

# Quantum fluctuations, decoherence of the mean field, and structure formation in the early Universe

E. Calzetta

*Instituto de Astronomía y Física del Espacio and Facultad de Ciencias Exactas y Naturales,  
University of Buenos Aires, Argentina*

B. L. Hu

*Department of Physics, University of Maryland, College Park, Maryland 20742  
and School of Natural Sciences, Institute for Advanced Study, Princeton, New Jersey 08540*

(Received 10 May 1995)

We examine from first principles one of the basic assumptions of modern quantum theories of structure formation in the early Universe, i.e., the conditions upon which fluctuations of a quantum field may transmute into classical stochastic perturbations, which grew into galaxies. Our earlier works have discussed the quantum origin of noise in stochastic inflation and quantum fluctuations as measured by particle creation in semiclassical gravity. Here we focus on decoherence and the relation of quantum and classical fluctuations. Instead of using the rather *ad hoc* splitting of a quantum field into long and short wavelength parts, the latter providing the noise which decoheres the former, we treat a nonlinear theory and examine the decoherence of a quantum mean field by its own quantum fluctuations, or that of other fields it interacts with. This is an example of how a quantum system can be viewed as effectively open and decoheres through its own dynamics. The model we use to discuss fluctuation generation has the inflaton field coupled to the graviton field. We show that when the quantum to classical transition is properly treated, with due consideration of the relation of decoherence, noise, fluctuation, and dissipation, the amplitude of density contrast predicted falls in the acceptable range without requiring a fine-tuning of the coupling constant of the inflaton field. The conventional treatment which requires an unnaturally small  $\lambda \approx 10^{-12}$  in a  $\lambda\phi^4$  inflaton field stems from a basic flaw in naively identifying classical perturbations with quantum fluctuations.

PACS number(s): 98.80.Cq, 03.65.Sq, 05.40.+j, 04.62.+v

## I. INTRODUCTION AND SUMMARY

In this paper we shall examine, starting from first principles, under what circumstances the fluctuations of a quantum field transmute into classical, stochastic fluctuations. To do so we shall analyze the relationship between the phenomena of dissipation, fluctuation, noise, and decoherence [1], first in an interacting scalar field theory in flat space-time [2], and then in the more complex but realistic case of a scalar field interacting with gravitons in an expanding universe.

The main motivation for this work is to develop the necessary tools to analyze the quantum to classical transition of primordial density fluctuations in the early Universe. Indeed, this is the third of a series of papers by the authors and collaborators on the quantum statistical theory of structure formation. The first one [3] calls into question conventional treatments of this issue, and focuses on decoherence and the quantum origin of noise in stochastic inflation. The second one [4] explains how noise and fluctuations originate from particle creation in semiclassical gravity, and casts doubt on the conventional practice of simplistically identifying quantum and classical fluctuations. In this paper we will discuss how a quantum mean field can be decohered by its own quantum fluctuations, and turn into a classical field with a stochastic component which is hitherto unnoticed or ignored in quantum field theory. We will also explain how

a proper treatment of quantum and classical fluctuations can lead to a much improved prediction of the density contrast in the inflationary cosmology.

### A. Outstanding issues in the quantum theories of galaxy formation

Let us begin by placing the present discussion within the larger context of theories of structure formation in the Universe. A standard mechanism for galaxy formation is the amplification of primordial density fluctuations by the evolutionary dynamics of space-time [5,6]. In the lowest order approximation the gravitational perturbations (scalar perturbations for matter density and tensor perturbations for gravitational waves) obey linear equations of motion. Their initial values and distributions are stipulated, generally assumed to be a white noise spectrum. In these theories, fashionable in the 1960s and 1970s, the primordial fluctuations are classical in nature. The standard model of Friedmann-Lemaître-Robertson-Walker (FLRW), where the scale factor of the Universe grows as a power of cosmic time, generates a density contrast which turns out to be too small to account for the observed galaxy masses. The observed nearly scale-invariant spectrum [7] also does not find any easy explanation in this model [8,9].

In the inflationary cosmology of the 1980s [10–12] a constant vacuum energy density of a quantum field  $\Phi$ , the

inflaton, drives the universe into a phase of exponential expansion, with the scale factor  $a(t) = a_0 \exp(Ht)$ , where  $H = \dot{a}/a$  is the Hubble expansion rate, assumed to be a constant for the de Sitter phase of the evolution (a dot over a quantity stands for a derivative with respect to cosmic time). The scalar field  $\Phi$  evolves according to the equation

$$\ddot{\Phi} + 3H\dot{\Phi} + V'[\Phi] = 0, \quad (1.1)$$

where the potential  $V[\Phi]$  can take on a variety of forms, such as  $\Phi^4$  (Guth's original "old" inflation via tunneling [10], or Linde's chaotic inflation via rolling down the potential [13]), Coleman-Weinberg ("new" inflation [11,12]), or an exponential form (for power law inflation [14]).<sup>1</sup> Consider the "eternal inflation" stage where the Universe has locally a de Sitter geometry, with a constant Hubble radius (de Sitter horizon)  $l_h = H^{-1}$ . The physical wavelength  $l$  of a mode of the inflaton field is  $l = p^{-1} = a/k$ , where  $k$  is the wave number of that mode. As the scale factor increases exponentially, the wavelengths of many modes can grow larger than the horizon size. After the end of the de Sitter phase, the Universe begins to reheat and turns into a radiation-dominated Friedmann universe with power law expansion  $a(t) \sim t^n$ . In this phase, the Hubble radius grows much faster than the physical wavelength, and some inflaton modes will reenter the horizon. The fluctuations of these long-wavelength inflaton modes that go out of the de Sitter horizon and later come back into the FLRW horizon play an important role in determining the large scale density fluctuations of the early Universe, which in time seeded the galaxies [15].

With the exponential expansion in the de Sitter phase, any classical primordial inhomogeneity will likely be redshifted to insignificance by the time the relevant modes leave the horizon, and one may wonder where such fluctuations could arise. In stochastic inflation, Starobinsky [16] and others observed that the inflaton field driving inflation is itself subject to quantum fluctuations, which may provide the seeds for structure formation. They assume that the inflation field is divided into two parts at every instant according to their physical wavelengths, i.e.,

$$\Phi(x) = \phi(x) + \psi(x). \quad (1.2)$$

The first part  $\phi$  (the "system field") consists of field modes whose physical wavelengths are longer than the de Sitter horizon size  $l > \epsilon H$ , or  $p < \epsilon H$  ( $\epsilon \approx 1$ ). The second part  $\psi$  (the "environment field") consists of field

modes whose physical wavelengths are shorter than the horizon size, or  $p > \epsilon H$ . Inflation continuously shifts more and more modes of the environment field into the system, stretching their physical wavelengths beyond the de Sitter horizon size. It is often stated that this process generates an effective interaction between system and environment, and the system field would then be randomly driven by the unknown environment field, developing stochastic fluctuations which are the required primordial fluctuations. We have doubts on this mechanism, noting that a free field cannot generate noise.

While this overall picture is generally agreeable, not least because of its qualitative depictive power (it makes present day structures correspond to near-Planckian scales early enough in the inflationary period, whereby the physics of these fluctuations is expected to be mostly model independent), it has some basic shortcomings, like the oversimplified treatment of the quantum to classical transition and the unnecessarily special role it ascribes a system-environment split. As we shall show below, in taking these conceptual points seriously one can significantly improve on the quantitative predictions on inflationary models. Let us now discuss these issues in more detail, as an introduction to the main body of this paper.

### 1. How quantum fields acquire classical stochastic behavior: Decoherence

Consider the model of a free, massless, minimally-coupled inflaton field. Using the separation (1.2), the equation of motion for the system field  $\phi$  is given by

$$\ddot{\phi}(t) + 3H\dot{\phi} + V'(\phi) = \xi(t), \quad (1.3)$$

where, according to authors of [16],  $\xi$  is a white noise originating from the high frequency modes of the bath field  $\psi$  with properties,

$$\langle \xi(t) \rangle = 0, \quad \langle \xi(t)\xi(t') \rangle \simeq \delta(t - t'). \quad (1.4)$$

The common belief is that the short wavelength field modes (the bath) contribute a white noise source to a classical Langevin equation governing the long-wavelength (system) field modes. A Fokker-Planck equation can also be derived which depicts the evolution of the probability distribution of the scalar field  $\mathcal{P}(\phi, t)$ . Much recent effort is devoted to the solution of this Langevin equation or its related Fokker-Planck equation for descriptions of the inflationary transition and galaxy formation problems. Although this scenario leads to the prediction of an essentially scale-free distribution of density fluctuations, consistent with the observational data [7], and in spite of continued efforts, no satisfactory implementation of these ideas has been proposed so far.

Note that in transforming a quantum field theoretic problem to a classical stochastic mechanics problem as in here, two basic assumptions are made: (1) the low frequency scalar field modes (the system field) behave clas-

<sup>1</sup>As we shall discuss in more detail below, agreement with the average amplitude of primordial energy density fluctuations requires, in the conventional approaches, that the scalar potential has a flat plateau, which generally is only possible if the potential is fine-tuned for that purpose. For at least this reason, none of the implementations of inflation proposed so far is regarded as totally satisfactory.

sically, and (2) the high frequency quantum field modes (the environment field) behave like a white noise.

Most previous researchers seem to hold the view that the first condition is pretty obvious [17] and that the second condition can be easily proven. One of us [18] challenged this view and called attention to the need for building a sounder foundation to the quantum theory of structure formation. A rigorous program of investigation was outlined in [3] with quantum open system concepts [19] and the influence functional formalism [20]. It was stressed that on the issue of quantum to classical transition, one needs to consider the decoherence process [21,22], and on the issue of noise, one needs to trace its origin to the quantum field interactions and the coarse-graining measures involved. These two issues are interrelated, as the noise in the environment is what decoheres the system and endows it with a classical stochastic dynamics.

Technically, the dynamics of the system is described by an influence action, which is generally both complex and nonlocal (it becomes local for the rather special case of an Ohmic bath, but this is unimportant to our present concerns). The imaginary part of this influence action is related to both decoherence and spontaneous fluctuations in the system variables; thus, decoherence is always associated with noisiness [23,24]. The nonlocal part is associated with dissipation, and it is related to the imaginary part through the fluctuation-dissipation relation [25–27]. Thus, in a nonlinear theory, decoherence, fluctuation, and dissipation are interrelated aspects of the same phenomenon [1,23,4]. We can visualize this as dissipation representing the average action of the bath on the system, while fluctuation describing the departures from the average. The nonlinear interaction also creates correlations, whose severing upon tracing of the bath degrees of freedom induces decoherence.<sup>2</sup>

In [3] a model of two interacting fields representing the system and the bath is used to derive the (functional) Langevin equation and the correlator of the (colored) noise. Further work need be carried out in finding solutions to these stochastic equations for galaxy formation considerations [29]. A recent work along lines similar to ours is that of Buryak [30].

Given the complexity of the quantum to classical tran-

sition issue, one may be tempted, as is indeed the case for most papers on this topic, to forget all about it, simply expand the quantum inflaton field in any suitable set of modes, and identify the density profile with the amplitude of those modes. However, some careful thought will reveal this position to be untenable. To begin with, extracting the physically observable field variable out of the basic quantum one is not always trivial; both are related through the renormalization process [31]. Besides, while one can describe the quantum fluctuations in the inflaton field as a coherent superposition of localized fluctuations, this does not imply a physical inhomogeneity, because different fluctuations are not mutually exclusive, and the quantum state is homogeneous [32]. Only when these fluctuations become mutually exclusive, through the process of decoherence, and some of them are realized, by the equivalent of some “measurement” process, will it be proper to speak of inhomogeneity in the Universe. In other words, a quantum field may be expanded in any set of basic modes (for example, Minkowsky or Rindler modes in flat space-time), but only one preferred set may describe the observable (classical) density fluctuations. Which mechanism gives that particular set its special character is a physical question (not unlike which criterium picks out the preferred pointer basis in environment-induced state reduction [21]), and should better be answered on the basis of the dynamics of the system itself.

## 2. Coarse graining a noninteracting field cannot generate noise

Most discussions on the origin of primordial fluctuations in the literature are confined to a free scalar field propagating on a fixed geometrical background. This cannot, as we argued in [3] using stochastic inflation as an example, generate any noise, and without noise the system cannot decohere and become classical. It also misses out on all the interesting phenomena associated with changes of correlations in the system due to nonlinearities. (Even those who properly account for the mixing of matter and gravitational degrees of freedom, say, by employing gauge invariant rather than canonical variables [6], often stop at the linearized level, thus missing the dynamical contribution to decoherence and the evolution of correlations.) To compound the situation, most people would agree that the initial quantum state of the field should be read out of a Hartle-Hawking-like “wave function of the Universe” [33,34], which predicts lack of correlations among different modes. This leaves the inflation practitioner with only two alternatives, namely, either consider several free scalar fields and add a mixing matrix [35], so that the relevant degrees of freedom are not those that diagonalize the Hamiltonian, or else consider a time-dependent system-bath split, so that the correlations are carried by the modes themselves, as they switch labels from “bath mode” to “system mode” or vice versa [16,35]. It should be observed that the first alternative detracts from the predictive power of the model, by introducing the elements of the mixing matrix as so

<sup>2</sup>In order to prevent misunderstanding, let us observe that in the case of a quantum oscillator following an accelerated trajectory and coupled to a quantum scalar field, where the influence action may be nontrivial even if the Hamiltonian is quadratic [28], the Hamiltonian is not diagonal if expressed in terms of “oscillator” and “field” degrees of freedom; actually, the transformation from these “naive” modes to those that diagonalize the Hamiltonian is nonanalytic in the coupling strength among field and oscillator, so we cannot even speak of a “weakly coupled” regime. Therefore, in this case we may say that the nontriviality of the influence action is induced by an arbitrary partition of the degrees of freedom in relevant and otherwise.

many new free parameters. In the second approach, the whole issue of structure formation seems to hang on the special way one labels the modes which define the system. This subjective element we find rather uneasy. There is a third alternative, which is to assume that decoherence never occurred, at least at long enough wavelengths [36]. This we regard as an evasive way out. We would prefer to see the decoherence of a system as a consequence of its own dynamics.

### 3. Overproduction of density contrast and the fine-tuning problem

In addition to the problem of deriving a classical stochastic equation from quantum field theory, there is also the outstanding problem of overamplification of density contrast and unnatural constraining of the field parameters. Recall that one distinct advantage of inflation is that it provides a natural explanation of the scale-invariant Harrison-Zeldovich spectrum [7]. But the excess amplitude in the density contrast is still an unresolved problem. The density contrast  $\delta\rho/\rho$  can be shown to be related to the fluctuations of the scalar field  $\delta\Phi$  approximately by [15]

$$\frac{\delta\rho}{\rho} \approx \frac{H\delta\Phi}{\langle\dot{\Phi}\rangle}, \quad (1.5)$$

where angular brackets denote averaging over some spatial range. In the conventional treatment (where quantum fluctuations are treated in the same capacity as classical fluctuations), for the density contrasts to be within  $10^{-5}$  when the modes enter the horizon, the coupling constant in the Higgs field (of, say, a  $\lambda\phi^4$  theory in the standard grand unified models) has to be fine-tuned to an unnaturally small value ( $\lambda \sim 10^{-12}$ ).

In summary, there are two sets of outstanding issues: (1) How the long-wavelength modes become classical, and the quantum fluctuations develop into classical perturbations, and (2) how to get the correct order of magnitude for the density contrast without assuming an unnatural value for the field parameters.

We shall show in this paper that these two issues are related to each other: decoherence of long-wavelength modes by short wavelength modes (as done in [3]), or a mean field by its quantum fluctuations (done here), gives rise to a classical stochastic evolution with noise properly determined by the coupling between these two sectors. We shall consider a more realistic model of gravitons coupled to the inflaton, and show that a correct treatment of the relationship between quantum and classical fluctuations can provide a much improved estimate of the density contrast without “fine-tuning.” We shall also investigate a different type of system-environment split, namely, that between the mean field and its fluctuations. This is in the spirit of the background field method used frequently in quantum field theory. It is particularly relevant to improvements beyond the mean field results in phase transition problems [37]. We will also use this split

as an example (the simplest) of the correlation hierarchy discussed elsewhere in the context of decoherence [38].

It should be clear from the above that the proper identification of the relevant system and its environment is an essential part of the analysis of fluctuation generation in the early Universe. This identification reflects an arbitrary choice made freely by the “observer.” Contrary to this conventional view, we note that on any given physical situation there are only a few meaningful ways to identify the relevant system, which are prescribed by the dynamics and the limitations of observation. Let us clarify this important point, as a way to approach our main concerns.

### B. Our approach: Decoherence of a nonlinear quantum field by its own quantum fluctuations

The criteria for choosing a particular subsystem for special treatment (calling it relevant and the rest irrelevant [39] is already a preferential treatment), i.e., the definition of an open system, is, to us, as important a physical issue as describing the evolution of the open system. (For a general discussion, see [18,40].) The possibility of successfully identifying a relevant system within a complex physical problem hinges on the decoupling of some degrees of freedom from the rest. If the complete system is divisible into two sectors (subsystems) with significant difference in their characteristic time, frequency, energy, mass, length or interaction scales, then one can view one as the (open) system and the other as the environment. An example of mass discrepancy is the case of quantum cosmology [33], where the much heavier Planck mass makes it possible to treat the gravitational sector differently via the Born-Oppenheimer approximation. Decoherence of the “massive” gravitational sector by the “lighter” matter field sector can lead to the emergence of classical space-times in the semiclassical gravity regime [34]. Another example is the separation of “slow-fast” variables [41]. On the slow time scale, only the average action of the fast degrees of freedom affects the relevant slow modes in an appreciable way. Factoring in the asymptotic behavior of the fast variables, one can express their average influence in terms of the slow variables themselves, thus obtaining an effectively closed (and generally irreversible) dynamics for the latter [39].

While there are many ways to split a complex system into a system proper and an environment, only a few of these lead to physically interesting theories. For example, not only the system proper should include everything of interest to this or that particular observer, but also the dynamics of the system proper should admit a closed, self-consistent description (with some degree of stochasticity). This requires that the system and bath to be weakly coupled, the system being robust against the perturbation induced by its environment. Thus the issue of the proper system-bath split in a definite situation is not to be answered by the consideration of the observer’s interests alone. On the contrary, the answer should be

rooted in the physics of the system and the observational context.<sup>3</sup>

### 1. *Nonlinear fields and correlation dynamics*

As an open system is identifiably or dynamically separated from its environment, decoherence occurs as it habitually interacts only with the averaged environmental degrees of freedom *if and only if* there are nontrivial correlations between the system and environment variables. These correlations, in turn, may have a dynamical origin, which requires nonlinear interactions between system and bath, or else they may be present already in the initial conditions [43].

Focusing on the correlational aspects, we have proposed earlier [38] that a natural way to partition a closed system is by way of the correlation functions, defining the system as a subset of the Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) (classical) or Dyson (quantum) hierarchies of correlation functions. The process of reducing the full dynamics to the autonomous dynamics of the subsystem is usually described as “truncation” and “factorization.” In the Boltzmann molecular dynamics case this involves truncating the BBGKY hierarchy at a correlation order and assuming that this order of correlation function can be written as a direct product of the lower order ones, known as the molecular chaos assumption. The actual state of the environmental modes, however, is never quite equal to their truncated value; the small discrepancy is fed back into the evolution of the system degrees of freedom as noise. Effective autonomy of the relevant system is needed for the stability and robustness which are defining properties of classicality; but it is undermined constantly by the effect of noise and fluctuations, which, as we have seen before [1,23], is instrumental to decoherence and the emergence of classical behavior. Because dissipation and noise are two aspects of the same underlying physics, they are linked by consistency relations, known categorically as the fluctuation-dissipation relations [25], which underly the theory of fluctuations in the stochastic, kinetic, and hydrodynamic regimes [44]. (Under equilibrium conditions, these relations take the form of the famous Green-Kubo formulas [25]. The existence of such a relation in nonequilibrium conditions is explored in [4,27].) Such is the necessary dynamical balance which prevails in the quantum-classical interface.

In what follows, we shall concentrate on nonlinear theories, and seek to understand the generic conditions for a certain subset of the degrees of freedom to decouple from the rest, while being decohered and randomly driven by the remaining degrees of freedom. Our goal is to show how these processes actually occur in interacting field

theories, and apply the results to fluctuation generation in the early Universe which is only a manifestation of this universal phenomenon.

### 2. *Mean field and quantum fluctuations*

In field theory applications, the different scales are usually associated to the masses of the different particles, the mass being, in natural units, the inverse correlation scale [45]. In the presence of spontaneous symmetry breaking, another set of scales appear, associated with the development of phase transition on one hand, and of quantum fluctuations around the instantaneous value of the mean field on the other [37]. In addition to these, there is an intrinsic scale separation associated with nonlinear quantum field theories, which arises because the physical, observable excitations of the field are “dressed” by a cloud of virtual, microscopic quantum fluctuations. Thus we can distinguish between the scale associated with the physical or dressed excitations, and that associated with the microscopic, elementary fluctuations [46]. The decoherence of quantized dressed excitations is the main focus of this paper.

The description of the quantum to classical transition in terms of the decoherence of the dressed field has several advantages over the conventional procedure of splitting the field modes by hand into relevant and irrelevant, the most important being that in this approach we do not have to prejudge the importance of the different modes. Thus, for example, in the actual application to fields in de Sitter space, we shall not require any *a priori* consideration on the behavior of the different excitations on horizon crossing. These considerations are sometimes hard to justify on a rigorous basis, since the de Sitter horizon is an observer-dependent construction, with no objective geometrical meaning. Moreover, our approach turns out to be just the simplest of a hierarchy of increasingly accurate descriptions of the field, where not only the dressed field but also other composite operators are retained as relevant. We have presented the details of the full approach elsewhere [38].

### 3. *Organization of this paper*

The paper is organized as follows. In Sec. II we discuss the decoherence of fluctuations in the dressed field of a self-interacting, symmetry breaking field theory in flat space. For simplicity, we shall only consider fluctuations around the false vacuum state, rather than the phase transition in full generality. Our objective is to lay down the basic elements of our approach, putting strong emphasis on the physical processes linking dissipation, noise and decoherence to each other. Section III applies the formalism above to a massless minimal field in a de

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<sup>3</sup>There are more sophisticated ways to define an open system, such as by the partition of either physical or phase space into relevant and irrelevant sectors (see, e.g., [42]).

Sitter background. This can be taken as a simple model describing the physics of fluctuations in the inflaton field in the early stage of inflation. Despite its appearance, the theory is nonlinear, because of the coupling of the scalar field to gravitons. Of course, since we do not allow for correlations to be present in the initial state, nonlinearity is a necessary condition for decoherence. The minimal coupling of field and gravitation is the only nonlinear term which does not detract from the predictive power of the model. In Sec. IV we discuss the main consequences of our findings.

A word about notations. We shall consider throughout a real scalar field in flat space-time. The signature shall be  $(-, + + +)$ . Fourier transforms are defined as

$$A(x) \equiv \int \frac{d^4 k}{(2\pi)^4} e^{ikx} A(k), \quad (1.6)$$

where  $kx = k_\mu x^\mu = -\omega t + \vec{k} \cdot \vec{x}$ . Always  $\omega = k^0$ . In the case of translation-invariant kernels, we shall also define

$$A(x, x') \equiv \int \frac{d^4 k}{(2\pi)^4} e^{ik(x-x')} A(k). \quad (1.7)$$

We shall assume that all interactions are adiabatically switched off in the distant past, where the state of the field is the  $IN$  vacuum. At finite times, the state of the field will have evolved due to the influence of a nontrivial background field. The notation  $\langle \rangle \equiv \langle IN || IN \rangle$  will always denote an expectation value with respect to this state evolved from  $IN$  vacuum. As a particular case, when the background field vanishes the  $IN$  vacuum persists; we shall identify expectation values taken at zero background as  $\langle \rangle_0$ .

In curved space-time we shall use Misner-Thorne-Wheeler (MTW) conventions throughout [47]. We shall only consider fields on a fixed background de Sitter geometry, or rather, that part of de Sitter space which can be described as a spatially flat FLRW Universe. In this case the role of  $IN$  vacuum shall be filled by the massless limit of the de Sitter invariant vacuum. Again, we shall use  $\langle \rangle$  and  $\langle \rangle_0$  to denote expectation values at finite or vanishing background field, respectively.

Later on, we shall have opportunity for computing variational derivatives of various objects. The basic formula is

$$\frac{\delta \phi(x)}{\delta \phi(y)} = \delta(x, y), \quad (1.8)$$

where  $\delta(x, y)$  denotes the covariant Dirac distribution, defined from

$$\int d^4 y \sqrt{-g(y)} \delta(y, x) f(y) = f(x). \quad (1.9)$$

## II. CLASSICAL BEHAVIOR OF QUANTUM FIELDS

In this section we shall discuss the quantum to classical transition in a nonlinear quantum field theory, taking as a working example the emergence of classical stochastic behavior in a  $g\phi^3$  scalar field in flat space-time. We shall adopt the consistent histories approach to quantum mechanics [22], considering coarse-grained histories whose constitutive fine-grained configurations are small departures from a given mean field. This mean field may be interpreted as the physical quantum field, dressed by the microscopic quantum fluctuations around it. We shall show that, in the limit where the allowed variations of the field are small enough, the decoherence functional is largely insensitive to the details of the “window function” defining the coarse-graining procedure. Moreover, in this limit these histories are consistent among themselves, in a sense to be made precise below. The decoherence of these “quantum mean field” histories is closely related to the phenomena of noise and dissipation also present in the theory.

### A. Wave equation for fluctuations in the quantum mean field

We consider a scalar field theory with action

$$S[\Phi] = \int d^4 x \left\{ -\frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - V(\Phi) \right\}, \quad (2.1)$$

where the potential is<sup>4</sup>

$$V(\Phi) = c\Phi + \frac{1}{2} m^2 \Phi^2 - \frac{1}{6} g \Phi^3. \quad (2.2)$$

The Heisenberg equations of motion are identical in form to the classical field equation

$$-\square \Phi(x) + \frac{dV}{d\Phi(x)} = 0. \quad (2.3)$$

To identify the quantum mean field  $\phi$ , we write  $\Phi = \phi + \varphi$ ,

<sup>4</sup>Of course, more general forms are also possible, but this one is convenient, for example, to study the onset of first order phase transitions. For renormalization purposes, it is necessary to include a quartic term as well; we shall ignore this, assuming that the corresponding coupling constant vanishes after all necessary subtractions have been carried out. See, e.g., [48].

where  $\varphi$  represents small quantum fluctuations<sup>5</sup> around  $\phi$ . Thus,  $\varphi$  obeys the linearized equation

$$-\square\varphi + m^2\varphi - g\phi\varphi = 0. \quad (2.4)$$

Subtracting this from the full equation, we find the equation for the quantum mean field

$$-\square\phi + c + m^2\phi - \frac{1}{2}g\phi^2 - \frac{1}{2}g\langle\varphi^2\rangle = \frac{1}{2}g(\varphi^2 - \langle\varphi^2\rangle), \quad (2.5)$$

where  $\langle\rangle$  denotes the vacuum expectation value of the  $\varphi$  field, in the background provided by the  $\phi$  field. Later on, we shall use the notation  $\langle\rangle_0$  to single out the expectation value computed at zero background field.

Comparing (2.5) with the original Heisenberg equation (2.3) we notice that the presence of the  $\varphi$  field has modified the inertia of the  $\phi$  field. We interpret the solutions to (2.5) as describing the dynamics of real excitations, surrounded by a cloud of virtual  $\varphi$  quanta. We shall consider the  $\phi$  field as the system, and  $\varphi$  as its environment. This procedure is meaningful insofar as the details of the  $\varphi$  excitations are either irrelevant or inaccessible, or both. In particular, we should be able to average out “fast” variables such as the phase of the  $\varphi$  field, or equivalently, to assume that these phases are actually random. In this regime, the right-hand side of (2.5) becomes small. When neglected, then (2.5) admits a solution with  $\phi \equiv 0$ , the so-called “false vacuum,” provided

$$c = \frac{1}{2}g\langle\varphi^2\rangle_0 = \frac{1}{2}g\Delta_F(x, x), \quad (2.6)$$

where

$$\Delta_F(x, x') = \int \frac{d^4k}{(2\pi)^4} e^{ik(x-x')} \frac{-i}{(k^2 + m^2 - i\epsilon)} \quad (2.7)$$

is the Feynman propagator of quantum fluctuations around the false vacuum. Henceforth,  $\Delta$  will always denote the Green function of a microscopic quantum fluctuation  $\varphi$ , that is, expectation values of binary products of  $\varphi$  field operators. In the following, we shall be concerned as well with the propagation of perturbations in the dressed field  $\phi$  itself, which may also be described in terms of Green functions (for which we shall use capital  $G$  letters).

Assuming that  $\phi$  remains small, we can linearize the

left-hand side of (2.5), to obtain the wave equation for the propagation of small fluctuations in the quantum mean field. The right-hand side is assumed to be already small, and therefore is evaluated at the false vacuum value  $\phi = 0$ . Thus we obtain

$$-\square\phi(x) + m^2\phi(x) - \frac{1}{2}g \int d^4x' \frac{\delta\langle\varphi^2\rangle(x)}{\delta\phi(x')} \Big|_{\phi=0} \phi(x') = gj(x), \quad (2.8)$$

where

$$j(x) \equiv \frac{1}{2}\{\varphi^2(x) - \langle\varphi^2\rangle_0(x)\}. \quad (2.9)$$

For latter use, let us call

$$\frac{\delta\langle\varphi^2\rangle(x)}{\delta\phi(x')} \Big|_{\phi=0} = -2gD(x, x') \quad (2.10)$$

and observe the elementary identity

$$D(x, x') = \{\text{Im}\Delta_F(x, x')\}^2\theta(t - t'). \quad (2.11)$$

While we have the necessary data to compute this kernel explicitly, it is actually more conducive for our purposes to observe that, because of Lorentz invariance and the analytic properties associated with time ordering, the square of the Feynman function admits a Lehmann representation

$$\Delta_F^2(x, x') = -i \int \frac{d^4k}{(2\pi)^4} e^{ik(x-x')} \int_0^\infty \frac{ds h(s)}{(s + k^2 - i\epsilon)}, \quad (2.12)$$

where the function  $h$  is positive and vanishes for  $s \leq s_0$ ,  $s_0$  being a positive threshold [an actual evaluation yields  $h = \sqrt{1 - (4m^2/s)}\theta(s - 4m^2)$ ]. We find immediately

$$D(x, x') = \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} e^{ik(x-x')} \int_0^\infty \frac{ds h(s)}{-(k + i\epsilon)^2 - s} \quad (2.13)$$

[we use the notation  $(k + i\epsilon)^2 = -(\omega + i\epsilon)^2 + \vec{k}^2$ ]. We may now write down the equation of motion for the fluctuations of the quantum mean field

$$\{-\square_x + m^2\}\phi(x) + g^2 \int d^4x' D(x, x')\phi(x') = gj(x), \quad (2.14)$$

where the right-hand side is given by (2.9). Obviously the expectation value of the driving force vanishes, but its higher momenta do not. In particular, we find

$$\langle\{j(x), j(x')\}\rangle \equiv 2N(x, x') = \text{Re}[\Delta_F(x, x')]^2. \quad (2.15)$$

More explicitly,

<sup>5</sup>The only requirement for the decomposition  $\Phi = \phi + \varphi$  is that  $\varphi$  is small compared to  $\phi$ ; both are quantum in nature. This is the case when there exists some discrepancy in space, time, or energy scales in two sectors described by the fields. The more precise meaning of  $\phi$  and  $\varphi$ , in relation to the conventionally defined classical mean field and quantum fluctuation field will become clear in the conceptual framework of consistent or decoherent history interpretation of quantum mechanics. See footnote at the end of Sec. II B.

$$N(x, x') = \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} e^{ik(x-x')} \pi h(-k^2). \quad (2.16)$$

The kernel  $N$ , or rather, its Fourier transform  $\mathcal{N} = h$ , also plays a distinguished role with respect to dissipation in this model. To begin with, let us observe that if

$$s_0 + \frac{g^2}{2} \int ds \frac{\mathcal{N}(s)}{(s-s_0)} > m^2 > \frac{g^2}{2} \int ds \frac{\mathcal{N}(s)}{s} \quad (2.17)$$

(which is largely satisfied within one loop accuracy) then this theory admits stable one-particle asymptotic states of mass  $M^2 < s_0$ , where

$$M^2 + \frac{g^2}{2} \int ds \frac{\mathcal{N}(s)}{(s-M^2)} = m^2. \quad (2.18)$$

The properties of the quantum fluctuations of the mean field  $\phi$  are largely determined by the retarded propagator  $G_r$ , defined through its Fourier components

$$G_r(k) = \left\{ [(k+i\epsilon)^2 + m^2] - \frac{g^2}{2} \int_0^\infty \frac{ds \mathcal{N}(s)}{s + (k+i\epsilon)^2} \right\}^{-1}. \quad (2.19)$$

Or else, isolating the pole at  $-k^2 = M^2$ ,

$$G_r(k) = \frac{B}{[(k+i\epsilon)^2 + M^2]} + \frac{g^2}{2} \int_0^\infty \frac{ds \mathcal{N}(s)}{[(k+i\epsilon)^2 + s]} |G_r(s)|^2, \quad (2.20)$$

where

$$B^{-1} = 1 + \frac{g^2}{2} \int ds \frac{\mathcal{N}(s)}{(s-M^2)^2} \quad (2.21)$$

and  $G_r(s)$  means the propagator evaluated at any momentum  $k$  with  $k^2 = -s$ . Equation (2.20) implies that a perturbation of the quantum mean field propagates as an elementary free scalar field with mass  $M^2$ , superimposed to a continuous spectrum of fields with masses ranging from  $s_0$  to  $\infty$ . The on-shell oscillations, with  $k^2 = -M^2$ , are undamped. Above the  $s_0$  threshold, however, oscillations are damped, as it can be seen from the  $\phi$  field self-energy developing a positive imaginary part. As we can see from (2.19), this imaginary part is again given by the function  $h(-k^2)$ . The same conclusion may be derived from the Feynman propagator for the  $\phi$  field, which, assuming vacuum initial conditions, is obtained from the retarded propagator by analytical continuation  $(k+i\epsilon)^2 \rightarrow (k^2-i\epsilon)$ . As usual, this absorptive part in the field self-energy is associated with the emission probability of real  $\varphi$  quanta. Indeed, we have shown in [48] that the total amount of energy dissipated from the quantum mean field is exactly the mean energy carried away by the created particles.

The dual role of the  $N$  kernel in both fluctuation and dissipation, far from being an accident, follows from the fluctuation-dissipation theorem. Indeed, always assuming vacuum initial conditions, we may derive the

Hadamard function for the quantum mean fields (e.g., from the Kubo-Martin-Schwinger (KMS) condition at zero temperature [49]) to be

$$G_1(k) = 2\pi \left\{ B\delta(k^2 + M^2) + \left( \frac{g^2}{2} \right) h(-k^2) |G_r(-k^2)|^2 \right\}. \quad (2.22)$$

Since on-shell fluctuations are undamped, we may assume that the on-shell contribution to the Hadamard function was already present in the initial conditions for the mean field. However, such an interpretation would be untenable above threshold, because there the  $\phi$  field is damped, and the memory of initial conditions is eventually lost. On the other hand, in this part of the spectrum we have  $\phi(k) = gG_r(k)j(k)$ , so the fluctuations in the driving force induce fluctuations in the mean field by an amount

$$G_1(k) = 2g^2 |G_r(k)|^2 \mathcal{N}(k). \quad (2.23)$$

Comparing this with (2.22) we conclude that the force self-correlation must indeed be given by (2.16). The connection between these fluctuations and particle creation is equally straightforward: While dissipation describes the mean effects of particle creation, the source  $j$  accounts for the deviation of the actual number of created particles from this mean. The relationship between fluctuation and particle creation is explored in full in Ref. [4].

It is interesting to observe that the structure of the Hadamard kernel (2.22) as the sum of on-shell and off-shell contributions, the latter being related to dissipation, suggests that these fluctuations may be regarded as independent. Should there be several decay channels for the quantum mean field, then each would provide a further term to the Hadamard function, so that the fluctuation-dissipation balance may hold.

## B. Decoherence of the mean field by its quantum fluctuations

So far we have derived the wave equation for the quantum mean field  $\phi$ . The equation of motion (2.14) admits  $c$ -number solutions only under the Hartree-Fock approximation  $j \sim 0$ . We now proceed to study under what circumstances, if any, the dressed field is able to shed its quantum nature. We adopt to this end the consistent histories approach to quantum mechanics [22].

The basic tenet of this view of quantum mechanics is that quantum evolution may be considered as a result of the coherent superposition of virtual fine-grained histories, each carrying full information on the state of the system at any given time. If we adopt the ‘‘natural’’ procedure of specifying a fine-grained history by defining the value of the field  $\Phi(x)$  at every space-time point, these field values being  $c$  numbers, then the quantum mechanical amplitude for a given history is  $\Psi[\Phi] \sim e^{iS[\Phi]}$ , where



$S$  is the classical action evaluated at that particular history. These histories are virtual because there exists interference between pairs of histories. The strength of these effects is measured by the “decoherence functional”

$$\mathcal{D}[\Phi, \Phi'] \sim \Psi[\Phi]\Psi[\Phi']^* \sim e^{i(S[\Phi]-S[\Phi'])}. \tag{2.24}$$

On the other hand, actual physical observations refer only to “coarse grained” histories, where several fine-grained histories are bundled together. A coarse-grained history is defined, generally speaking, by a “filter function”  $\alpha$ , which determines which fine-grained histories belong to the superposition, and their relative phases. For example, we may have a system with two degrees of freedom  $x$  and  $y$ , and define a coarse-grained history by specifying the values  $x_0(t)$  of  $x$  at all times. Then the filter function is  $\alpha[x, y] = \prod_{t \in R} \delta[x(t) - x_0(t)]$ . The quantum mechanical amplitude for the coarse-grained history is defined as

$$\Psi[\alpha] = \int D\Phi e^{iS} \alpha[\Phi]. \tag{2.25}$$

We assume that the relevant information on the quantum state has been encoded into the initial conditions for the paths in the integration domain. The decoherence functional for two coarse-grained histories is [22]

$$\mathcal{D}[\alpha, \alpha'] = \int D\Phi^1 D\Phi^2 e^{i(S(\Phi^1) - S(\Phi^2))} \alpha[\Phi^1] \alpha'[\Phi^2]^*. \tag{2.26}$$

The two histories  $\Phi^1$  and  $\Phi^2$  are not independent: they must assume identical values on a  $t = T = \text{const}$  surface in the far future. Decoherence means physically that the different coarse-grained histories making up the full quantum evolution acquire individual reality, and may therefore be assigned definite probabilities in the classical sense. Therefore, as long as we remain within the accuracy afforded by the coarse-graining procedure, we may disregard the quantum nature of our system, and describe the dynamics as the self-consistent evolution of  $c$ -number variables.

For our particular application, we wish to consider as a single coarse-grained history all those fine-grained ones where the full field  $\Phi$  remains close to a prescribed quantum mean field configuration  $\phi$ . Thus the filter function  $\alpha_\phi(\Phi)$  takes the form

$$\alpha_\phi(\Phi) = \int DJ e^{iJ(\Phi - \phi)} \alpha_\phi(J), \tag{2.27}$$

where  $\alpha_\phi(J)$  is a smooth function (we explicitly exclude, however, the case  $\alpha_\phi \equiv \text{const}$ , where there is no coarse graining at all). In (2.27) we use the summation convention over continuous indices: i.e.,

$$J\Phi \equiv \int d^4x J(x)\Phi(x). \tag{2.28}$$

The decoherence functional between two of these “mean field” histories is then

$$\begin{aligned} \mathcal{D}[\alpha_\phi, \alpha_{\phi'}] &= \int DJDJ' e^{i\{W[J, J'] - (J\phi - J'\phi')\}} \\ &\times \alpha_\phi[J] \alpha_{\phi'}[J']^*, \end{aligned} \tag{2.29}$$

where

$$e^{iW[J, J']} = \int D\Phi D\Phi' e^{i(S[\Phi] - S^*[\Phi'] + J\Phi - J'\Phi')} \tag{2.30}$$

is precisely the closed-time-path (CTP) generating functional [50]. Since the filter functions are smooth, we may evaluate the integrals over the  $J$ 's by saddle point methods, thus obtaining

$$\mathcal{D}[\alpha_\phi, \alpha_{\phi'}] = \mathcal{C}(\phi, \phi') e^{i\Gamma[\phi, \phi']}. \tag{2.31}$$

We recognize that  $\Gamma$  is the closed-time-path effective action, and  $\mathcal{C}$  is a slowly varying prefactor:

$$\begin{aligned} \mathcal{C}(\phi, \phi') &\sim \alpha_\phi[-\Gamma, \phi] \alpha_{\phi'}[\Gamma, \phi']^* \\ &\times \left\{ \text{Det} \left[ \frac{\delta^2 \Gamma}{\delta \phi^a(x) \delta \phi^b(x')} \right]^{1/2} \right\}, \end{aligned} \tag{2.32}$$

where  $a, b = 1, 2$ , (e.g.,  $\phi^1 = \phi$ ,  $\phi^2 = \phi'$ ). Equation (2.31), which establishes the connection between the decoherence functional for “mean field” histories and the closed-time-path effective action, is a major result reported here. Of course, it is only a particular case of the more general “correlation” histories discussed in [38]. For simplicity, we shall ignore the prefactor in what follows.

The evaluation of the closed-time-path effective action is standard. To one-loop accuracy it is given by [51,48]

$$\Gamma[\phi^a] = S[\phi^a] + \frac{i}{2} \ln \text{Det} \frac{\delta^2 S}{\delta \phi^a \delta \phi^b}, \tag{2.33}$$

where  $S[\phi^a]$  is taken to mean  $S[\phi] - S[\phi']^*$  (complex conjugation applies if an  $i\epsilon$  term has been included to enforce the boundary conditions), and the “internal” index  $a$  is lowered with the “metric”  $g_{ab} = \text{diag}(1, -1)$ . Functionally expanding  $\Gamma$  in powers of  $\phi^a$ , and retaining only up to quadratic terms, we get

$$\Gamma[\phi^a] = \frac{1}{2} \int d^d x d^d x' \{-[\phi](x) \tilde{D}(x, x') \{\phi\}(x') + ig^2[\phi](x) N(x, x') [\phi](x')\}, \quad (2.34)$$

where  $[\phi] = (\phi^1 - \phi^2)$ ,  $\{\phi\} = (\phi^1 + \phi^2)$ ,

$$\tilde{D}(x, x') = \{-\square_x + m^2\} \delta(x - x') + g^2 D(x, x') \quad (2.35)$$

and the “dissipation” ( $D$ ) and “noise” ( $N$ ) kernels are defined in Eqs. (2.11) and (2.15), respectively.

As discussed in the Introduction, fluctuations, namely, the presence of the driving force (2.9) on the right-hand side of the wave equation (2.14) and decoherence, namely, the suppression of the decoherence functional (2.31) between different mean field histories both depend on one and the same kernel  $N$ , related to the positive imaginary part of the effective action, and are therefore revealed as aspects of the same phenomenon. Note that both effects vanish if the cubic interaction is switched off, revealing the essential role played by nonlinearity in this problem. In turn, the presence of fluctuations is associated with the back reaction of particle creation and thereby to dissipation: the two effects are linked by the fluctuation-dissipation theorem. This manifests the interrelation of decoherence, noise, and dissipation [4,52]. As have been shown earlier [20], the equations generated by the effective action (2.34) are equivalent to the linearized mean field equations coupled to a stochastic Gaussian source  $gj$ , the noise kernel  $N$  being the autocorrelator of the source  $j$ . Comparing this with the full equations (2.8), (2.9) we see that this equation lies in between mean field theory, where the source is simply ignored, and the full quantum theory. This approximation, moreover, successfully captures the main property of the driving term, namely its mean square value (2.15). To fully account for non-Gaussian statistics, we must go to higher loops and also include more complex correlation functions, employing the more general methods described in [38].

By repeating the arguments in the previous subsection, we see that the mean-squared value of the decohered quantum mean field, as driven by the stochastic source, is again given by (2.23). It is clear that this amounts to only a fraction of the full quantum fluctuations, given by the Hadamard function (2.22). Thus seeking the amount of classical fluctuations subsequent to the quantum to classical transition by simply equating the classical and quantum correlators, without a further analysis of the decoherence process, is definitely unwarranted, unless it is meant as a simple order of magnitude estimate. As shown elsewhere [4], this fluctuation is related to the uncertainty in the number of created particles from the dynamical quantum mean field.

Clearly, there is much more to be done to achieve a full understanding of the quantum to classical transition in this model. For our present concerns, however, we are satisfied with the observation that the quantum mean fields may decohere through interaction with quantum fluctuations around them, developing random classical

fluctuations in the process.<sup>6</sup>

This phenomenon may only occur in a nonlinear theory, and it is independent of any *a priori* partition of fields or modes into relevant and irrelevant. Besides these, there is a rather powerful and comprehensive theory describing it, built with well-proven techniques from nonequilibrium quantum field theory [20,50]. We shall now turn our attention to the problem of fluctuation generation in the early Universe, and show it may be understood with the same set of basic principles.

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<sup>6</sup>Thus in the consistent or decoherent history conceptual framework, we see a clearer rationale and justification in the decomposition of  $\Phi = \phi + \varphi$ .  $\phi$  represents the local value of the field measured within one particular history, while  $\varphi$  represents the deviations of this field from the actual or total field  $\Phi$ . What makes  $\varphi$  nonvanishing is that the histories are coarse grained. The important observation explained in this section (Sec. IIB) is that as long as the deviations stay at the order of magnitude of the quantum fluctuations, then the decoherence functional is largely independent of the details of the decomposition. Specifically, the properties of the deviations are defined by the window function  $\alpha$  in (2.25). But at the level of the decoherence functional, only the prefactor (which we discarded) depends on  $\alpha$ ; the exponential term is universal. So the only condition on the  $\alpha$  functions is that the prefactor (2.32) be slowly varying. In other words,  $\alpha$  should not be too narrow (too little coarse graining) nor too wide (excessive coarse graining). Within these limits, a family of sufficiently different  $\phi$ 's actually represent consistent histories, so treating it as a *c*-number field is justified. (This, incidentally, is the reason why one can couple it to a classical gravitational field, as we will do in Sec. III.)

Concretely, our discussion here shows that for a wide range of coarse graining based on approximated histories of the field, the decoherence functional takes the form (2.31), with  $\Gamma$  given by (2.34). The most likely histories (in the sense of what histories contribute most to the path integrals representing the expectation values of observables) are those where the measured field  $\phi$  obeys the Langevin equation (2.14) with noise kernel given by (2.15). The end result is that the quantum mean  $\phi$  field is decohered into a classical mean field and the part  $\varphi$  which measures its deviations from the actual value  $\Phi$  is regarded as quantum fluctuations around this mean field.

It is clear that our definition of the mean field from the consistent or decoherent history viewpoint is conceptually different from the conventional view, which defines  $\phi$  as some kind of average on the full field. It is true that for reasons given above, the ensemble average of our field is equal to the quantum average of the full field, but the rationale is different. In fact, the conventional approach to structure formation by treating quantum fluctuations naively as classical fluctuations misses the whole point about decoherence which gives rise to the classical mean field, and the expedient practice of taking the vacuum expectation values of the quantum fields amounts to throwing away the baby with the water, since it suppresses quantum fluctuations altogether.

### III. QUANTUM FLUCTUATIONS AND DENSITY PERTURBATIONS IN DE SITTER UNIVERSE

In this section we shall turn our attention to quantum fields in de Sitter space-time. Our goal is to describe, within the theory developed in the previous section, how a quantum scalar field loses its quantum coherence, and undergoes stochastic fluctuations in the process. During the inflationary period, when the space-time can be approximated by the de Sitter geometry and the inflaton field described as a free, massless scalar field, this may be seen as a model for the generation of primordial fluctuations in the early Universe.

Here we shall seek a dynamical origin for decoherence (rather than imposing a relevance criterium by hand). As we have seen in the previous section, decoherence from an uncorrelated initial state can only occur in a nonlinear theory. On the other hand, adding a self-coupling to the inflaton field, even leaving aside the stringent conditions imposed by the requirement of “successful inflation,” necessarily implies the inclusion of new parameters into the model, making it correspondingly less compelling. Therefore we are led to consider the only available parameter-free source of nonlinearity, namely, the gravitational couplings of the inflaton. To appeal to quantum effects of the gravitational field immediately evokes a number of difficulties arising from the nonrenormalizability of general relativity. In this work we shall sidestep this issue, by considering only one loop effects. Moreover, as in the previous section, we shall not carry through the renormalization procedure explicitly, but rather assume that the theory has already been rendered finite by adding suitable counterterms to the classical action. In fact, we shall base our analysis on the Einstein-Hilbert form of the action, without including higher order terms which could arise in the renormalization process. This procedure is fully justified at the scales of interest [53]. Another feature of quantum gravity which we shall sidestep is the gauge character of the gravitational field. To highlight the physical ideas in our approach, we shall take the simple-minded view of fixing the gauge at the classical level, considering only quantum fluctuations of the “physical” degrees of freedom [54]. We shall present the results of a more complete calculation elsewhere.

Thus we shall consider a theory involving two quantum fields, the gravitational field  $g_{\mu\nu}^q$  and the inflaton field  $\Phi$ . The classical action functionals are given, respectively, by

$$S_g = m_p^2 \int d^4x \sqrt{-g^q} \{R - 2\Lambda\} \quad (3.1)$$

and

$$S_f = -\frac{1}{2} \int d^4x \sqrt{-g^q} \partial_\mu \Phi \partial^\mu \Phi. \quad (3.2)$$

As in the previous section, we shall consider coarse-grained histories defined by the values of the quantum mean fields  $g_{\mu\nu}$  and  $\phi$ . The decoherence functional is related to the CTP effective action as in (2.31). Following the usual prescription for the computation of the

one-loop effective action, we write both fields in terms of the quantum mean (or dressed) fields and their perturbations:

$$g_{\mu\nu}^q = g_{\mu\nu} + h_{\mu\nu} \quad (3.3)$$

and

$$\Phi = \phi + \varphi. \quad (3.4)$$

To make sure that the fluctuations are physical, we work with the transverse traceless gauge in synchronous coordinates, namely,

$$h_\mu^\mu = h_{\mu;\nu}^\nu = 0; \quad h_\mu^0 = 0, \quad (3.5)$$

where indices are raised and lowered with the background metric, and the derivative is taken with the background Levi-Civita connection [54]. Observe that the classical equations of motion admit a solution with

$$g_{\mu\nu} = (H\tau)^{-2} \eta_{\mu\nu} \quad (3.6)$$

and with vanishing field. Here  $\tau \leq 0$ , where  $\tau \equiv \int dt/a$  is the conformal time,  $\Lambda = 3H^2$ , and  $\eta_{\mu\nu}$  is the flat space-time metric. (This solution, of course, represents only one-half of de Sitter space-time [55].) Since we are not concerned at this moment with the stochastic fluctuations of the gravitational field itself (see [4] in this connection), we shall compute the noise and dissipation kernels for this value of the gravitational background, leaving only  $\phi$  arbitrary.<sup>7</sup>

Continuing with the computation of the dissipative and stochastic elements in the dynamics of the inflaton, we should expand the classical action in powers of the perturbations, and retain terms only up to quadratic orders. This is of course equivalent to computing the full nonlinear equation, and linearizing afterwards, as we did in Sec. II, but in a more complex theory, this approach is more efficient. Beginning with the scalar action for simplicity, we obtain three kinds of terms which are independent of, linear, and quadratic in  $\varphi$ , respectively. The term which does not contain  $\varphi$  is necessarily quadratic in both  $h_\nu^\mu$  and  $\phi$ . To one loop accuracy it only appears in “tadpole” graphs, with no relationship to the nonlocal part of the noise and dissipation kernels, and we will not consider it

<sup>7</sup>Note that the metric field  $g_{\mu\nu}$  also has fluctuations and, in general, there is no clear separation between the metric and matter field fluctuations. Indeed, linearized gravitational perturbations behave like massless minimally coupled scalar fields. One can use the gauge-invariant formalism of Bardeen and others [6] to treat the gravitational fluctuations. However, since we are interested here specifically in the nonlinear effects, for simplicity, we have ignored the gravitational fluctuations since we regard the behavior of linear fluctuations to the scalar field as representative of both the matter and gravitational field fluctuations. Nonlinear gravitational fluctuations can likely generate qualitatively new effects and should merit more attention, but the present state of knowledge is not adequate to treat them in an intelligent way.

further. The part quadratic in  $\varphi$  defines the propagator for these microscopic fluctuations. It takes the form

$$S_f^{(2)} = \frac{1}{2} \int d\tau d^3x (H\tau)^{-2} [(\varphi')^2 - (\vec{\nabla}\varphi)^2], \quad (3.7)$$

where the prime stands for a  $\tau$  derivative. The  $\varphi h$  cross term is the source of dissipation, decoherence, and noise in this model. For physical gravitational perturbations (that is, those obeying  $h_0^0 = h_i^0 = h_i^i = h_{i,j}^j = 0$ ), it is given by

$$S_f^{(1,1)} = - \int d\tau d^3x (H\tau)^{-2} \phi_{,ij} h_i^j \varphi, \quad (3.8)$$

where Latin indices run from 1 to 3.

Expanding the Einstein-Hilbert action we can read out the free graviton propagator.<sup>8</sup> The quadratic terms in the action are

$$S_g^{(2)} = \frac{m_p^2}{4} \int d\tau d^3x (H\tau)^{-2} (h_j^i h_i^j - h_{j,k}^i h_{i,k}^j). \quad (3.9)$$

However, the graviton components  $h_j^i$  are not independent, since they are linked through the gauge conditions. It is convenient to write the graviton field explicitly in terms of the independent physical degrees of freedom, as in [54]

$$S_f^{(1,1)} = - \left( \frac{1}{m_p} \right) \int \frac{d\tau d^3x d^3y}{(H\tau)^2} (\phi_{,ij} \varphi) (\tau, \vec{x}) \{ G_j^{+i} (\vec{x} - \vec{y}) h^+ (\tau, \vec{y}) + G_j^{\times i} (\vec{x} - \vec{y}) h^\times (\tau, \vec{y}) \}. \quad (3.16)$$

While it is possible to derive the effective action for this model, in order to find the noise and dissipation kernels it is simplest to proceed from the equations of motion, as given in Sec. II A. Let us begin with the Heisenberg equation of motion for the inflaton field

$$\square \Phi - \frac{(H\tau)^2}{m_p} \partial_{ij}^2 \int d^3y \varphi (\tau, \vec{x}) \{ G_j^{+i} (\vec{x} - \vec{y}) h^+ (\tau, \vec{y}) + G_j^{\times i} (\vec{x} - \vec{y}) h^\times (\tau, \vec{y}) \} = 0. \quad (3.17)$$

It is clear that in the absence of a nontrivial background field, the expectation value

$$\langle \varphi (\tau, \vec{x}) h^{+, \times} (\tau, \vec{y}) \rangle |_{\phi=0} \equiv 0. \quad (3.18)$$

Thus the linearized equation for the mean field reads

$$\square \phi (x) - \int \frac{d^4x'}{(H\tau')^4} D (x, x') \phi (x') = 0, \quad (3.19)$$

where

$$D (x, x') = \frac{(H\tau)^2}{m_p} \partial_{ij}^2 \int d^3y \left\{ G_j^{+i} (\vec{x} - \vec{y}) \frac{\delta}{\delta \phi (x')} \langle \varphi (\tau, \vec{x}) h^+ (\tau, \vec{y}) \rangle |_{\phi=0} + (+ \leftrightarrow \times) \right\}. \quad (3.20)$$

<sup>8</sup>Note that although a self-coupling  $\lambda \Phi^4$  field is considered, the noise kernel (3.42) is calculated here only from the vertex  $S_f^{1,1}$  in (3.8). The vertex induced from the self-coupling is ignored because the self-coupling contribution is much smaller than the gravitational contribution. As we shall see [in the paragraph after (4.11)], with  $\lambda \sim 10^{-6}$  the self-coupling contributes a term of the order  $10^{-12}$ , much below the gravitational contribution. In any case, our calculation only aims at a parameter-free lower bound. Inclusion of self-interaction would only add noise to this. For a more complete calculation, see [61].

$$h_j^i (\tau, \vec{x}) = \left( \frac{1}{m_p} \right) \int d^3y \{ G_j^{+i} (\vec{x} - \vec{y}) h^+ (\tau, \vec{y}) + G_j^{\times i} (\vec{x} - \vec{y}) h^\times (\tau, \vec{y}) \} \quad (3.10)$$

where

$$G_j^{+i} (\vec{x} - \vec{y}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}(\vec{x}-\vec{y})} A_{\vec{k}j}^{+i} \quad (3.11)$$

and a similar formula exists for the cross ( $\times$ ) polarization. The  $A$  matrices obey

$$A_{\vec{k}i}^{+i} = k_i A_{\vec{k}j}^{+i} = A_{\vec{k}i}^{\times i} = k_i A_{\vec{k}j}^{\times i} = 0, \quad (3.12)$$

$$A_{\vec{k}j}^{+i} A_{-\vec{k}l}^{+j} = A_{\vec{k}j}^{\times i} A_{-\vec{k}l}^{\times j} = \delta_l^i - \frac{k^i k_l}{k^2}, \quad (3.13)$$

$$A_{\vec{k}j}^{+i} A_{-\vec{k}i}^{\times j} = 0. \quad (3.14)$$

The graviton action thus splits into two parts:

$$S_g^{(2)} = S^+ + S^\times \quad (3.15)$$

each being the action for a massless, real scalar field, (3.7). We also find

Comparing the mean field equation with the Heisenberg equation, we obtain the equation for the dressed fluctuations

$$\square \phi (x) - \int \frac{d^4x'}{(H\tau')^4} D (x, x') \phi (x') = \frac{(H\tau)^2}{m_p} j (x), \quad (3.21)$$

where in principle  $j$  represents the composite operator

$$j (x) = \int d^3y \partial_{ij}^2 \varphi (\tau, \vec{x}) \{ G_j^{+i} (\vec{x} - \vec{y}) h^+ (\tau, \vec{y}) + G_j^{\times i} (\vec{x} - \vec{y}) h^\times (\tau, \vec{y}) \}. \quad (3.22)$$

(Here we have used the transversal character of the  $G$  tensors.) As we discussed in detail in Sec. II, upon decoherence we can think of  $j$  as a classical stochastic source, whose autocorrelation is given by the noise kernel

$$N(x, x') = \langle j(x) j(x') \rangle_c = \frac{1}{2} \langle \{j(x), j(x')\} \rangle_0. \quad (3.23)$$

To compute the quantum expectation values in Eqs. (3.20) and (3.23), we expand the quantum field operators in terms of the destruction and creation operators, as in

$$\varphi(\tau, \vec{x}) = iH \int \frac{d^3k}{(2\pi)^{3/2}} \frac{e^{i\vec{k}\vec{x}}}{k\sqrt{2k}} \left\{ a_{\vec{k}} f_k(\tau) + a_{-\vec{k}}^\dagger f_k^*(\tau) \right\} \quad (3.24)$$

and perform similar expansions for the graviton amplitudes. All three scalar fields  $\varphi$ ,  $h^+$ , and  $h^\times$  are expanded in terms of the same modes

$$f_k(\tau) = e^{-ik\tau} [1 + ik\tau], \quad (3.25)$$

which are related to the Fulling-Davies vacuum, this being the natural choice of quantum state in this problem [56]. The Wronskian of these modes is

$$f_k^*(\tau) f_k'(\tau) - f_k(\tau) f_k^{*\prime}(\tau) = -2ik^3\tau^2. \quad (3.26)$$

### A. Dissipation and dynamics of free mean fields

Before continuing with the discussion of the fluctuations in the quantum mean field, we want to first analyze the solutions to the source-free mean field equation, (3.19). Concretely, our goal is to establish the dissipative character of this equation, to be able later to analyze the fluctuations in terms of the fluctuation-dissipation relation.

As we show in the Appendix, the dissipation kernel is conveniently written as

$$D(x, x') = \frac{H^4}{m_p^2} \theta(\tau - \tau') (H\tau)^2 (H\tau')^2 \int \frac{d^3k}{(2\pi)^3} \times e^{i\vec{k}(\vec{x}-\vec{x}')} D_k(\tau, \tau'), \quad (3.27)$$

where

$$D_k(\tau, \tau') = \frac{1}{(2\pi)^3} \int \frac{d^3p}{2p^3} \frac{d^3q}{2q^3} \delta(\vec{p} + \vec{q} - \vec{k}) \times \Theta(\vec{p}, \vec{q}) F_{pq}(\tau, \tau') \quad (3.28)$$

and

$$\Theta(\vec{p}, \vec{q}) = p_i p_j p_l p_m \left[ A_{\vec{q}j}^{+i} A_{-\vec{q}l}^{+j} + A_{\vec{q}j}^{\times i} A_{-\vec{q}l}^{\times j} \right], \quad (3.29)$$

$$F_{pq}(\tau, \tau') = 2 \text{Im} [f_p^*(\tau) f_q^*(\tau) f_p(\tau') f_q(\tau')]. \quad (3.30)$$

Since the background is spatially homogeneous, we

can also expand the quantum mean field in terms of its Fourier modes:

$$\phi(\tau, \vec{x}) = H \int \frac{d^3k}{(2\pi)^{3/2}} \frac{e^{i\vec{k}\vec{x}}}{k\sqrt{2k}} \phi_k(\tau). \quad (3.31)$$

The time dependent amplitude  $\phi_k(\tau)$  can always be written as

$$\phi_k(\tau) = \alpha_{\vec{k}} f_k(\tau) + \beta_{\vec{k}} f_k^*(\tau). \quad (3.32)$$

Imposing the auxiliary condition

$$\phi_k'(\tau) = \alpha_{\vec{k}} f_k'(\tau) + \beta_{\vec{k}} f_k^{*\prime}(\tau), \quad (3.33)$$

and keeping the Wronskian (3.26) in mind, we find

$$\alpha_k(\tau) = \frac{i}{2k^3} \left[ f_k^*(\tau) \left( \frac{\phi_k'(\tau)}{\tau^2} \right) - \phi_k(\tau) \left( \frac{f_k^{*\prime}(\tau)}{\tau^2} \right) \right], \quad (3.34)$$

$$\beta_k(\tau) = -\frac{i}{2k^3} \left[ f_k(\tau) \left( \frac{\phi_k'(\tau)}{\tau^2} \right) - \phi_k(\tau) \left( \frac{f_k^{*\prime}(\tau)}{\tau^2} \right) \right]. \quad (3.35)$$

In the absence of dissipation, the coefficients  $\alpha$  and  $\beta$  would be constant. In the presence of the dissipation kernel, they become functions of time, with evolution equations

$$\tau^2 \alpha_k'(\tau) = \frac{i}{2k^3} \frac{H^4}{m_p^2} f_k^*(\tau) \int^\tau \frac{d\tau'}{(H\tau')^2} \times D_k(\tau, \tau') \phi_k(\tau'), \quad (3.36)$$

$$\tau^2 \beta_k'(\tau) = -\frac{i}{2k^3} \frac{H^4}{m_p^2} f_k(\tau) \int^\tau \frac{d\tau'}{(H\tau')^2} \times D_k(\tau, \tau') \phi_k(\tau'). \quad (3.37)$$

We are interested in the solution where  $\alpha \rightarrow 1$  and  $\beta \rightarrow 0$  in the far past. In a first approximation, we may substitute  $\phi_k$  by its free value  $f_k$  in Eqs. (3.36) and (3.37). In particular, we obtain, for the total integrated change in the amplitude of the mode  $\sigma_k \equiv |\alpha_k|^2$ ,

$$\Delta\sigma_k = -\frac{2H^6}{m_p^2 k^3} \int \frac{d\tau}{(H\tau)^2} \frac{d\tau'}{(H\tau')^2} \theta(\tau - \tau') \times D_k(\tau, \tau') \text{Im} [f_k(\tau') f_k^*(\tau)]. \quad (3.38)$$

Substituting the value of  $D_k$ , this yields

$$\Delta\sigma_k = -\frac{H^2}{m_p^2 (2\pi)^3 k^3} \int \frac{d^3p}{2p^3} \frac{d^3q}{2q^3} \delta(\vec{p} + \vec{q} - \vec{k}) \times \Theta(\vec{p}, \vec{q}) \left\{ |F_{kpq}|^2 - |G_{kpq}|^2 \right\}, \quad (3.39)$$

where

$$F_{k pq} = \int \frac{d\tau}{\tau^2} f_k(\tau) f_p^*(\tau) f_q^*(\tau), \quad (3.40)$$

$$G_{k pq} = \int \frac{d\tau}{\tau^2} f_k(\tau) f_p(\tau) f_q(\tau). \quad (3.41)$$

Clearly, the more rapidly oscillatory function  $G$  will be much smaller than  $F$ , and we shall neglect it in what follows. It should be observed, besides, that when  $k = p + q$ , the condition  $\vec{k} = \vec{p} + \vec{q}$ , enforced by the delta function in (3.39), implies that  $\vec{p}$  and  $\vec{q}$  are collinear, in which case  $\Theta(\vec{p}, \vec{q}) = 0$ . Because of this, the integrand in  $F_{k pq}$  is always oscillatory, and the integral is independent of the lower limit of integration.

To summarize, if we ignore the effects of fluctuations, we must conclude that the mean field is dissipated by its interaction with the environment, losing an amount  $\Delta\sigma_k$  of its original amplitude (and an equal amount of its original Klein-Gordon charge) over the de Sitter period of cosmic evolution. Since on the other hand zero point fluctuations cannot disappear, we should expect that an equal amount will be provided by the environment, now under the guise of random driving force, so that the fluctuation-dissipation balance may be kept. We turn now to investigate this issue.

### B. Noise and the fluctuation-dissipation balance

Let us return to the full dynamics of the quantum mean field, as described by (3.21). We see that, besides the dissipative terms just analyzed, the field is coupled to a random source  $j$ , whose mean square value is given by the noise kernel (3.23). Substituting the mode decomposition (3.24), it is straightforward to find the explicit expression

$$N(x, x') = H^4 \int \frac{d^3 k}{(2\pi)^3 2k^3} \frac{d^3 p}{(2\pi)^3 2p^3} e^{i(\vec{k}+\vec{p})(\vec{x}-\vec{x}')} \times \Theta(\vec{k}, \vec{p}) \text{Re} [f_k(\tau) f_p(\tau) f_k^*(\tau') f_p^*(\tau')]. \quad (3.42)$$

More interesting than the noise kernel are the fluctuations induced on the quantum mean field  $\phi$  itself. From the point of view of the theory of primordial fluctuations in the Universe, the most relevant quantity is the mean square value of the fluctuations at a given time, which are given by<sup>9</sup>

<sup>9</sup>The homogeneous term in  $\phi$  as a solution to the Langevin equation is ignored here. The rationale lies in the belief that inflation can only happen if the initial field is homogeneous on scales larger than the horizon [57]. Assuming that inflation once did happen, as most authors on this subject do, purports observational evidence that the field in the patch which inflated was very smooth initially, at least in the very short scales much below the horizon of interest to later observers, thus rendering the homogeneous solution negligible. The actual happening depends on the semiclassical physics between the post-Planckian and the preinflationary [grand unified theory (GUT)] epochs. See [58].

$$\langle \phi(\tau, \vec{x}) \phi(\tau, 0) \rangle_c = \frac{1}{m_p^2} \int \frac{d^4 r_1}{(H\tau_1)^2} \frac{d^4 r_2}{(H\tau_2)^2} G_r[(\tau, \vec{x}), r_1] \times G_r[(\tau, 0), r_2] N(r_1, r_2). \quad (3.43)$$

In a first approximation, we may use free retarded propagators for the Green functions

$$G_r(x, x_1) = -iH^2 \theta(\tau - \tau_1) \int \frac{d^3 k}{(2\pi)^3 2k^3} e^{i\vec{k}(\vec{x}-\vec{x}_1)} \times \{f_k(\tau) f_k^*(\tau_1) - f_k(\tau_1) f_k^*(\tau)\}. \quad (3.44)$$

The result is

$$\langle \phi(\tau, \vec{x}) \phi(\tau, 0) \rangle_c = \int d^3 k e^{i\vec{k}\vec{x}} |\delta\phi_k|^2(\tau), \quad (3.45)$$

where

$$|\delta\phi_k|^2 = \frac{H^2}{(2\pi)^3 2k^3} F(\tau, k) \quad (3.46)$$

and

$$F(\tau, k) = \frac{H^2}{(2\pi)^3 m_p^2 k^3} \int \frac{d^3 q}{2q^3} \frac{d^3 p}{2p^3} \delta(\vec{k} - \vec{p} - \vec{q}) \Theta(\vec{q}, \vec{p}) \times |\{f_k(\tau) F_{k pq}^*(\tau) - f_k^*(\tau) G_{k pq}^*(\tau)\}|^2 \quad (3.47)$$

[ $\Theta$  was defined in (3.29),  $F$  and  $G$  in (3.39)]. If, as in the previous subsection, we neglect  $G$  compared to  $F$ , this reduces to the simple result

$$F(\tau, k) = -\Delta\sigma_k |f_k(\tau)|^2. \quad (3.48)$$

This result, of course, is exactly what we should expect from the fluctuation-dissipation arguments. The environment injects into the system exactly the amount of fluctuations necessary to maintain consistency with equilibrium against the tendency of the mean field to dissipate away. We could say that the environment returns as noise what it had previously absorbed as dissipation; the important point is that in the process these fluctuations have been degraded from coherent quantum fluctuations to incoherent stochastic ones. This reprocessing of part of the quantum mean field by the environment is the physical content of the decoherence process.

An important consequence of (3.48) is that only a fraction, and indeed a very small part, of the total zero point fluctuations may ever become classical, and thus contribute to structure formation, unless the Hubble parameter  $H$  is of the order of the Planck mass. This is the crucial point relevant to the cosmological problem, and therefore deserves to be elaborated in some detail.

## IV. GENERATION OF FLUCTUATIONS IN INFLATIONARY COSMOLOGY

So far we have presented a comprehensive framework to study the transmutation of quantum into classical fluctuations in nonlinear field theories, and applied it to a scalar field propagating on a de Sitter background while

interacting with gravitons. Let us now apply these results to the problem of the generation of primordial fluctuations during the inflationary era. We shall contrast the well-accepted results in the literature with that obtained by our method.

For simplicity, we follow the Guth-Pi treatment [59] for the density contrast derived from quantum fluctuations in the inflaton field. It is estimated to be

$$\frac{\delta\rho}{\rho} \approx \frac{H\delta\phi_k}{\langle\dot{\Phi}\rangle}, \quad (4.1)$$

where the right-hand side is evaluated at the time a given mode “leaves” the de Sitter horizon, i.e., when  $k\tau \sim 1$ . As a viable example, we shall adopt the “chaotic” inflation [13] model, where the inflaton field self-interacts with a  $\lambda\Phi^4$  effective potential. During the inflationary period, the vacuum energy dominates the stress energy tensor of the field, and the inflaton slowly rolls down the potential well. Because of the first assumption, the Hubble parameter becomes

$$H^2 \sim \frac{\lambda\Phi^4}{m_p^2} \quad (4.2)$$

(in this discussion, we shall systematically ignore factors of order unity). Because of the second assumption, the equation of motion for the field is

$$\dot{\Phi} + \sqrt{\lambda}m_p\Phi = 0, \quad (4.3)$$

with solution

$$\Phi(t) = \Phi_0 e^{-\sqrt{\lambda}m_p t} \quad (4.4)$$

and

$$H(t) = \frac{\sqrt{\lambda}\Phi_0^2}{m_p} e^{-2\sqrt{\lambda}m_p t}, \quad (4.5)$$

where we have placed the origin of cosmic proper time at the beginning of inflation. Both assumptions break down when  $\Phi \sim m_p$ , so we estimate the length of the inflationary period  $\Delta t$  as

$$e^{\sqrt{\lambda}m_p \Delta t} \sim \frac{\Phi_0}{m_p} \quad (4.6)$$

and the number of  $e$ -foldings

$$n = \int^{\Delta t} H dt \sim \left(\frac{\Phi_0}{m_p}\right)^2. \quad (4.7)$$

A satisfactory resolution of the horizon problem demands  $n \geq 60$ . This implies that the variation of  $H$  over an  $e$ -folding is small.

With these inputs, we may compute the spectrum of primordial fluctuations. In the conventional treatment, where the full quantum fluctuations of the inflaton are seen as contributing to structure formation,  $\delta\phi_k$  is read directly out of the mode expansion as [60]

$$|\delta\phi_k|_{\text{naive}}^2 = \frac{H^2}{(2\pi)^3 2k^3}. \quad (4.8)$$

Therefore

$$\frac{\delta\rho}{\rho} \sim \frac{H^2}{\dot{\Phi}} \sim \sqrt{\lambda} \left(\frac{\Phi}{m_p}\right)^3. \quad (4.9)$$

The physically most relevant modes are those which leave the horizon late in the inflationary period, when  $\Phi \sim m_p$ . For these modes, the observational constraint  $\delta\rho/\rho \sim 10^{-6}$  at decoupling leads to a severe bound on the inflaton interactions  $\lambda \sim 10^{-12}$ . This is one of the outstanding puzzles in inflationary cosmology.

If we compare our results for the semiclassical fluctuations with the usual estimates in the literature, we find they differ by the presence of the  $F$  factor defined in Eq. (3.47). Closer examination reveals that the integral defining  $F(\tau, k)$  depends on its arguments only through the combination  $k\tau$ , and as the mode “leaves the horizon” it becomes a dimensionless constant.<sup>10</sup> (Of course, if we take the defining expression at face value, this constant would be infinite, but, since the divergence is only logarithmic, after suitable ultraviolet and infrared cutoffs are introduced, the physical result shall be of order one.) Therefore we simply obtain

$$F \sim \left(\frac{H}{m_p}\right)^2 \sim \lambda \left(\frac{\Phi}{m_p}\right)^4. \quad (4.10)$$

And, for “short” wavelength modes,

$$\frac{\delta\rho}{\rho} \sim \lambda. \quad (4.11)$$

This correction modifies the above bound on  $\lambda$  by six orders of magnitude, i.e., we have  $\delta\rho/\rho \sim 10^{-6}$ , with  $\lambda \sim 10^{-6}$ . This represents a dramatic reduction in the fine-tuning required by the model; in fact, this value of  $\lambda$  is consistent with the inflaton taking part in non-Abelian gauge interactions with a coupling constant of  $10^{-2}$ , while the older estimate would require one to shield the inflaton unnaturally from radiative corrections. On the other hand, the value of  $\lambda$  is not so high as to make the coupling of the inflaton with its own fluctuations prevail over the gravitational couplings considered here.

As we have illustrated in this paper, with proper con-

<sup>10</sup>In the function  $F$  defined in Eq. (3.47), the function  $f_k$  depends only on  $k\tau$  [Eq. (3.25)]. Therefore  $F_{kpq}$  [Eq. (3.40)] scales as  $\tau^{-1}$ , or as  $k$  at horizon crossing, since  $k\tau \sim 1$ , while  $\Theta$  [Eq. (3.29)] scales as  $k^4$ . On the other hand, the  $\delta$  function scales as  $k^{-3}$ , and there is another  $k^{-3}$  explicitly in front of the integral. Thus naive power counting shows that  $F$  [Eq. (3.47)] is dimensionless. Since all the possible dependence on  $H$  and  $m_p$  are explicitly given, it either depends only on the dimensionless combination  $k\tau$ , or else on a regularization scale or cutoff. However, the latter can only happen logarithmically, since this is the degree of divergence of the integral.

sideration of decoherence and noise for quantum fields, the possibility of developing successful inflationary scenarios within moderately nonlinear field theories has far-reaching consequences. Not only can we place the inflaton field in the proper ranges of conventional high energy physics in the treatment of fluctuations, but also better implement the inflaton dynamics [62], entropy generation [63], and reheating problems [64]. We are continuing this line of investigation on these outstanding issues.

## ACKNOWLEDGMENTS

This work was supported in part by the National Science Foundation under Grants Nos. PHY91-19726, INT91-02088 (USA), and by CONICET, UBA, and Fundación Antorchas (Argentina). Thanks are due to N. Deruelle, Andrew Matacz, and C. Stephens for discussions on different aspects of this work. B.L.H. acknowl-

edges support from the General Research Board of the Graduate School of the University of Maryland and the Dyson Visiting Professor Fund at the Institute for Advanced Study, Princeton. Part of this work was done while he visited the Newton Institute for Mathematical Sciences at the University of Cambridge during the Geometry and Gravity program, and the physics department of the Hong Kong University of Science and Technology.

## APPENDIX

### 1. Dissipation kernel

Let us begin with the calculation of the dissipation kernel (3.20). In order to compute the variational derivatives, we must consider the equations for the  $\varphi$  and  $h$  fields; namely,

$$\square\varphi(x) = \frac{(H\tau)^2}{m_p} \phi_{,ij}(x) \int d^3z \{G_j^{+i}(\vec{x} - \vec{z}) h^+(\tau, \vec{z}) + (+ \leftrightarrow \times)\}, \quad (\text{A1})$$

$$\square h^+(\tau, \vec{y}) = \frac{(H\tau)^2}{m_p} \int d^3z \{G_j^{+i}(\vec{y} - \vec{z}) [\phi_{,ij}\varphi](\tau, \vec{z})\}, \quad (\text{A2})$$

and a similar equation for the cross ( $\times$ ) polarization. Taking variational derivatives of both sides of these equations, we find

$$\square \left. \frac{\delta\varphi(x)}{\delta\phi(x')} \right|_{\phi=0} = \frac{(H\tau)^2}{m_p} \partial_{ij}^2 \delta(x, x') \int d^3z \{G_j^{+i}(\vec{x}' - \vec{z}) h^+(\tau, \vec{z}) + (+ \leftrightarrow \times)\}, \quad (\text{A3})$$

$$\square \left. \frac{\delta h^+(\tau, \vec{y})}{\delta\phi(x')} \right|_{\phi=0} = \frac{(H\tau')^2}{m_p} \delta(\tau, \tau') G_j^{+i}(\vec{y} - \vec{x}') \partial_{ij}^2 \varphi(x'). \quad (\text{A4})$$

These equations may be solved as

$$\left. \frac{\delta\varphi(x)}{\delta\phi(x')} \right|_{\phi=0} = \frac{(H\tau')^2}{m_p} \partial_{ij}^2 \Delta_{\text{ret}}(x, x') \int d^3z \{G_j^{+i}(\vec{x}' - \vec{z}) h^+(\tau, \vec{z}) + (+ \leftrightarrow \times)\}, \quad (\text{A5})$$

$$\left. \frac{\delta h^+(\tau, \vec{y})}{\delta\phi(x')} \right|_{\phi=0} = \frac{(H\tau')^2}{m_p} \int d^3z \Delta_{\text{ret}}[(\tau, \vec{y}), (\tau', \vec{z})] G_j^{+i}(\vec{z} - \vec{x}') \partial_{ij}^2 \varphi(x'). \quad (\text{A6})$$

The remaining steps consist of substituting these in (3.20) and computing the quantum expectation values with the help of the mode decomposition (3.24). These straightforward manipulations shall be omitted.

### 2. The propagator approach

Let us recall the action for the microscopic  $\varphi$  field

$$S_f^{(2)} = \frac{1}{2} \int d\tau d^3x (H\tau)^{-2} [(\varphi')^2 - (\vec{\nabla}\varphi)^2], \quad (\text{A7})$$

where the prime stands for a  $\tau$  derivative. We derive from it the microscopic Feynman propagator [65]

$$\Delta_F(x_1, x_2) = - \left( \frac{H}{2\pi} \right)^2 \left[ \frac{\tau_1 \tau_2}{(\tau^2 - r^2 - i\epsilon)} + \frac{1}{2} \ln(r^2 - \tau^2 + i\epsilon) - 2c \right], \quad (\text{A8})$$

where  $\tau = \tau_1 - \tau_2$ ,  $\vec{r} = \vec{x}_1 - \vec{x}_2$ , and  $c$  is an undetermined constant. The retarded propagator  $\Delta_r = -2 \text{Im} \Delta_F \theta(\tau)$



and the Hadamard propagator  $\Delta_1 = 2 \text{Re}\Delta_F$  also follow.

The graviton Feynman propagator is a symmetric bitensor  $\Delta_{j_1, j_2}^{i_1, i_2}(x_1, x_2)$ . Out of symmetry considerations, it must take the form [66]

$$\begin{aligned} \Delta_{jl}^{ik} = & \Delta_{PP} P_j^i P_l^k \\ & + \Delta_{PQ} (P_j^i Q_l^k + Q_j^i P_l^k) \\ & + \tilde{\Delta}_{PQ} (P^{ik} Q_{jl} + P_l^i Q^k j + Q^{ik} P_{jl} + Q_l^i P_j^k) \\ & + \Delta_{QQ} Q_j^i Q_l^k \\ & + \tilde{\Delta}_{QQ} (Q^{ik} Q_{jl} + Q_l^i Q_j^k), \end{aligned} \quad (\text{A9})$$

where

$$P^{ij} = \frac{(r^i r^j)}{r^2}, \quad (\text{A10})$$

$$Q^{ij} = \delta^{ij} - P^{ij}. \quad (\text{A11})$$

The restrictions on physical gravitons imply a number of identities the graviton propagator must satisfy; namely,

$$\Delta_{il}^{ik} = \Delta_{jl, i}^{ik} = 0. \quad (\text{A12})$$

These identities allow us to write all the biscalar coefficients in terms of  $\Delta_{PP}$ , concretely,

$$\begin{aligned} \Delta_{PQ} &= -\frac{1}{2} \Delta_{PP}, \\ \tilde{\Delta}_{PQ} &= \left( \frac{3}{4} + \frac{r^2}{2} \frac{d}{dr^2} \right) \Delta_{PP}, \\ \Delta_{QQ} &= \left[ -\frac{9}{8} - 3r^2 \frac{d}{dr^2} - \left( r^2 \frac{d}{dr^2} \right)^2 \right] \Delta_{PP}, \\ \tilde{\Delta}_{QQ} &= \left[ \frac{11}{8} + 3r^2 \frac{d}{dr^2} + \left( r^2 \frac{d}{dr^2} \right)^2 \right] \Delta_{PP}. \end{aligned} \quad (\text{A13})$$

Moreover, the graviton propagator is linked to the scalar one by

$$\Delta_{ki}^{ik} = \left( \frac{8}{m_p^2} \right) \Delta_F, \quad (\text{A14})$$

which provides a connection between  $\Delta_{PP}$  and  $\Delta_F$ : namely,

$$\Delta_{PP} = \frac{16}{3m_p^2} \int_0^1 du [u - u^4] \Delta_F(u\vec{r}). \quad (\text{A15})$$

In our approach here, the correlator of decohered fluctuations is not to be guessed *a priori*, but should rather be deduced from the analysis of the noise kernel induced by the inflaton-graviton coupling. Our task is simplified by the observation that this model has the same structure as the  $g\Phi^3$  theory from the previous section, only we now deal with a multicomponent field. It is therefore immediate to write down the noise kernel

$$N(x_1, x_2) = \text{Re}\Sigma(x_1, x_2) \quad (\text{A16})$$

and the dissipation kernel

$$D(x_1, x_2) = 2 \text{Im}\Sigma(x_1, x_2) \theta(\tau_1 - \tau_2), \quad (\text{A17})$$

where

$$\Sigma(x_1, x_2) = \frac{2}{(H\tau_1)^2 (H\tau_2)^2} [\partial_i \partial_j \partial_k \partial_l (\Delta_F \Delta_{jl}^{ik})] (x_1, x_2). \quad (\text{A18})$$

An explicit evaluation yields

$$\begin{aligned} \Sigma(x_1, x_2) = & \frac{68}{(2\pi)^4 m_p^2 \tau_1^4 \tau_2^4} \frac{1}{(Z-1)^3} \left\{ (Z-3) \left[ (1-Z)^{-1} - \ln[\tau_1 \tau_2 (1-Z)] - 2c \right] \right. \\ & + 4b^2 \frac{(Z-5)}{(Z-1)^2} \left[ \frac{21}{100} - \frac{a^2}{15} - \frac{1}{3b} - \left( \frac{a^4}{5} + \frac{a^2}{b} \right) [1 - a \text{arctanh}(a^{-1})] \right. \\ & + \left. \frac{3c}{10} + \left( \frac{1}{4} + \frac{1}{2b} \right) \ln \left( 1 - \frac{1}{a^2} \right) - \frac{3}{20} \ln[\tau_1 \tau_2 (1-Z)] \right] \\ & \left. + \frac{2(Z-4)}{(Z-1)} \left[ -b - 2bc - (b+2) \ln \left( 1 - \frac{1}{a^2} \right) + b \ln[\tau_1 \tau_2 (1-Z)] \right] \right\}, \end{aligned} \quad (\text{A19})$$

where  $Z - 1 = \tau^2 - r^2/2\tau_1\tau_2$ ,  $b = r^2/\tau_1\tau_2$ , and  $a^2 = \tau^2/r^2$  (we assume  $\tau^2$  has a small negative imaginary part to obtain the correct time-ordering property). Observe that  $Z$  is de Sitter invariant, while  $a$  and  $b$  are not.

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