Gottfried sum rule and the ratio F_2^n/F_2^p

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We describe the nucleon as a bound state of three constituent objects, called "valons," which themselves have structure. At high enough Q^2 it is the valon structure, governed by QCD, which is probed and, thus, the nucleon structure is described in terms of its partonic distributions, while at low Q^2 the nucleon is described in terms of its valon distributions, independent of a probe and controlled by nonperturbative QCD. The implications of this phenomenological model, then, are applied to the New Muon Collaboration (NMC) data for F_2^n/F_2^p and on the Gottfried sum rule. It is shown that the model successfully reproduces the experimental value of the Gottfried sum rule $S_G[0 \le x \le 1, \langle Q^2\rangle = 4 \text{ GeV}^2] = 0.243$, consistent with the experimental result as well as the ratio F_2^n/F_2^p down to the lowest x value.

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I. INTRODUCTION

The New Muon Collaboration (NMC) [1,2] experiment and the Fermilab E665 [3] experiment have provided us with the ratio of the structure functions F_2^n/F_2^p obtained in deep-inelastic scattering of muons on hydrogen and deuterium targets. The data cover the kinematical range down to $x = 0.004$ and Q^2 as low as 0.3 GeV^2 for the NMC and ($x = 0.000\,05, Q^2 = 0.03$ GeV^2) for the E665 experiment. As is well known, the first moment of $F_2(x)$ in the parton model is equal to the sum of the electric charge squares times the momentum distributions of all quark and antiquark parton flavors in the target:

$$\int_0^1 dx \frac{F_2(x)}{x} = \sum_i e_i^2 q_i(x) \ , \qquad (1)$$

where $q_i(x)$ is the momentum distribution of quark flavor i. The difference of the first moment (1) measured with a proton and neutron target is

$$S_G \equiv \int_0^1 dx \frac{F_2^p(x) - F_2^n(x)}{x} \ . \qquad (2)$$

If one computes the above difference in (2) with the assumption that sea quark and antiquark distributions for u and d quarks are the same, then one arrives at $S_G = \frac{1}{3}$. This value of Eq. (2) is known as the Gottfried sum rule. The NMC data [1,2] instead lead to a value

$$S_G = 0.235 \pm 0.026 \ . \qquad (3)$$

This value is obtained at $\langle Q^2\rangle = 4$ GeV^2 by extrapolating the value of the measured x region,

$$S_G[0 \le x \le 0.8, \langle Q^2\rangle = 4 \text{ GeV}^2] = 0.227 \pm 0.021 \ , \qquad (4)$$

to the full $0 \le x \le 1$ range. Since there is no free neutron available, the neutron structure function is extracted from the difference between deuteron and proton structure functions neglecting nuclear shadowing.

The discrepancy between the Gottfried sum-rule value of $\frac{1}{3}$ and the experimental value given in Eq. (3) has generated considerable theoretical activities. It is attributed to the excess of d quarks in the sea over u quarks [4] by some authors and to the nuclear medium effect by the others [5]. In the latter case the nuclear shadowing phenomena is considered to set in at small x which changes the quark distribution in the nucleon as compared to the free nucleon. If nuclear shadowing is taken into account it gives yet a smaller value of the S_G around 0.19 [5] which is far smaller than what is measured.

In this paper we will offer an alternative approach which will regenerate NMC results on the ratio F_2^n/F_2^p and data from the Fermilab E665 experiment [3] on the ratio of cross sections which is measured at much lower x variables ($x = 0.000\,05$; $Q^2 = 0.03$ GeV^2) than in the NMC experiment. We will show that the results of the model nicely accommodate the Gottfried sum-rule value measured by the NMC. This will be done in the context of the so-called "valon model" [6], described briefly in the next section.

II. THE VALON MODEL

The valon description of the nucleon was developed by Hwa [6,7] some years ago and was successfully used [8] to describe the contribution of constituents of the proton to its spin in order to understand the so-called *spin crisis*. Encouraged by this we have applied the same model with an appropriate assumption relevant to the Gottfried sum-rule issue. Below we will give a brief description of the valon model. An extensive review of the model can be found in [6], and references cited therein.

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A valon is defined to be a dressed valence quark in QCD with cloud of gluons and sea quarks which can be resolved by high-Q^2 probes. In a scattering process the virtual emission and absorption of gluons in a valon become bremsstrahlung and pair creation. The structure function of a valon is determined by gluon bremsstrahlung and pair creation in QCD. At sufficiently low Q^2 the internal structure of a valon cannot be resolved and hence it behaves as a valence quark.

Let $G_{v/h}(y)$ describe the valon distribution in a hadron and $F^N(x,Q^2)$ denote the structure function of nucleon. Denoting the valon structure function by $f^v(z,Q^2)$, then the two structure functions are related by the convolution theorem as

$$F^N(x,Q^2)=\sum_v\int_x^1 dy G_{v/n}(y) f^v\left(\frac{x}{y},Q^2\right), \quad (5)$$

where $F^v(z,Q^2)$ is described accurately by the leading-order results in QCD [6], and its moments expressed completely in terms of evolution parameter $S = \ln[\ln(Q^2/\Lambda^2)/\ln(Q_0^2/\Lambda^2)]$.

According to the valon picture of the nucleon, unpolarized parton distributions are given as [6,7]

$$xu_v(x)=6.3385(1-x)^{3.3346}x^{0.734}, \quad (6a)$$

$$xd_v(x)=2.271\,22(1-x)^{3.574}x^{0.636}, \quad (6b)$$

$$xG(x)=0.749(1-x)^{4.6977}+1.7636(1-x)^{11.9583} +8.1155(1-x)^{44.7342}, \quad (6c)$$

$$xu_{\rm sea}(x)=xd_{\rm sea}(x)=0.934e^{-5.6x}(1-x)^{2.59}, \quad (6d)$$

$$xs_{\rm sea}(x)=0.168e^{-5.6x}(1-x)^{2.59}, \quad (6e)$$

where $q_v(x)$, $G(x)$, and $q_{\rm sea}(x)$ refer to valence quark, gluon, and sea quark distributions in the proton, respectively. These distributions are evaluated for $Q^2 = 4$ GeV^2. The valon distributions in the proton, $U_p(y)$ and $D_p(y)$, which are Q^2 independent [6] and governed by nonperturbative QCD, are given as

$$U_p(y)=7.98y^{0.65}(1-y)^2, \quad (7a)$$

$$D_p(y)=6.01y^{0.35}(1-y)^{2.3}. \quad (7b)$$

The fact that the bound-state problem of the nucleon can be described by three constituent quarks with internal structure implies that spatially they do not overlap appreciably. This picture of the nucleon in terms of three valons is similar to the usual picture of deuteron in terms of two nucleons. At low Q^2 the resolution is low and the description of the deuteron as a bound state of two nucleons is adequate and the medium to long range of the deuteron wave function summarizes the bound-state problem. The short-range part of the deuteron wave function is complicated and is intimately related to the nucleon structure [6].

III. CALCULATION OF GOTTFRIED SUM RULE

The NMC data are taken at the kinematic range $0.004 < x < 0.8$ and $0.4 < Q^2 < 190$ GeV^2, and the results are given for an averaged $Q^2 = 4$ GeV^2. At very low Q^2, usually corresponding to low x, we are not able to use parton distributions of Eq. (6) to evaluate structure functions, according to Eq. (1), within the valon picture. If Q^2 is low, then the probe is not able to resolve the structure of the constituent; i.e., in the low Q^2 the probe γ^* cannot distinguish the partonic structure of a nucleon and, hence, the parton model is inapplicable. Therefore, in evaluating the integral in Eq. (2) we divide x into two regions: (a) for $x < 0.07$ we use valon distribution in calculating structure functions (the low-resolution region, corresponding to low Q^2); and (b) for $x > 0.07$, where we describe the structure functions in terms of the parton distributions. This is the only assumption that we have made in the following calculations. In what follows, the calculation of the Gottfried sum rule is done without nuclear shadowing considered; however, we will comment on the shadowing phenomena later regarding the ratio F_2^n/F_2^p.

Using Eq. (2) along with parton distributions of Eq. (6) we get

$$\int_{0.07}^1 dx\left(\frac{F_2^p(x)-F_2^n(x)}{x}\right)_{\rm parton}=0.2616, \quad (8)$$

and, for the range of $0 < x < 0.07$, from the valon distribution of Eq. (7) we have

$$\int_0^{0.07} dx\left(\frac{F_2^p(x)-F_2^n(x)}{x}\right)_{\rm valon}=-0.018\,95\,. \quad (9)$$

Now, the Gottfried sum can be obtained combining Eqs. (8) and (9) and we get

$$S_G=\int_0^1 dx\frac{F_2^p(x)-F_2^n(x)}{x}=0.2427\,. \quad (10)$$

This result is evidently in good agreement with the result in Eq. (3). In Fig. 1, we also present the ratio F_n^2/F_p^2 calculated using the model. The experimental values from the NMC [1,2] and E665 [3] experiments are also presented. As one can see from Fig. 1, the model calculation

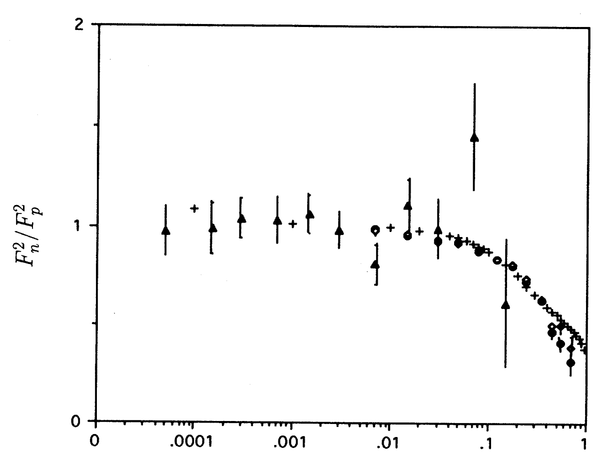

FIG. 1. Ratio of F_2^n/F_2^p vs x from the model calculation (+), and the results of NMC measurements of [1,2] (circles), and Fermilab E665 (triangles) [3]. Errors are statistical only.

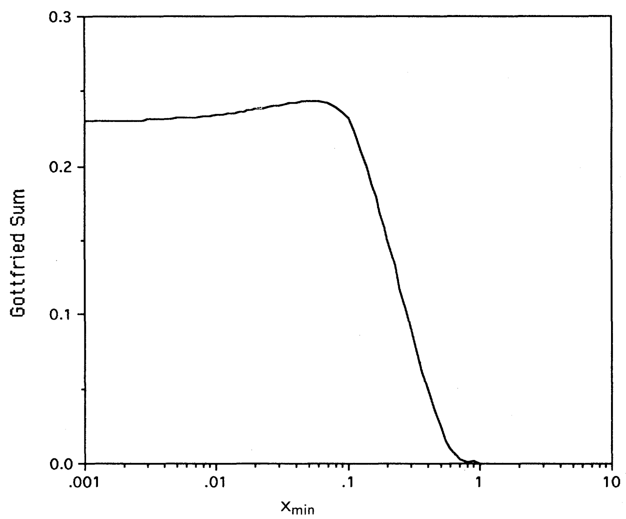

FIG. 2. $\int F_2^p(x) - F_2^n(x)dx/x$ vs $x_{\min}$ for the Gottfried sum rule calculated from the model. For details see the text.

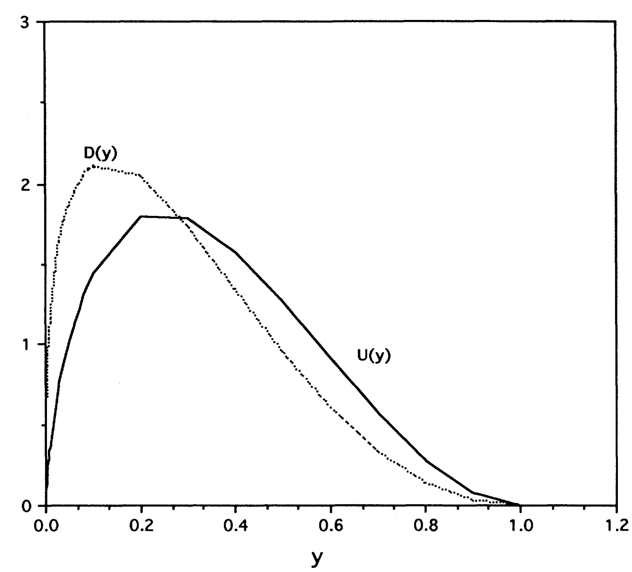

FIG. 3. The valon distribution vs y of Eq. (7).

is in perfect agreement with both experiments.

It is interesting to investigate the impact of nuclear shadowing on our results. It is known that a gluon shadowing at small x will alter the gluon distribution at higher x values. Similarly, quark and antiquark annihilation and creation which shadows at some low x value alters those partonic distributions at other x values. In this respect one also leads to consider antishadowing effects. We have chosen the nuclear shadowing model of Mueller and Qiu [9] to investigate the nuclear shadowing effects within our model. Since the model of [10] is based on the gluon recombination mechanism as a cause for nuclear shadowing it is necessary to multiply the sea quark distribution by the shadowing factor $R_s(x, A)$ given as

$$R_s(x,A) = \begin{cases} 1, \quad x_n < x < 1, \\ 1 - 0.1(A^{1/3} - 1)\left(\frac{1}{x} - \frac{1}{x_n}\right)\left(\frac{1}{x_A} - \frac{1}{x_n}\right), \quad x_A < x < x_n, \\ 1 - 0.1(A^{1/3} - 1), \quad 0 < x < x_A, \end{cases} \tag{11}$$

where $x_n = 1/(2R_n m_N)$ is the onset point for shadowing and $x_A = 1/(2R_A m_N)$ the saturation point for shadowing. For the deuteron with a radius 4.23 fm and a nucleon of radius of 1 fm, we have $x_n = 0.1$ and $x_A = 0.02$.

Therefore we divide x into three intervals to include shadowing effects: namely, (i) $0.1 < x < 1$, (ii) $0.02 < x < 0.1$, and (iii) $0 < x < 0.02$. The contributions to S_G from various intervals of x, with the shadowing factor included for $x < 0.1$, are obtained as

$$S_1 = [0.02 < x < 0.1] = 0.077\,89, \tag{12a}$$

$$S_2 = [0 < x < 0.02] = 0.009\,359, \tag{12b}$$

$$S_3 = [0.1 < x < 1] = 0.232\,78, \tag{12c}$$

and hence, $S_G = S_1 + S_2 + S_3 = 0.320\,02$, which is much different from what the NMC measurement for S_G implies. This investigation suggests that the violation of the Gottfried sum rule may not be due to the nuclear shadowing effect, at least in the Qiu [10] model for shadowing, or, to put it in other words, its impact on lightest nucleus, the deuteron, is negligible; while for a heavier nucleus it is appreciable. Of course this is not a drastic conclusion since, based on geometrical arguments, only the nucleon inside of the cubic angle $\Delta\Omega = \pi R_n^2/R_D^2$ is possible to shadow the other nucleon in the deuteron, where R_D is the deuteron radius. In Fig. 1, we have included the shadowing effect for the two lowest x points in our model.

In Fig. 2, we have plotted S_G as a function of x from our model for $Q^2 = 4$ GeV^2. We have also recalculated our results for a higher value of $Q^2 = 11$ GeV^2 and found that the features presented here persist, and $x = 0.07$ can be lowered due to the higher Q^2 probe.

IV. CONCLUSION

In this paper we have shown that within the valon model of [6] and with the assumption that there is a threshold for the Q^2 value of the probe in order to resolve the partonic structure of the nucleon, one can nicely accommodate the NMC [1,2] and E665 [3] data and pin down the origin of the Gottfried sum-rule violation. It appears to us that while the nuclear shadowing effect, which is inherited from the famous EMC effect, in fact is significant for heavier nuclear targets it is also negligible for the deuterium target. The physics at issue here seems to be a low Q^2 phenomena rather than a low x issue. This can be justified in the sense that the data are taken at very low Q^2 for the smaller x bins.

Finally, we have plotted in Fig. 3 the valon distributions for $U(y)$ and $D(y)$ of Eq. (7) which is broader than a δ function at $y = \frac{1}{3}$. This implies that the valons are not loosely bound, the assumption that played a role in this analysis.

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