Fluctuation corrections to bubble nucleation

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The fluctuation determinant which determines the preexponential factor of the transition rate for minimal bubbles is computed for the electroweak theory with $\sin\Theta_W = 0$. As the basic action we use the three-dimensional high-temperature action including, in addition to temperature-dependent masses, the $T\Phi^3$ one-loop contribution which makes the phase transition first order. The results show that this term (which has then to be subtracted from the exact result) gives the dominant contribution to the one-loop effective action. The remaining correction is of the order of, but in general larger than, the critical bubble action. The results for the Higgs field fluctuations are compared with those of an approximate heat kernel computation by Kripfganz, Laser, and Schmidt; good agreement is found for small bubbles and strong deviations for large thin-wall bubbles.

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I. INTRODUCTION

The electroweak phase transition is at present the object of extensive investigations [1]. If the phase transition is first order, which is possibly the case if the mass of the Higgs boson is not too large, the phase transition occurs via bubble nucleation. Bubble nucleation can have various consequences for cosmology in the early universe. The possibility of baryogenesis in bubble walls has been investigated recently by many authors (see, e.g., [2, 3]); reheating after the phase transition could be mediated by bubble nucleation and subsequent coalescence, the creation of inhomogeneities by bubble formation could be observable (see, e.g., [4–6] for representative discussions of the physics of bubble nucleation and growth).

Bubble nucleation is described usually within the reaction rate theory formulation of Langer [7] or, equivalently, the semiclassical approach to quantum field theory by Callan and Coleman [8, 9]. This formulation requires the existence of a saddle point in configuration space, the minimal bubble, with one unstable mode, possible zero modes, and real frequency fluctuation modes. The leading term in the tunneling rate is given by the negative exponential of the minimal bubble action; the corrections arise from integrating out the fluctuations in the Gaussian approximation, leading to a fluctuation determinant prefactor whose negative logarithm is the one-loop effective action. If the leading approximation is good, this prefactor should be of order 1; substantial prefactors have, however, been found in the case of the sphaleron transition, both from bosonic [10–12] and fermionic [13] fluctuations. It is therefore of interest to investigate how strongly these prefactors modify the leading-order approximation to the bubble nucleation rate.

Here we present an exact computation of the bosonic fluctuation determinant of the critical bubble. As the basic action is determined by the usual Higgs potential with just one minimum at the classical expectation value, some fluctuation effects have to be included already at the tree level in order to allow for minimal bubble solutions. The exact fluctuation determinant should then reproduce those in order to justify this modification of the leading-order action. Following the basic work of Coleman and Weinberg [14] such modified actions have been proposed by many authors [6, 15–17] and used to describe the bubble nucleation in leading order. To be specific we use here the one given by Dine *et al.* [6] which was also the basis of a recent approximate computation of the one-loop Higgs fluctuations by Kripfganz, Laser, and Schmidt [18].

The plan of this paper is as follows. In the next section we will introduce the model and set up the basic relations for the bubble nucleation rate. In Sec. III we will discuss the structure of the fluctuation operator, in particular its partial-wave decomposition. The computation of its determinant, based on a very useful theorem, will be described in Sec. IV. In the final section we will present some results and conclusions.

II. BASIC RELATIONS

The three-dimensional high-temperature action is given, in the formulation by Dine *et al.* [6], by

$$S_{\rm ht} = \frac{1}{g_3(T)^2}\int d^3x\left[\frac{1}{4}F_{ij}F_{ij} + \frac{1}{2}(D_i\Phi)^\dagger(D_i\Phi) + V_{\rm ht}(\Phi^\dagger\Phi) + \frac{1}{2}A_0\left(-D_iD_i + \frac{1}{4}\Phi^\dagger\Phi\right)A_0\right]. \qquad (2.1)$$

Here the coordinates and fields have been rescaled as [19]

$$\vec{x} \to \frac{\vec{x}}{gv(T)}, \quad \Phi \to v(T)\Phi, \quad A \to v(T)A\,. \qquad (2.2)$$

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The vacuum expectation value $v(T)$ is defined as

$$v^2(T) = \frac{2D}{\lambda_T}(T_0^2 - T^2) . \tag{2.3}$$

T_0 is the temperature at which the high-temperature potential $V_{\rm ht}$ changes its extremum at $\Phi = 0$ from a minimum at $T > T_0$ to a maximum at $T < T_0$. The temperature-dependent coupling of the three-dimensional theory is defined as

$$g_3(T)^2 = \frac{gT}{v(T)} . \tag{2.4}$$

In terms of the zero-temperature parameters we have $m_W = gv_0/2$, $m_H = \sqrt{2\lambda}v_0$ with $v_0 = 246$ GeV, and we use the definitions of Dine *et al.* [6], modified by setting $\Theta_W = 0$ and therefore $m_W = m_Z$:

$$D = (3m_W^2 + 2m_t^2)/8v_0^2 ,$$

$$E = 3g^3/32\pi ,$$

$$B = 3(3m_W^4 - 4m_t^4)/64\pi^2 v_0^4 ,$$

$$T_0^2 = (m_H^4 - 8v_0^2 B)/4D , \tag{2.5}$$

$$\lambda_T = \lambda - 3\left(3m_W^4 \ln\frac{m_W^2}{a_B T^2} - 4m_t^4 \ln\frac{m_t^2}{a_F T^2}\right) \bigg/ 16\pi^2 v_0^4 . \tag{2.6}$$

In terms of these parameters the high-temperature potential is given by

$$V_{\rm ht}(\Phi^\dagger\Phi) = \frac{\lambda_T}{4g^2}\left((\Phi^\dagger\Phi)^2 - 2\Phi^\dagger\Phi - \frac{4E}{\lambda_T v(T)}(\Phi^\dagger\Phi)^{3/2}\right) . \tag{2.7}$$

The rescaling Eq. (2.2) with the scale $v(T)$ makes sense only for $T < T_0$. On the other hand the high-temperature potential has, before rescaling, a secondary minimum at $|\Phi| = \tilde{v}(T)$ with

$$\tilde{v}(T) = \frac{3ET}{2\lambda} + \sqrt{\left(\frac{3ET}{2\lambda}\right)^2 + v^2(T)} . \tag{2.8}$$

This minimum is degenerate with the one at $\Phi = 0$ at a temperature defined implicitly by

$$T_C = T_0/\sqrt{1 - E^2/D\lambda_{T_C}} . \tag{2.9}$$

T_C marks the onset of bubble formation by thermal barrier transition. In the work of Hellmund *et al.* [20] and Kripfganz *et al.* [18] the vacuum expectation value of the broken symmetry phase $\tilde{v}(T)$ is chosen for the rescaling of the fields; i.e., in Eq. (2.2), $v(T)$ is replaced by $\tilde{v}(T)$, and the high-temperature coupling constant Eq.(2.4) is redefined analogously and denoted[1] as $\tilde{g}_3(T)$. By this change of scale the high-temperature potential changes as well; it becomes[2] [20]

$$V_{\rm ht}(\Phi^\dagger\Phi) = \frac{\lambda_T}{4g^2}\{(\Phi^\dagger\Phi)^2 - \epsilon(T)(\Phi^\dagger\Phi)^{3/2} + [\tfrac{3}{2}\epsilon(T) - 2]\Phi^\dagger\Phi\} , \tag{2.10}$$

with

$$\epsilon(T) = \frac{4}{3}\left(1 - \frac{v(T)^2}{\tilde{v}(T)^2}\right) . \tag{2.11}$$

The action and its rescaling differ slightly from that of Hellmund *et al.* [20] and of Kripfganz *et al.* [18]. In contrast with the former we do not mimic the influence of a Debye mass by decoupling the longitudinal degrees of freedom. In contrast with the second one we include only the Φ^3 contribution of the gauge field and would-be Goldstone degrees of freedom as in Ref. [16]. This form of the Φ^3 contribution was found to yield a good approximation for the exact results in the case of the sphaleron [10, 11], at least in the case $m_H/m_W \ll 1$. We will find, indeed, that this term dominates the effective action.

The process of bubble nucleation is, within the approach of Langer [7] and Coleman and Callan [8, 9], followed by the work of Affleck [21], Linde [22], and others, described by the rate

$$\Gamma/V = \frac{\omega_-}{2\pi}\left(\frac{\tilde{S}}{2\pi}\right)^{3/2} \exp(-\tilde{S})\, \mathcal{J}^{-1/2} . \tag{2.12}$$

Here $\tilde{S}$ is the high-temperature action, Eq. (2.1), with the new rescaling, minimized by a classical minimal bubble configuration (see below), and $\mathcal{J}$ is the fluctuation determinant which describes the next-to-leading part of the semiclassical approach and which will be defined below; its logarithm is related to the one-loop effective action by

$$S_{\rm eff}^{1-l} = \frac{1}{2}\ln\mathcal{J} . \tag{2.13}$$

Finally ω_- is the absolute value of the unstable mode frequency.

The classical bubble configuration is described by a vanishing gauge field and a real spherically symmetric Higgs field $\Phi(r) = |\Phi|(r)$ which is a solution of the Euler-Lagrange equation

$$-\Phi''(r) - \frac{2}{r}\Phi'(r) + \frac{dV_{\rm ht}}{d\Phi(r)} = 0 , \tag{2.14}$$

with the boundary conditions

$$\lim_{r\to\infty}\Phi(r) = 0 \text{ and } \Phi'(0) = 0 . \tag{2.15}$$

[1]Our notation differs from the one of Refs. [18, 20].

[2]We do not introduce a tilde for the rescaled fields.

This differential equation can be solved numerically, e.g., by the shooting method. The solution will be denoted as $H_0(r)$.

III. FLUCTUATION ANALYSIS

In terms of the action S the fluctuation operator is defined generally as

$$\mathcal{M}_{ab} = \frac{\delta^2 S}{\delta\phi_a \delta\phi_b}|_{\phi=\phi_{\rm bubble}}, \tag{3.1}$$

where ϕ_a stands for the various gauge and Higgs field components and $\phi_{\rm bubble}$ is the field configuration of the minimal bubble. An analogous derivative, taken at $\phi = \phi_{\rm vac} \equiv 0$, defines the vacuum fluctuation operator $\mathcal{M}^0$. In both configurations the gauge fields vanish, and the Higgs field is given by

$$\mathbf{\Phi} = H_0(r)\begin{pmatrix}0\\1\end{pmatrix} \tag{3.2}$$

in the bubble configuration and vanishes in the vacuum.

The fluctuation determinant $\mathcal{J}$ appearing in the rate formula is defined by

$$\mathcal{J} = \frac{\det''\mathcal{M}}{\det\mathcal{M}^0} . \tag{3.3}$$

Here the symbol det$''$ denotes the determinant with removed translation zero modes and with the unstable mode frequency replaced by its absolute value.

The analysis of fluctuations of the minimal bubble can be related to a similar analysis performed recently for the electroweak sphaleron without gauge fixing in Ref. [23] and in the 't Hooft–Feynman background gauge in Ref. [11]. We will use this latter analysis. One can take over the fluctuation operator with two modifications which represent at the same time essential simplifications: The high-temperature effective potential has to be modified from the one in Eq. (2.7) to the one in Eq. (2.10); the sphaleron and the broken symmetry vacuum configurations are replaced by the bubble and the symmetric vacuum configurations defined above.

Furthermore we use here [see Eq. (2.2)] for the coordinates the scale $(g\tilde{v})^{-1}$ instead of the scale $M_W^{-1} = 2/gv$ used in Ref. [11].

The expansion of gauge and Higgs fields reads then [11]

$$W_\mu^a = a_\mu^a ,$$
$$\mathbf{\Phi} = (H_0 + h + \tau^a\phi^a)\begin{pmatrix}0\\1\end{pmatrix} . \tag{3.4}$$

Here the fields denoted with small letters, a_μ^a, h, and ϕ_a, are the fluctuating fields.

Before we discuss fluctuations we have to fix the gauge. We work here in the 't Hooft–Feynman background gauge. The gauge conditions read

$$\mathcal{F}_a = \partial_\mu a_a^\mu + \frac{1}{2}H_0\phi_a = 0 . \tag{3.5}$$

The total gauge-fixed action S_t is obtained from the high-temperature action by adding to it the gauge-fixing action

$$S_{\rm GF} = \frac{1}{\tilde{g}_3^2(T)}\int d^3x \frac{1}{2}\mathcal{F}_a\mathcal{F}_a \tag{3.6}$$

and the Faddeev-Popov action

$$S_{\rm FP} = \frac{1}{\tilde{g}_3^2(T)}\int d^3x \eta^\dagger\left(-\Delta + \frac{H_0^2}{4}\right)\eta . \tag{3.7}$$

It is the action $S_t = S_{\rm ht} + S_{\rm GF} + S_{\rm FP}$ which has to be used in the definition of the fluctuation operator (3.1).

The Hilbert space of fluctuations decomposes into subspaces defined by the symmetries of the background field. The fluctuation operators given below have been derived from those of Ref. [11]. This analysis was based on a K spin basis ($\vec{K} = \vec{J} + \vec{I}$). Alternatively one might have used here simply an analysis based on ordinary spin, i.e., an expansion where the Higgs field, the Faddeev-Popov field, and the time components of the gauge fields are expanded with respect to spherical harmonics and the space components of the gauge fields with respect to vector spherical harmonics $\hat{x}Y_l^m$, $r\nabla Y_l^m$ and $\vec{L}Y_l^m$.

The electric components of the gauge field and the isovector (would-be Goldstone) components of the Higgs field form a coupled (3×3) system. The fluctuation operator can be written in the form $\mathbf{M} = \mathbf{M}^0 + \mathbf{V}$. The free operator $\mathbf{M}^0$ is diagonal. It consists of free partial-wave Klein-Gordon operators

$$\mathbf{M}^0 = -\frac{d^2}{dr^2} - \frac{2}{r}\frac{d}{dr} + \frac{l_n(l_n+1)}{r^2} + m_n^2 , \tag{3.8}$$

with masses m_n given by $(0, 0, m_H)$, respectively, for the three components and with centrifugal barriers corresponding to angular momenta l_n given analogously by $(l+1, l-1, l)$. The nonvanishing components of the potential are

$$V_{11} = V_{22} = H_0^2/4 ,$$
$$V_{33} = H_0^2/4 + (\lambda_T/4g^2)(4H_0^2 - 3\epsilon H_0) ,$$
$$V_{13} = V_{31} = -\sqrt{\frac{l+1}{2l+1}}\frac{dH_0}{dr} , \tag{3.9}$$
$$V_{23} = V_{32} = \sqrt{\frac{l}{2l+1}}\frac{dH_0}{dr} . \tag{3.10}$$

For $l = 0$ the second component is absent due to the vanishing of the vector spherical harmonic $r\nabla Y_0^0$. These amplitudes have a triple degeneracy due to isospin besides the ordinary degeneracy $(2l+1)$ from spin.

The fluctuation operator for the scalar part of the Higgs field is given by

$$\mathbf{M} = -\frac{d^2}{dr^2} - \frac{2}{r}\frac{d}{dr} + \frac{l(l+1)}{r^2} + m_H^2 + V_{44}(r) ,$$
$$V_{44} = \frac{\lambda_T}{4g^2}(12H_0^2 - 6\epsilon H_0) , \tag{3.11}$$
$$m_H^2 = \frac{\lambda_T}{4g^2}(3\epsilon - 4) .$$

This channel being an isosinglet, its degeneracy is just $(2l+1)$.

The time components of the gauge fields, the Faddeev-Popov fields, and the magnetic components of the vector potentials all satisfy the same equation

$$\mathbf{M}_l \psi_5 = \omega^2 \psi_5 \ . \tag{3.12}$$

It consists of a free massless partial-wave Klein-Gordon operator

$$\mathbf{M}^0 = -\frac{d^2}{dr^2} - \frac{2}{r}\frac{d}{dr} + \frac{l(l+1)}{r^2} \tag{3.13}$$

and a potential

$$V_{55} = \frac{H_0^2}{4} \ , \tag{3.14}$$

which vanishes exponentially as $r \to \infty$. There is no $l = 0$ component of the magnetic vector potential since the vector spherical harmonic $\vec{L}Y_l^m$ vanishes. In the fluctuation determinant all of these contributions cancel, and only the s-wave Faddeev-Popov contribution survives, due to the lack of its magnetic counterpart. It is triply degenerate due to isospin and has to be subtracted.

The partial-wave decomposition of the fluctuation operator decomposes also its determinant:

$$\ln \mathcal{J} = \sum (2l+1) \ln \mathcal{J}_l \ . \tag{3.15}$$

We now need a method for computing numerically the determinants of the partial-wave fluctuation operators. Such a method has been developed recently by Kiselev and Baacke [29] and will be presented briefly in the following section.

IV. THE FLUCTUATION DETERMINANT OF THE ELECTROWEAK BUBBLE

A very fast method for computing fluctuation determinants is based on a theorem on functional determinants; references to earlier work and an elegant proof are given in Ref. [24]. Generalized to a coupled $(n \times n)$ system it can be stated in the following way

Let $\mathbf{f}(\nu, r)$ and $\mathbf{f}^0(\nu, r)$ denote the $(n \times n)$ matrices formed by n linearly independent solutions $f_i^\alpha(\nu, r)$ and $f_i^{\alpha 0}(\nu, r)$ of

$$(\mathbf{M}_{ij} + \nu^2 \delta_{ij}) f_j^\alpha(\nu, r) = 0 \tag{4.1}$$

and

$$(\mathbf{M}_{ij}^0 + \nu^2 \delta_{ij}) f_j^{\alpha 0}(\nu, r) = 0 \ , \tag{4.2}$$

respectively, with regular boundary conditions at $r = 0$. The lower index denotes the n components, and the different solutions are labeled by the Greek upper index. Let these solutions be normalized such that

$$\lim_{r \to 0} \mathbf{f}(\nu, r) [\mathbf{f}^0(\nu, r)]^{-1} = \mathbf{1} \ . \tag{4.3}$$

Then the following equality holds:

$$\mathcal{J}(\nu) \equiv \frac{\det(\mathbf{M} + \nu^2)}{\det(\mathbf{M}^0 + \nu^2)} = \lim_{r \to \infty} \frac{\det \mathbf{f}(\nu, r)}{\det \mathbf{f}^0(\nu, r)} \ , \tag{4.4}$$

where the determinants on the left-hand side are determinants in functional space, and those on the right-hand side are ordinary determinants of the $n \times n$ matrices defined above. If the theorem is applied at $\nu = 0$, it yields the desired ratio of fluctuation determinants $\mathcal{J} \equiv \mathcal{J}(0)$. The consideration of finite values of ν is necessary in the discussion of zero modes.

The theorem has found previously some applications in $(1+1)$-dimensional models [25–27]; the generalization to partial waves and to finite-temperature computations, using the analytic properties of Jost functions, has been given by Bochkarev [28]. It has been developed into a numerical method for computing fluctuation determinants for three-dimensional systems, including renormalization, in [29, 30]. These numerical applications have shown that the method yields very precise results and in addition is very fast.

In the numerical application the solutions f_n^α were written as [31]

$$f_n^\alpha(r) = [\delta_n^\alpha + h_n^\alpha(r)] i_{l_n}(\kappa_n r) \ , \tag{4.5}$$

with the boundary condition $h_n^\alpha(r) \to 0$ as $r \to 0$. The values l_n and $\kappa_n = \sqrt{m_n^2 + \nu^2}$ depend on the channel as specified in the previous section. This way one generates a set of linearly independent solutions which near $r = 0$ behave like the free solution as required by the theorem which then takes the form

$$\mathcal{J}(\nu) = \lim_{r \to \infty} \det\{\delta_n^\alpha + h_n^\alpha(r)\} \ . \tag{4.6}$$

The functions $h_n^\alpha(r)$ satisfy the differential equation [31]

$$\frac{d^2}{dr^2} h_n^\alpha(r) + \left(\frac{2}{r} + 2\kappa_n \frac{i'_{l_n}(\kappa_n r)}{i_{l_n}(\kappa_n r)}\right) \frac{d}{dr} h_n^\alpha(r) = V_{nn'}(r) [\delta_{n'}^\alpha + h_{n'}^\alpha(r)] \frac{i_{l_{n'}}(\kappa_{n'} r)}{i_{l_n}(\kappa_n r)} \ , \tag{4.7}$$

which can also easily be used for generating the functions h_n^α order by order in V. In particular, if this differential equation is truncated by leaving out the term $h_{n'}^\alpha$ on the right-hand side, one generates the first-order contribution to h which is the tadpole term. For more technical details we refer to Refs. [29, 30].

With the partial-wave fluctuation operators given in the previous section the application of the theorem to the case of the electroweak bubble is straightforward. Some points to be considered are the subtraction of the divergent tadpole graphs, double counting of gauge and would-be Goldstone fluctuations, removing the translation zero mode, and removing a particular gauge zero mode. We will discuss these briefly. We will add also some remarks on details of the numerical computation.

A. Tadpole diagrams

The high-temperature three-dimensional theory has only linear divergences of the form of tadpole diagrams which renormalize the mass term of the Higgs field. They have to be subtracted in the numerical computation to obtain finite results. This was done in each partial wave, for which the tadpole contribution may be computed [29, 30] either by solving a truncated differential equation or [31] as an analytic expression using the partial-wave Green function. After these contributions have been subtracted, the partial-wave contributions converge as $1/l^2$ and have a finite sum.

Of course this contribution has to be added back, after having been regularized and renormalized. Part of these diagrams has already been taken into account in the renormalization of the four-dimensional theory and in giving the vacuum expectation value (2.3) of the Higgs field a quadratic temperature dependence. Some terms linear in the temperature survive, however, and contribute [10, 11, 32] (after dividing by the temperature) to the one-loop effective action, i.e., the logarithm of the fluctuation determinant. If the mass of the field in the loop is m_i and its coupling to the external field is described by the potential V_i, their contribution to the effective action is given by $-m_i/8\pi \int d^3x V_i(r)$. The fluctuating gauge fields have vanishing mass and do not contribute. However, we receive contributions from the fluctuating Higgs fields. The mass circulating in the loop is then m_H which is, including the temperature dependence and rescaling, given by Eq. (3.11). The potentials are V_{33} with triple isospin degeneracy and V_{44}. So we have to restitute the terms

$$S^{1\text{L}}_{g\text{-tad}} = -\frac{m_H}{2}\int dr r^2 \left(\frac{3}{4}H_0^2 + 3\frac{\lambda_T}{4g^2}(4H_0^2 - 3\epsilon H_0)\right) \tag{4.8}$$

for the gauge fields and

$$S^{1\text{L}}_{h\text{-tad}} = -\frac{m_H}{2}\int dr r^2 6\frac{\lambda_T}{4g^2}(2H_0^2 - \epsilon H_0) \tag{4.9}$$

for the Higgs field to the one-loop effective action.

We should like to remark on two slight inconsistencies of this procedure. The first one concerns our choice of the high-temperature action. We have adopted the action of Dine *et al.* [6] since it sets a certain standard and since it has been used also in Ref. [18] to which we want to compare part of our results (and which shares the inconsistency). In this action the T^2 term does not include the contribution of the Higgs loop tadpole. This can be seen from the coefficient D in Eqs. (2.6) which should include a term m_H^2 in addition to $3m_W^2 + 2m_t^2$. The contribution was neglected already in Ref. [17], "taking the Higgs boson sufficiently light." Since the expression is dominated by the top quark contribution whose fluctuations are not included at all here, this omission may be tolerated at the present level of accuracy. In a more refined analysis it should and can easily be remedied.

The second point is the fact that we have included already one-loop effects into the tree-level action, so that part of our computation is now at the two-loop level, without constituting a complete and systematic two-loop analysis. This applies in particular also to the tadpole terms for which this could be a more severe problem since they are the finite remnants of divergent graphs. We can appeal here only to an argument, common in many perturbative calculations, that possible inconsistencies are of higher order and acceptable at an intermediate level as they will be cured in a complete higher-order analysis.

B. Double counting of gauge field fluctuations

As mentioned in Sec. II we are working with an action that contains already the part of the one-loop effective potential induced by integrating out the gauge field and would-be Goldstone boson fluctuations. These are present in the temperature scale factors and couplings and appear especially in the high-temperature effective potential as the term proportional to Φ^3. While the T^2 contribution to the vacuum expectation value (2.3) comes from the tadpole diagrams and has been taken into account along with these, the Φ^3 term is contained in our exact one-loop effective action. In order to avoid double counting it, this term has to be subtracted from our numerical results. The incorporation of this term into the tree-level action was necessary in order to obtain a first-order phase transition and bubble solution. If this was a good leading-order approximation the gauge field action should be well approximated by this term. This is indeed the case (see below) but this also implies that the remaining gauge and would-be Goldstone field contributions are small differences of large terms, and that they cannot therefore be expected to be very precise.

C. Translation zero mode and unstable mode

Translation invariance is broken by the classical solution, and so a zero mode appears. It occurs in the $l = 1$ partial wave of the fluctuation operator of the isoscalar part of the Higgs field. It is easily removed using the prescription given in [29]: One applies the theorem mentioned above at finite ν and defines

$$\mathcal{J}_{l=1,\text{Higgs}} = \lim_{\nu\to 0}\lim_{r\to\infty}\left(\frac{\psi_4(\nu,r)}{\nu^2 i_l(\kappa r)}\right), \tag{4.10}$$

where $\kappa = \sqrt{\nu^2 + m_H^2}$. Removing three eigenvalues $\omega^2 = 0$ gives the fluctuation determinant $\mathcal{J}$ the dimension $(\text{energy})^{-6}$. The rate gets then a dimension $(\text{energy})^3 = 1/(\text{length})^3$. An additional dimension energy=1/time comes from the unstable mode prefactor [see (2.12)]. The numerical computation is based on energy units $g\tilde{v}(T)$ which are given in the tables below.

The unstable mode makes the determinant of the *p*-wave contribution negative. Replacing it by its absolute value means just to revert the sign of the determinant before taking the logarithm. We note that, in contrast to Ref. [18] and in analogy to Refs. [10, 11], we do not remove the zero mode from the fluctuation determinant.

D. Gauge zero mode

Though we have imposed a gauge condition, there is one residual gauge degree of freedom. It is analogous to a constant gauge function for the free theory. Indeed in the latter case a constant gauge potential $\Lambda(\vec{x}) = g_0$ does not contribute to the vector potential and is therefore not eliminated by the gauge condition $\partial_\mu a^\mu = 0$. In the case of the bubble background field there is a similar *but nontrivial* mode which satisfies the background gauge condition and is therefore not eliminated by it. It manifests itself as a zero mode in the electric system for $l = 0$. The form of this mode (and the fact that it is really an exact zero mode) was found after extended numerical experiments. It is given by a gauge function $g(r)$ which satisfies the same differential equation as the electric and Faddeev-Popov modes, Eq. (3.12), i.e.,

$$g'' + \frac{2}{r}g' - \frac{H_0^2}{4}g = 0 \, . \tag{4.11}$$

With regular boundary conditions at $r = 0$, $g(r)$ becomes constant as $r \to \infty$, in analogy to the free case. Then the functions

$$\begin{aligned}\psi_1(r) &= -2g'(r) \, , \\ \psi_3(r) &= H_0 g(r)\end{aligned} \tag{4.12}$$

satisfy the coupled system for the electric modes at $l = 0$ which is given explicitly by

$$\begin{aligned}\psi_1'' + \frac{2}{r}\psi_1' - \frac{2}{r^2}\psi_1 &= \frac{H_0^2}{4}\psi_1 - H_0'\psi_3 \, , \\ \psi_3'' + \frac{2}{r}\psi_3' - m_H^2\psi_3 &= \frac{H_0^2}{4}\psi_3 + \frac{\lambda_T}{4g^2}(4H_0^2 - 3\epsilon H_0)\psi_3 \\ &\quad -H_0'\psi_1 \, .\end{aligned} \tag{4.13}$$

It can be checked easily that this gauge zero mode satisfies the background gauge condition (3.5) and is therefore not eliminated by it. Since this zero mode is not due to a symmetry broken by the classical solution as the translation mode, it cannot be handled in the usual way. On the other hand we observe that precisely for the s-wave the Faddeev-Popov contribution has survived; furthermore, to each Faddeev-Popov mode with finite energy, i.e., a solution of

$$\psi_5'' + \frac{2}{r}\psi_5' - \frac{H_0^2}{4}\psi_5 = -\omega_\alpha^2\psi_5 \, , \tag{4.14}$$

there is a solution of the electric s-wave system constructed exactly as that for the gauge zero mode, i.e., Eq. (4.12), with g replaced by ψ_5. So there is a cancellation of all electric modes of this type with the corresponding Faddeev-Popov ones, except for the mode with $\omega_\alpha^2 = 0$. There *is* of course a solution of the Faddeev-Popov equation at this energy, but it is "singular" at infinity, going to a constant there. The corresponding mode in the electric system is normalizable, however, since only its derivative is involved in ψ_1 and its product with the exponentially decreasing function H_0 in ψ_3. The cancellation between the s-wave electric modes (4.12) and the Faddeev-Popov ones can be extended therefore to the zero mode if the boundary condition at $r \to \infty$ for the latter ones is replaced by $\psi_5'(r) \to 0$. This can be done in analogy with the procedure described in the previous section by computing the fluctuation determinant of the Faddeev-Popov mode at finite ν via

$$\mathcal{J}_{l=0,\mathrm{FP}}(\nu) = \lim_{r\to\infty}\left(\frac{\psi_5'(\nu,r)}{i_l'(\nu r)}\right) \, . \tag{4.15}$$

Then the Faddeev-Popov system at $l = 0$ exhibits a zero mode as well; the limit

$$\lim_{\nu\to 0}[\ln \mathcal{J}_{l=0,el}(\nu) - \ln \mathcal{J}_{l=0,FP}(\nu)] \tag{4.16}$$

is finite and defines the s-wave part of the logarithm of the fluctuation determinant. This way the Faddeev-Popov term cancels all unwanted longitudinal electric modes for $l = 0$, including the one with frequency zero. We note that the change of boundary condition as $r \to \infty$ affects only the s wave and only for massless fields. The definition (4.15) yields results identical to the usual one (4.4) if $l \neq 0$ and/or the fields are massive.

E. Some numerical details

The analysis was performed as described in previous publications [29, 30]. Contributions of angular momenta up to $l_{\max} = 30$ were computed numerically; the higher ones were included by performing a power fit $Al^{-2} + Bl^{-3} + Cl^{-4}$ through the last five computed contributions and by adding a corresponding sum from $l_{\max}$ to ∞. This was done already at lower values of l, treating the highest included angular momentum as the actual value of $l_{\max}$. The resulting expressions were found to be independent of l within typically four significant digits for $l > 20$.

A more subtle point is the extrapolation to $r = \infty$ implied in Eq. (4.4). In the previous analyses [29, 30] the fields had finite mass and the approach to $r = \infty$ was exponential. For the massless fields the Bessel functions $i_l(\kappa r)$ are replaced by $r^l/(2l+1)!!$ and the functions h_1^α and h_2^α approach their asymptotic value only as $h_\infty + \mathrm{const}/r$. The extrapolation was performed using this ansatz. An exception occurs in the electric p-wave system, where h_2^α picks up a logarithmic dependence on r due to the cross term with $h_3^\alpha(r)$ on the right-hand side (RHS) of Eq. (4.13) which decreases only as $1/r^2$. However, this logarithmic dependence being strictly proportional to $\delta_3^\alpha + h_3^\alpha(\infty)$, i.e., to the third row of the matrix, it does not contribute in the determinant, as also observed numerically.

The tadpole contributions were computed in two ways, once by solving a truncated differential equation as described in [29, 30] and performing the analogous extrapolation, and once as an integral using the partial-wave Green function. In comparing the two results the extrapolation was found, for the tadpole contributions, to be reliable to four significant digits typically.

Judging the accuracy of the results from the stability with respect to varying extrapolations as $r \to \infty$ and for large l, we would think that the purely numerical part is accurate to 1%. The restituted tadpole contributions are given by expressions (4.8) and (4.9) whose evaluation implies simple numerical integrals; they can be considered as exact analytic expressions. This restitution implies no delicate cancellations. However, even with a precision of 1% for the numerical results the final values of the gauge field contribution have substantial errors since the numerical part plus the tadpole contribution is almost canceled by the analytic Φ^3 contribution. Unfortunately, in contrast to the sphaleron computation [11], the cancellation ist not merely one between two analytic expressions, the tadpole and Φ^3 contributions, but between the numerical results and the analytic Φ^3 contribution.

V. RESULTS AND CONCLUSIONS

The numerical results are given in Tables I to IV. Here Table I is based on the values of Higgs and gauge boson masses $m_H = m_W = 80.2$ GeV, and a value of the top mass of $m_t = 170$ GeV. For the vacuum expectation of the Higgs field we used $v_0 = 246$ GeV and for the gauge coupling the value $g = 0.6516$. For the computation of Table II the values $m_H = 60$ GeV, $m_t = 170$ GeV were used. Table III corresponds to values $m_H = 60$ GeV and $m_t = 140$ GeV, Table IV to values $m_H = 80.2$ GeV and $m_t = 140$ GeV; these latter tables are presented in order to compare with results obtained in Ref. [18] using the heat kernel expansion. The values for the temperature chosen correspond to ten equidistant steps of the quantity $\epsilon(T)$, defined in Eq. (2.11), between the onset of bubble nucleation at $\epsilon = 2$ and the critical temperature T_0 where bubble nucleation ends at $\epsilon = 4/3$. This choice is equivalent to the choice of Kripfganz *et al.* [18] who parametrize this range of temperatures by a variable y taking values between 0 and 1. Since Kripfganz *et al.* use a somewhat different effective potential, the relation between y and ϵ is not precise; it is essentially given by $y = 3 - 2\epsilon$ which we use as a definition of "our" y. At small y the bubbles are large with thin walls, and for $y \simeq 1$ the bubbles are small and have thick walls.

Tables I and II are split into a part (a) which contains the essential parameters for the minimal bubble and, in the last column, the nucleation rate R *without* fluctuation corrections. The part (b) contains the fluctuation corrections, i.e., the one-loop effective action. The results for $m_t = 170$ GeV are given separately for the isoscalar part of the Higgs field as S_h^{1L} and for the system of would-

TABLE I. (a) Parameters of the minimal bubbles for $m_H = 60$ GeV and $m_t = 170$ GeV. The results are given as a function of temperature in equidistant steps of the variable ϵ [Eq. (2.11)]. $\tilde{v}(T)$ is the temperature-dependent vacuum expectation value of Eq. (2.3), λ_T the temperature-dependent renormalized Φ^4 coupling. $\tilde{S}$ is the minimal bubble action (or energy divided by T). ω_-^2 is the square of the frequency of the unstable mode, given in units of $g\tilde{v}(T)^2$. The last column contains the logarithm of the nucleation rate *without* the one-loop corrections. (b) $S_{h-\text{num}}^{1L}$ is the one-loop of the isoscalar part of the Higgs field as obtained in the numerical analysis. S_h^{1L} is the total Higgs part of the one-loop effective action, obtained from $S_{h-\text{num}}^{1L}$ by adding the tadpole contribution $S_{h-\text{tad}}^{1L}$. $S_{g-\text{num}}^{1L}$ is the one-loop gauge and would-be Goldstone field action obtained by the numerical analysis; S_g^{1L} is again obtained by including the tadpole contribution. The next colum gives the Φ^3 term as included into the high-temperature action. ΔS_g^{1L} is the gauge field action after subtraction of this Φ^3 contribution. $\Delta S_{\text{eff}}^{1L}$ is the total effective action after removing the Φ^3 contribution.

T [GeV]	ϵ	y	$\tilde{v}$ [GeV]	$10^2\lambda_T$ (a)	$\tilde{S}$	$10^3\omega_-^2$	$\ln(R[\text{GeV}^4])$
94.557	1.933	0.1	48.82	3.309	1114.2	-0.1947	-1098.7
94.529	1.866	0.2	50.53	3.310	278.47	-0.7911	-264.21
94.495	1.800	0.3	52.37	3.311	121.40	-1.819	-107.83
94.455	1.733	0.4	54.33	3.312	65.218	-3.338	-52.13
94.405	1.663	0.5	56.47	3.314	37.875	-5.438	-25.20
94.347	1.600	0.6	58.72	3.316	22.686	-7.814	-10.45
94.276	1.533	0.7	61.19	3.319	12.891	-9.760	-1.223
94.191	1.466	0.8	63.85	3.322	6.3659	-9.928	4.423
94.089	1.400	0.9	66.75	3.325	2.1002	-7.007	7.028
y	$S_{h-\text{num}}^{1L}$	S_h^{1L}	$S_{g-\text{num}}^{1L}$	S_g^{1L} (b)	Φ^3	$\Delta S_{\text{gauge}}^{1L}$	$\Delta S_{\text{eff}}^{1L}$
0.1	-389.2	505.0	-85974	-102494	-104089	1595	2100
0.2	-81.21	133.6	-8457	-9952	-10170	218	351.6
0.3	-28.77	64.25	-1984	-2285.5	-2338.3	52.83	117.1
0.4	-11.78	28.30	-663.6	-739.33	-757.91	18.59	46.89
0.5	-4.41	28.85	-263.44	-277.53	-287.56	10.03	38.88
0.6	-7.54	23.04	-118.53	-113.26	-121.51	8.25	31.29
0.7	1.48	19.64	-54.93	-42.63	-51.13	8.50	28.14
0.8	3.11	17.71	-24.85	-10.04	-19.51	9.47	27.18
0.9	4.79	16.96	-10.04	5.53	-5.14	10.67	27.63

TABLE II. (a) The same as Table I(a) for $m_H = m_W = 80.2$ GeV and $m_t = 170$ GeV. (b) The same as Table I(b) for $m_H = m_W = 80.2$ GeV and $m_t = 170$ GeV.

(a)

T [GeV]	ϵ	y	$\tilde{v}$ [GeV]	$10^2\lambda_T$	$\tilde{S}$	$10^3\omega_-^2$	$\ln(R[\text{GeV}^4])$
115.725	1.933	0.1	39.75	4.973	601.83	-0.294	-587.87
115.702	1.866	0.2	41.16	4.974	151.04	-1.190	-138.32
115.675	1.800	0.3	42.67	4.975	65.944	-2.733	-53.91
115.642	1.733	0.4	44.29	4.976	35.314	-5.030	-23.76
115.602	1.663	0.5	46.05	4.977	20.560	-8.173	- 9.42
115.555	1.600	0.6	47.91	4.978	12.329	-11.73	- 1.61
115.498	1.533	0.7	49.96	4.980	7.066	-14.65	3.09
115.428	1.466	0.8	52.19	4.982	3.450	-14.88	5.82
115.345	1.400	0.9	54.49	4.984	1.145	-10.51	6.64

(b)

y	$S^{1L}_{h-\text{num}}$	S^{1L}_h	$S^{1L}_{g-\text{num}}$	S^{1L}_g	Φ^3	$\Delta S^{1L}_{\text{gauge}}$	$\Delta S^{1L}_{\text{eff}}$
0.1	-389.27	501.83	-44642	-54007	-56066	2060	2561
0.2	-81.76	132.93	-4463.4	-5236.8	-5513.5	276.7	409.6
0.3	-29.40	63.66	-1067.5	-1194.4	-1270.6	76.21	139.9
0.4	-12.33	39.39	-362.64	-377.30	-409.36	32.06	71.45
0.5	-5.01	28.22	-148.57	-137.31	-155.97	18.66	46.88
0.6	-1.36	22.42	-69.30	-52.00	-66.02	14.04	36.46
0.7	0.87	19.03	-33.64	-15.43	-27.78	12.35	31.38
0.8	2.51	17.09	-16.35	1.26	-10.56	11.82	28.91
0.9	4.20	16.36	-7.70	8.91	- 2.80	11.71	28.07

be Goldstone fields and gauge fields ("gauge field contribution" for short) as S^{1L}_g, respectively. We also give separately the parts which were obtained by the numerical analysis described in Sec. IV, denoted as $S^{1L}_{h\text{-num}}$ and $S^{1L}_{g\text{-num}}$, respectively. The difference between S^{1L}_h and $S^{1L}_{h\text{-num}}$ is the tadpole contibution $S^{1L}_{h\text{-tad}}$ of Eq. (4.9), and analogously for the gauge field. Note that the tadpole contributions to the Higgs field are substantial. The gauge field contribution S^{1L}_g contains the Φ^3 part discussed in the previous section. The numerical value of this term is given in the column labeled "Φ^3." This term should be close to the gauge field contribution, and indeed it is. So the basic action used for computing the bubble profiles represents a reasonable approximation to the exact one-loop effective action. The gauge field contribution has to be reduced by this term since it would be double counted otherwise. The net gauge field contribution is denoted as ΔS^{1L}_g and given in the last column. The correction to the rate can be simply obtained as a factor $\exp(-\Delta S^{1L}_{\text{eff}})$ where $\Delta S^{1L}_{\text{eff}} = S^{1L}_h + \Delta S^{1L}_g$. The dimension energy^3, here in units of $g\tilde{v}$, is already included in the minimal bubble rate R. One sees that the fluctuations lead to a substantial suppression of the nucleation rate.

While the result that the effective action for the gauge fields is well approximated by the effective potential, i.e.,

TABLE III. Comparison of the Higgs field effective action with approximate results by Kripfganz *et al.* for $m_H = 60$ GeV amd $m_t = 140$ GeV. The first entries are as defined in the previous tables. The quantity A is the Higgs part of the fluctuation determinant with removed unstable mode. Our results are compared to the one of Ref. [18], marked with the subscript KLS.

T [GeV]	ϵ	$\tilde{S}$	$10^3\omega_-^2$	$\tilde{v}$ [GeV]	S^{1L}_{Higgs}	$\ln\left(\frac{A}{T^4}\right)$	$\ln\left(\frac{A}{T^4}\right)_{\text{KLS}}$
97.311	1.933	1336.1	-0.1686	56.88	505.2	-513.40	-115.
97.268	1.866	336.57	-0.6965	58.87	134.2	-141.56	-82.0
97.216	1.800	146.77	-1.601	61.00	64.60	-71.40	-50.6
97.153	1.733	78.407	-2.955	63.30	40.20	-46.54	-36.7
97.078	1.663	45.935	-4.777	65.75	29.13	-35.06	-29.5
96.988	1.600	27.934	-6.891	68.38	23.23	-28.83	-25.4
96.879	1.533	15.554	-8.613	71.24	19.83	-25.15	-23.0
96.748	1.466	7.6707	-8.758	74.34	17.89	-23.01	-21.8
96.590	1.400	2.5103	-6.155	77.70	17.18	-22.30	-22.0

TABLE IV. The same as Table III for $m_H = 80.2$ GeV and $m_t = 140$ GeV.

T	ϵ	$\tilde{S}$	$10^3\omega_-^2$	$\tilde{v}$ [GeV]	$S^{1L}_{\rm Higgs}$	$\ln\left(\frac{A}{T^4}\right)$	$\ln\left(\frac{A}{T^4}\right)_{\rm KLS}$
123.815	1.933	598.02	-0.296	43.12	490.0	-500.0	-89.
123.783	1.866	153.96	-1.175	44.63	132.83	-142.0	-81.4
123.744	1.800	67.333	-2.695	46.26	63.712	-72.32	-52.0
123.697	1.733	36.144	-4.949	48.01	39.485	-47.61	-38.3
123.640	1.666	21.072	-8.035	49.92	28.306	-36.06	-31.0
123.572	1.600	12.563	-11.58	51.95	22.428	-29.84	-26.8
123.490	1.533	7.1533	-14.44	54.16	19.047	-26.18	-24.4
123.391	1.466	3.5336	-14.67	56.57	17.118	-24.06	-23.1
123.270	1.400	1.1142	-10.25	59.22	16.363	-23.30	-23.3

the Φ^3 term, is very rewarding a less comfortable feature appears if one compares the one-loop effective action with the tree level action $\tilde{S}$. If the saddle point approximation which forms the basis of transition rate formula (2.12) is justified, then the one-loop action should be smaller than the tree-level one. This is not the case. Large one-loop corrections were found already by Kripfganz *et al.* [18] when computing the one-loop effective action for the Higgs field fluctuation only. We compare our results to theirs in Tables III and IV. Since these authors define the fluctuation determinant differently—they remove the unstable mode—we give, besides our result $S^{1L}_{\rm Higgs}$, the expression $\ln(A/T^4)$ where A is the square root $\mathcal{J}^{-1/2}$ of the fluctuation determinant with translation *and* unstable modes removed. The results are close to each other for small ϵ or $y \simeq 1$, i.e., for small thick-wall bubbles. For small y, i.e., for large thin-wall bubbles, our exact results are systematically larger than the approximate ones of Ref. [18]. The question of finding reliable analytic estimates is certainly an interesting one, especially the order in which the terms of the heat kernel expansion are summed. In [30] it was found that a summation by the number of derivatives ("derivative expansion") yields very precise results if the mass of the fluctuation is much larger than the inverse size of the background field configuration. In Ref. [18] the terms are summed with respect to powers of the heat kernel time. The deviation at small y could be due to the fact that large bubbles with thin walls have a very substantial derivative contribution. On the other hand we differ already by the choice of the effective potentials used to compute the bubble profiles; one would therefore expect differences in the one-loop corrections that go even with the volume of the bubbles which becomes large at small y. It will be interesting to make a more systematic analysis of various analytical approaches. The comparison of the Higgs effective action with the leading minimal bubble action is less favorable than found in Ref. [18]. This is even more the case if the gauge loops are included, as one sees from the previous Tables I and II.

In conclusion we state three essential features of our results.

The one-loop effective action is substantial, of the order of and larger than the leading-order minimal bubble action. This sheds some doubt on the applicability of the semiclassical transition rate theory.

The sign of the one-loop effective action is such that the transition rate is suppressed.

The one-loop "Φ^3" contribution which has been incorporated into the basic effective potential is reproduced rather well by the one-loop action. This means that this term in the effective *potential* describes relevant features of the effective *action*.

It will be interesting to pursue this subject further. Certainly the fact that the one-loop correction is larger than the classical action is troublesome. But it means essentially that we have not minimized the correct action when computing the bubble profile. Especially the Φ^3 term is not the exact one loop effective potential. For small y, i.e., in the thin-wall limit, this results in one-loop corrections that go as the volume while the leading-order bubble action goes only as the surface, the profile being a saddle point of that action. So the effective potential has to be improved and used to adjust the profile and the critical temperature. We think that the optimal way to handle these problems is to undertake a self-consistent extremalization of the sum of classical and one-loop actions. Furthermore, in order to be more realistic the effect of the fermion determinant should be studied and included.

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