence that string theory fails to meet one very important assumption of the nice-slice argument—the assumption of locality in the low-energy nice-slice theory. To do so, we construct a low-energy nice-slice theory using light-front string field theory, and calculate the commutators of low-energy nice-slice fields. It turns out that nice-slice fields behind the horizon do not commute with nice-slice fields in front of the horizon. The idea that operators behind the horizon fail to commute with operators in front of the horizon has been advocated previously by several authors [2,3,5,6]. The conclusion is that there must exist nonlocal states in the low-energy nice-slice theory. It will be shown that to leading order in the string coupling, these states are highly excited strings stretched between points on the nice slice.

We present most of our calculations in the two Appendixes. This allows us to focus better on ideas and results in the main text. Our calculations are, strictly speaking, only valid in the limit of very large black hole mass which allows us to neglect the effects of the local curvature in the region of interest. We also only work to leading order in string perturbation theory which further restricts the range of validity of the calculations. In spite of these technical limitations, our results demonstrate a basic difference between light-front string theory and quantum field theory.

Before proceeding, it is important to discuss the various scales of size and energy which occur in the discussion of black hole evaporation. The largest energies, which we will call *trans-Planckian*, are vastly larger than the Planck mass M_P . For a black hole of mass M , the trans-Planckian energies involved can be of order $M_P \exp(GM^2)$. Recall that according to the conventional analysis of Hawking radiation [7], the outgoing radiation originates in incoming vacuum fluctuations which become deformed by the black hole geometry. After a time of order $G^2 M^3$, an appreciable fraction of the black hole energy has been radiated away, and information is expected to appear in the evaporation products [8]. At this stage, the outgoing Hawking radiation is associated with infalling modes with trans-Planckian energies of order $M_P \exp(GM^2)$.

It is generally felt, however, that a correct understanding of Hawking evaporation should not require knowledge of trans-Planckian physics. According to black hole complementarity, this is correct for observers who remain strictly outside the horizon. For such observers, a description in terms of a stretched horizon composed of Planck scale degrees of freedom should suffice. Using coordinates which lie partly behind the horizon, however, we will find that trans-Planckian degrees of freedom are important. In particular, an understanding of the redundancy of degrees of freedom on either side of the horizon requires a correct treatment of these extremely high-energy degrees of freedom. In free field theory, the trans-Planckian modes are associated with distance and time scales of order $l_P \exp(-GM^2)$. In the nice-slice argument, it is not necessary to assume that free-field theory

is valid for the trans-Planckian modes, but it is assumed that they can be localized on some scale small compared to the overall geometry, such as l_P . We shall see that in string theory, the relevant trans-Planckian modes correspond to distances of order $l_P \exp(GM^2)$. The nonlocality induced by trans-Planckian modes can thus extend to very large distances.

In the membrane picture the degrees of freedom contained in the region of order the Planck or string scale from the event horizon comprise the stretched horizon. These degrees of freedom store and thermalize information from the viewpoint of the external observer [9,10], and they are responsible for the entropy of the black hole [2,11,12]. Finally, there are low-energy modes well below the Planck or string scale, which correspond to the energies of Hawking particles that escape from the black hole. The nice-slice argument suggests that only these low-energy modes are essential to a complete understanding of Hawking evaporation. We will argue that this is not the case in string theory.

II. THE NICE-SLICE ARGUMENT

Consider the process by which a collection of particles of total mass M gravitationally collapses to form a black hole. In the absence of any net angular momentum, the black hole becomes approximately spherically symmetric in a short time after the collapse process. As long as we are only considering processes with lifetimes short compared to $G^2 M^3$, we can approximate the geometry of the system by the Schwarzschild solution. The nice-slice argument begins by introducing a family of Cauchy surfaces which foliate the geometry. The surfaces should avoid regions of strong spacetime curvature and yet cut through the infalling matter and the outgoing Hawking radiation so that both sets of particles have low energy in the local frame of the slice. Also, in order to ensure that short distance physics does not creep into the description through the choice of coordinates, we require that the slices be everywhere smooth, with small extrinsic curvature compared to any microscopic scale. For convenience, we will choose surfaces that agree with surfaces of constant Schwarzschild time far from the black hole. Such a family of surfaces will henceforth be designated “nice slices.” While it is seldom spelled out, the existence of such a set of surfaces is implicitly assumed in much of the existing literature on black hole evaporation. The first explicit construction of nice slices that we are aware of was carried out by Wald [13].

To construct an example of a family of nice slices, we begin with the Schwarzschild black hole in Kruskal-Szekeres coordinates, as shown in Fig. 1. These coordinates are related to the usual Schwarzschild coordinates r and t by the transformation

$$-UV = 16G^2 M^2 \left(\frac{r}{2GM} - 1 \right) e^{r/2GM}, \quad (2.1)$$

$$-V/U = e^{t/2GM}.$$

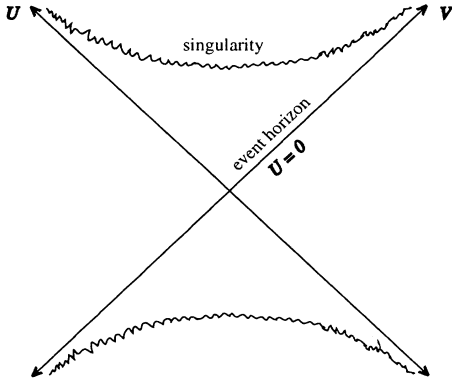


FIG. 1. Kruskal diagram of static black hole solution.

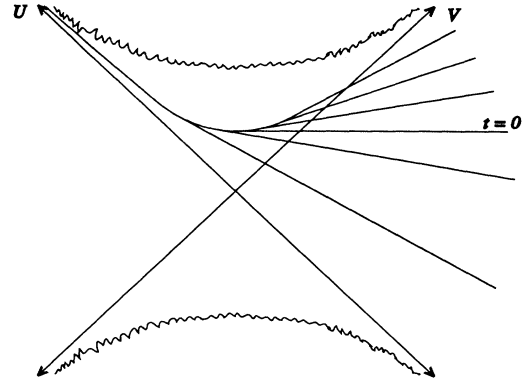


FIG. 2. A family of nice slices.

The singularity is at $UV = 16G^2M^2$, and the event horizon is the surface $U = 0$.

We first construct a spacelike surface composed of two pieces joined smoothly at the surface $U = V$. The first piece is the hyperbola which satisfies the condition $UV = R^2$ for $V < U$. The constant R is assumed to be large by comparison with any microscopic scale, but should not be so large that the surface is anywhere near the singularity. Later, we will choose R to be fixed and send $M \rightarrow \infty$. The second piece of the nice slice is defined by the line satisfying $U + V = 2R$ for $V > U$. The resulting surface is asymptotic to the surface defined by $t = 0$.

The nice slice we have constructed can be pushed forward and backward in time by using the symmetry under the boostlike operations

$$\begin{aligned} U &\rightarrow U' = Ue^{-t/4GM}, \\ V &\rightarrow V' = Ve^{t/4GM}. \end{aligned} \quad (2.2)$$

Since the nice slices are asymptotic to surfaces of constant Schwarzschild time, they can be parametrized by t . The full set of nice slices can then be written

$$\begin{aligned} UV &= R^2, \quad V < e^{t/2GM}U, \\ e^{t/4GM}U + e^{-t/4GM}V &= 2R, \quad V > e^{t/2GM}U, \end{aligned} \quad (2.3)$$

and is shown in Fig. 2. The join between the line segment and the hyperbola on each slice should be smoothed to avoid having a large extrinsic curvature there.

A Hamiltonian may be defined by introducing a vector field v which is orthogonal to the nice slices. The nice-slice Hamiltonian H_{NS} is the generator of motions along this vector field, and maps the state of the system on one slice to a state on a neighboring slice by means of the Schrödinger equation

$$i\partial_t|\psi\rangle = H_{\text{NS}}|\psi\rangle. \quad (2.4)$$

Since the vector field v is not a Killing vector field, the Hamiltonian H_{NS} is time dependent, so there is no con-

served nice-slice energy. If the mass M of the black hole and the value of R are very large, however, the rate of change of the nice-slice energy is very small, and becomes adiabatic as M and R tend to infinity.

The three-momentum of an infalling particle as it crosses a given slice can be defined by projecting the four-momentum vector onto the slice. It is easily seen that the spatial momentum of an ordinary particle remains small throughout its entire journey toward the hyperbola $UV = R^2$. This is in sharp contrast to the situation in Schwarzschild coordinates, where the three-momentum of a particle diverges as it approaches the horizon. The outgoing Hawking quanta also have low momenta as they cross each nice slice.

So far we have been discussing a classical background geometry, but in order to address the issue of information loss we should include the effect of black hole evaporation. This introduces an important new feature into the problem. Our nice slices in the static classical geometry never get closer to the singularity than the hyperbola $UV = R^2$, but when the semiclassical back-reaction is included the black hole has a finite lifetime and eventually the part of a slice that extends into the black hole interior will encounter large curvature. The best one can do in this case is to construct a family of slices that avoid the region of strong gravitational coupling for as long as possible. This is not a serious drawback. If we start with a sufficiently large black hole then even after 99.99% of the energy has evaporated the black hole will still be large and a set of nice slices can still be found. By starting with a sufficiently large black hole, we can thus follow the evolution long enough to have most of the total Hawking radiation already emitted, and well separated from the black hole region, by the time our nice slices run into strong curvature.

It is therefore natural to make the assumption that the entire history of the black hole (except for the short time when the Hawking temperature exceeds the cutoff scale) can be described by a low-energy effective quantum field theory defined on a slowly varying background geometry. The cutoff length is chosen sufficiently large that gravitational fluctuations can be ignored altogether. We

emphasize that no trans-Planckian frequencies have entered into the discussion. In fact the adiabatic theorem implies that only degrees of freedom of very low energy ($E \sim 1/M$) get excited from their ground state in the Hawking emission process. The construction seems to leave no room for stringy short distance behavior, or any other modification of field theory, to influence the details of the Hawking radiation.

The nice-slice argument can be combined with another argument, the “no quantum Xerox principle” [1], to show that the information carried by infalling matter cannot be stored on the stretched horizon. Consider a late time slice Σ , and partition it into two portions Σ_{in} and Σ_{out} , with Σ_{in} containing the region behind the horizon, while Σ_{out} contains the horizon and the region outside the black hole. Since the low-energy nice-slice theory is a local quantum field theory on a time-dependent but classical background geometry, low-energy field operators will commute when their arguments are spacelike related (as measured by the background metric). This means that we can form a complete set of commuting observables using local fields defined on Σ_{in} and Σ_{out} , and thus the Hilbert space of states on the slice Σ factorizes into a tensor product

$$\mathcal{H}(\Sigma) = \mathcal{H}(\Sigma_{\text{in}}) \otimes \mathcal{H}(\Sigma_{\text{out}}). \quad (2.5)$$

The evolution operator defines a linear map from states in the Hilbert space of initial configurations of infalling matter to states in $\mathcal{H}(\Sigma)$.

It is now easy to give the argument for information loss. The “no quantum Xerox principle” states that the process of linear evolution cannot faithfully replicate quantum information in two separate sets of commuting degrees of freedom. If the infalling information is completely and faithfully recorded in the states of $\mathcal{H}(\Sigma_{\text{in}})$ (as is widely believed), then little or no information can be found in the states of $\mathcal{H}(\Sigma_{\text{out}})$. This conclusion would apply to both the Hawking radiation itself and to the stretched horizon.

III. THE NICE SLICE AND STRING FIELD THEORY

Although the nice-slice argument seems very general, we will present evidence from string theory that it fails. A key assumption that goes into the nice-slice argument is that the underlying microscopic theory is approximately a local field theory. Specifically, it assumes that any non-locality which results from the nonzero size of strings is limited to small spacelike separations, but this is incorrect, as we shall see later on.

The analysis begins by considering two spacetime points x_1 and x_2 , which lie on a fixed nice slice Σ_t corresponding to Schwarzschild time t . The time t is chosen large, but not so large that an appreciable amount of evaporation has occurred. The point x_2 lies behind the horizon, and could be chosen to lie on the hyperbola $UV = R^2$. It may be thought of as a point near the trajectory of a low-energy particle which has fallen through

the horizon at some early time. Point x_1 lies outside the event horizon and should be associated with the stretched horizon. According to the principle of black hole complementarity, an observer who remains permanently outside the black hole sees the infalling information stored in this region. The point x_1 may be chosen according to the following procedure. First, consider the nice slice Σ_0 , and pick a point outside the event horizon, for example, the point $U = U_0 < 0$, $V = V_0 > 0$. Now push the point forward in time along a timelike Killing vector until it arrives at the point

$$U_1 = U_0 e^{-t/4GM}, \quad (3.1)$$

$$V_1 = V_0 e^{t/4GM},$$

which lies on Σ_t . As t increases, the spacelike separation between x_1 and x_2 grows like $\exp(t/4GM)$.

We will assume that some form of string field theory allows us to obtain component fields $\phi_A(x)$ for each mass eigenstate of the string. A low-energy effective field $\hat{\phi}_A(y)$ can then be defined by

$$\hat{\phi}_A(y) = \int d^D x f(x-y) \phi_A(x), \quad (3.2)$$

where the test function f is assumed to be smooth enough to eliminate variations of ϕ_A on scales smaller than some cutoff scale ε in an appropriate local frame. If a low-energy observer at point (U_0, V_0) uses a test function f , then a low-energy observer at point x_1 uses a test function which is obtained from the test function f by boosting along the timelike Killing vector. Using the notation of Eq. (3.1), this function is

$$\begin{aligned} f(x-x_1) \\ = f(e^{t/4GM}(U-U_1), e^{-t/4GM}(V-V_1), \vec{x}-\vec{x}_1). \end{aligned} \quad (3.3)$$

The assumption of locality of the nice-slice theory can be tested by calculating the commutators of low-energy nice-slice fields obtained from light-front open bosonic string field theory. For technical reasons, we are unable to carry out such a calculation in the Schwarzschild geometry. In the limit of large black hole mass, however, the spacetime region of interest is well approximated by flat Minkowski space, where the computations are straightforward.

Using light-front coordinates

$$x^\pm = \frac{1}{\sqrt{2}}(x^0 \mp x^{D-1}), \quad (3.4)$$

in which the Minkowski line element is

$$ds^2 = -2dx^+ dx^- + \delta_{ij} dx^i dx^j, \quad i, j \in \{1, \dots, D-2\}, \quad (3.5)$$

the light-front time x^+ corresponds to U , the longitudinal direction x^- corresponds to V , and the horizon becomes the planar lightlike hypersurface $x^+ = 0$. A set of nice

slices can be constructed in Minkowski spacetime in exactly the same way as the black hole spacetime, except that we must replace asymptotic time t by the Rindler time $\omega = t/4GM$.

In Appendix A we calculate the matrix element

$$M(1, 2; 3) = \langle 0 | [\Phi(1), \Phi(2)] | 3 \rangle, \quad (3.6)$$

to leading order in the string coupling g . It gives the overlap of the commutator of two light-front open bosonic string fields with state $|3\rangle$. To leading order in g , the commutator creates a single string state. The matrix element

$$M_{AB}(1, 2; 3) = \langle 0 | [\phi_A(x_1), \phi_B(x_2)] | 3 \rangle, \quad (3.7)$$

for the commutator of any two mass eigenstate fields $\phi_A(x)$ can be obtained from Eq. (3.6) by folding in with the appropriate transverse string wave functions. We show that there exist matrix elements of this form which are nonzero even when $x_1 - x_2$ is spacelike. Finally, we obtain matrix elements of the commutator of low-energy nice-slice fields

$$\hat{M}_{AB}(1, 2; 3) = \langle 0 | [\hat{\phi}_A(x_1), \hat{\phi}_B(x_2)] | 3 \rangle, \quad (3.8)$$

by integrating the mass eigenstate fields against test functions appropriate for nice-slice observers at x_1 and x_2 .

Because the spacelike separation between x_1 and x_2 grows exponentially with ω , one would ordinarily expect the matrix elements (3.8) to tend to zero very rapidly with ω . Indeed, for the commutator of tachyon component fields with a spectator tachyon, this is what we find. This does not give a good indication of the “size” of the commutator, however, because for large ω the commutator creates a high-mass string state which has very little overlap with any fixed-mass spectator state. A better way to obtain the magnitude of the commutator is to multiply the matrix element (3.8) by its complex conjugate and sum over spectator states; in other words, to calculate a matrix element of the form

$$\hat{M}(1, 2; 1', 2') = \langle 0 | [\hat{\phi}_1(x_1), \hat{\phi}_2(x_2)] [\hat{\phi}_2(x'_2), \hat{\phi}_1(x'_1)] | 0 \rangle. \quad (3.9)$$

A detailed calculation of these matrix elements to leading order in g is presented in Appendix B, but we shall describe the basic points of the calculation here. The matrix element (3.9) has the form of a sum of finite time scattering amplitudes, which can be calculated using the first quantized formalism. The discussion is simplified if we Fourier transform the amplitude to momentum space. Since the spectator states are all single string states, if we define $s = -(p_1 + p_2)^2$ and $t = -(p_2 - p'_2)^2$, then the relevant string world sheet describes an s -channel diagram. One finds that as ω increases, the dominant contribution to the matrix element comes from the region where the longitudinal momenta p_1^+ and $p_1'^+$ are of order $e^{-\omega}$, and the s variable is large and positive. Heuristically, what happens is the scattering amplitudes are driven into the Regge region, where they behave as $s^{\alpha(t)}$. In open string

theory, $\alpha(t) = \alpha't + 1$ is the leading Regge trajectory. Because of the Regge behavior, matrix elements of the form (3.9) do not decrease as $\omega \rightarrow \infty$, but stay approximately constant. Moreover, since the behavior is dominated by the leading Regge intercept, the extension to closed bosonic strings is straightforward. The matrix element for tachyon component fields in the closed string theory actually grows like e^ω .

One might have argued that the behavior found above should be expected, since the spectrum of the bosonic string contains a tachyon. It should be clear from the above discussion, however, that the existence of the tachyon has no bearing on the result. In fact, if one canonically quantizes a tachyon field in Minkowski space by imposing that the field commute with itself on some spacelike surface, the Lorentz transformation properties of scalar fields guarantee that the field will commute with itself on all spacelike surfaces. It is the Regge behavior of strings, or in other words the existence of the infinite tower of massive states to which the commutator can couple, which governs the behavior of the matrix element (3.9) as $\omega \rightarrow \infty$. This behavior will therefore also be present in the tachyon-free superstring case.

The above results have important consequences for the nice-slice theory. The effective fields $\hat{\phi}_A(x_1)$ and $\hat{\phi}_B(x_2)$ are, by construction, low-energy fields as measured by nice-slice observers. Since these fields belong to the algebra of operators of the regulated nice-slice theory, then so must their commutator. Our explicit computation shows, however, that the commutator (to leading order in the string coupling g) is an operator which creates a single excited string which is stretched between x_1 and x_2 . The mass squared of this string state is of order $s \sim e^\omega/\alpha'$, and eventually becomes trans-Planckian as ω gets large. Nevertheless, the nice-slice energy of the configuration remains low, and this state must therefore be regarded as an unavoidable part of the nice-slice theory, which could never have been discovered if we had first truncated the theory to the massless fields.

Let us consider how it is possible for configurations to have low values of the nice-slice energy, and yet have huge trans-Planckian masses. Consider a state with a massless particle of low nice-slice energy at each of the points x_1 and x_2 . Let the four-momentum of the particle at x_2 be

$$q_2 = (q_2^-, q_2^+, \vec{q}_2), \quad (3.10)$$

with the individual components of q_2 being smaller than some cutoff momentum. The four-momentum of a low nice-slice energy particle at x_1 is given by

$$q_1 = (e^\omega q_1^-, e^{-\omega} q_1^+, \vec{q}_1), \quad (3.11)$$

with the components q_1^μ being smaller than the cutoff momentum. Although the nice-slice energy of this configuration is small, the Minkowski mass of the state rapidly becomes trans-Planckian as ω increases:

$$m^2 = -(q_1 + q_2)^2 \sim e^\omega q_1^- q_2^+. \quad (3.12)$$

Generally, a state of low nice-slice energy will have very large mass squared if it is spread over a large distance.

It is disturbing at first sight to find not only nonvanishing, but large values for the commutator of fields with spacelike separated arguments. Note, however, that this effect is only encountered under very extreme kinematic circumstances. In Appendix A 2, it is shown that if one considers component fields appropriate for low-energy *Minkowski* observers, then the matrix elements (3.8) are given by the usual field-theoretic formulas, and are therefore suppressed when the fields are spacelike separated. This is simply because the invariant mass squared of this process is always much smaller than $1/\alpha'$, so the commutator cannot couple to the higher mass, extended string states.

IV. DISCUSSION AND CONCLUSIONS

Having obtained the commutators of nice-slice fields from string field theory, we are now in a position to understand their significance for the nice-slice argument. In this section we will argue that the assumption of locality of the low-energy nice-slice theory is not valid in string theory. Evidence for this will be obtained in two ways. First, we compare the commutators of nice slice fields derived from string field theory to their field theory counterparts, and find that the commutators in string theory exhibit far more nonlocality. Second, we argue that mathematical consistency of the low-energy theory necessitates the inclusion of extremely nonlocal states corresponding to very massive strings.

A. Comparing commutators

Let us begin our comparison by examining the commutators of local fields more closely. For the moment, let us ignore gravity, and focus on quantum field theory on a fixed background spacetime. The statement of causality is that no local physical signal can propagate faster than light. For a theory of quantum fields on a (well-behaved) manifold with a fixed metric, there exists a well defined light cone for each point, and causality is implemented by requiring the *gauge invariant* local fields to commute when their arguments are spacelike related. There is no such requirement for gauge variant fields. For example, it is shown in Appendix A 2 that in Yang-Mills theory the commutator $[A_i^a(x_1), A_j^b(x_2)]$ of two transverse vector fields in light-front gauge fails to vanish when $x_1 - x_2$ is spacelike. There exist gauge invariant local fields, however, such as $\text{tr}[F^2]$, and these fields do commute at spacelike separation.

When gravity is included, the situation is more tricky. Since the symmetry group of the theory includes diffeomorphisms, there simply are no local invariants, and one cannot place any restrictions on the commutators of strictly local fields. This can be said another way. In a quantum theory of gravity, it is impossible to say *a priori* whether two points x and y are spacelike related, so one cannot impose the condition $[\phi(x), \phi(y)] = 0$ as an operator equation. One can only calculate the matrix el-

ements of the commutator in a state of the gravitational field.

Now let us return to string theory and the nice slice. The nice-slice argument assumes that gravitational fluctuations can be neglected. Since gravity enters open string theory only at one-loop level, the calculations of commutators of nice-slice fields obtained from open string field theory can be directly compared to their counterparts obtained from quantum field theory in a fixed background.

Consider first the commutator of two uncharged tachyon fields. In ordinary quantum field theory, the transformation properties of scalar fields (tachyonic or not) guarantee that if a scalar field commutes with itself on one spacelike surface, then it commutes with itself on all spacelike surfaces. Open string field theory, on the other hand, gives a nonvanishing commutator. Moreover, its magnitude stays roughly constant in time on the nice slice. We have thus found an example where gauge invariant fields fail to commute at spacelike separation in string theory.

The comparison of the commutator of two nice-slice vector fields is more subtle. Direct computation shows that the magnitude of this commutator stays roughly constant in both quantum field theory and in string theory. In Yang-Mills theory in light-front gauge, the matrix element (3.9) remains roughly constant because the A^- component of the vector field is a nonlocal functional of the transverse components. This nonlocality enters the commutator of the transverse components at order g . As was mentioned previously, no significance is attached to this, because there exist gauge invariant functionals of the gauge fields for which the commutator vanishes.

In string theory, the matrix element (3.9) for non-Abelian vector fields contains the expression obtained in Yang-Mills theory (as it must), but also contains additional terms. These extra terms also remain roughly constant, but do so because of the Regge behavior of strings, not because of the nonlocality introduced by the longitudinal components of the gauge fields. In this case, then, the most we can say is that string theory introduces additional nonlocality which arises for entirely different reasons.

In closed string theory, even more extreme effects occur. For example, the analog of the matrix element (3.9) for closed string tachyonic fields increases exponentially in time. Note that this additional degree of nonlocality is due to the shift in the intercept of the leading Regge trajectory from 1 to 2. In other words, it is due to the presence of the graviton.

Finally, one should compare the above findings to those obtained from string S -matrix calculations [14]. There one finds a degree of nonlocality far smaller than that present in the light-front string field commutator. Although certain S -matrix elements computed in [14] do display nonlocal effects over macroscopic separations, these amplitudes were found to be highly suppressed. This is to be expected, otherwise an observer crossing an event horizon would necessarily experience a large scale violation of the equivalence principle. It is an important open question whether an interpolating field could

be constructed which is “more local” than the light-front string fields.

B. The structure of the nice-slice theory

The comparison of the commutators of low-energy nice-slice fields in quantum field theory and in string theory has provided evidence that the assumption of locality of the nice-slice theory is not valid. Now, let us turn our attention to the mathematical structure of the low-energy nice-slice theory. The assumption of locality in the nice-slice argument is an assumption about the result of truncating a high-energy theory, which includes gravity, to the low nice-slice energy modes. This involves an order of operations. The order of operations that is implicit in presenting the nice-slice argument is to first truncate the high-energy theory down to a system of low-mass fields, and then to write a theory of these fields on the nice slice.

The appropriate order of operations, however, is to first write down the high-energy theory on the nice slice, and then to truncate the system to the low nice-slice energy modes. Let us postulate string field theory as the high-energy theory, and consider the structure of the low-energy nice-slice theory derived from it. The set of low-energy nice-slice degrees of freedom certainly contains the set of fields obtained by smearing the low-mass component fields of the string field theory with appropriate test functions. Let us see what else might enter the theory. One requirement we must impose is that the operator algebra of the low-energy theory is closed under commutation. Thus, consider the commutator of two tachyon fields. In open string field theory, the magnitude of this commutator (3.9) remains roughly constant in time, while in closed string field theory it grows like e^ω . The commutator couples strongly to the intermediate states that dominate the amplitude, and these states must be included in the low-energy nice-slice theory. Moreover, the Regge behavior of the amplitude (3.9) shows that these states consist of single strings stretched between x_1 and x_2 . The mass squared of these states grows like e^ω , and becomes trans-Planckian as ω increases. Nevertheless, the nice-slice energy of these states is low, so they cannot be excluded. The conclusion is that the low-energy nice-slice theory derived from string field theory must contain more than the usual low-mass fields. It must also contain highly nonlocal states of extremely massive strings stretched over macroscopic distances.

The assumption of approximate locality of the low-energy nice-slice theory is thus seen to be violated in string theory. The Hilbert space of the theory cannot be factorized into a product of a space of states inside the horizon with a space of states outside the horizon, and the argument for information loss breaks down.

It should be noted, of course, that we have in no way proved the conjecture of black hole complementarity. We have simply showed that the usual argument for information loss is no argument at all in light-front string theory. It is expected that information only begins to come out

of an evaporating black hole when the black hole area has reduced to half its original value, at a time of order $G^2 M^3$ [8]. Our perturbative calculation, however, is only valid for timescales of order $GM \ln(g)$ [14,15]. If we were to go to higher order in perturbation theory, the singularity would begin to manifest itself after a time of order $GM \ln(GM^2)$, and at present we do not have a theory which allowed us to deal with this problem.

Given the results obtained here, it is difficult to imagine how string theory could be formulated in terms of a $(D - 1)$ -dimensional set of local degrees of freedom, at least without incorporating an enormous amount of gauge symmetry. Further evidence for this view has been given previously by numerous authors [15–18].

ACKNOWLEDGMENTS

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APPENDIX A: THE COMMUTATOR OF OPEN BOSONIC STRING FIELDS

1. Calculation of the commutator

We employ the formalism of light-front open bosonic string field theory [19–22], and work in $(D = 26)$ -dimensional Minkowski spacetime with light-front coordinates

$$x^\pm = \frac{1}{\sqrt{2}}(x^0 \mp x^{D-1}). \quad (\text{A1})$$

The coordinate x^+ is the time coordinate, and in the light-front gauge we fix the string coordinate $X^+ = x^+$. The $D - 2$ transverse string coordinates are expanded as

$$\vec{X}(\sigma) = \vec{x} + 2 \sum_{l=1}^{\infty} \vec{x}_l \cos(l\sigma). \quad (\text{A2})$$

We are interested in the following commutator $[\Phi_H(1), \Phi_H(2)]$, where $\Phi_H(i) = \Phi_H(x_i^+, x_i^-, \vec{X}_i(\sigma))$ are light-front open bosonic string field operators in the Heisenberg picture. We will suppress gauge indices for the most part in the following. In the free theory, the commutator of mass eigenstate fields vanishes at spacelike separation, which is the kinematical situation we are interested in [23,24]. When string interactions are included, however, the commutator no longer vanishes. This was established in [25] for fields that are spacelike separated in the transverse direction but lie on the same light-front time slice. In the following, this result is generalized to spacelike separated fields on different light-front time slices by evaluating matrix elements of the commutator to leading nontrivial order in a perturbative expansion in the string coupling g .

The light-front Hamiltonian can be written as $H =$

$H_0 + \sum_{i=1}^{\infty} g^i V_i$, where H_0 is the Hamiltonian for noninteracting string fields, and the leading order interaction term V_1 is the standard cubic string coupling. In the interaction picture, the string fields have the expansion $\Phi_I = \Phi_a + \Phi_c$ with

$$\Phi_a(x^+, x^-, \vec{X}(\sigma)) = \int \frac{d^{D-2}p}{(2\pi)^{D-2}} \int_0^\infty \frac{dp^+}{4\pi p^+} \times \sum_{\{\vec{n}_i\}} e^{ip \cdot x} f_{\{\vec{n}_i\}}(\vec{x}_i) A(p^+, \vec{p}, \{\vec{n}_i\}), \quad (\text{A3})$$

where $p \cdot x = -p^- x^+ - p^+ x^- + \vec{p} \cdot \vec{x}$, and $\Phi_c = \Phi_a^\dagger$. The light-front energy of a string state is given by

$$p^-(p^+, \vec{p}, \{\vec{n}_i\}) = \frac{\vec{p}^2 + 2 \sum_{l,i} \ln_l^i + m_0^2}{2p^+}, \quad (\text{A4})$$

and the $f_{\{\vec{n}_i\}}(\vec{x}_i)$ are wave functions for the modes of transverse oscillation of the string. In our conventions, $\alpha' = \frac{1}{2}$. The mode operators obey the canonical commutation relations

$$\begin{aligned} & [A(p^+, \vec{p}, \{\vec{n}_i\}), A^\dagger(p'^+, \vec{p}', \{\vec{n}'_i\})] \\ &= 2p^+ (2\pi)^{D-1} \delta(p^+ - p'^+) \delta^{D-2}(\vec{p} - \vec{p}') \delta_{\{\vec{n}_i\}, \{\vec{n}'_i\}}. \end{aligned} \quad (\text{A5})$$

Consider the matrix element

$$M(1, 2; 3) = \langle 0 | [\Phi_H(1), \Phi_H(2)] | 3 \rangle, \quad (\text{A6})$$

where $|0\rangle$ is the vacuum state and $|3\rangle$ is a spectator state, which is necessary in order to have a nonvanishing contribution at first order in the string interaction. Written in the interaction picture, the matrix element (A6) becomes

$$\begin{aligned} M(1, 2; 3) &= {}_I \langle 0 | \Phi_I(1) U_I(x_1^+, x_2^+) \Phi_I(2) | 3; x_2^+ \rangle_I \\ &\quad - (1 \leftrightarrow 2) \end{aligned} \quad (\text{A7})$$

where U_I is the interaction picture time evolution operator. Using the Feynman-Dyson expansion for U_I , M can be expanded in powers of the string coupling g as $M = \sum_{i=0}^{\infty} g^i (M^{(i)})$. The zeroth order term is simply a matrix element of the commutator of two interaction picture string fields,

$$M^{(0)}(1, 2; 3) = \langle 0 | [\Phi_I(1), \Phi_I(2)] | 3 \rangle, \quad (\text{A8})$$

which vanishes for the kinematics we are interested in [23,24].

Consider the next term in the expansion for M . It will prove convenient to choose $|3\rangle$ to be an eigenstate of H_0 . After some algebra, the first order term can be written

$$M^{(1)}(1, 2; 3) = ig \int_{x_1^+}^{x_2^+} dx^+ \langle 0 | \{ \Phi_I(1) V_1(x^+) \Phi_I(2) + \Phi_I(2) V_1(x^+) \Phi_I(1) - \Phi_I(1) \Phi_I(2) V_1(x^+) \} | 3 \rangle. \quad (\text{A9})$$

Separating the vertex into terms with given numbers of creation and annihilation operators $V_1 = V_{1aaa} + V_{1aac} + V_{1acc} + V_{1ccc}$, one finds that the matrix element is nonvanishing only when the spectator is a single string state,

$$|3\rangle = A^\dagger(p_3^+, \vec{p}_3, \{\vec{n}_{l,3}\}) |0\rangle, \quad (\text{A10})$$

and the only terms which contribute are

$$\langle 0 | \{ \Phi_a(1) V_{1aac}(x^+) \Phi_c(2) + \Phi_a(2) V_{1acc}(x^+) \Phi_c(1) - \Phi_a(1) \Phi_a(2) V_{1acc}(x^+) \} | 3 \rangle. \quad (\text{A11})$$

The leading order contribution to the matrix element (A9) with a single string spectator is given by

$$\begin{aligned} M^{(1)}(1, 2; 3) &= (2\pi)^{D-1} g \left(\prod_{r=1}^2 \int \frac{d^{D-2}p_r}{(2\pi)^{D-2}} \int_{-\infty}^{\infty} \frac{d\alpha_r}{4\pi|\alpha_r|} \sum_{\{\vec{n}_{l,r}\}} f_{\{\vec{n}_{l,r}\}}(\{\vec{x}_{l,r}\}) \right) \\ &\quad \times F(\alpha_1, \alpha_2) \left(\sum_{r=1}^3 p_r^- \right)^{-1} \left[\exp\left(-ix_1^+ \sum_{r=1}^3 p_r^-\right) - \exp\left(-ix_2^+ \sum_{r=1}^3 p_r^-\right) \right] \\ &\quad \times \exp\left(i \sum_{r=1}^2 p_r \cdot x_r\right) \delta\left(\sum_{r=1}^3 \alpha_r\right) \delta^{D-2}\left(\sum_{r=1}^3 \vec{p}_r\right) [\tilde{V}_1(1; 2; 3) + \tilde{V}_1(2; 1; 3)], \end{aligned} \quad (\text{A12})$$

where \tilde{V}_1 is the three-string vertex in momentum space and in the occupation number basis $\{\vec{n}_{l,r}\}$. In the above, we have defined $|\alpha_r| = 2p_r^+$ and $p_r^- = (\vec{p}_r^2 + m_r^2)/\alpha_r$. The sign of α_r is positive (negative) for incoming (outgoing) strings. We have also defined

$$F(\alpha_1, \alpha_2) = \Theta(\alpha_1)\Theta(-\alpha_2) + \Theta(-\alpha_1)\Theta(\alpha_2) - \Theta(-\alpha_1)\Theta(-\alpha_2), \quad (\text{A13})$$

where Θ is the Heaviside function, the three terms corresponding to the three terms in the matrix element (A11).

The first question we want to address is whether this matrix element vanishes when the string fields in the commutator are at spacelike separation, as it would, for example, if we were dealing with a local field theory of scalar fields. For this purpose, let us specialize to the case of tachyon component fields and a tachyon spectator state. This simplifies the analysis considerably, as the momentum space representation of the three tachyon vertex reduces to

$$\tilde{V}_1(\alpha_1, \vec{p}_1; \alpha_2, \vec{p}_2; \alpha_3, \vec{p}_3) = \exp\left(\frac{\tau_0}{2} \sum_{r=1}^3 p_r^-\right), \quad (\text{A14})$$

where $\tau_0 = \sum_{r=1}^3 \alpha_r \ln|\alpha_r|$. We can then write

$$\begin{aligned} \langle 0|[T(x_1), T(x_2)]|3\rangle &= 2(2\pi)^{D-1} g \left(\prod_{r=1}^2 \int \frac{d^{D-2} p_r}{(2\pi)^{D-2}} \int_{-\infty}^{\infty} \frac{d\alpha_r}{4\pi|\alpha_r|} \right) F(\alpha_1, \alpha_2) \\ &\times \left(\sum_{r=1}^3 p_r^- \right)^{-1} \left[\exp\left(-ix_1^+ \sum_{r=1}^3 p_r^-\right) - \exp\left(-ix_2^+ \sum_{r=1}^3 p_r^-\right) \right] \\ &\times \delta\left(\sum_{r=1}^3 \alpha_r\right) \delta^{D-2}\left(\sum_{r=1}^3 \vec{p}_r\right) \exp\left(\frac{\tau_0}{2} \sum_{r=1}^3 p_r^-\right) e^{ip_1 \cdot x_1 + ip_2 \cdot x_2}. \end{aligned} \quad (\text{A15})$$

If the cubic string vertex (A14) were polynomial in the longitudinal momenta α_r , the matrix element (A15) would vanish for $x_1 - x_2$ spacelike, by the usual contour deformation argument [26]. The vertex is not polynomial, however, so the usual cancellation between terms fails, leaving behind a nonvanishing answer for the matrix element.

This is an important sign that the nice-slice argument may fail in light-front string theory. We must, however, do more work to establish this result. On the one hand, we need to show that the commutator is significantly different from zero for spacelike separated fields on a nice slice, while at the same time this effect should be very much suppressed under the kinematic conditions found in everyday experiments at low energies. These issues will each be addressed in what follows.

2. Correspondence with low-energy Minkowski field theory

Local quantum field theory in Minkowski space provides a very good description of the low-energy world we observe, and any unified theory should reproduce the structure of local Minkowski field theory for kinematic situations appropriate to low-energy Minkowski observers. This structure includes the requirement that the commutator of gauge-invariant local fields must vanish at spacelike separation. In this section, we show that the low-energy fields obtained from light-front string field theory satisfy this requirement.

Consider a Minkowski observer, whose measuring apparatus is sensitive to frequencies of order $E \ll 1/\sqrt{\alpha'}$. Such an observer will describe physics by a set of low-energy fields $\hat{\Phi}_A(x)$, which can be obtained from the component fields ϕ_A of light-front string field theory by integrating $\phi_A(y)$ against a test function $f(x-y)$. The Fourier components $\tilde{f}(q)$ of f have support only for $q^\mu \lesssim E$.

Let us consider the vector field, which clearly enters the low-energy Minkowski theory. The interaction picture vector field is expanded as

$$A_\mu^b(x) = \sum_{\lambda=1}^{24} \int \frac{d^{D-2} p}{(2\pi)^{D-2}} \int \frac{dp^+}{4\pi p^+} [\epsilon_\mu(\lambda) a(p^+, \vec{p}, \lambda, b) e^{ip \cdot x} + \text{H.c.}], \quad (\text{A16})$$

where the $\epsilon_\mu(\lambda)$ are polarization vectors which correspond to the polarization states λ and b is the relevant group index. In the light-front gauge, $A^+ = 0$, and A^- is expressed as a nonlocal function of the $D-2$ transverse components A^i . The commutation relations for A^- will therefore be nonlocal even in free field theory. For this reason, we restrict ourselves to commutators of the transverse fields, which do vanish in free field theory. The matrix element involving massless vectors can now be calculated using Eq. (A12), and we obtain

$$\begin{aligned} \langle 0|[\hat{A}_i^a(x_1), \hat{A}_j^b(x_2)]|3, c\rangle &= (2\pi)^{D-1} g f^{abc} \left(\prod_{r=1}^2 \int \frac{d^{D-2} p_r}{(2\pi)^{D-2}} \int_{-\infty}^{\infty} \frac{d\alpha_r}{4\pi|\alpha_r|} \right) \\ &\times \frac{F(\alpha_1, \alpha_2)}{\sum_{r=1}^3 p_r^-} \delta\left(\sum_{r=1}^3 \alpha_r\right) \delta^{D-2}\left(\sum_{r=1}^3 \vec{p}_r\right) \tilde{V}_1(1, i; 2, j; 3) e^{ip_1 \cdot x_1 + ip_2 \cdot x_2} \\ &\times \left[\tilde{f}_1(-p_2 - p_3) \tilde{f}_2(p_2) \exp\left(-ix_1^+ \sum_{r=1}^3 p_r^-\right) - \tilde{f}_1(p_1) \tilde{f}_2(-p_1 - p_3) \exp\left(-ix_2^+ \sum_{r=1}^3 p_r^-\right) \right]. \end{aligned} \quad (\text{A17})$$

Consider the first term in the square brackets. The function \tilde{f}_2 constrains $p_2^\mu \sim E$, and the function \tilde{f}_1 constrains the sum $p_2^\mu + p_3^\mu \sim E$, which implies that $p_3^\mu \sim E$ as well. A similar argument holds for the second term. In other words, the commutator of two low-energy operators is itself a low-energy operator, and does not couple to states of high energy. Therefore, for the matrix element (A17) to be non-negligible, the state $|3, c\rangle$ must be a low-energy state, which we will select to be a vector boson polarized along the k direction. The three vector boson vertex is

$$\tilde{V}_1(1, i; 2, j; 3, k) = \left[\delta_{ij} \frac{\mathcal{P}_k}{\alpha_3} + \delta_{jk} \frac{\mathcal{P}_i}{\alpha_1} + \delta_{ki} \frac{\mathcal{P}_j}{\alpha_2} + 2\alpha' \frac{\mathcal{P}_i \mathcal{P}_j \mathcal{P}_k}{\alpha_1 \alpha_2 \alpha_3} \right] \exp\left(\frac{\tau_0}{2} \sum_{r=3}^3 p_r^-\right), \quad (\text{A18})$$

where $\vec{\mathcal{P}} = \alpha_1 \vec{p}_2 - \alpha_2 \vec{p}_1$ is cyclically symmetric. We can now substitute this expression into Eq. (A17).

Since the test functions restrict the allowed momenta to be of order $E \ll 1/\sqrt{\alpha'}$, we keep only the leading terms in α' . The only dependence on α' comes from the vertex (A18), both from the explicit dependence shown and through τ_0 , which is properly written

$$\tau_0 = \alpha' \sum_{r=3}^3 \alpha_r \ln(\alpha' \alpha_r^2). \quad (\text{A19})$$

Dropping the test functions, the leading term of the commutator is

$$\begin{aligned} \langle 0 | [\hat{A}_i^a(x_1), \hat{A}_j^b(x_2)] | 3, c, k \rangle &= (2\pi)^{D-1} g f^{abc} \left(\prod_{r=1}^2 \int \frac{d^{D-2} p_r}{(2\pi)^{D-2}} \int_{-\infty}^{\infty} \frac{d\alpha_r}{4\pi|\alpha_r|} \right) \\ &\times \frac{F(\alpha_1, \alpha_2)}{\sum_{r=1}^3 p_r^-} \delta\left(\sum_{r=1}^3 \alpha_r\right) \delta^{D-2}\left(\sum_{r=1}^3 \vec{p}_r\right) e^{ip_1 \cdot x_1 + ip_2 \cdot x_2} \\ &\times \left[\delta_{ij} \frac{\mathcal{P}_k}{\alpha_3} + \delta_{jk} \frac{\mathcal{P}_i}{\alpha_1} + \delta_{ki} \frac{\mathcal{P}_j}{\alpha_2} \right] \left[\exp\left(-ix_1^+ \sum_{r=1}^3 p_r^-\right) - \exp\left(-ix_2^+ \sum_{r=1}^3 p_r^-\right) \right], \quad (\text{A20}) \end{aligned}$$

which is the same result as that obtained from Yang-Mills theory in the light-front gauge.

The commutator (A20) does not vanish for $x_1 - x_2$ spacelike, because the integrand is nonpolynomial in the longitudinal momenta α_r . This does not violate the rules of local field theory, however, because the fields A_i^a are not gauge invariant. It is only the commutators of gauge invariant local fields, such as $\text{tr} F^2$, which are required to vanish when the arguments of the fields are spacelike related. This ensures that a gauge invariant signal cannot propagate faster than the speed of light. What we have shown is that the commutator of low-energy gauge fields obtained from light-front string field theory is exactly the same as that predicted by low-energy field theory. In other words, the commutator of low Minkowski energy fields does not acquire any *additional* nonlocality in string theory. Therefore, one may construct the usual gauge invariant fields from the A_μ^a , and they will commute at spacelike separations.

Light-front string field theory has thus passed an important test. Had we found that light-front string field theory introduces additional nonlocality into the commutator of low-energy fields, it would have indicated a catastrophic breakdown in the low-energy predictions of the theory, and the commutator (and perhaps the theory itself) could not be taken seriously. As it stands, the theory avoids introducing any additional nonlocality at low energy, and we can proceed to study the predictions it makes for higher energy.

3. The tachyon commutator on a nice slice

In Eq. (A15), we considered the overlap of the state $[T(x_1), T(x_2)]|0\rangle$ with a single tachyon state. We would now like to consider the behavior of this matrix element when the fields $T(x_1)$ and $T(x_2)$ are replaced with low-energy nice-slice fields $\hat{T}(x_1)$ and $\hat{T}(x_2)$. Following an analysis similar to that in Appendix A 2, it is easy to see that the test functions defined by Eqs. (3.2) and (3.3) restrict $p_2^- \sim E$, as before, but now $p_3^- \sim \alpha'^{-1/2} e^\omega$. The momentum conservation δ functions restrict α_3 and p_3^+ to be of order E , so the only way to make p_3^- large is to have a very large mass squared. Therefore, the three tachyon amplitude goes to zero quickly as ω increases.

This does not mean that the commutator of two tachyons is “small” but simply that its overlap with single tachyon states is. This is not surprising in view of our expectation that the commutator creates a high-mass extended string state from the vacuum. The aim of Appendix A 1 was only to establish that the commutator of string fields can be nonvanishing at spacelike separation and we focused on the case of a tachyon spectator for simplicity. In Appendix B we will evaluate matrix elements which involve a sum over spectator states and should therefore pick up the dominant overlap with the commutator.

APPENDIX B: THE MAGNITUDE OF THE COMMUTATOR

1. A four point amplitude involving commutators

In this appendix we present the calculation of the matrix element

$$C(4; 3; 2; 1) = \langle 0 | [\Phi_H(4), \Phi_H(3)] [\Phi_H(2), \Phi_H(1)] | 0 \rangle. \quad (\text{B1})$$

As before, we will use perturbation theory to calculate C to leading nontrivial order in g . Inserting a complete set of states $\{|\gamma\rangle\}$, Eq. (B1) can be written

$$C(4; 3; 2; 1) = \sum_{\gamma} M(4, 3; \gamma) M(1, 2; \gamma)^*, \quad (\text{B2})$$

where M is given by Eq. (A6). The first term which does not vanish identically when the centers of mass of the string fields are spacelike related is the second order term

$$C^{(2)}(4; 3; 2; 1) = \sum_{\gamma} M^{(1)}(4, 3; \gamma) M^{(1)}(1, 2; \gamma)^*. \quad (\text{B3})$$

It was stated in Appendix A that $M^{(1)}(1, 2; \gamma)$ vanishes unless $|\gamma\rangle$ is a single string state. Thus we can perform the sum, using the earlier results (A9) and (A11). This leads to a sum of nine terms. Any one of these terms can be isolated by integrating the expression against appropriate functions of the longitudinal momenta, so they cannot cancel one another. We shall present the calculation for only one of those terms, the term in which $\alpha_r > 0$ for all r :

$$C^{(2)}(4; 3; 2; 1) = g^2 \int_{x_4^+}^{x_3^+} dx'^+ \int_{x_1^+}^{x_2^+} dx''^+ \langle 0 | \Phi_a(4) \Phi_a(3) V_{1acc}(x'^+) V_{1aac}(x''^+) \Phi_c(1) \Phi_c(2) | 0 \rangle. \quad (\text{B4})$$

Transforming to a momentum and occupation number representation [the measure being as in the first line of (A12)], the matrix element is

$$\tilde{C}^{(2)}(4; 3; 2; 1) = g^2 \int_{x_4^+}^{x_3^+} dx'^+ \int_{x_1^+}^{x_2^+} dx''^+ \langle 0 | A(4) A(3) V_{1acc}(x'^+) V_{1aac}(x''^+) A^\dagger(1) A^\dagger(2) | 0 \rangle. \quad (\text{B5})$$

The time and operator orderings in the matrix element (B5) differ, but we may calculate first the Euclidean time ordered matrix element and obtain the needed Minkowski result by analytic continuation. The fields will be at Euclidean times σ_r^0 and the interactions at σ'^0, σ''^0 , with $\sigma_{3,4}^0 > \sigma'^0 > \sigma''^0 > \sigma_{1,2}^0$. The matrix element in Eq. (B5) is then given by

$$\left(\prod_{s=1}^4 2\alpha_s^+ \right)^{1/2} 2\pi \delta(\alpha_1 + \alpha_2 - \alpha_3 - \alpha_4) \mathcal{P}(1, 2, 3, 4; \sigma'^0, \sigma''^0), \quad (\text{B6})$$

where \mathcal{P} is the functional integral on the string world sheet shown in Fig. 3 (the other cyclic ordering, 1243, does not contribute to the Regge behavior of interest).

It is convenient to take the external times $|\sigma_r^0|$ to be large. The inverse of the mapping

$$\rho(z) = \sigma^0 + i\sigma^1 = \alpha_1 \ln(z) + \alpha_2 \ln(z-x) - \alpha_3 \ln(1-z) + \bar{\rho} \quad (\text{B7})$$

then takes the string world sheet Σ to the upper half z plane minus four small semicircles surrounding the points $0, x, 1$, and ∞ . The parameters x and $\bar{\rho}$ in the mapping are determined implicitly in terms of the interaction times σ'^0, σ''^0 by the condition that $\partial_z \rho = 0$ at the interaction points:

$$0 = \frac{\alpha_1}{z} + \frac{\alpha_2}{z-x} + \frac{\alpha_3}{1-z}. \quad (\text{B8})$$

Solving this (quadratic) equation for z', z'' , the equations $\rho(z') = \sigma'^0 + i\pi\alpha_3$, $\rho(z'') = \sigma''^0 + i\pi\alpha_2$ give two relations between σ'^0, σ''^0 and $x, \bar{\rho}$. Note that the difference of these relates x to $\delta = \sigma'^0 - \sigma''^0$. Strings 1, 2, 3, 4 are mapped to the semicircles, with respective radii

$$\begin{aligned} r_1 &= x^{-\alpha_2/\alpha_1} e^{(\sigma_1^0 - \bar{\rho})/\alpha_1}, \\ r_2 &= x^{-\alpha_1/\alpha_2} (1-x)^{\alpha_3/\alpha_2} e^{(\sigma_2^0 - \bar{\rho})/\alpha_2}, \\ r_3 &= (1-x)^{\alpha_2/\alpha_3} e^{(\bar{\rho} - \sigma_3^0)/\alpha_3}, \\ r_4 &= e^{(\bar{\rho} - \sigma_4^0)/\alpha_4}, \end{aligned} \quad (\text{B9})$$

r_4 being the radius in the universe coordinate z^{-1} . Then

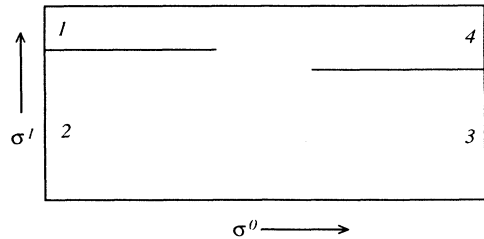


FIG. 3. String diagram for four-point amplitude.

$$\begin{aligned} \mathcal{P}(1, 2, 3, 4; \sigma'^0, \sigma''^0) &= J(\alpha_s, \sigma_s^0, \sigma'^0, \sigma''^0) \mathcal{P}'(1, 2, 3, 4; \sigma'^0, \sigma''^0) \\ &= J(\alpha_s, \sigma_s^0, \sigma'^0, \sigma''^0) \left\langle \prod_{s=1}^4 r_s^{h_s} \mathcal{V}_s \right\rangle. \end{aligned} \quad (\text{B10})$$

Here \mathcal{P}' is the path integral on the upper half-plane minus semicircles and J is the determinant from the conformal mapping. In the second line the small semicircles have been replaced by the corresponding vertex operators. The semicircles correspond to vertex operators renormalized at scale r_s ; the factor $r_s^{h_s}$ with h_s the weight of \mathcal{V}_s relates these to normal ordered vertex operators. It can be thought of as arising from the radial evolution from radius r to a standard radius 1. To simplify the analysis, we will consider the case of four tachyon component fields, for which Eq. (B10) is

$$\mathcal{P}(1, 2, 3, 4; \sigma'^0, \sigma''^0) = J(\alpha_s, \sigma_s^0, \sigma'^0, \sigma''^0) (2\pi)^{D-2} \delta^{D-2}(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4) x^{\vec{p}_1 \cdot \vec{p}_2} (1-x)^{-\vec{p}_2 \cdot \vec{p}_3} \prod_{s=1}^4 r_s^{\vec{p}_s^2/2}. \quad (\text{B11})$$

The determinant J can now be determined by comparison with the on-shell four-point function, using the fact that J is independent of the transverse momenta:

$$J(\alpha_s, \sigma_s^0, \sigma'^0, \sigma''^0) = \left(\prod_{s=1}^4 2\alpha_s^+ \right)^{-1/2} \frac{dx}{d\delta} x^{\alpha_1/\alpha_2 + \alpha_2/\alpha_1} (1-x)^{-\alpha_3/\alpha_2 - \alpha_2/\alpha_3} \exp \left\{ \sum_{s=1}^4 (-\sigma_s^0 + \bar{\rho}) \theta_s / \alpha_s \right\}, \quad (\text{B12})$$

with $\theta_s = +1$ for $s = 1, 2$ and $\theta_s = -1$ for $s = 3, 4$. The first factor cancels the noncovariant factor in the matrix element (B6).

The result for the matrix element (B5) is

$$\begin{aligned} \tilde{C}^{(2)}(4; 3; 2; 1) &= g^2 (2\pi)^{D-1} \delta(\alpha_1 + \alpha_2 - \alpha_3 - \alpha_4) \delta^{D-2}(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4) \\ &\times \int_{x_4^+}^{x_3^+} dx'^+ \int_{x_1^+}^{x_2^+} dx''^+ \frac{dx}{d\delta} x^{p_1 \cdot p_2} (1-x)^{-p_2 \cdot p_3} \exp \left\{ \sum_{s=1}^4 (ix_s^+ - \bar{\rho}) p_s^- \theta_s \right\}. \end{aligned} \quad (\text{B13})$$

The straightforward continuation $\sigma_s^0 \rightarrow ix_s^+$ has been carried out. The continuation $\sigma'^0, \sigma''^0 \rightarrow ix'^+, ix''^+$ is also implicit; this makes x and $\bar{\rho}$ complex. The matrix element (B13) cannot be evaluated in closed form. Solving the conformal mapping to express all quantities in terms of x and σ'^0 , the integral over σ'^0 can be carried out but the result is a rather complicated integral over x , with end points that are determined only implicitly. However, the important behavior will be determined simply from scaling.

2. The behavior of the commutator of nice-slice fields

Low-energy nice-slice fields are defined as in Sec. III. As before, we consider the case of four tachyon component fields. We treat the mass squared of the tachyon as a small positive parameter; we can do this, for example, by giving the external tachyons momenta in a complicated direction. This is ultimately justified by considering superstring theory where the calculations will yield essentially the same qualitative results. The commutator in free superstring field theory was studied in Ref. [24] where further discussion of this issue can be found.

We smear the fields $T(1)$ and $T(4)$ with test functions appropriate to position y_1 , and the fields $T(2)$ and $T(3)$ with test functions appropriate to position y_2 . The am-

plitude under consideration is thus

$$\hat{C} = \langle 0 | [\hat{T}(y_1), \hat{T}(y_2)] [\hat{T}(y_2), \hat{T}(y_1)] | 0 \rangle. \quad (\text{B14})$$

This is the position-space matrix element $C(4; 3; 2; 1)$ folded into

$$\begin{aligned} &f(e^\omega(x_1^+ - y_1^+), e^{-\omega}(x_1^- - y_1^-), \vec{x}_1 - \vec{y}_1) f(x_2 - y_2) \\ &\times f(x_3 - y_2) f(e^\omega(x_4^+ - y_1^+), e^{-\omega}(x_4^- - y_1^-), \vec{x}_4 - \vec{y}_1). \end{aligned} \quad (\text{B15})$$

Thus,

$$\alpha_1, \alpha_4, x_1^+, x_4^+, \propto e^{-\omega} \quad (\text{B16})$$

with $\alpha_2, \alpha_3, x_2^+, x_3^+$ and the transverse quantities approaching constants. It is straightforward to read off the scaling from the result (B13). The momenta p_1^- and p_4^- are large, $O(e^\omega)$, so the phase factor is highly oscillatory and damps the integral unless

$$x'^+ - x_4^+ = O(e^{-\omega}), \quad x''^+ - x_1^+ = O(e^{-\omega}). \quad (\text{B17})$$

This in turn implies that $\delta, \bar{\rho}$, and $1-x$ are $O(e^{-\omega})$, while $dx/d\delta$ approaches a constant. The measures for the convolution and the Fourier transform are covariant and so constant under the scaling. The only scaling then

comes from the ranges of the time integrals and the factor $(1-x)^{-p_2 \cdot p_3}$, with the result

$$\hat{C} \sim e^{\omega(p_2 \cdot p_3 - 2)} = e^{\omega t/2} = e^{\omega(\alpha(t)-1)} \quad (\text{B18})$$

in terms of $t = -(p_2 - p_3)^2$ and the tachyon Regge trajectory $\alpha(t) = \frac{1}{2}t + 1$. Now $p_2 - p_3$ is in general a spacelike vector, so $t < 0$, but in all cases $|t| \ll 1/\alpha' = 2$, so the magnitude of the commutator stays essentially constant as $\omega \rightarrow \infty$. This behavior is completely different than that exhibited by the commutator of two local scalar fields, which vanishes when the fields are spacelike separated.

3. Extension to closed bosonic strings and superstrings

Our calculation was only carried out at tree level, in part because the inconsistency of bosonic string theory prevents a further analysis. Since gravity does not appear in the open bosonic string at tree level, we do not expect to see gravitational effects in the commutator calculated above. The closed bosonic string does contain gravity at tree level, however, so it is of interest to repeat the calculation for this case. The result is easily anticipated: since the amplitude is dominated by the Regge behavior of the strings, we simply need to replace the open string Regge trajectory $\alpha(s) = \frac{1}{2}s + 1$ by the closed string trajectory $\alpha(s) = \frac{1}{2}s + 2$. Explicit calculation of this amplitude shows that this is indeed correct. For the closed string,

then, the amplitude actually grows like e^ω . Superstrings exhibit the same behavior, although the calculation is considerably more complicated.

4. A limiting case

For completeness we evaluate the commutator (B13) in a limit where it is possible to obtain an analytic expression, namely $\alpha_1, \alpha_4 \rightarrow 0$ with other quantities held fixed. Solving for the interaction points gives

$$z' = \frac{\alpha_2(1-x)}{\alpha_4} + O(\alpha_{1,4}^0), \quad z'' = \frac{\alpha_1 x}{\alpha_2(1-x)} + O(\alpha_{1,4}^2),$$

$$\sigma'^0 = \bar{\rho} - \alpha_4 \ln \frac{\alpha_4}{\alpha_2 e(1-x)} + O(\alpha_{1,4}^2), \quad (\text{B19})$$

$$\sigma''^0 = \bar{\rho} + \alpha_2 \ln x + \alpha_1 \ln \frac{\alpha_1 x}{\alpha_2 e(1-x)} + O(\alpha_{1,4}^2),$$

$$x = e^{-\delta/\alpha_2} + O(\alpha_{1,4}).$$

We have kept some second order terms which are needed because some of the exponents in the commutator are of order $\alpha_{1,4}^{-1}$. Inserting the values (B19) into the commutator (B13) gives

$$\begin{aligned} \tilde{C}^{(2)}(4; 3; 2; 1) &= g^2 (2\pi)^{D-1} \delta(\alpha_1 + \alpha_2 - \alpha_3 - \alpha_4) \delta^{D-2}(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4) \\ &\quad \times e^{2 - (\vec{p}_1^2 + \vec{p}_4^2)/2} \alpha_2^{1 - (\vec{p}_1^2 + \vec{p}_4^2)/2} \alpha_1^{-1 + \vec{p}_1^2/2} \alpha_4^{-1 + \vec{p}_4^2/2} \\ &\quad \times \int_{x_4^+}^{x_3^+} dx'^+ \int_{x_1^+}^{x_2^+} dx''^+ \{ x^{-1 + (\vec{p}_1 + \vec{p}_2)^2/2} (1-x)^{-\vec{p}_1 \cdot \vec{p}_4} \\ &\quad \times e^{ip_1^- (x_1^+ - x''^+) + ip_2^- (x_2^+ - x''^+) - ip_3^- (x_3^+ - x'^+) - ip_4^- (x_4^+ - x'^+)} \}. \end{aligned} \quad (\text{B20})$$

The light-cone energies p_1^-, p_4^- are $O(\alpha_{1,4}^{-1})$ and so the time integrals are highly oscillatory and are dominated by the lower end points. Thus,

$$\begin{aligned} \tilde{C}^{(2)}(4; 3; 2; 1) &= g^2 (2\pi)^{D-1} \delta(\alpha_1 + \alpha_2 - \alpha_3 - \alpha_4) \delta^{D-2}(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4) \\ &\quad \times e^{2 - (\vec{p}_1 - \vec{p}_4)^2/2} \alpha_2^{1 - (\vec{p}_1^2 + \vec{p}_4^2)/2} \alpha_1^{\vec{p}_1^2/2} \alpha_4^{\vec{p}_4^2/2} \frac{(ix_4^+ - ix_1^+)^{-\vec{p}_1 \cdot \vec{p}_4}}{(1 - \vec{p}_1^2/2)(1 - \vec{p}_4^2/2)} e^{ip_2^- (x_2^+ - x_1^+) - ip_3^- (x_3^+ - x_4^+)}. \end{aligned} \quad (\text{B21})$$

Under simultaneous scaling of $\alpha_{1,4}$ and $x_{1,4}^+$ one recovers the behavior found previously.

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