Random surface representation for Einstein quantum gravity

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(Received 6 September 1994; revised manuscript received 28 April 1995)

We propose a random surface functional solution for the Wheeler-DeWitt quantum Einstein gravity constraint in the Ashtekar-Sen field coordinates.

PACS number(s): 04.60.Ds, 11.15.Tk, 11.25.Sq

I. INTRODUCTION

The Ashtekar-Sen proposal of a new set of complex SU(2) coordinates for the Einstein action [1] has become very promising at the quantum level by allowing explicit formal loop space solutions for the Wheeler-DeWitt equation without a cosmological term [2]. However, string structure from these loop space solutions was not found, as opposed to the loop space framework for Yang-Mills confining gauge theories [3,4,6,10].

In this paper we propose to overcome this problem by solving directly the Wheeler-DeWitt equation with a zero cosmological constant by means of a string theory functional integral possessing intrinsic SU(2) color degrees of freedom [3,6]. We show in Sec. II that this proposed random surface functional satisfies the Wheeler-DeWitt equation and the diffeomorphism constraint. In Sec. III we present a topological wave equation for this quantum gravity string theory inspired in our previous work in bag representations for QCD [SU(∞)] [3]. Finally, in Appendix A and Appendix B we clarify some calculations presented in the bulk of this paper.

II. THE RANDOM SURFACE WAVE FUNCTIONAL

Let us start our analysis by considering the problem of associating a wave functional for an arbitrary self-intersecting random surface S with boundary $C_{xx} = \{C_\mu(\sigma),\ 0 \le \sigma \le 2\pi,\ C_\mu(0) = C_\mu(2\pi) = x\}$ and possessing SU(2) color degrees of freedom interacting with an external SU(2) connection $A^i_\mu(x)\lambda_i$. Here λ_i denote the SU(2) generators in the fundamental representation.

The surface S is characterized by two fields: first, by the usual (bosonic) vector position $X_\mu(\xi)$, $\xi \in D$, where D is the appropriate parameter associated to the surface $X_\mu(D) = S$. In addition, we have the surface SU(2) color variable $g(\xi)$ which belongs to the fundamental SU(2) group. The intrinsic metric properties of S are represented by two-dimensional (2D) metric fields $h_{ab}(\xi)$ in Polyakov's formalism for random surfaces [4].

The classical action for this color SU(2) surface is given in Polyakov's formalism [4]:

$$S = S_0 + S_1^{(B)} \tag{1}$$

with

$$S_0 = \tfrac{1}{2}\int_D d^2\xi(\sqrt{h}h^{ab}\partial_a X^\mu \partial_b X_\mu)(\xi) + \mu^2 \int_D d^2\xi(\sqrt{h})(\xi)\ , \tag{1a}$$

$$S_1^{(B)} = \frac{1}{4\pi M}\int_D d^2\xi[\sqrt{h}\,\mathrm{Tr}^{(c)}(g^{-1}\partial_a g)^2](\xi) + 4\pi i \Gamma_{\mathrm{WZ}}[g]\ , \tag{1b}$$

where $\Gamma_{\mathrm{WZ}}[g]$ denotes the two-dimensional Wess-Zumino functional. Its existence, together with the integer m in the written SU(2) σ model, allows us to consider the more suitable fermionic equivalent action for $S_1^{(B)}$:

$$S_1^{(F)} = \int_D [\sqrt{h}\overline{\psi}(i\gamma^a\nabla_a)\psi](\xi)d^2\xi\ , \tag{2}$$

where the two-dimensional Dirac field $\psi(\xi)$ belongs to the fermionic fundamental SU(2) representation.

At this point, the simplest action taking into account the interaction with the external Ashtekar-Sen connection is given by

$$S^{\mathrm{int}}[\psi(\xi),\overline{\psi}(\xi);A_\mu(x)] = ie\int_D (\sqrt{h}\overline{\psi}[\gamma^a\partial_a X^\mu A^l_\mu(X^\chi)\lambda_l]\psi)(\xi)d^2\xi\ . \tag{3}$$

It is instructive to point out that the interaction Eq. (3) written in terms of the bosonic SU(2) variable $g(\xi)$ was presented in Ref. [5].

The complete classical interacting action Eqs. (1)–(3) is invariant under the gauge transformations

$$A_\mu(X^\chi(\xi)) \to (l^{-1}A_\mu l + l^{-1}\partial_\mu l)[X^\chi(\xi)]\ ,$$

$$\psi(\xi) \to l[X^\chi(\xi)]\psi(\xi)\ , \tag{4}$$

$$\overline{\psi}(\xi) \to \overline{\psi}(\xi)l^{-1}[X^\chi(\xi)]\ ,$$

where $l(x) \in$SU(2).

We shall now use Eqs. (1) and (3) to propose the following random surface fermionic functional integral as a surface Wilson loop [3,6]:

$$W_{ab}[C_{xx}, S, A_\mu] = \mathrm{Tr}^{(c)}\left\{\int D^c[\psi(\xi)]D^c[\overline{\psi}(\xi)][\psi_a(0,0)\overline{\psi}_b(2\pi,0)] \times \exp[-(S_0 + S_1^{(F)} + S^{\mathrm{int}})]\right\} . \tag{5}$$

Notice that our random surface phase factor proposed above is a 2×2 matrix in the flat domain $D(a,b=1,2)$, since it is a kind of *two-dimensional spinor propagator on D.*

The covariant fermion functional integral is defined by the functional element of volume associated with the following functional Riemann metric with the fermion fields satisfying the Neumann boundary condition:

$$||\delta\psi||^2 = \int_D [\sqrt{h}(\delta\psi\delta\overline{\psi})](\xi)d^2\xi . \tag{6}$$

The quantum surface functional will be defined by the Nambu-Goto functional integral over the $X_\mu(\xi)$ variables as written in Ref. [7], Eq. (23), with the Dirichlet boundary condition $\partial S = C_{xx}$ and in the orthonormal surface coordinates:

$$\sum_{C_{xx}}\left\{\sum_{\{S\};\partial S=C_{xx}} W_{ab}[S, C_{xx}, A_\mu]\right\} = \Phi_{ab}[A_\mu] , \tag{7}$$

where

$$\sum_{C_{xx}} \equiv \int_{C_\mu(0)=C_\mu(2\pi)=x_\mu} D^F[C_\mu(\sigma)] \times \exp\left(-\tfrac{1}{2}\int_0^{2\pi} \dot{C}^\mu(\sigma)^2 d\sigma\right) \tag{7a}$$

is the *x-dependent* loop average and

$$\sum_{\{S\},\partial S=C_{xx}} = \langle\,\rangle = \int_{X^\mu(\sigma,0)=C^\mu(\sigma)}\left(\prod_{(\xi,\mu)} dX^\mu(\xi)\right)\exp\left(-\tfrac{1}{2}\int d\xi^+ d\xi^- [\partial_+ X^\mu \partial_- X_\mu](\xi^+,\xi^-)\right) \times \exp\left\{-\frac{26-(4+3)}{48\pi}\int_D d\xi^+ d\xi^- \left(\frac{(\partial_+^2 X^\mu)(\partial_- X_\mu)(\partial_-^2 X^\mu)(\partial_+ X_\mu)}{[(\partial_+ X^\mu)(\partial_- X_\mu)]^2}\right)(\xi^+,\xi^-)\right\} \tag{7b}$$

denotes the correct way to sum random surfaces with the weight given by the Nambu-Goto action which is obtained from Polyakov's covariant path integrals by imposing the constraint $h_{ab}(\xi) = (\partial_a X^\mu \partial_b X_\mu)(\xi)$ at the quantum level [see Eq. (1) in Ref. [7]].

Let us show that Eq. (7) satisfies formally the Wheeler-DeWitt constant [2] integrated over the three-dimensional manifold $M \subset R^4$ ([8] Chap. 3):

$$\int_M d^3x\, \epsilon^{ijk} F^i_{\mu\nu}(x)\frac{\delta}{\delta A^i_\mu(x)}\frac{\delta}{\delta A^j_\nu(x)}\Phi_{ab}[A] \equiv 0 . \tag{8}$$

It is a straightforward calculation to show that

$$\int_M d^3x\, \epsilon^{ijk} F^i_{\mu\nu}(x)\frac{\delta}{\delta A^j_\mu(x)} W_{ab}[S, C_{xx}, A]$$

$$= \int_D d^2\xi\, \sqrt{h(\xi)}\int_D d^2\xi \sqrt{h(\xi')}[\partial^c X^\mu(\xi)\partial^d X^\nu(\xi')]\delta^{(3)}(X^\chi(\xi) - X^\chi(\xi'))\epsilon^{ijk}F^i_{\mu\nu}(X^\chi(\xi))$$

$$\times \mathrm{Tr}^{\mathrm{color}}\left\{\int D^c[\psi(\xi)]D^c[\overline{\psi}(\xi)]\{\psi_a(0,0)[\overline{\psi}(\xi)\gamma_c\lambda^j\psi(\xi)\overline{\psi}(\xi')\partial_d\psi(\xi')]\overline{\psi}_b(2\pi,0)\}\right.$$

$$\left.\times \exp[-(S_0 + S_1^{(F)} + S^{\mathrm{int}})]\right\} . \tag{9}$$

In the context of the random surfaces sum Eq. (8) we have that the $X_\mu(\xi)$ functional integral leads us to the following condition in the perturbative expansion for Eq. (7) [9,13]:

$$\langle(\partial^C X^\mu(\xi)\delta^{(3)}(X^\chi(\xi)-X^\chi(\xi'))\partial^d X^\nu(\xi')F_{\mu\nu}(X(\xi))\rangle$$

$$= \langle\delta_{\mu\nu}(\partial^c X^\mu(\xi)\delta^{(3)}(X^\chi(\xi)-X^\chi(\xi'))\partial^d X^\nu(\xi')) \times F_{\mu\nu}(X(\xi))\rangle \ . \quad (10)$$

As a consequence of $F_{\mu\nu}(X(\xi))$ being antisymmetric in the (μ,ν) indices we get the result Eq. (8).

It is interesting to point out that only in the condition of non-self-intersection surfaces $X_\mu(\xi)=X_\mu(\xi')\rightarrow \xi=\xi'$ does one obtain that Eq. (5) is solution of the integrated Wheeler-DeWitt equation.

In order to satisfy automatically general coordinate invariance on the three-dimensional (3D) manifold M,

$$\delta x^\mu = \epsilon^\mu(x^\chi) =^\epsilon (x^\mu) \ , \quad (11)$$

with $\varepsilon^\mu(x)$ being the vector field generator of an element of $G_{\text{diff}}(M)$, one could consider formally the functional integral over $G_{\text{diff}}(M)$ of the action piece of our proposed solution Eq. (5) involving the random surface S coordinates: namely,

$$\overline{S}[X^\mu(\xi),\varepsilon^\mu(x)] = \tfrac{1}{2}\int_D d^2\xi(\sqrt{h}h^{ab}\partial_a[{}^\varepsilon X^\mu](\xi)\partial_b[{}^\varepsilon X^\mu](\xi) + ie\int_D d^2\xi\sqrt{h(\xi)}\overline{\psi}(\xi)(\gamma_a\partial^a[{}^\varepsilon X^\mu]\overline{A}^l[{}^\varepsilon X^\chi]\lambda_l) \times\psi(\xi) \ . \quad (12)$$

We have, thus, the $G_{\text{diff}}(M)$ invariant solution

$$\tilde{\Phi}_{ab}[\overline{A}_\mu] = \sum \Phi_{ab}[\overline{A}_\mu,\varepsilon] \ ,$$

$$\varepsilon^\mu(x)\in G_{\text{diff}}(M) \quad (13)$$

where the sum over the field generators $\varepsilon^\mu(x)$ on M must be weighted with the noncompact formal Haar measure associated with $G_{\text{diff}}(M)$.

It is important to remark that the diffeomorphism constraint [2] imposed on our proposed surface Wilson loop

$$\int_M d^3x\, C_\mu^{(x)}\{W_{ab}[S,C_{xx},A]\} = \int_M d^3x\left(F^i_{\mu\nu}(x)\frac{\delta}{\delta A^i_\nu(x)}(W_{ab}[S,C_{xx},A])\right) \quad (14)$$

is exactly given by the "Lorentz force" acting on the surface vector position $X_\mu(\xi)$: i.e.,

$$\int_M d^3x\, C_\mu^{(x)}\{W_{ab}[S,C_{xx},A]\} = \text{Tr}^{(\text{color})}\left\{\int D^c[\psi(\xi)]D^c[\overline{\psi}(\xi)][\psi_a(0,0)\overline{\psi}_b(2\pi,0)]\exp\{-(S_0+S_1^{(F)}+S^{\text{int}})\} \times\int_D d^2\xi\,\sqrt{h(\xi)}F^i_{\mu\nu}(X^\chi(\xi))[\partial^j X^\nu(\xi)][\overline{\psi}(\xi)\gamma_j\lambda_i\psi(\xi)]\right\} \ , \quad (15)$$

which is zero if we impose that the surface S is the minimal surface bounded by C_{xx} or if one considers the surface kinetic term (1a) identically zero [11]. At this point it is worth remarking that Eq. (13) automatically vanishes under operation of Eq. (14) by the way it was constructed, preserving general coordinate invariance on M.

Let us remark that a similar procedure may be used to make the Smolin-Jacobson loop space solutions covariant under diffeomorphism of the loops by means of introduction of an additional one-dimensional metric $\varepsilon(\xi)$ (see Ref. [6]).

Finally we point out that in the case when the random surface S degenerates to its boundary $X_\mu(\xi)\rightarrow C_\mu(\sigma)$, our surface Wilson loop will be given by the usual Wilson Loop used by Jacobson and Smolin in Ref. [2] (Appendix B).

III. THE RANDOM SURFACE TOPOLOGICAL WAVE EQUATION

Before turning to the construction of a random surface topological wave equation for Eq. (5) similar to that of Ref. [11] we follow the usual procedures of quantum theory by defining the average of a general quantum observable $\Theta[A^i_\mu(x)\lambda_i]$ by the Chern-Simon functional integral (see Appendix A):

$$\langle\langle\Theta[A_\mu]\rangle\rangle = \int\left(\prod_{x\in M} dA_\mu(x)\right)\delta^{(F)}\left(\frac{\partial}{\partial x_\mu}F^{\mu\nu}(A)\right) \times\exp\left\{-\int_M \text{Tr}^{\text{color}}[A\wedge dA+\tfrac{2}{3}A\wedge A\wedge A](X)d^3x\right\}\Theta[A_\mu] \ . \quad (16)$$

Note that the use of Eq. (16) for averages is automatically diffeomorphism and gauge invariant due to the Chern-Simon functional measure used to weight the functional space of the Chern-Simon connections $A^i_\mu(x)$ [2,12].

Let us, thus, proceed as in non-Abelian gauge theories [6,11] by considering the invariance under translations of the Feynman measure

$$\prod_{x\in M}\left[dA_\mu\left(\frac{\partial}{\partial x_\mu}F^{\mu\nu}(A)\right)\right]$$

(x) for the connection $A_\mu(x)$ average of our proposed surface Wilson loop for Einstein quantum gravity:

$$0=\int\left(\prod_{x\in M}dA^i_\mu(x)\right)\delta^{(F)}\left(\frac{\partial}{\partial X_\mu}F^{\mu\nu}(A)\right)\times\frac{\delta}{\delta A^i_\mu(x)}\left\{\exp\left(-\int_M d^3x(A\wedge dA+\tfrac{2}{3}A\wedge A\wedge A)\right)\right\}\times\mathrm{Tr}^{\mathrm{color}}\{W_{ab}[S,C_{xx},A_\mu]\lambda_i\}\ . \quad (17)$$

The $A^i_\mu(x)$ functional variation of the Chern-Simon weight produces the results (see Ref. [11])

$$\langle\langle\mathrm{Tr}^{\mathrm{color}}(\varepsilon^{\mu\chi\beta}F_{\chi\beta}(A(x))W_{ab}[S,C_{xx},A_\mu])\rangle\rangle\ . \quad (18)$$

Now a straightforward calculation for the $A^i_\mu(x)$ functional variation of the surface Wilson loop yields the expression [3]

$$\begin{aligned}\lambda^i\frac{\delta}{\delta A^i_\mu(x)}&\{\mathrm{Tr}^{(c)}(W_{ab}[S,C_{xx},A_\mu]\lambda_i)\}\\&=-\int_D d^2\sigma\,\delta^{(3)}(x_\mu-X_\mu(\sigma_1,\sigma_2))\partial_A X^\mu(\sigma_1,\sigma_2)\\&\quad\times\langle\langle W_{aa_1}[S_1,C_{X(0),X(\sigma_1)},A_\mu]\rangle\rangle\\&\quad\times(\gamma^A)_{a_1a_2}\langle\langle W_{a_2b}[S_2,C_{X(\sigma_1),X(2\pi)},A_\mu]\rangle\rangle\ ,\end{aligned}\quad (19)$$

where the end point of the loop C_{xx} is defined to coincide with the x argument of the Chern-Simon field in the functional variation in the result written above. S_1 and S_2 are, respectively, defined by the restriction of the mapping $X_\mu(\xi_1,\xi_2)$ for the (split domains $D=D_1\cup D_2$:

$$S_1=X_\mu(D_1),\quad S_2=X_\mu(D_2) \quad (20)$$

with

$$D_1=\{(\xi_1,\xi_2)\ ;\ 0\le\xi_1\le\sigma_1\ ;\ -\infty<\xi_2<\infty\}\ ,$$

$$D_2=\{(\xi_1,\xi_2)\ ;\ \sigma_1\le\xi_1\le2\pi\ ;\ -\infty<\xi_2<\infty\}\ . \quad (21)$$

Here we have taken the domain D in Eq. (1) as an infinite rectangle.

It is now convenient to multiply both sides of Eq. (17), after using Eqs. (18) and (19), by the loop current density,

$$J^\mu_{ma}(C_{xx})=\int_D\delta^{(3)}(x_\mu-X_\mu(\bar\xi_1,0))\partial_B X^\mu(\bar\xi,0)(\gamma^B)\times\sqrt{h(\bar\xi)}d\bar\xi_1 d\bar\xi_2 \quad (22)$$

and integrate out the obtained result relative to the $x\in M$ variable. By taking into account the diffeomorphism constraint Eqs. (14) and (15) we finally get our topological wave equation

$$\left\{\int_D d^2\sigma\sqrt{h(\sigma)}\int_{C_{xx}}d\bar\xi_1\sqrt{h(\bar\xi_1,0)}\delta^{(3)}(X_\mu(\bar\xi_1,0)-X_\mu(\sigma_1,\sigma_2))\partial_A X^\beta(\bar\xi_1,0)\partial_B X^\mu(\sigma_1,\sigma_2)\epsilon_{\mu\chi\beta}\times(\gamma^B)_{ma}W_{aa_1}[S_1,C_{x(0),x(\sigma_1)},A_\mu](\gamma^A)_{a_1a_2}W_{a_2b}[S_2,C_{x(\sigma_1),x(2\pi)},A_\mu]\right\}=0\ . \quad (23)$$

At this point the reader should compare Eq. (23) with our proposed similar topological equation for pure loop Jacobson-Smolin quantum gravity functionals [Eq. (9) of Ref. [11] with zero area variation]. As a result of Eq. (23), we conjecture that after integrating the two-dimensional fermion SU(2) degrees of freedom in Eq. (5) we should get topological invariants for some kind of "braid groups" for surfaces on M and paralleling similar results for the Jacobson-Smolin loop quantum gravity solutions [12,9].

The important point to discuss now is the possibility of representing quantum gravity observables by interpreting physically the loop C_{xx} in the argument of our proposed Eq. (5) as the M manifold projected closed space-time world line of a pair of matter field excitations, as is usually done in the loop space framework for QCD with random loops [6]. In order to implement this idea, we consider the generating functional for pairs of left-handed space-time fermions:

$$Z[J(x^{\tilde{A}})] = \det\left[\gamma^{\tilde{M}}\left(\frac{\partial}{\partial x^{\tilde{M}}} + \omega_{\tilde{M}}^{\tilde{a}\tilde{b}}\sigma_{\tilde{a}\tilde{b}}(1+\gamma_5)\right) + J(x^{\tilde{A}})\right] . \quad (24)$$

It is instructive to point out that Green functions for correlations among these pairs are given by the $J(x^{\tilde{A}})$ functional differentiations of Eq. (23), where $x^{\tilde{A}}$ denotes an arbitrary point of the four-dimensional space-time $N = M \times [0, V]$ [14] which will be taken for simplicity as a cylinder with base M. At this point, we follow the QCD loop space formalism by writing the fermionic functional determinant Eq. (23) by means of pairs of closed trajectories $L_{X_{\tilde{M}}X_{\tilde{M}}}$ on the cylinder space-time N:

$$Z[J(X^{\tilde{M}})] = \exp\left\{-\sum_{L_{X_{\tilde{M}}X_{\tilde{M}}}\subseteq N} \mathrm{Tr}\mathbb{P}\left[\exp\left(i\oint_{L_{X_{\tilde{M}}X_{\tilde{M}}}} \omega_{\tilde{M}}^{\tilde{a}\tilde{b}}\sigma_{\tilde{a}\tilde{b}}(1+\gamma_5)dX^{\tilde{M}}\right)\right]\right\}\exp\left(\oint_{L_{X_{\tilde{M}}X_{\tilde{M}}}} J\, ds\right) \quad (25)$$

Now if we consider the gravity quantum average of Eq. (24),

$$\langle\langle Z[J]W_{ab}[S, C_{xx}, A_\mu]\rangle\rangle , \quad (26)$$

and take *into account that the four-dimensional spin connection for left-handed spinors restricted to the embedded base three-dimensional manifold M coincides with the Asthekar-Sen Connection* $A_\mu^i(x)$, we should identify the M-projected space-time loop $L_{X\tilde{M}X\tilde{M}}$ with the three-dimensional loop C_{xx} and yielding the averaged four-dimensional spinors generating functional Eq. (25) as a random surface scalar vertex generator projected on the N-manifold boundary [4,6,10,11].

ACKNOWLEDGMENTS

This research was supported by CNPq—Brazil.

APPENDIX A

In this Appendix we briefly sketch another argument leading to Eq. (10) in the case of nontrivial self-intersecting random surfaces. In order to simplify the analysis we consider the average Eq. (10) in the Gaussian case written in momentum space for the involved self-intersecting vertices:

$$\tfrac{1}{2}\langle\partial_a X^\mu(\xi)\exp\{ik^x[X_x(\xi) - X_x(\xi')]\}\partial_b X^\nu(\xi') \times \exp\{iP^\beta[X_\beta(\xi) + X_\beta(\xi')]\}\rangle_{\mathrm{surface}}\tilde{F}_{\mu\nu}(P^\beta) , \quad (A1)$$

where

$$F_{\mu\nu}(A)(x^\alpha) = \int_M d^3x\, e^{iP^\beta x_\beta}[F_{\mu\nu}(A)](x^\alpha). \quad (A2)$$

In writing Eq. (A1) we have taken into account that the Ashtekar-Sen connection is an object defined in the extrinsic space M, so it is a single-valued function as an object in the random surface: namely,

$$F_{\mu\nu}(X^\beta(\xi)) = F_{\mu\nu}(X^\beta(\xi')) \quad (A3)$$

for any coordinate ξ on the random surface parameter domain.

By considering a power expansion in the extrinsic momenta variables (P^α, K^α) for the vertices in Eq. (A1), one obtains the generic form for this object:

$$(P_\nu K_\mu - K_\nu P_\mu)\tilde{F}_{\mu\nu}(P)\Phi_{ab}(P^\chi, k^\chi, \xi, \xi') , \quad (A4)$$

where $\Phi_{ab}(P^\chi, k^\chi, \xi, \xi')$ denotes the random surface vector position contractions which are a Lorentz scalar object in the extrinsic space M.

At this point we note that the motion equations hold true for our proposed quantum gravity functionals [Eq. (5)]:

$$\frac{\partial}{\partial x_\mu}F_{\alpha\mu}(x) = \epsilon^{\alpha\nu\beta}\partial_\nu j_\beta(x) , \quad (A5)$$

where the random surface current is given explicitly by [see Eq. (3)]

$$j_\beta(x) = ie\int_D d^2\xi\, \delta^{(3)}(x^\alpha - X^\alpha(\xi))(\overline{\psi}\gamma^a\partial_a X^\mu\psi)(\xi) . \quad (A6)$$

The vanishing of Eq. (A5) is a direct consequence of the identity

$$\begin{aligned}\frac{\partial}{\partial x_\alpha}j_\beta(x) &= -ie\int_D d^2\xi\int_D d^2\overline{\xi}\left[\frac{\delta}{\delta X_\alpha(\overline{\xi})}\delta^{(3)}(x^\alpha - X^\alpha(\xi))\right](\overline{\psi}\gamma^a\partial_a X^\mu\psi)(\xi)\\ &= -ie(\delta^{\mu\alpha})\int_D d^2\xi\,\delta^{(3)}(x^\alpha - X^x(\xi))(\overline{\psi}\gamma^a\psi)\int_D \partial_a(\delta^{(2)}(\xi - \overline{\xi}))d^2\overline{\xi} . \end{aligned} \quad (A7)$$

APPENDIX B

In this Appendix we obtain the expression of our proposed quantum gravity stringy state in terms of a generalized supersymmetric loop Jacobson-Smolin functional.

In order to show this result, we first integrate out the two-dimensional intrinsic Dirac fields in Eq. (5) and write the 2D fermionic functional determinant in terms of Grassmanian trajectories on random surface S (embedded on the 3D manifold M) as in Ref. [10] Eq. (1) $[C_\mu^F(\sigma,\theta)=C_\mu(\sigma)+i\theta\psi_\mu(\sigma)]$

$$\ln\det[i\gamma^a(\partial_a+A_\mu^l(X^\chi(\xi))\partial_a X^\mu(\xi)\lambda_l)]$$
$$=+\tfrac{1}{2}\int_0^\infty\frac{dt}{t}\int_M d^3x\int_{C_\mu(0)=C_\mu(t)=x_\mu}D^F[\psi_\mu(\sigma),\psi_\mu^*(\sigma)]$$
$$\times\exp\left\{-\int_0^t d\sigma\left[\frac{1}{2}\left(\frac{d}{d\sigma}C_\mu(\sigma)\right)^2-\dot{\psi}_\mu(\sigma)\psi_\mu^*(\sigma)\right]\right\}W[A_\mu^l,C_\mu^F(\sigma,\theta)]\ , \tag{B1}$$

where we have introduced our proposed QCD Grassmanian Wilson loop defined now by the SU(2) Ashtekar-Sen gauge field, namely,

$$W^{(F)}[A_\mu,C_\mu(\sigma,\theta)]$$
$$=\mathrm{Tr}^{\mathrm{color}}\mathbb{P}\left\{\exp\left(-\int_0^t d\sigma\int d\theta A_\mu^i(C_\mu(\sigma)+i\theta\psi_\mu(\sigma))\right.\right.$$
$$\left.\left.\times\lambda_i DC_\mu^F(\sigma,\theta)\right)\right\}\ . \tag{B2}$$

We can interpret Eq. (B1), after introducing it in Eq. (7), as expressing our random surface quantum gravity state as a kind of coherent packet of Grassmanian Jacobson-Smolin functionals Eq. (B2). It is worth calling attention to the fact that in the case of nonfluctuating loops $C_\mu(\sigma)$, and with "frozen" Grassmanian degrees of freedom $\theta\equiv 0$, our string solution reduces to the *usual Jacobson-Smolin remark that Wilson loops defined by the Ashtekar-Sen connection satisfy the quantum gravity Wheeler-DeWitt equation* [2,6].

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