Constraints from inflation on scalar-tensor gravity theories

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We show how observations of the perturbation spectra produced during inflation may be used to constrain the parameters of general scalar-tensor theories of gravity, which include both an inflaton and dilaton field. An interesting feature of these models is the possibility that the curvature perturbations on superhorizon scales may not be constant due to nonadiabatic perturbations of the two fields. Within a given model, the tilt and relative amplitude of the scalar and tensor perturbation spectra gives constraints on the parameters of the gravity theory, which may be comparable with those from primordial nucleosynthesis and post-Newtonian experiments.

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I. INTRODUCTION

The most convincing explanation for the flatness, isotropy, and homogeneity of the observed universe is the inflationary scenario [1]. Moreover, the most compelling evidence for this model is the prediction of a nearly scale-invariant distribution of Gaussian perturbations. If these are indeed the origin of the perturbations observed in the microwave background sky, and of the initial inhomogeneities from which galaxies formed, then they could provide our earliest glimpse of the physics of the early universe and, in particular, of the effective theory of gravity at that time.

In this paper we consider the possible constraints that can be placed upon the allowed gravity theory during inflation. Precision tests of gravity in the solar system severely constrain the effective gravity theory today [2], while predictions from primordial nucleosynthesis have been used to restrict scalar-tensor deviations from general relativity since the radiation-dominated era [3]. Our aim is to push back those limits to a still earlier epoch, i.e., inflation.

We will do this within the context of general scalar-tensor gravity theories, which involve a massless dilaton or Brans-Dicke field [4,5]. These provide a well-defined class of theories against which to test the predictions of general relativity. Almost all attempts to produce a renormalizable quantum theory of gravity seem to include scalar fields nonminimally coupled to the spacetime curvature. The low-energy effective action of string theory, for instance, involves a dilaton field coupled to the Ricci curvature [6]. Scalar fields coupled directly to the curvature tensor appear in all dimensionally reduced gravity theories, and their influence on cosmological models was first seriously considered by Jordan [7]. Gravity Lagrangians including terms of higher order in the Ricci scalar can also be cast as scalar-tensor theories [8,9] with appropriate scalar potentials.

Recently, Damour and Nordtvedt [10] have pointed out that during a dust-dominated cosmological era, scalar-tensor theories of gravity tend to approach general relativity at late times even in the absence of a potential for the Brans-Dicke field. They realized that this would occur during most of the history of the universe, when the trace of the energy-momentum tensor drives the scalar field. However, this mechanism is not effective during the radiation dominated era [11]. The approach to general relativity can be parametrized by the Brans-Dicke parameter ω, which determines the ratio of the tensor to scalar couplings to matter, and tends to infinity in the general relativistic limit. Considering a wide class of theories where ω is some arbitrary function of the gravitational coupling, Damour and Nordtvedt [10] calculate how far toward the general relativistic limit the universe might be expected to evolve (i.e., how large ω becomes) after the radiation era. They find that it should be simply related to the number of e-foldings, $N = \ln(a_0/a_{\rm eq})$ since matter-radiation equality.

Assuming most of the expansion of the universe to date occurred during the inflationary era, one expects the same approach to general relativity to occur during inflation [12]. This has been recently discussed in the context of string theory by Damour and Vilenkin [13]. Thus, apart from setting the scene for the conventional hot big bang, by producing a spatially flat, isotropic, homogeneous (but slightly perturbed) metric, it is tempting to suggest that inflation may also produce general relativity as the low-energy effective theory of gravity, even if it started as a generic scalar-tensor theory at the Planck scale [14].

We show in this paper that this can indeed occur. We give the generalization to scalar-tensor gravity of the inflationary slow-roll parameters in general relativity and show that the vanishing of these parameters corresponds to the usual post-Newtonian limit of Einstein gravity. We calculate the scalar and tensor perturbations produced during inflation in a general scalar-tensor theory and evaluate the relative amplitude and slope of the spectra in terms of the slow-roll parameters. While it is difficult to obtain model-independent bounds on the scalar-tensor parameters, we will study a particular model of chaotic inflation for which strong bounds can be given that are comparable with the post-Newtonian and nucleosynthesis limits.

II. SCALAR–TENSOR GRAVITY THEORIES

The scalar-tensor field equations are derived from the action [5]

$$S = \int d^4x\sqrt{-\bar{g}}[f(\phi)\bar{R} - \tfrac{1}{2}\bar{g}^{ab}\phi_{,a}\phi_{,b} - U(\phi) + \bar{\mathcal{L}}_{\rm matter}] , \qquad (2.1)$$

where $\bar{R}$ is the usual Ricci curvature scalar and $16\pi f(\phi)$ is the Brans-Dicke field. Thus the gravitational coupling strength (Newton's constant G in general relativity) is determined here by the dynamical variable $f(\phi)$. In the particular case of Brans-Dicke gravity, $f(\phi) = \phi^2/8\omega$, where ω is the Brans-Dicke parameter, and we recover general relativity in the limit $\omega \to \infty$. More general scalar-tensor theories with different choices of $f(\phi)$ correspond to the case where $\omega(f)$ is a function of f.

The potential $U(\phi)$ is the generalization of the cosmological constant Λ in general relativity. Perturbation spectra produced in models of inflation driven by a potential for the Brans-Dicke field have recently been discussed by Kaiser [15], while the bounds on the allowed mass have been discussed by Damour and Vilenkin [13] and Steinhardt and Will [16]. Such a potential is often introduced to fix the value of Brans-Dicke field at late times, however we shall show that this is not necessary in order to attain general relativity as a cosmological attractor. In what follows we will leave $f(\phi)$ as a free function but consider only models in which $U(\phi)$ is zero. They correspond to flat directions in the scalar potential of string effective theories [17]. In the absence of nonperturbative effects, the dilaton remains massless. We will work in this approximation and will discuss possible extensions in future work.

Matter is minimally coupled to the metric $\bar{g}_{ab}$ and thus test particles follow geodesics in this frame, which we refer to as the Jordan frame. However it is often useful to write the action in terms of the conformally related Einstein metric [18]

$$g_{ab} \equiv e^{-2a(\psi)}\,\bar{g}_{ab} , \qquad (2.2)$$

where the conformal factor is defined as $e^{-2a} \equiv 2\kappa^2 f(\phi)$, in terms of which the action in Eq. (2.1) takes the Einstein-Hilbert form with a fixed gravitational constant $G \equiv \kappa^2/8\pi$, and the Brans-Dicke field can be written as a scalar field ψ with a canonical kinetic term in the new matter Lagrangian

$$\mathcal{L}_{\rm matter} = -\tfrac{1}{2}g^{ab}\psi_{,a}\psi_{,b} + e^{4a(\psi)}\bar{\mathcal{L}}_{\rm matter} . \qquad (2.3)$$

There are now explicit interactions between this field and the original matter fields whose energy-momentum tensors are therefore not necessarily conserved with respect to g_{ab} [18,19]. We will define a dimensionless parameter [10], $\alpha(\psi) \equiv \kappa^{-1}(da/d\psi)$, which characterizes the scalar-tensor theory: α^2 specifies the ratio of the dilaton and graviton couplings to matter. A given choice of function $a(\psi)$, or equivalently $f(\phi)$, determines $\alpha(\psi)$. In particular, it is related to the Brans-Dicke parameter ω by

$$2\alpha^2 = \frac{1}{2\omega+3} . \qquad (2.4)$$

For a linear $a(\psi) = \alpha\kappa\psi$ with constant α, we recover Brans-Dicke theory.[1]

Present-day observational tests constrain the post-Newtonian parameters γ and β [2], written in terms of α and $\alpha'(\psi) \equiv \kappa^{-1}(d\alpha/d\psi)$ as

$$\gamma = 1 - \frac{4\alpha^2}{1+2\alpha^2} , \qquad \beta = 1 + \frac{2\alpha^2\alpha'(\psi)}{1+2\alpha^2} , \qquad (2.5)$$

which are constrained by present-day solar system tests to be [2] $|\gamma - 1| < 0.002$ and $|4\beta - \gamma - 3| < 0.001$ [22]. Therefore,

$$\alpha^2 < 5\times 10^{-4} , \qquad 4\alpha^2|1+2\alpha'| < 10^{-3} . \qquad (2.6)$$

There are similar constraints from primordial nucleosynthesis [3]. These constrain possible variations of the Planck mass during and after the radiation dominated era. Our aim is to go beyond the radiation era and try to constrain possible deviations from general relativity during inflation.

III. SCALAR-TENSOR INFLATION

In this section we will analyze the classical evolution of the scalar fields during inflation. The inflaton field, σ, minimally coupled in the Jordan frame, with a self-interaction potential $V(\sigma)$ gives an explicit matter Lagrangian to consider in a scalar-tensor cosmology. The total matter Lagrangian in the Einstein frame including the Brans-Dicke field is then

$$\mathcal{L}_{\rm matter} = -\tfrac{1}{2}g^{ab}\psi_{,a}\psi_{,b} - \tfrac{1}{2}e^{2a(\psi)}\,g^{ab}\sigma_{,a}\sigma_{,b} - e^{4a(\psi)}\,V(\sigma) . \qquad (3.1)$$

We see by inspecting the potential term in the Lagrangian that σ will evolve toward a minimum of $V(\sigma)$ while ψ evolves toward a minimum of $a(\psi)$ or, equivalently, a zero of $\alpha(\psi)$. However, from Eq. (2.4) we see that a zero of $\alpha(\psi)$ requires that $\omega \to \infty$. That is, general relativity will generically be an attractor during the cosmological evolution, if $a(\psi)$ possesses a minimum.

The field equations for the fields σ and ψ in a spatially flat Friedmann-Robertson-Walker metric are then

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[1] Our notation coincides with that of Damour-Gibbons-Gundlach [20] and Starobinsky-Yokoyama [21] for constant $-2\alpha = \gamma = \beta/2$. Note that Damour and Nordtvedt [10] define a dimensionless field $\varphi \equiv \kappa\psi/\sqrt{2}$, and thus their parameter α differs from ours by a factor $\sqrt{2}$.

$$\ddot\sigma + 3H\dot\sigma = -\,2\alpha\kappa\,\dot\psi\,\dot\sigma - e^{2a}\,V'(\sigma)\ , \tag{3.2}$$

$$\ddot\psi + 3H\dot\psi = -\,\alpha\kappa\Big(e^{4a}\,4V - e^{2a}\,\dot\sigma^2\Big)\ , \tag{3.3}$$

$$\dot H = -\,\frac{\kappa^2}{2}\left(\dot\psi^2 + e^{2a}\,\dot\sigma^2\right)\ , \tag{3.4}$$

and the Hamiltonian constraint

$$H^2 = \frac{\kappa^2}{6}\left(\dot\psi^2 + e^{2a}\,\dot\sigma^2 + e^{4a}\,2V\right)\ . \tag{3.5}$$

The condition for inflation to occur in the Einstein frame $|\dot H| < H^2$ is thus, see Eqs. (3.4) and (3.5),

$$\dot\psi^2 + e^{2a}\,\dot\sigma^2 < e^{4a}\,V(\sigma)\ . \tag{3.6}$$

A. Slow-roll inflation

We will work in the slow-roll approximation in both scalar fields. In principle this is not a necessary constraint: one of the fields might roll quickly to the minimum of its potential and then the problem reduces to single field inflation, either the familiar chaotic inflation in general relativity (for $\dot\psi = 0$) or old extended inflation in Brans-Dicke (for $\dot\sigma = 0$). However, we would like to consider the more general case in which both fields slow roll [23–25]. In this case, the general field equations can be written as first-order equations:

$$3H^2 \simeq \kappa^2 e^{4a}\,V(\sigma)\ , \tag{3.7}$$

$$3H\dot\sigma \simeq -\,e^{2a}\,V'(\sigma)\ , \tag{3.8}$$

$$3H\dot\psi \simeq -\,4\alpha\kappa\,e^{4a}\,V(\sigma)\ . \tag{3.9}$$

Neglecting the other terms in the equations of motion amounts to the assumptions

$$\max\left\{\dot\psi^2,\ \alpha^2\dot\psi^2,\ e^{2a}\,\dot\sigma^2\right\} \ll e^{4a}\,V(\sigma)\ , \tag{3.10}$$

$$|\ddot\sigma| \ll |H\dot\sigma| \qquad \text{and} \qquad |\ddot\psi| \ll |H\dot\psi|\ . \tag{3.11}$$

Having written down first-order equations for the evolution of the fields we can turn the slow-roll assumptions based on values of the fields' derivatives into consistency equations in terms of the parameters of the theory:

$$\epsilon_\sigma \equiv \frac{1}{2\kappa^2}\left(\frac{V'(\sigma)}{V(\sigma)}\right)^2\ ,\quad \eta_\sigma \equiv \frac{1}{\kappa^2}\,\frac{V''(\sigma)}{V(\sigma)}\ , \tag{3.12}$$

$$\epsilon_\psi \equiv 8\alpha^2(\psi)\ ,\quad \eta_\psi \equiv 4\alpha'(\psi) - 16\alpha^2(\psi)\ . \tag{3.13}$$

The consistency equations for slow-roll inflation are then $\left\{e^{-2a}\,\epsilon_\sigma,\ e^{-2a}\,|\eta_\sigma|,\ \epsilon_\psi,\ |\eta_\psi|\right\} \ll 1$. The first two conditions are just the expected generalization to scalar-tensor gravity of the slow-roll conditions for an inflaton field in general relativity. Notice, however, that if $\dot a = \kappa\alpha\dot\psi \le 0$ during the subsequent evolution of the universe (i.e., $a > 0$), the conditions on ϵ_σ and η_σ are relaxed compared to the general relativistic case, where $a(\psi) = 0$ throughout. The last two parameters are not present in general relativity and arise here by requiring that both the Brans-Dicke and the inflaton field slow roll.

We have defined the slow-roll parameters ϵ and η by extending the usual definition of these parameters for a single field [26] to the separable potential for the two fields $U(\sigma,\psi) = V(\sigma)\,e^{4a(\psi)}$. It is intriguing to note that the limit of vanishing slow-roll parameters for the ψ field coincides with the general relativistic weak-field limit in the post-Newtonian parametrization of the scalar-tensor gravity theory [2], see Eq. (2.5).

In calculating the rate of change of quantities with respect to different comoving scales, it is useful to write down the relation between the number of e-foldings from the end of inflation and the values of the scalar fields:

$$N = -\int_{t_e} H dt \ = \frac{\kappa}{4}\int_{\psi_e}\frac{d\psi}{\alpha(\psi)}$$

$$= \kappa^2\int_{\sigma_e}\frac{V(\sigma)}{e^{2a}V'(\sigma)}\,d\sigma\ . \tag{3.14}$$

Our present horizon crossed outside the Hubble scale about 50-60 e-foldings before the end of inflation. In fact, the precise number depends logarithmically on the energy scale during inflation and the efficiency of reheating, and so is weakly model dependent.

IV. DENSITY PERTURBATIONS

Inflation is the only known mechanism that solves the horizon and homogeneity problems. However, the main observational constraint on inflationary models is the spectrum of density perturbations that they produce. Strictly speaking, observations of perturbations in the microwave background, or of the large-scale structure in our patch of the universe, only provides an upper limit on the level of density perturbations, which could perhaps be produced by some other source of inhomogeneities. Nonetheless, the apparently Gaussian and nearly scale-invariant nature of the perturbations are natural properties of perturbations due to quantum fluctuations of the inflaton field during inflation.

In the case of a single slow-rolling field, only adiabatic perturbations are possible. Any fluctuation in the field must produce a fluctuation in the local curvature. However, in the presence of two coupled fields we must also consider the effect of isocurvature (or entropy) perturbations between the two fields. In particular this can lead to the breakdown of the usual assumption that the curvature perturbation is frozen-in on scales greater than the Hubble length ($k_{\rm ph} < H$). It is important to allow for such effects if we hope to constrain the possible role of a variable gravitational coupling.

Our calculations extend those of Starobinsky and Yokoyama [21] who considered the particular case of inflation in Brans-Dicke gravity. As we shall see, their results are readily extended to more general scalar-tensor theories.

A. Perturbed field equations

In this section we will consider the linear perturbations around the homogeneous background fields, $\sigma(t)+\delta\sigma(t,x^i)$, $\psi(t)+\delta\psi(t,x^i)$, with a perturbed metric in the longitudinal gauge

$$ds^2 = -\left[1+2\Phi(t,x^i)\right]dt^2 + R^2(t)\left[1-2\Phi(t,x^i)\right]\delta_{ij}dx^i dx^j \ . \tag{4.1}$$

We can study the evolution of each Fourier mode (whose physical wave numbers we denote by $k_{\rm ph}$) separately, since they decouple in the linear approximation. The perturbed field equations then yield the following expressions to first order:

$$\ddot{\delta\sigma} + 3H\dot{\delta\sigma} + k_{\rm ph}^2\delta\sigma + e^{2a}V''(\sigma)\delta\sigma = 4\dot\sigma\dot\Phi + -2e^{2a}V'(\sigma)\Phi - 2\alpha\kappa\left(\dot\sigma\dot{\delta\psi}+\dot\psi\dot{\delta\sigma}\right) -2\alpha'(\psi)\kappa^2\dot\sigma\dot\psi\,\delta\psi - 2\alpha\kappa\, e^{2a}V'(\sigma)\delta\psi \ , \tag{4.2}$$

$$\ddot{\delta\psi} + 3H\dot{\delta\psi} + k_{\rm ph}^2\delta\psi + \alpha'(\psi)\kappa^2\left(e^{4a}4V - e^{2a}\dot\sigma^2\right)\delta\psi + 2\alpha^2\kappa^2\left(e^{4a}8V - e^{2a}\dot\sigma^2\right)\delta\psi = 4\dot\psi\dot\Phi - 8\alpha\kappa e^{4a}V\Phi - 2\alpha\kappa\left[e^{4a}2V'(\sigma)\delta\sigma - e^{2a}\dot\sigma\dot{\delta\sigma}\right] , \tag{4.3}$$

$$\ddot\Phi + 4H\dot\Phi + \left(\dot H + 3H^2\right)\Phi = \frac{\kappa^2}{2}\left[\dot\psi\dot{\delta\psi} + e^{2a}\dot\sigma\dot{\delta\sigma} - e^{4a}V'(\sigma)\delta\sigma + \left(e^{2a}\dot\sigma^2 - e^{4a}4V\right)\alpha\kappa\,\delta\psi\right] , \tag{4.4}$$

together with the energy and momentum constraints

$$\dot\Phi + H\Phi = \frac{\kappa^2}{2}\left(\dot\psi\delta\psi + e^{2a}\dot\sigma\delta\sigma\right) , \tag{4.5}$$

$$3H\dot\Phi + \left(\dot H + 3H^2\right)\Phi + k_{\rm ph}^2\Phi = -\frac{\kappa^2}{2}\left[\dot\psi\dot{\delta\psi} + e^{2a}\dot\sigma\dot{\delta\sigma} + e^{4a}V'(\sigma)\delta\sigma + \left(e^{2a}\dot\sigma^2 + e^{4a}4V\right)\alpha\kappa\,\delta\psi\right] . \tag{4.6}$$

A very useful quantity for the study of perturbation spectra is the curvature perturbation on hypersurfaces of fixed energy density [27,28,31]:

$$\zeta \equiv -\frac{H^2}{\dot H}\left(\Phi + H^{-1}\dot\Phi\right) + \Phi \ , \tag{4.7}$$

on scales much larger than the Hubble length. Combining Eqs. (4.4)–(4.6), and using the equations of motion, we find an exact expression for the time variation of ζ:

$$\dot\zeta = k_{\rm ph}^2\frac{H}{\dot H}\Phi - H\left[\frac{d}{dt}\left(\frac{e^{2a}\dot\sigma^2 - \dot\psi^2}{e^{2a}\dot\sigma^2+\dot\psi^2}\right) + \dot C\right] \times\left(\frac{\delta\psi}{\dot\psi} - \frac{\delta\sigma}{\dot\sigma}\right) , \tag{4.8}$$

where $\dot C = 2\alpha\kappa\dot\psi e^{4a}\dot\sigma^4/(e^{2a}\dot\sigma^2+\dot\psi^2)^2$ is due to the frictional damping of the σ field by ψ.

In the single field case (where one of the fields is held fixed) the right-hand side of Eq. (4.8) vanishes in the limit $k_{\rm ph}\to 0$, and thus ζ remains constant on scales exceeding the Hubble length [28]. This allows one to determine the large-scale curvature perturbation at the end of inflation simply by equating it with the perturbation when the mode first crossed outside the Hubble scale. However this is true in general only for adiabatic perturbations and need no longer hold in the presence of two fields [29,21].

This is due to the entropy perturbation [30]

$$\tau\delta S = \dot H\left[\frac{d}{dt}\left(\frac{e^{2a}\dot\sigma^2-\dot\psi^2}{e^{2a}\dot\sigma^2+\dot\psi^2}\right) + \dot C\right]\left(\frac{\delta\psi}{\dot\psi} - \frac{\delta\sigma}{\dot\sigma}\right) . \tag{4.9}$$

The first term in the square brackets will be present whenever two fields are evolving but the second term, $\dot C$, would not be present if both fields had standard kinetic terms. It is clear that for adiabatic modes $\delta\psi/\dot\psi = \delta\sigma/\dot\sigma$ (perturbations along the classical trajectory) ζ remains constant on super-Hubble scales, but in general the curvature perturbation at horizon crossing cannot be equated with that at the end of inflation, due to the nonadiabatic terms.

Note that all the above perturbed equations are exact and we have not yet invoked the slow-roll approximation. In the following we shall solve for the evolution of ζ to lowest order in the slow-roll parameters.

We will assume that the curvature perturbation will be conserved on super-Hubble scales through reheating and the subsequent radiation and matter dominated eras. Recently, it has been emphasized [31] that the curvature perturbation ζ is conserved at transitions in the equation of state across a boundary at a fixed energy density. This is the case when perturbations are adiabatic and the end of inflation must coincide with a particular energy density. However this is not necessarily the case in the presence of two fields. As we can see from Eq. (4.8), our assumption that only adiabatic perturbations exist at the end of inflation requires that the motion of one of the fields dominates. This will usually be the case, especially when inflation ends because one of the fields' kinetic energy becomes comparable to the potential energy. The slow-roll parameter ϵ for one of the fields becomes of order unity while the other remains small. It is possible that both slow-roll parameters become large at the same point, but this seems to be unlikely *a priori.*

The perturbation at the end of inflation can then be equated with that at reentry if its subsequent evolution remains adiabatic. Any variation of the Brans-Dicke field after the end of inflation could invalidate this assumption. Nucleosynthesis limits [3] suggest this does not occur at temperatures below about an MeV and in the absence of a potential for the Brans-Dicke field, $\dot{\psi} = 0$ is a stable solution during a radiation dominated era for arbitrary scalar-tensor gravity theories [11]. However this could be altered by the presence of an explicit potential term and the consequences would require careful investigation.

B. Short-wavelength limit

For large values of $k_{\rm ph} \gg H$ we can neglect the potential terms in the perturbed field equations (4.3) and (4.2) and they reduce to those for massless fields (i.e., $e^{-2a}|\eta_\psi|,\ |\eta_\sigma| \ll 1$). Thus, to lowest order in the slow-roll parameters, the expectation values of the perturbations as they cross outside the Hubble radius ($k_{\rm ph} \simeq H_*$) are given by Gaussian random variables with $\langle|\delta\sigma_*|^2\rangle = e^{-2a_*}H_*^2/2k^3$, $\langle|\delta\psi_*|^2\rangle = H_*^2/2k^3$, where k is the comoving wave number. Note that, while the field ψ is minimally coupled in the Einstein frame, the σ field is minimally coupled in the Jordan frame and therefore it is the conformally transformed Hubble constant (to lowest order) that determines its amplitude at Hubble crossing [32].

We shall denote the spectrum of a quantity A by $\mathcal{P}_A(k) \equiv \frac{4\pi k^3}{(2\pi)^3}\langle|A|^2\rangle$, as defined in [33]. Thus we have

$$\mathcal{P}_{\delta\sigma} \simeq e^{-2a_*}\left(\frac{H_*}{2\pi}\right)^2 , \tag{4.10}$$

$$\mathcal{P}_{\delta\psi} \simeq \left(\frac{H_*}{2\pi}\right)^2 . \tag{4.11}$$

C. Long-wavelength limit

For slowly varying ($\dot{\Phi} \ll H\Phi$), long-wavelength ($k_{\rm ph} \to 0$) modes, to lowest order in the slow-roll parameters, Eqs. (4.5), (4.2), and (4.3) reduce to

$$\Phi \simeq -\,2\alpha\,\kappa\,\delta\psi - \frac{1}{2}\frac{V'(\sigma)}{V}\,\delta\sigma , \tag{4.12}$$

$$3H\dot{\delta\psi} \simeq 4\alpha'(\psi)\,\kappa^2\,e^{4a}\,V\,\delta\psi , \tag{4.13}$$

$$3H\dot{\delta\sigma} \simeq -\,e^{2a}\left(\frac{V'(\sigma)}{V}\right)'V\,\delta\sigma + 2\alpha\,\kappa\,e^{2a}\,V'(\sigma)\,\delta\psi . \tag{4.14}$$

Note that for constant α we recover Starobinsky and Yokoyama's results [21].

Using Eqs. (3.8) and (3.9), the last two equations can be integrated to give the evolution of fluctuations in the scalar fields at long wavelengths:

$$\delta\psi \simeq -\frac{4\alpha}{\kappa}Q_1 , \tag{4.15}$$

$$\delta\sigma \simeq \frac{1}{\kappa^2}\frac{V'(\sigma)}{V}\left(Q_2 - e^{-2a}Q_1\right) , \tag{4.16}$$

and thus

$$\begin{aligned}\Phi &\simeq 8\alpha^2 Q_1 - \frac{1}{2\kappa^2}\left(\frac{V'(\sigma)}{V}\right)^2\left(Q_2 - e^{-2a}Q_1\right)\\ &= \epsilon_\psi Q_1 - \epsilon_\sigma\left(Q_2 - e^{-2a}Q_1\right) ,\end{aligned} \tag{4.17}$$

where Q_1 and Q_2 are constants of integration, chosen to coincide with those introduced by Starobinsky and Yokoyama [21]. It will be convenient to define a new constant $Q_3 \equiv Q_1 e^{-2a_*} - Q_2$, so that Q_1 and Q_3 are independent Gaussian random variables whose values, for a given Fourier mode, are determined by the amplitude of $\delta\sigma_*$ and $\delta\psi_*$ at horizon crossing (when $k_{\rm ph} = H_*$). Thus they have expectation values

$$\mathcal{P}_{Q_1} = \frac{e^{4a_*}\,\kappa^4\,V_*}{24\pi^2\,\epsilon_\psi^*} , \tag{4.18}$$

$$\mathcal{P}_{Q_3} = \frac{e^{2a_*}\,\kappa^4\,V_*}{24\pi^2\,\epsilon_\sigma^*} . \tag{4.19}$$

From Eqs. (4.5) and (4.7) we have, during slow roll [23,24], in the long wavelength limit,

$$\zeta \simeq H\,\frac{\dot{\psi}\delta\psi + e^{2a}\dot{\sigma}\delta\sigma}{\dot{\psi}^2 + e^{2a}\dot{\sigma}^2} . \tag{4.20}$$

As shown in Eq. (4.8) this expression need not be constant. Substituting in our results for the long-wavelength modes of the scalar fields we have

$$\zeta \simeq \frac{\left[\epsilon_\psi + (e^{-2a} - e^{-2a_*})\epsilon_\sigma\right]Q_1 + \epsilon_\sigma Q_3}{\epsilon_\psi + e^{-2a}\epsilon_\sigma} . \tag{4.21}$$

If either of the scalar fields is fixed (ϵ_σ or ϵ_ψ identically zero) then we recover the single field results where ζ is constant (equal to Q_1 or Q_3, respectively).

The spectrum of density perturbations at the end of inflation $\mathcal{P}_{\zeta_e}(k)$ can be computed from (4.21):

$$\mathcal{P}_{\zeta_e} \simeq \left(\frac{\epsilon_\psi^e + (1-e^{-2a_*})\epsilon_\sigma^e}{\epsilon_\psi^e + \epsilon_\sigma^e}\right)^2 \mathcal{P}_{Q_1} + \left(\frac{\epsilon_\sigma^e}{\epsilon_\psi^e + \epsilon_\sigma^e}\right)^2 \mathcal{P}_{Q_3} \ . \tag{4.22}$$

In Sec. VI we study a particular model and give numerical results showing how and when the different terms dominate.

D. Gravitational wave perturbations

In addition to the scalar curvature perturbations that give rise to density perturbations, tensor or gravitational wave perturbations can also be generated from quantum fluctuations during inflation [34]. Since we have chosen to work in the Einstein conformal frame we can use the standard results for the evolution of tensor perturbations of the metric. The two independent polarizations evolve like minimally coupled massless fields with a spectrum [30,33]:

$$\mathcal{P}_g = \frac{2\kappa^4 e^{4a_*} V_*}{3\pi^2} \ . \tag{4.23}$$

Gravitational wave perturbations can contribute to the microwave background anisotropies only on the largest scales (scales larger than the Hubble scale at last-scattering, corresponding to about $> 1^\circ$ on the sky). Their contribution relative to scalar curvature perturbations is given by the ratio [33]

$$R \simeq \frac{3}{4}\frac{\mathcal{P}_g}{\mathcal{P}_{\zeta_e}} \ . \tag{4.24}$$

The rapid decay of the gravitational wave anisotropies on smaller scales is their most distinctive signature. In Sec. VI we will study a particular model and show how the ratio R changes with scale.

V. OBSERVATIONAL CONSTRAINTS

Having allowed for the possible evolution of the curvature perturbation ζ on super-Hubble scales during inflation, we will now restrict our analysis to those cases where ζ has become constant on observable scales by the end of inflation, i.e., entropy pertubations become negligible. This allows us to assume that ζ remains fixed on super-Hubble scales until it reenters the Hubble length at late times. We can then relate the curvature perturbation at the end of inflation to the density perturbation at reentry during the matter dominated era:

$$\delta_H^2 \simeq \tfrac{4}{25}\mathcal{P}_{\zeta_e} \ , \tag{5.1}$$

following the notation of [33].

In any model of inflation, the amplitude of the density perturbations depends upon the magnitude of the potential energy density relative to the Planck scale, which is essentially a free parameter. We will concentrate upon the variation of the amplitude of the curvature perturbations with comoving scale. At each point in the spectrum, the "tilt" is given by the spectral index n_s, where $n_s - 1 = d\ln\delta_H^2/d\ln k$. This can be evaluated within the slow-roll approximation, where the comoving wave number corresponds to a given scale at horizon crossing, $d\ln k \simeq -dN_*$, and thus, from Eq. (3.14),

$$n_s - 1 = \frac{d\ln\delta_H^2}{d\ln k} \simeq -\left(\frac{4\alpha_*}{\kappa}\frac{\partial}{\partial\psi_*} + \frac{e^{2a_*}V_*'}{\kappa^2 V_*}\frac{\partial}{\partial\sigma_*}\right)\ln\delta_H^2 \ . \tag{5.2}$$

A well-known result in general relativity, for slow-roll inflation with a single field, is that $n_s - 1 \simeq -6\epsilon_* + 2\eta_*$, a function solely of the slow-roll parameters at Hubble crossing [35,33]. Because ζ can evolve on super-Hubble scales during scalar-tensor inflation, ζ_e and thus δ_H^2 will in general depend upon parameters both at Hubble crossing and at the end of inflation. However, the latter will not change with comoving scale.

Large-angle microwave background experiments probe scales close to our observable horizon, which crossed outside the Hubble scale when $N \sim 60$ [33]. The Cosmic Background Explorer (COBE) two-year data constrains the spectral tilt to be in the range $0.7 < n < 1.7$ at the 1σ level [36], but future experiments should be able to constrain the tilt to within about 0.1 [37]. However, from observations of galaxy clustering we might hope to recover the primordial density perturbations on scales down to about 1 Mpc, which leave the Hubble scale at about $N \sim 50$. This could provide a much more precise determination of the tilt, though it is currently limited by uncertainties in other cosmological parameters such as the value of the Hubble constant or the type of dark matter. While a tilt of $0.7 < n < 0.8$ may have some advantages over $n = 1$ in an otherwise standard cold dark matter model, a tilt $n < 0.6$ is probably unacceptable [38,33].

The general expression for the tilt from Eq. (5.2) is rather complicated. We will only attempt to evaluate it in two general regions of parameter space and then specialize it to the case of an inflaton with a generic chaotic inflationary potential, for which we give numerical results.

Note that the tilt of the gravitational wave spectrum is just given by

$$n_g \simeq -16\alpha_*^2 - 2e^{2a_*}\epsilon_\sigma^* \ . \tag{5.3}$$

Unlike the approximate expressions for the scalar tilt which will be given below, this simple expression for n_g is valid in the whole range of parameter space. Moreover, since both terms on the right-hand side must be nonpositive, one could in principle give a direct constraint on α^2 completely independent of the form of the inflaton potential. However, the measurement of this slope will be exceedingly difficult. Tensor perturbations do not contribute to structure formation and in many inflationary models the observable effect of gravitational waves is completely negligible [33].

The main constraint coming from gravitational waves will be their relative amplitude, given by Eq. (4.24). If R becomes of order unity then, since independent Gaussian random variables add in quadrature, the amplitude of the scalar perturbations inferred from anisotropies of the microwave background on large scales is reduced by about 70%. As R increases, the allowed amplitude of scalar perturbations decreases, eventually becoming incompatible with structure formation. It is the combined effect of a tilted spectrum and the gravitational wave contribution that proves such a strong constraint on models of extended inflation [35].

Finally, note that if $n_s < 1$ then, on very large scales, the potential energy density relative to the Planck mass at Hubble crossing becomes so large that the amplitude of density perturbations is of order unity and the universe enters a self-reproducing regime, where the classical motion is dominated by quantum fluctuations [24,32,14]. Such inhomogeneities, even on superhorizon scales today, could be detected through the Grishchuk-Zel'dovich effect [39] unless they are on scales at least 500 times greater than our horizon [40]. This represents another constraint on any model. Sufficient inflation in the classical regime thus requires $N_{\max} > 66$.

A. Scalar-tensor extended inflation

Let us first consider the case where the Brans-Dicke field evolution dominates that of the inflaton at the end of inflation, $\epsilon_\psi^e \gg \epsilon_\sigma^e$. Then, for scales crossing outside the Hubble scale near the end of inflation, we have $\zeta_e \simeq Q_1$ and thus $\delta_H^2 \simeq (4/25)\mathcal{P}_{Q_1}$. This remains valid at scales for which $(\epsilon_\psi^e/\epsilon_\sigma^e)^2 \gg e^{-2a_*}\epsilon_\psi^*/\epsilon_\sigma^*$. It includes models of extended inflation [41] where the field σ is trapped in a metastable false vacuum so that $\epsilon_\sigma = 0$ (for any $\eta_\sigma > 0$) and where ζ remains fixed on super-Hubble scales. But this result for δ_H also includes perturbations for which $\epsilon_\sigma^* \geq \epsilon_\psi^*$, where there will be significant evolution of the curvature perturbation ζ when $k_{\rm ph} < H$ during inflation.

We then have

$$n_s - 1 \simeq -16\alpha_*^2 + 8\alpha_*' - 2e^{2a_*}\epsilon_\sigma^* \,. \tag{5.4}$$

When $\epsilon_\sigma^* = 0$ this expression generalizes the well-known result for extended inflation in Brans-Dicke models [33] to more general scalar-tensor theories. We see that, just as in general relativity, n_s need not always be less than unity. For instance, we can produce a Harrison-Zel'dovich spectrum ($n_s = 1$) by choosing a scalar-tensor theory where $\alpha' = 2\alpha^2$ corresponding to $a(\psi) = -(1/2)\ln(\psi/\psi_e)$. This is a particular realization of "intermediate inflation" [42].

More generally, as ϵ_σ is always non-negative, a lower bound on the tilt of the power spectrum then constrains the slow-roll parameters of the gravity theory, irrespective of the form of the inflaton potential.

The relative contribution of tensor and scalar perturbations to the microwave background anisotropies is given by Eq. (4.24), which in this limit yields

$$R \simeq 10^2\,\alpha_*^2 \,. \tag{5.5}$$

An upper limit on this ratio then constrains α^2 independently of α'.

B. Scalar-tensor chaotic inflation

In the opposite limit, $\epsilon_\sigma^e \gg \epsilon_\psi^e$, in which the evolution of the inflaton dominates that of the Brans-Dicke field at the end of inflation, we find $\zeta_e \simeq Q_3$ and thus $\delta_H^2 \simeq (4/25)\mathcal{P}_{Q_3}$. This result will hold for the last scales to leave the Hubble length during inflation. It remains valid on larger scales as long as

$$\left(1 - e^{-2a_*} + \frac{\epsilon_\psi^e}{\epsilon_\sigma^e}\right)^2 \ll \frac{\epsilon_\psi^*}{e^{2a_*}\epsilon_\sigma^*} \tag{5.6}$$

is satisfied, see Eq. (4.22). In these limits, the curvature perturbation is always due to fluctuations in the inflaton field σ, but there may still be evolution on super-Hubble scales due to the frictional damping by ψ. We find from Eq. (4.21) that $\zeta \simeq e^{2a}Q_3$, which coincides with the solution of $\dot\zeta \simeq \dot C\zeta$, given by Eq. (4.8) in this limit.

Given the above result for δ_H^2 we thus find that on sufficiently small scales the tilt will be given by

$$n_s - 1 \simeq e^{2a_*}\left(-6\epsilon_\sigma^* + 2\eta_\sigma^*\right) - 8\alpha_*^2 \,. \tag{5.7}$$

Note that η_σ can be positive or negative and thus could lead to a positive spectral tilt [33,43].

The larger effective gravitational constant at early times ($a_* > 0$) amplifies the tilt due to the changing shape of the inflaton potential, and its variation leads to an additional negative tilt. Any chance of constraining α_*^2 from observations of the tilt is clearly limited by uncertainty in the form of the inflaton potential. In the simplest case of chaotic inflation driven by a polynomial potential, $V(\sigma) = \lambda\sigma^{2n}/2n$, there is a simple relation between ϵ_σ, η_σ and the value of σ:

$$\epsilon_\sigma = \left(\frac{n}{2n-1}\right)\eta_\sigma = \frac{2n^2}{\kappa^2\sigma^2} \,. \tag{5.8}$$

This guarantees that $-6\epsilon_\sigma + 2\eta_\sigma$ is negative and thus the slope of the density perturbation spectrum in Eq. (5.7) is always $n_s < 1$. In arbitrary scalar-tensor theories, a lower limit on the slope then places an upper bound on α^2.

The ratio between the scalar and tensor contributions to the microwave background anisotropies reduces to the usual general relativistic case [33],

$$R \simeq 12\,e^{2a_*}\,\epsilon_\sigma^* \,. \tag{5.9}$$

When the condition given in Eq. (5.6) no longer holds and instead $(1 - e^{-2a_*}) \gg \epsilon_\psi^*/\epsilon_\sigma^*$, we find that $\zeta_e \simeq Q_1$. In this regime the results of Eqs. (5.4) and (5.5) apply. It is interesting to note that the naive calculation based on taking $\zeta \simeq \zeta_*$ does in fact give the correct result and, even in our careful analysis, ζ remains constant on super-Hubble scales. This is clearly seen in the Fig. 1, where we show the evolution of ζ after Hubble crossing in the

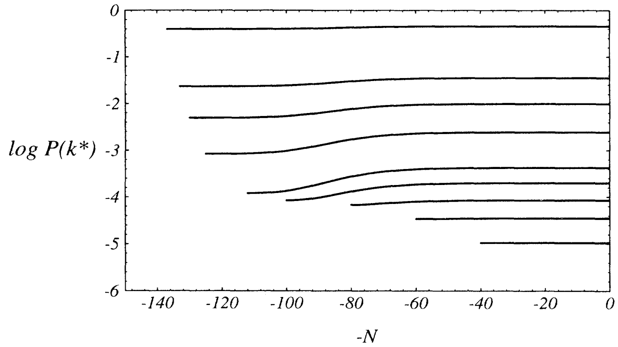

FIG. 1. The evolution of the amplitude of density perturbations after Hubble crossing for a range of comoving scales, k_*, as a function of the number N of e-folds from end of inflation, for the model described in Sec. VI. The logarithm is to base 10.

specific model of chaotic inflation discussed in the next section.

This occurs despite the fact that at Hubble crossing the curvature perturbation is due to the ψ field, $\zeta \simeq H\delta\psi/\dot{\psi}$, while by the end of inflation it appears as a perturbation in the σ field, $\zeta \simeq H\delta\sigma/\dot{\sigma}$. This is a consequence of the coupling between the two fields and the dependence of $\delta\sigma$ upon the evolution of $\delta\psi$ seen in Eq. (4.14). We do not expect this result to hold in general for two fields in general relativity. Moreover, for intermediate scales ζ does evolve on super-Hubble scales.

VI. NUMERICAL RESULTS

In this section we will try to show the main features discussed above with a particular example that includes both regimes. We will choose the arbitrary origin of the field ψ so that $\psi_e = 0$ at the end of inflation (when $a_e = 0$) and assume that $\alpha(\psi)$ can be approximated as a linear function during the latter stages of inflation (when observable scales cross outside the horizon); i.e., we take a Taylor expansion for $a(\psi)$ up to second order, as was done in [3]:

$$a(\psi) = a_1\,\kappa\psi + \frac{a_2}{2}\,(\kappa\psi)^2 \ , \qquad (6.1)$$

$$\alpha(\psi) = a_1 + a_2\,\kappa\psi \ , \qquad (6.2)$$

where a_2 and a_1 are constants and ψ is then given in terms of the number of e-foldings, using Eq. (3.14), as

$$\kappa\psi \simeq \frac{a_1}{a_2}\left[\exp\left(4a_2\,N\right) - 1\right] \ . \qquad (6.3)$$

Therefore we have

$$\alpha \simeq a_1 \exp\left(4a_2\,N\right) \ , \qquad (6.4)$$

and

$$e^{-2a} = 2\kappa^2 f(\phi) \simeq \exp\left[-\,\frac{a_1^2}{a_2}\,\left[\exp(8a_2\,N) - 1\right]\right] \ . \qquad (6.5)$$

Note that at the end of inflation ($N = 0$) we have the present value of the gravitational coupling, and our parameters a_1 and a_2 correspond to α and α', respectively, at the end of inflation.

For a simple inflaton potential $V(\sigma) = \lambda\sigma^4/4$, the slow-roll solution for σ, satisfying $\epsilon_\sigma^e = 1$, is

$$\kappa^2\sigma^2 \simeq 8 + \frac{\exp(a_1^2/a_2)}{a_2}\,\left[E_1(\alpha^2/a_2) - E_1(a_1^2/a_2)\right] \ , \qquad (6.6)$$

where $E_1(z)$ is the exponential integral function [44].

It will be convenient to define some new variables

$$x = \kappa\sigma, \quad y = \exp[-2\,a(\psi)] \ , \qquad (6.7)$$

whose classical evolution in the slow-roll approximation is shown in Fig. 2 for $a_2 = 10^{-2}$ and $a_1 = 10^{-2}$ and $a_1 = 10^{-3}$. Note that in both cases y, and thus the Planck mass, becomes essentially constant by the end of inflation.

If we also introduce $w \equiv 1 - y + 8a_1^2$, then in the limit $w_*^2 \ll y_* x_*^2 \alpha_*^2$, where all starred quantities are to be evaluated at $N = N_*$, we satisfy the condition given in Eq. (5.6) and we find $\zeta_e \simeq Q_3$. This must hold for the last modes to leave the Hubble scale at the end of inflation, as $y \to 1$ and $x \to 2\sqrt{2}$, for $8a_1^2 \ll 1$. For larger scales, when $w_*^2 \gg y_* x_*^2 \alpha_*^2$, we find $\zeta_e \simeq Q_1$. This is clearly demonstrated in Figs. 3(a) and 3(b).

The spectral tilt (5.2) at a scale which crosses outside the Hubble length when $N = N_*$, is given, at lowest order in the slow-roll parameters, by

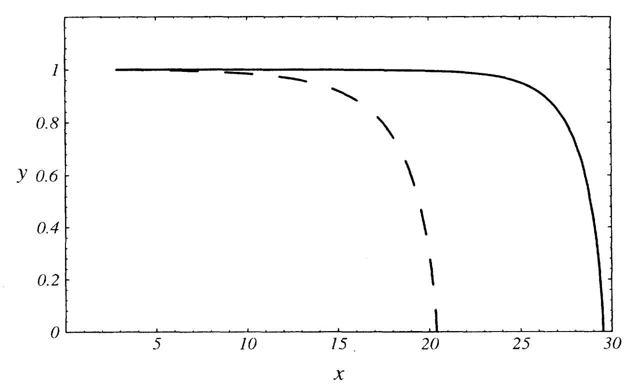

FIG. 2. Classical trajectory in the space of fields (x, y) defined in Eq. (6.7). The solid line corresponds to parameters $(a_1 = 10^{-3}, a_2 = 10^{-2})$, while the dashed line corresponds to $(a_1 = 10^{-2}, a_2 = 10^{-2})$.

$$n_s - 1 \simeq -8 \left[\frac{\left[2\alpha_*^2 - a_2 + (2/y_* x_*^2)\right] w_*^2 + 2\alpha_*^2 y_* w_* + \alpha_*^2 \left(3 + y_* x_*^2 \alpha_*^2\right)}{w_*^2 + y_* x_*^2 \alpha_*^2} \right] , \tag{6.8}$$

see Fig. 4. In Fig. 5 we have also computed the ratio of gravitational to scalar components:

$$R \simeq \frac{96\alpha_*^2(8a_1^2+1)^2}{w_*^2 + y_* x_*^2 \alpha_*^2} . \tag{6.9}$$

We thus recover the results of Eqs. (5.7) and (5.9) for $n_s - 1$ and R, respectively, on small scales, and on larger scales by Eqs. (5.4) and (5.5). The variation of both the tilt and the ratio R with Hubble crossing epoch, N_*, is shown in Figs. 4 and 5.

Observational constraints on R and the tilt of the perturbation spectrum can bound the values of the parameters a_1 and a_2, thus constraining deviations from general relativity as far back as N_* e-folds before the end of inflation. Figure 6 is a contour plot showing n_s for scales that left the horizon at $N_* = 60$, corresponding roughly to 6000 Mpc today, and thus the sort of scale constrained by observations of large-scale structure. We see that both a_1 and a_2 must be very small in order for the tilt of the spectrum to remain close to the general relativistic value of $n_s \simeq 0.95$. This reflects the need to keep the Planck mass essentially constant to avoid large departures from the Harrison-Zel'dovich ($n_s = 1$) spectrum. Figure 7 shows similar results for the contribution of gravitational wave perturbations at the same scale, $N_* = 60$.

VII. CONCLUSIONS

In this paper we have considered the constraints that may be placed upon the effective theory of gravity during a period of inflation in the early universe. We do this in the context of scalar-tensor theories, taking the coupling of the Brans-Dicke field to matter as an arbitrary function $\alpha(\psi)$, and neglecting any explicit potential for the dilaton field.

Present-day observational limits on the variation of the Brans-Dicke field are expressed as bounds on the post-Newtonian parameters of the theory. We have shown that the general relativistic limit of these parameters coincides with the vanishing of the corresponding slow-roll parameters for the Brans-Dicke field during inflation. Slow-roll inflation already requires the scalar-tensor theory to be close to the general relativistic limit. The observed spec-

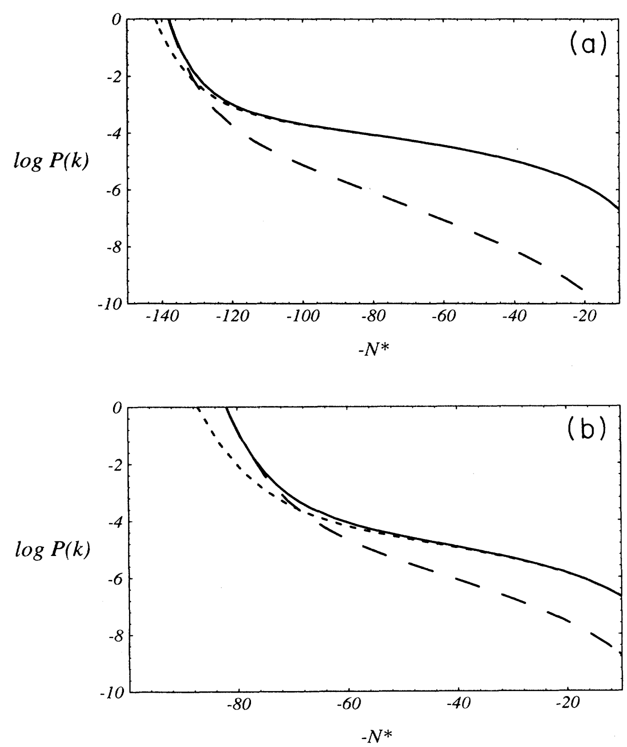

FIG. 3. (a) The solid line shows the spectrum of density perturbations at the end of inflation, $\mathcal{P}_{\zeta_e}(k)$, as a function of the number N_* of e-folds before the end of inflation, when the corresponding scale left the horizon, for parameters ($a_1 = 10^{-3}, a_2 = 10^{-2}$). The dashed line corresponds to $\mathcal{P}_{Q_1}(k)$ and the dotted line to $\mathcal{P}_{Q_3}(k)$. The logarithm is to base 10. (b) Same as in Fig. 2(a), but for parameters ($a_1 = 10^{-2}, a_2 = 10^{-2}$).

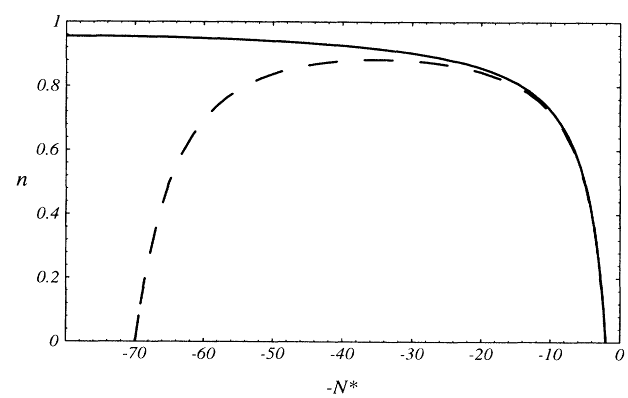

FIG. 4. The tilt $n(k)$ of the spectrum of density perturbations, as a function of the number N_* of e-folds before the end of inflation, when the corresponding scale left the horizon. The solid line corresponds to parameters ($a_1 = 10^{-3}, a_2 = 10^{-2}$), while the dashed line corresponds to ($a_1 = 10^{-2}, a_2 = 10^{-2}$).

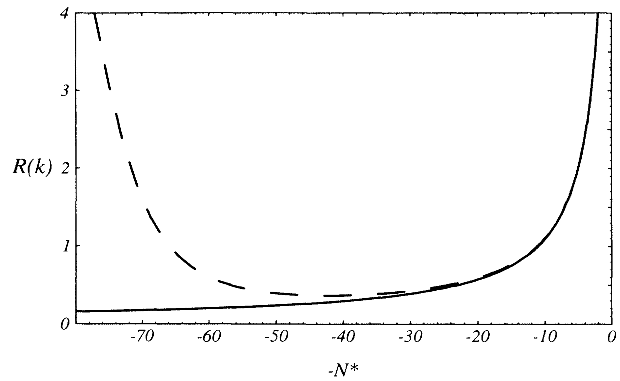

FIG. 5. The ratio $R(k)$ of tensor (gravitational waves) to scalar (density) perturbations, as a function of the number N_* of e-folds before the end of inflation, when the corresponding scale left the horizon. The solid line corresponds to parameters $(a_1 = 10^{-3}, a_2 = 10^{-2})$, while the dashed line corresponds to $(a_1 = 10^{-2}, a_2 = 10^{-2})$.

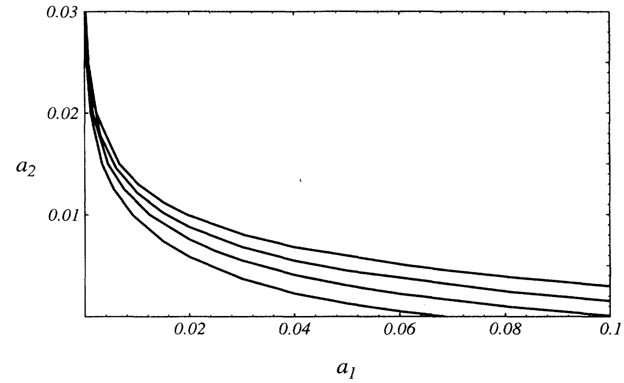

FIG. 7. Contour plot for the ratio R of tensor to scalar perturbations in the (a_1, a_2) parameter-space. The region below the curves is the allowed region for $R < 4, 2, 1$, and 0.5, from top to bottom. (a_1, a_2) corresponds to (α, α') at the end of inflation.

trum of density perturbations produced from quantum fluctuations in the inflaton and Brans-Dicke fields can then constrain just how large the deviation may be.

A careful calculation of the curvature perturbation ζ during inflation shows that some of the results, applicable to single-field inflation in general relativity, no longer apply. Due to the evolution of the two fields during inflation, there will be nonadiabatic perturbations, which can lead to the evolution of ζ on scales larger than the Hubble length. Therefore, the amplitude of ζ at reentry can no longer be equated with that at the time the scale left the horizon. This is a general feature of inflation with two fields. However, in scalar-tensor theories there are regimes for which ζ does remain constant outside the Hubble scale. In models where inflation ends as the inflaton field rolls to the minimum of its potential, we find two regimes for which ζ at the end of inflation is equal to that at Hubble crossing. It is only in an intermediate regime that the naive calculation breaks down.

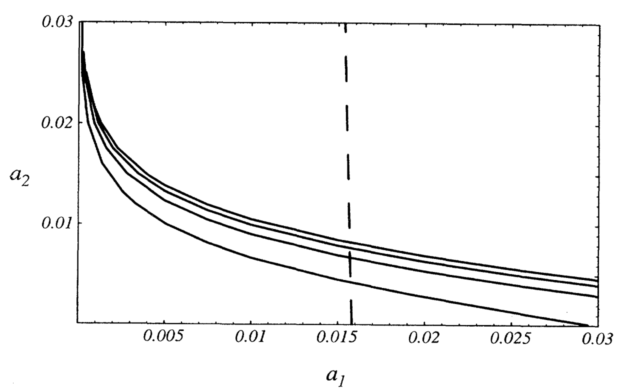

FIG. 6. Contour plot for the tilt n_s of the spectrum of density perturbations in the (a_1, a_2) parameter space. The region below the curves is the allowed region for $n > 0.6, 0.7, 0.8$, and 0.9, from top to bottom. (a_1, a_2) corresponds to (α, α') at the end of inflation. For comparison, the dashed line corresponds to the post-Newtonian bounds on (α, α') in Eq. (2.6).

We give expressions for the spectral slopes, n_s and n_g, and relative amplitude, R, of the scalar and tensor perturbations produced, in terms of the slow-roll parameters of a general scalar-tensor theory of gravity. Observational bounds then place constraints on these parameters. A possible signature of scalar-tensor inflation is the breakdown of the consistency relations predicted in single-field inflation [45].

It is important to emphasize that our ability to make quantitative predictions relies on our knowledge of the inflaton potential. In our specific example of a chaotic inflation model, we find that if we constrain the slope of the spectrum to be $n_s > 0.6$ we obtain bounds on α and α' at the end of inflation that are comparable with those from nucleosynthesis or solar-system tests. For example, for $\alpha < 0.015$ we require $\alpha' < 0.01$, which is much stronger than the corresponding post-Newtonian bound.

Future observations will be able to constrain n_s to within 0.1 [37] which would further improve the bounds on (α,α'), in the context of a given inflaton potential. Although far harder to measure, the tilt of the tensor perturbations, n_g, gives a model-independent bound on α.

Note added in proof. The issue of density perturbations from multiple field inflation has also been addressed in Refs. [46,47].

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[1] A.D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood, Chur, Switzerland, 1990).
[2] C.M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University Press, Cambridge, England, 1993).
[3] D. Kalligas, K. Nordtvedt, and R.V. Wagoner, "Nucleosynthesis constraints on scalar-tensor theories of gravitation," Stanford University Report No., SU-ITP-95/7, 1995 (unpublished).
[4] C. Brans and R.H. Dicke, Phys. Rev. **124**, 925 (1961).
[5] P.G. Bergmann, Int. J. Theor. Phys. **1**, 25 (1968); R.V. Wagoner, Phys. Rev. D **1**, 3209 (1970); K. Nordtvedt, Astrophys. J. **161**, 1059 (1970).
[6] E.S. Fradkin and A.A. Tseytlin, Nucl. Phys. **B261**, 1 (1985); C.G. Callan, D. Friedan, E.J. Martinec, and M.J. Perry, *ibid.* **B262**, 593 (1985).
[7] P. Jordan, Z. Phys. **157**, 112 (1959).
[8] P. Teyssandier and P. Tourrenc, J. Math. Phys. **24**, 2793 (1983).
[9] D. Wands, Class. Quantum Grav. **11**, 269 (1993).
[10] T. Damour and K. Nordtvedt, Phys. Rev. Lett. **70**, 2217 (1993); Phys. Rev. D **48**, 3436 (1993).
[11] J. D. Barrow, Phys. Rev. D **47**, 5329 (1993); J. P. Mimoso and D. Wands, *ibid.* **51**, 477 (1995).
[12] P.J. Steinhardt and F.S. Accetta, Phys. Rev. Lett. **64**, 2470 (1990); J.D. Barrow and K. Maeda, Nucl. Phys. **B341**, 294 (1990); J. García-Bellido and M. Quirós, Phys. Lett. B **243**, 45 (1990).
[13] T. Damour and A. Vilenkin, "String theory and inflation," Report No. IHES-P-95-26, hep-th/9503149, 1995 (unpublished).
[14] J. García-Bellido and D. Wands, Phys. Rev. D **52**, 5636 (1995).
[15] D. I. Kaiser, Phys. Rev. D **52**, 4295 (1995).
[16] P. J. Steinhardt and C. M. Will, Phys. Rev. D **52**, 628 (1995).
[17] J.A. Casas, J. García-Bellido, and M. Quirós, Nucl. Phys. **B361**, 713 (1991); J. García-Bellido and M. Quirós, *ibid.* **B368**, 463 (1992).
[18] R.H. Dicke, Phys. Rev. **125**, 2163 (1962).
[19] J.A. Casas, J. García-Bellido, and M. Quirós, Class. Quantum Grav. **9**, 1371 (1992); L.J. Garay and J. García-Bellido, Nucl. Phys. **B400**, 416 (1993).
[20] T. Damour, G. Gibbons, and C. Gundlach, Phys. Rev. Lett. **64**, 123 (1990); Phys. Rev. D **43**, 3873 (1991).
[21] A. A. Starobinsky and J. Yokoyama, "Density fluctuations in Brans-Dicke inflation," Report No. astro-ph/9502002, 1995 (unpublished).
[22] K. Nordtvedt, in *General Relativity*, Proceedings of the Seventh Marcel Grossmann Meeting on General Relativity, Stanford, California, 1994, edited by M. Keiser and R. Kallosh (World Scientifc, Singapore, 1995).
[23] S. Mollerach and S. Matarrese, Phys. Rev. D **45**, 1961 (1992).
[24] J. García-Bellido, A.D. Linde, and D.A. Linde, Phys. Rev. D **50**, 730 (1994).
[25] J. D. Barrow, Phys. Rev. D **51**, 2729 (1995).
[26] A. R. Liddle, P. Parsons, and J. D. Barrow, Phys. Rev. D **50**, 7222 (1994).
[27] J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, Phys. Rev. D **28**, 679 (1983).
[28] D. H. Lyth, Phys. Rev. D **31**, 1792 (1985).
[29] N. Deruelle, C. Gundlach, and D. Langlois, Phys. Rev. D **46**, 5337 (1992).
[30] V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger, Phys. Rep. **215**, 203 (1992).
[31] N. Deruelle and V. Mukhanov, Phys. Rev. D **52**, 5549 (1995).
[32] J. García-Bellido, Nucl. Phys. **B423**, 221 (1994).
[33] A. R. Liddle and D. H. Lyth, Phys. Rep. **231**, 1 (1993).
[34] L.F. Abbott and M.B. Wise, Nucl. Phys. **B244**, 541 (1984).
[35] A. R. Liddle and D. H. Lyth, Phys. Lett. B **291**, 391 (1992).
[36] K. M. Górski *et al.*, Astrophys. J. Lett. **430**, L89 (1994).
[37] J. R. Bond, R. Crittenden, R. L. Davis, G. Efstathiou, and P. J. Steinhardt, Phys. Rev. Lett. **72**, 13 (1994); L. Knox, "Determination of inflationary observables by CMB anisotropy experiments," Report No. astro-ph/9504054, 1995 (unpublished).
[38] N. Vittorio, S. Matarrese, and F. Lucchin, Astrophys. J. **328**, 69 (1988); A. R. Liddle, D. H. Lyth, and W. Sutherland, Phys. Lett. B **279**, 244 (1992); R. Cen, N. Y. Gnedin, L. A. Kofman, and J. P. Ostriker, Astrophys. J. Lett. **399**, L11 (1992); J. A. Peacock and S. J. Dodds, Mon. Not. R. Astron. Soc. **267**, 1020 (1994).
[39] L. Grischuk and Ya. B. Zel'dovich, Sov. Astron. **22**, 125 (1978).
[40] M. S. Turner, Phys. Rev. D **44**, 3737 (1991); L. Grischuk, *ibid.* **45**, 4717 (1992).
[41] D. La and P.J. Steinhardt, Phys. Rev. Lett. **62**, 376 (1989); C. Mathiazhagan and V. B. Johri, Class. Quantum Grav. **1**, L29 (1984).
[42] J. D. Barrow and A. R. Liddle, Phys. Rev. D **47**, R5219 (1993).
[43] A. D. Linde, Phys. Rev. D **49**, 748 (1994); E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart, and D. Wands, *ibid.* **49**, 6410 (1994); S. Mollerach, S. Matarrese, and F. Lucchin, *ibid.* **50**, 4835 (1994).
[44] M. Abramowitz and I.A. Stegun, *Mathematical Functions* (Dover, New York, 1972).
[45] E. J. Copeland, E. W. Kolb, A. R. Liddle, and J. E. Lidsey, Phys. Rev. D **48**, 2529 (1993); M. S. Turner, *ibid.* **48**, 5539 (1993).
[46] J.-C. Hwang, Astrophys. J. **375**, 443 (1991).
[47] D. S. Salopek, Phys. Rev. D **52**, 5563 (1995); M. Sasaki and E. D. Stewart, "A general analytic formula for the spectral index of the density perturbations produced during inflation," Report No. astroph/9507001, 1995 (unpublished).