$R + R^2$ gravity as $R+$ back reaction

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(Received 20 March 1995; revised manuscript received 11 September 1995)

The quadratic theory of gravity is a complicated constraint system. We investigate some consequences of treating quadratic terms perturbatively (higher derivative version of back reaction effects), which is consistent with the way the existence of quadratic terms was originally established (radiative loop effects and renormalization procedures which induced quadratic terms). We show that this approach overcomes some well-known problems associated with higher derivative theories, i.e., the physical gravitational degree of freedom remains unchanged from those of Einstein gravity. Using such an approach we first study the classical cosmology of $R + \beta R^2$ theory coupled to matter with a characteristic $\rho \propto a(t)^{-n}$ dependence on the scale factor. We show that for $n > 4$ (i.e., $p > \frac{1}{3}\rho$) and for a particular sign of β, corresponding to the nontachyon case, there is no big bang in the traditional sense. And, therefore, a contracting FRW universe ($k > 0$, $k = 0$, $k < 0$) will rebounce to an expansion phase without a total gravitational collapse. We then quantize the corresponding minisuperspace model that resulted from treating the βR^2 as a perturbation. We conclude that the potential $W(a)$, in the Wheeler-DeWitt equation $\left[-\partial^2/\partial a^2 + 2W(a)\right]\psi(a) = 0$, develops a repulsive barrier near $a \approx 0$ again for $n > 4$ (i.e., $p > \frac{1}{3}\rho$) and for the sign of β that corresponds to the nontachyon case. Since $a \approx 0$ is a classically forbidden region, the probability of finding a universe with a singularity ($a = 0$) is exponentially suppressed. Unlike the quantum cosmology of Einstein's gravity, the formalism has dictated an appropriate boundary (initial) condition. Classical and quantum analyses demonstrate that a minimum radius of collapse increases for a larger value of $|\beta|$. It is also shown that, to first order in β, the βR^2 term has no effect during the radiation ($p = \frac{1}{3}\rho$) and inflationary ($p = -\rho$) era. Therefore, a de Sitter phase can be readily generated by incorporating a scalar field.

PACS number(s): 04.50.+h, 98.80.Hw

I. INTRODUCTION

Since the discovery of the singularity theorem of Hawking and Penrose [1], the speculation of creating a nonsingular theory by incorporating the quantum property of gravity and/or using non-Einstein gravity has attracted some interest. Because of advances in quantum field theory, the two avenues of speculations seem to be mathematically related. That is, even if one starts with Einstein's gravity, the renormalization consideration dictates that the action for gravity must have terms that are quadratic in the Ricci tensor [2].

In quantum cosmology, canonical quantization is the preferred formalism. This is because, for a universe that is homogeneous and isotropic in large scales, there are no asymptotically "in" and "out" fields, which are necessary in order to implement the covariant quantization formalism. The task of identifying dynamical degrees of freedom for quadratic gravity has been reduced to solving the constraint system of Boulware [3]. Because of the technical difficulties of solving such constraints, quantum cosmology for quadratic gravity has been solved for only simple systems such as a vacuum [4,5].

In this paper, we explore the consequences of viewing $R + R^2$ gravity as R + perturbation (higher derivative version of back reaction) [6]. In essence, in this approach, the physical gravitational degree of freedom is not changed from that of Einstein gravity. It is shown that this view overcomes the bulk of the technical difficulties with the higher derivative content of quadratic gravity. In Sec. II, we argue that this view is also in agreement with the way the existence of quadratic terms was originally established (via renormalization procedures that treat quadratic terms perturbatively).

Using such an approach (Sec. III), we study the classical cosmology of $R + \beta R^2$ theory coupled to matter that has a characteristic $\rho \propto a(t)^{-n}$ dependence on the scale factor. We show that for $n > 4$ (i.e., $p > \frac{1}{3}\rho$) and for a particular sign of β, corresponding to the nontachyon case, there is no big bang in the traditional sense, and therefore, even for a close Friedmann-Robertson-Walker (FRW) metric, the universe will rebounce without a complete collapse.

In Sec. IV, we quantize the corresponding minisuperspace model, which resulted from treating the βR^2 as a back reaction. We conclude that the potential $W(a)$, in the Wheeler-DeWitt equation $\left[-\partial^2/\partial a^2 + 2W(a)\right]\psi(a) = 0$, develops a repulsive barrier near $a \approx 0$ again for $n > 4$ (i.e., $p > \frac{1}{3}\rho$) and for

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a particular sign of β, corresponding to the nontachyon case.

The sign conventions used in this paper are as follows. $g = (-,+,+,+)$, $R_{\mu\nu} - \frac{1}{2}g_{\mu,\nu}R = (+)\,8\pi G T_{\mu\nu}$, $(+)\,R(\mu,\nu) = \nabla_\mu\nabla_\nu - \nabla_\nu\nabla_\mu - \nabla_{[\mu,\nu]}$.

II. PRELIMINARIES

The most general quadratic action for gravity coupled to matter is

$$I = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}R - \int d^4x\Big[\beta_1 R^2 + \beta_2 R_{ab}R^{ab} + \beta_3 R_{abcd}R^{abcd}\Big] + I_{\text{matter}} + \text{surface term}\ . \quad (2.1)$$

We have formally included a surface term to cancel any boundary term that would result in applying the variational principle. By dimensional analysis, β_1, β_2, and β_3 are dimensionless. We will be interested in applying the formalism to a homogeneous and isotropic metric, i.e., the Weyl tensor vanishes $C_{abcd} = 0$ [2]. By definition of the Weyl tensor, $C_{abcd}C^{abcd} = R_{abcd}R^{abcd} - 2R_{ab}R^{ab} + \frac{1}{3}R^2$. This gives one relationship among the possible quadratic terms.

The second relationship is from the four-dimensional generalization of the Gauss-Bonnet formula [2]

$$R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd} = \text{exact derivative.} \quad (2.2)$$

The two relationships, combined with the fact that Euler Lagrange equations are unchanged by addition of an exact differential, allow any two of β_1, β_2, and β_3 to be set equal to zero in the action (2.1). We choose to set $\beta_3 = \beta_2 = 0$.

Upon variation of the metrics, the resulting Euler Lagrange equations are

$$\frac{1}{2}Rg_{ab} - R_{ab} + 16\pi G\beta\Big(\frac{1}{2}R^2 g_{ab} - 2RR_{ab} + 2R_{;\sigma}^{;\sigma}g_{ab} - 2R_{;a;b}\Big) = 8\pi G T_{ab}. \quad (2.3)$$

The trace of this equation is

$$6\times 16\pi G\beta R_{;\sigma}^{;\sigma} + R = 8\pi G T, \quad (2.4)$$

which reduces to a familiar form for $\beta = 0$.

For the $\beta = 0$ case, it is well known that both the left and the right side of (2.3) vanish under a covariant derivative (the right-hand side by local conservation of the energy-momentum tensor and the left-hand side by the Bianchi identity). It is interesting to note that this is true even for $\beta \neq 0$ (the additional term is also a covariant constant by virtue of the Bianchi identity). This consistency is expected since local conservation of the energy-momentum tensor and Bianchi identity is intimately related to reparametrization invariance of the action (2.1).

In this paper, we shall consider only the simplest metric, that of a spatially homogeneous and isotropic universe (i.e., FRW metric):

$$ds^2 = -dt^2 + a^2\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2\right), \quad (2.5)$$

with the standard energy-momentum tensor

$$T_{ab} = pg_{ab} + (p+\rho)U_aU_b\ , \qquad U^0 = 1, U^i = 0, i = 1,2,3. \quad (2.6)$$

Using

$$R = -6\left[\frac{\ddot a}{a} + \left(\frac{\dot a}{a}\right)^2 + \frac{k}{a^2}\right], \quad (2.7)$$

for the Ricci scalar, one gets, for the time-time component of (2.3),

$$\left(\frac{\dot a}{a}\right)^2 + \frac{k}{a^2} - \frac{16\pi G\beta}{3}\left(\frac{1}{2}R^2 + 6R\frac{\ddot a}{a} - 2\frac{\partial_t\,(a^3\partial_t R)}{a^3} + 2\partial_t^2 R\right) = \frac{8\pi G\rho}{3}, \quad (2.8)$$

and, for any one of the space-space components,

$$2\frac{\ddot a}{a} + \left(\frac{\dot a}{a}\right)^2 + \frac{k}{a^2} - 16\pi G\beta\left\{\frac{1}{2}R^2 + 2R\left[\frac{\ddot a}{a} + 2\left(\frac{\dot a}{a}\right)^2 + \frac{2k}{a^2}\right] - 2\frac{\partial_t\,(a^2\partial_t R)}{a^2}\right\} = -8\pi G p. \quad (2.9)$$

As is evident, for $\beta \neq 0$, (2.8) is transformed from a first order to a third order, and (2.9) is transformed from a second order to a fourth order differential equation.

Even on a classical level, there are pathological problems with higher derivative theories [6,7]. One is the need for additional initial conditions to completely specify a system, and whether solutions obtained by solving (2.8) and (2.9) for $\beta \neq 0$ have well-behaved properties as $\beta \to 0$, and the existence of runaway solutions. A review of motivation for studying quadratic gravity (i.e., renormalization considerations) seems to offer an alternate method of handling higher derivative theories of gravity.

In perturbative covariant quantization, even if one starts with Einstein's action coupled to matter as the bare action,

$$I_{\text{bare}} = \int \sqrt{-g}R + I_{\text{matter}}\ , \quad (2.10)$$

by quantum loop effects from both gravity and matter, the effective action acquires terms with higher derivatives

$$I_{\text{effective}} = I_{\text{bare}} + \int \Big[\alpha_1 R^2 + \alpha_2 R_{ab} R^{ab} + \alpha_3 R_{abcd} R^{abcd} \Big] \tag{2.11}$$

with divergent α_1, α_2, and α_3. The precise nature of the divergence depends on the choice for the matter field.

The perturbative renormalization prescription is to add terms to the bare action to precisely cancel these infinities: i.e.,

$$I_{\text{bare}} \to \int \sqrt{-g} R + I_{\text{matter}} + I_{\text{counterterm}} \;, \tag{2.12}$$

$$I_{\text{counterterm}} = \int [(\beta_1 - \alpha_1) R^2 + (\beta_2 - \alpha_2) R_{ab} R^{ab} + (\beta_3 - \alpha_3) R_{abcd} R^{abcd}]. \tag{2.13}$$

This renders the resulting effective action finite, which may be used for semiclassical analysis. (For a fuller discussion of unitarity and renormalizability, please see [2,7].) The crucial observation is that the physical degrees of freedom for asymptotic "in" and "out" fields were those of Einstein action. Moreover, even when the bare action had higher derivative counterterms (2.12), the higher derivative terms were treated perturbatively and not on an equal footing with the Einstein action. This is in marked contrast with [3]. The details of our proposed procedures will be shown in Sec. III. Our proposal for gravity is similar to the procedure of Jaen *et al.* [6] for reducing a general Lagrangian system with arbitrary higher derivatives into a second-order differential system.

A further support of this interpretation of theories in which higher derivatives were induced can be found in classical electrodynamics. For a pedagogical review, please see Barut [8]. Consider a nonrelativistic harmonic oscillator. It obeys the Newton law:

$$m\ddot{\mathbf{R}} = \mathbf{F_{ext}} = -m\omega^2 \mathbf{R}. \tag{2.14}$$

If the particle is also charged, then the accelerated particle emits radiation and in turn must effect change on the motion of the particle. One can take account of this radiative loss of energy by an effective radiative back reaction force

$$m\ddot{\mathbf{R}} = \mathbf{F_{ext}} + \mathbf{F_{rad}} \tag{2.15}$$

with

$$\mathbf{F_{rad}} = \frac{2e^2}{3c^3} \dddot{\mathbf{R}} \;. \tag{2.16}$$

The resulting equation of motion is changed from second to third order. Besides the necessity of additional initial conditions, (2.15) also has an unphysical runaway solution. For example, if $\mathbf{F_{ext}} = \mathbf{0}$, then there should be no acceleration and hence no radiative loss, i.e., $\ddot{\mathbf{R}} = \mathbf{0}$. Yet it is straightforward to verify that (2.15) admits an unphysical solution $\ddot{\mathbf{R}} = \mathbf{C} e^{\frac{t}{\tau}}$ with $\tau = 2e^2/3mc^3$.

For a weak radiative loss, there are well-known approximate methods of handling these problems, which result in physically acceptable solutions. For the simple harmonic oscillator (SHO) problem, weak radiative loss means that the solution should be harmonic to first order, $\dddot{\mathbf{R}} \approx -\omega^2 \dot{\mathbf{R}}$. Upon substitution, the resulting effective equation of motion is returned to the original second order: i.e.,

$$\ddot{\mathbf{R}} + \omega^2 \tau \dot{\mathbf{R}} + \omega^2 \mathbf{R} = \mathbf{0}. \tag{2.17}$$

For a general $\mathbf{F_{ext}}$, one can still eliminate unphysical solutions by using an integration factor and surgically choosing initial conditions for $\ddot{\mathbf{R}}(0)$. In either case, the lesson is that the induced higher derivative forces were treated as back reactions which did not increase the physical degree of freedom.

The analogy is even stronger for the action at a distance treatment of classical electrodynamics. In this case, the back reaction force is not deduced by balancing energy but by a dynamical process. Here the back reaction force can be split into contributions from near and far field produced by the particle. The back reaction force from the far field is a relativistic generalization of (2.16), and the back reaction force from the near field results in classical mass renormalization.

The similarity between induced higher derivatives (via radiative processes) in classical electrodynamics and the present problem with gravity is obvious. Therefore, using these as motivations and possibly even as justifications, we shall also treat the higher derivative terms perturbatively when implementing the canonical quantization procedure (i.e., Wheeler-DeWitt equation). The rest of the paper can be categorized as consequences of such an approach.

III. CLASSICAL EVOLUTION OF $R + R^2$ GRAVITY COUPLED TO MATTER

In (2.3) and (2.4), we are interested in treating contributions from R^2 as a perturbation. We will use β as a dimensionless expansion parameter and study the first order correction to the equation of motion. The value of β is of course unknown, but in order to implement the perturbation method we will also have to assume that β is small. Hopefully, the results obtained by assuming small β will only be amplified by a larger β. We will return to this issue at the end.

By (2.4), $R = 8\pi G T + O(\beta)$ and $R_{ab} = -8\pi G \left(T_{ab} - \frac{1}{2} T g_{ab}\right) + O(\beta)$. Therefore Eq. (2.3) is

$$\frac{1}{2} R g_{ab} - R_{ab} = \tilde{G} T_{ab} + 2\tilde{G}^2 \beta \bigg(\frac{1}{2} \tilde{G} T^2 g_{ab} - 2\tilde{G} T T_{ab} - 2T_{;\sigma}^{;\sigma} g_{ab} + 2T_{;a;b} \bigg) + O(\beta^2). \tag{3.1}$$

We have introduced the notation $\tilde{G} \equiv 8\pi G$.

Using (2.5)–(2.7), to first order in β the time-time component is

$$\left(\frac{\dot a}{a}\right)^2+\frac{k}{a^2}=\frac{1}{3}\tilde G\rho-2\tilde G^2\beta\left(\frac{1}{2}\tilde G(3p-\rho)(p+\rho)+2\frac{\dot a}{a}\partial_t(3p-\rho)\right). \tag{3.2}$$

And any one of the space-space components gives the single equation

$$2\frac{\ddot a}{a}+\left(\frac{\dot a}{a}\right)^2+\frac{k}{a^2}=-\tilde G p+\tilde G^2\beta\left(\tilde G(3p-\rho)(p+\rho)-\frac{4\partial_t(a^2\partial_t(3p-\rho))}{a^2}\right). \tag{3.3}$$

The matter sector must satisfy local conservation of the energy-momentum tensor

$$\frac{d(\rho a^3)}{da}=-3pa^2. \tag{3.4}$$

For $\beta=0$, it is well known that solutions obtained by (3.2) and (3.4) automatically satisfy (3.3). As pointed out in Sec. II, this is guaranteed even for $\beta\neq 0$ by reparametrization invariance of the action (2.1). Therefore we shall proceed to solving (3.2) with (3.4).

From the form of (3.2), one can immediately extract several conclusions. First, to first order in β, the R^2 has no contribution in the radiation era because of $p_{\rm rad}=-\frac{1}{3}\rho_{\rm rad}$. Second, if the energy-momentum tensor ever becomes dominated by an almost constant potential of a scalar field $[-p_\phi\approx\rho_\phi\approx V(\phi)\approx\text{const}]$ then we can again conclude that the contribution from R^2 vanishes (to first order in β). Therefore, a possible de Sitter phase in $R+\beta R^2$ with a scalar field ϕ should be more or less identical with a de Sitter phase in standard Einstein gravity. It is of course a separate question whether one can generate an inflationary phase in $R+R^2$ gravity without fine-tuning.

In comparison, Mijic *et al.* [9] and Page *et al.* [10] have studied the large β range and concluded that, even for a vacuum, gravity alone can generate an inflationary phase in $R+R^2$ gravity.

Now let us assume that during any epoch in the evolution of the universe the universe is dominated by a matter with a characteristic dependence on the scale factor (i.e., $\rho=\rho_0/a^n$), $n=3$ for the matter era, $n=4$ for the radiation era, and $n=0$ for an inflationary era, etc. From conservation of energy and momentum tensor (3.4), we get $p=[(n-3)/3]\rho_0/a^n$.

Therefore

$$(3p-\rho)(p+\rho)=\frac{1}{3}n(n-4){\rho_0}^2 \tag{3.5}$$

and

$$\frac{\dot a}{a}\partial_t(3p-\rho)=-n(n-4)\left(\frac{\dot a}{a}\right)^2\frac{\rho_0}{a^n}=-n(n-4)\left(-\frac{k}{a^2}+\frac{1}{3}\tilde G\rho\right)\frac{\rho_0}{a^n}+O(\beta). \tag{3.6}$$

After simplifying, we get

$$\dot a^2+2U(a)=0\ , \tag{3.7}$$

$$2U(a)=k\left(1+4n(n-4)\tilde G^2\beta\frac{\rho_0}{a^n}\right)-\frac{1}{3}\tilde G\frac{\rho_0 a^2}{a^n}\left(1+3n(n-4)\tilde G^2\beta\frac{\rho_0}{a^n}\right)+O(\beta^2)\ , \tag{3.8}$$

which can be interpreted as an equation describing a particle with a unit mass in a potential $U(a)$. For $\beta=0$, the form of $U(a)$ is shown in Fig. 1. As expected, depending on the sign of curvature of three-space ($k>0, k<0, k=0$) the universe evolves as a bound state, unbounded state, or as a critically opened state, respectively.

Now, for $\beta\neq 0$, the contributions from R^2 depend crucially on the sign of β. First, we note that $\beta<0$ corresponds to the nontachyon case. This can be readily deduced from (2.4). As noted by [3–6], $\phi\equiv R$ evolves like a scalar field with $m^2=-1/6\times 16\pi G\beta$. Therefore, $\beta<0$ is needed to eliminate tachyons. Mijec *et al.* [9], Stelle [11], Teyssandier and Tourrenc [12], Barrow and Ottewill [13], and Mazzitelli and Rodrigues [14] have noted that $\beta<0$ is necessary otherwise the Hubble parameter grows without bound.

Confining ourselves to $\beta<0$, notice that, for $n>4$ (i.e., $p>\frac{1}{3}\rho$), $U(a)$ develops a potential barrier near $a\approx 0$ (Fig. 2). The interpretation is straightforward. For such a case, a contracting FRW universe ($k>0, k=0, k<0$) will rebounce to an expansion phase without a total gravitational collapse. The graph of region $p>\frac{1}{3}\rho$ is shown in Fig. 3(a). For comparison, the strong energy condition for an isotropic and homogeneous system [15], which predicts the existence of a singularity, gives $\rho+3p\geq 0$ and $\rho+p\geq 0$, Fig. 3(b). As is obvious from Figs. 3(a) and 3(b), most of the parameter space (ρ,p) which is predicted to have a singularity in Einstein gravity does not have a singularity in $R+\beta R^2$ gravity.

Several comments are in order. First, the strong energy condition comes from study of Raychaudhuri's equation, which describes how a congruence of timelike geodesics deviate from one another. Indeed, the appropriate strong energy condition for $R+R^2$ is different from that of Einstein gravity and can be shown to be in agreement with

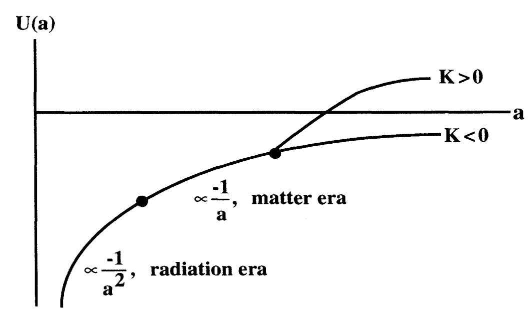

FIG. 1. $U(a)$ for $k>0$ and $k<0$ universe, $\beta=0$.

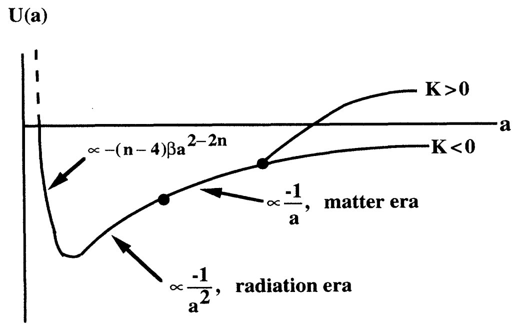

FIG. 2. $U(a)$ $n > 4, \beta < 0$.

the present result [16].

Second, Page [10], and Coule and Madsen [17] have studied a vacuum model and noted a bounce solution for only $k < 0$. The difference from the present work is more than a vacuum versus a nonvacuum model.

As explained in detail in Sec. II, in our approach, we have treated the βR^2 as a back reaction on Einstein gravity [6]. In essence, the physical gravitational degree of freedom has not been changed from that of Einstein gravity. This approach has the advantage of having a smooth limit as $\beta \to 0$, and avoids the pathological situation with the necessity for "extra" initial conditions in higher derivative theories.

On the other hand, the method of [10,17] is straightforward yet the field content and gravitational degree of freedom are different from those of Einstein gravity, and the existence of the limit as $\beta \to 0$ is questionable.

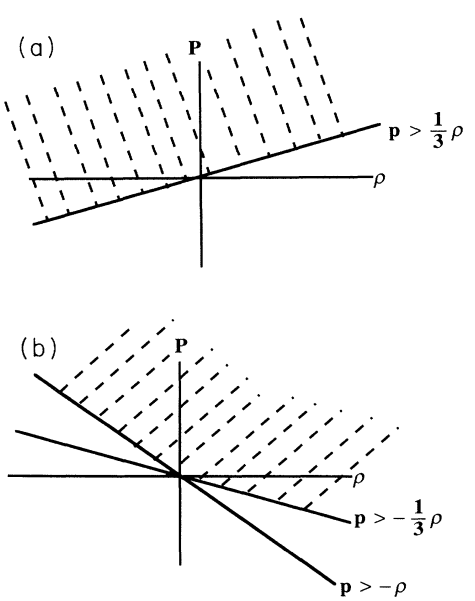

FIG. 3. Graph of region for $p > \frac{1}{3}\rho$ (a). Graph of regions $p \geq -\frac{1}{3}\rho$ and $p \geq -\rho$, for strong energy condition (b).

IV. QUANTIZATION OF MINISUPERSPACE MODEL

The prediction from classical analysis is indeed interesting, but near the Planck time a quantum analysis is needed. There has been some literature on quantizing the minisuperspace model for $R+R^2$ gravity [4,5]. In the literature, because of the technical difficulties of quantizing higher derivative theories, only very simple cases (i.e., no coupling to matter) have been considered. With our proposal of treating the R^2 term as a perturbation, we can readily address more realistic systems with matter.

Even though R^2 contains terms with higher time derivatives than in Einstein's action, we are treating it as a perturbation. Therefore the physical degrees of freedom (e.g., canonical momenta) are determined by Einstein's action alone [6].

For the FRW metric, the only gravitational degree of freedom is the scale factor $a(t)$. Therefore the canonical momentum conjugate to $a(t)$ is [18]

$$\pi_a \equiv \frac{\delta I_G}{\delta \dot{a}} = -\frac{3V_3}{4\pi G k^{3/2}} a\dot{a} = -\frac{3\pi}{2Gk^{3/2}} a\dot{a}. \tag{4.1}$$

In terms of π_a, the time-time component of (3.2) is

$$\pi_a^2 + 2W(a) = 0 \ , \tag{4.2}$$

$$2W(a) \equiv \left(\frac{12\pi^2}{\tilde{G}}\right)^2 \frac{2U(a)a^2}{k^3}. \tag{4.3}$$

$U(a)$ was defined in (3.8).

We will not need the expression for the Wheeler-DeWitt equation in its most general form. To quantize the system, we replace $\pi_a \to -i\partial/\partial a$ in (4.2) to get

$$\left[-\frac{\partial^2}{\partial a^2} + 2W(a)\right] \psi(a) = 0. \tag{4.4}$$

Because we are interested in semiclassical analysis, we have neglected the factor ordering parameter [19–21]. A comment is in order. Rigorously, there should also be a "kinetic" term $\partial^2/\partial\phi^2$ representing quantum fluctuation of some matter field. Indeed, the analysis gets quite

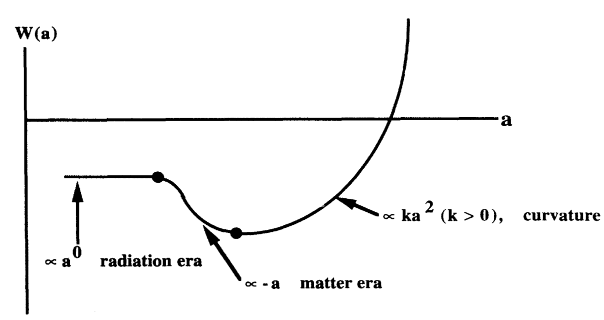

FIG. 4. $W(a)$ for $k > 0$ universe, $\beta = 0$.

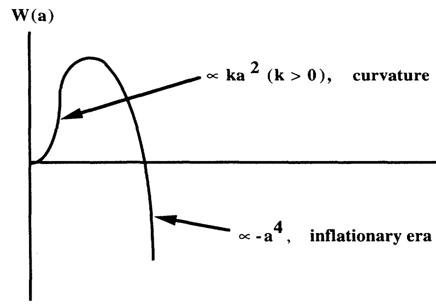

FIG. 5. $W(a)$ for very early universe during inflationary era, $n \approx 0$, $\beta = 0$.

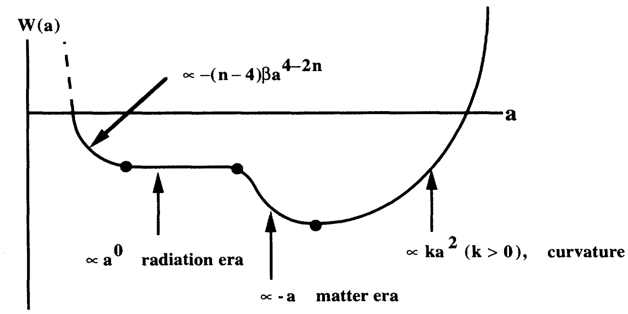

FIG. 6. $W(a)$ for $n > 4, \beta < 0$.

involved and it will be addressed in subsequent work. Here, we shall be satisfied with a particular "semiclassical" analysis in which only the gravitational sector is treated quantum mechanically [18, 22–25].

As before, we will assume that during any epoch in the evolution of universe the universe is dominated by a single matter with $\rho = \rho_0/a^n$. For example, the scalar field conformally coupled to gravity would be $n = 4$, a massive quantum field would be $n = 3$, and during a possible inflationary era $\rho_\phi \approx V(\phi) \approx$ const or $n = 0$, etc.

For $\beta = 0$ and closed ($k > 0$), the resulting Schrödinger equation resembles the quantum mechanical description of unit mass in a bound state potential $W(a)$ [18,22–24] (Fig. 4). The form of $W(a)$ for a universe which has undergone a standard inflationary phase (via a scalar field) is shown in Fig. 5. Since $a = 0$ is the boundary of a physically allowed region, a boundary condition (initial condition) $\psi(a = 0)$ must be specified to completely describe a system [26–29].

Now, for $\beta \neq 0$, the situation is similar to what we have discovered by classical analysis. Notice that again for $\beta < 0$ and for $n > 4$ (i.e., $p > \frac{1}{3}\rho$) $W(a)$ develops a potential barrier near $a \approx 0$ (Fig. 6).

The physical interpretation of ψ is unclear in quantum cosmology, but if one may interpret $[\psi \propto \exp(-|\,|)]$ as an indication of small probability, then our analysis indicated that a model with $p > \frac{1}{3}\rho$ and $\beta < 0$ will have a very small probability of a big bang or total recollapse. Notice that, unlike Einstein's gravity, there is no issue of boundary conditions. There is a shortcoming. In Fig. 6, the dotted region is precisely when the first order approximation breaks down.

V. CONCLUSION

In this paper, we have studied $R+\beta R^2$ by treating the R^2 term as a perturbation. For the FRW metric with matter, using the classical analysis, we have shown that for $n > 4$ (i.e., $p > \frac{1}{3}\rho$) and for a particular sign of β that corresponds to the nontachyon case, there is no big bang in the traditional sense. And therefore a recollapsing FRW closed universe will rebounce without a complete collapse.

From quantization of the corresponding minisuperspace model, we have shown that the potential $W(a)$, in the Wheeler-DeWitt equation $\left[-\partial^2/\partial a^2 + 2W(a)\right]\psi(a) = 0$, develops a repulsive barrier near $a \approx 0$ again for $n > 4$ (i.e., $p > \frac{1}{3}\rho$) and for the sign of β that corresponds to the nontachyon case. Since $a \approx 0$ is now strictly a classically forbidden region, the probability of finding a universe with a singularity ($a = 0$) is exponentially suppressed.

And in closing we can address the effects of a larger value of β. From (3.7), (3.8), and Fig. 2, and (4.2), (4.3), and Fig. 6, the minimal classical radius of collapse and the size of the classically forbidden region increase with larger $|\beta|$, respectively.

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