Possible implications of the atmospheric, the Bugey, and the Los Alamos neutrino experiments

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A combined analysis of the terrestrial neutrino experiments and the Kamiokande observation of the atmospheric neutrino anomaly is performed under the assumption of the existence of dark-matter-mass neutrinos, as suggested by the recent Los Alamos experiment. In the three-flavor mixing scheme of neutrinos it is shown that the constraints from these experiments are so strong that the patterns of mass hierarchy and flavor mixing of neutrinos are determined almost uniquely depending upon the interpretation of the atmospheric neutrino anomaly.

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There has been indirect evidences accumulating for nonvanishing masses and flavor mixings of neutrinos. The evidence includes, the solar neutrino deficit [1] which may be interpreted by either the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism [2] or vacuum neutrino oscillation [3], both being based upon the notion of flavor mixing. The second piece of the evidence in the list is the atmospheric neutrino anomaly first observed by the Kamiokande experiment [4] and subsequently confirmed by other detectors [5,6], which strongly indicates the large-angle flavor mixing of neutrinos.

The recent announcement of the discovery of a nonzero neutrino mass by the liquid scintillator neutrino detector (LSND) experiment [7,8] at Los Alamos may have brought the first direct evidence for neutrino masses and flavor mixing. The experiment may have observed the neutrino oscillation $\bar{\nu_\mu} \to \bar{\nu_e}$ with oscillation parameters $\Delta m^2 \simeq 0.2$–100 eV^2 and $\sin^2 2\theta \simeq 10^{-3} - 4 \times 10^{-2}$, if interpreted by the two-flavor mixing scheme. The result may be marginally compatible with the earlier results obtained by the Los Alamos [9] and the BNL experiments [10] and by the KARMEN Collaboration experiment [11]. Clearly the result, if confirmed by the continuing runs, has tremendous implications for particle physics and cosmology [12].

In this paper we try to extract the implications of the possible existence of the dark-matter-mass neutrinos, as suggested by the LSND result, in the light of the experimental information from underground, reactor, and accelerator experiments. We first observe, as many authors do [12], that one cannot explain the above three phenomena simultaneously by the three-flavor mixing scheme without introducing sterile neutrinos. This is simply due to the fact that the three-flavor scheme cannot accommodate three hierarchically different mass scales, $\Delta m^2 \simeq 0.2$–100 eV^2 for LSND, $\Delta m^2 \simeq 10^{-2}$ eV^2 for the atmospheric neutrino anomaly, and $\Delta m^2 \simeq 10^{-6}$–$10^{-5}$ eV^2 ($\simeq 10^{-10}$ eV^2) for the MSW (vacuum mixing) solution of the solar neutrino problem.

We derive the constraints imposed on neutrino masses and mixing angles via a combined analysis of reactor and the accelerator data and the atmospheric neutrino anomaly under the assumption that at least one of the neutrinos has a mass which falls into the mass range 1–10 eV which is appropriate for the hot component of cosmological dark matter. This assumption will be referred to as the assumption of dark-matter-mass neutrinos (DMMN's) hereafter. While this assumption itself generically allows the mass pattern of three almost degenerate neutrino states (to ~ 0.1 eV for our purpose) we do not consider this option in this paper because it cannot accommodate the possible oscillation events in the LSND experiment.

In this paper we take the following attitude for the present status of the LSND experiment. Since the experiment is still in the preliminary stage we do not consider the rate reported in [8] as the final value. In particular, the rate seems to depend strongly on how the fiducial volume is cut [13]. A better understanding of the background would be required to really determine the rate of oscillation events. Therefore, we interpret the present data as indicating interesting candidate events for the channel $\bar{\nu_\mu} \to \bar{\nu_e}$, but the oscillation probability is poorly determined. In this paper we tentatively assume that it is less than or equal to $\sim 10^{-3}$.

We employ the mixing scheme based on three-generation neutrinos, as beautifully confirmed by the experiments at the CERN e^+e^- collider LEP [14]. It will be demonstrated that it is essential to use the three-flavor mixing scheme, rather than optional use of various two-flavor mixings, for drawing correct interpretation of the data. We will also consider the restrictions imposed by the neutrinoless double β decay [15].

Amazingly, the constraints imposed by a minimal set of data, the atmospheric and the Bugey [16] experiments, and the assumption of DMMN's are so restrictive as to determine the masses and the mixing patterns of three-flavor neutrinos. Only a few patterns are allowed: (A) light "ν_e" and almost degenerate strongly mixed heavy "ν_μ" and "ν_τ," and its mass-inverted version, or (B) light "ν_τ" and almost degenerate strongly mixed heavy "ν_e" and "ν_μ," and its mass-inverted one. The choice of the solutions (A) or (B) is dictated by the interpretation of

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the atmospheric neutrino anomaly. The pattern (A) follows if we interpret the atmospheric neutrino anomaly as due to the $\nu_\mu \to \nu_\tau$ oscillation, while (B) results if it is due to the $\nu_\mu \to \nu_e$ oscillation.

We should mention that the combination with atmospheric neutrino data is preferred by our theoretical prejudice over the alternative one with the solar neutrino solution in the restricted framework of three-flavor neutrinos. It is natural to introduce sterile neutrinos to accommodate the third set of experimental data left over in both cases. It is, however, difficult to explain the atmospheric neutrino anomaly by introducing mixing with sterile neutrinos. In doing so one encounters trouble with the light-element nucleosynthesis [17]. Nonetheless, we should remark that combinations with neither atmospheric nor solar neutrino data are compelling. The analysis of the alternative combination is presented elsewhere [18].

We make use of one crucial aspect of the atmospheric neutrino data in our analysis; namely, the Kamiokande group recently provided a new data set called the multi-GeV sample [19]. They consist of the events with higher energy, $\gtrsim 1.33$ GeV, than the previously reported data. The important feature of the new data is that, because of the higher energy, the path-length dependence of the oscillation probability can be probed by measuring the zenith-angle dependence. It is striking that it can be perfectly fitted by neutrino oscillation with mixing parameters $\Delta m^2 \simeq 10^{-2}$ eV^2 and $\sin^2 2\theta \simeq 1$ [19]. Such quantitative agreement with the zenith-angle dependence is the strongest support for the neutrino oscillation interpretation of the atmospheric neutrino anomaly.

We classify the hierarchy of the neutrino masses into the following two types:

$$\text{(a)}\ \ m_3^2 \approx m_2^2 \gg m_1^2; \qquad \text{(b)}\ \ m_1^2 \gg m_2^2 \approx m_3^2. \tag{1}$$

Here the symbols $\approx$ and $\gg$ imply differences of $\sim 10^{-2}$ eV^2 and $\sim$ 1–100 eV^2, respectively. Throughout the analysis in this paper the relative magnitude of the masses connected by $\approx$ does not matter. The economy and convenience of treating the quite different mass patterns (a) and (b) on the same footing stems from the characteristic feature of the neutrino oscillation phenomenon that it does not distinguish between normal and inverted mass hierarchies. The patterns (a) and (b) can be distinguished when we address the constraint from double β decay [15]. The other types of mass hierarchies which are obtained by permuting 1, 2, and 3 will automatically be taken care of because they merely represent relabeling the mass eigenstates.

We derive the approximate formulas by taking into account the mass hierarchies and the experimental parameters of the three experiments. To this end we introduce the neutrino mixing matrix U which relates the flavor and the mass eigenstates as $\nu_\alpha = U_{\alpha i}\nu_i$, where the flavor index α runs over e, μ, and τ and the mass eigenstate index i runs over 1 – 3. We assume the CP invariance in the present analysis. As a convenient parametrization of the matrix U we use the standard form of the Kobayashi-Maskawa matrix advocated in [14], which is now adopted for the neutrino mixing matrix. We use c_{ij} and s_{ij} as shorthand notations for $\cos\theta_{ij}$ and $\sin\theta_{ij}$, respectively. We note that the three real angles can all be made to lie in the first quadrant by an appropriate redefinition of neutrino phases.

We write down the oscillation probability of neutrinos of energy E after traversing the distance L with use of the notation $\Delta m_{ij}^2 = |m_i^2 - m_j^2|$. The oscillation probability which corresponds to the LSND experiment is approximately given by

$$\begin{aligned} P(\bar{\nu_\mu} \to \bar{\nu_e}) &= 4c_{12}^2 c_{13}^2 (s_{12}c_{23} + c_{12}s_{23}s_{13})^2 \\ &\quad \times \sin^2\left(\frac{\Delta m_{12}^2}{4E}L\right), \end{aligned} \tag{2}$$

where two terms with $\Delta m_{12}^2 \approx \Delta m_{13}^2$ (which differ only by 10^{-2} eV^2) are combined and the term with Δm_{23}^2 is ignored. The former procedure can be neatly done by utilizing the orthogonality relation of the mixing matrix. The latter approximation is completely legitimate because the term is smaller than others by a factor of 10^{-4}–10^{-8} owing to the mass hierarchy $\Delta m_{23}^2/\Delta m_{12}^2 \simeq 10^{-2}$–$10^{-4}$.

If the atmospheric neutrino anomaly is attributed to the $\nu_\mu \to \nu_\tau$ oscillation the relevant formula is

$$\begin{aligned} P(\nu_\mu \to \nu_\tau) &= 2(s_{12}c_{23} + c_{12}s_{23}s_{13})^2 (s_{12}s_{23} - c_{12}c_{23}s_{13})^2 \\ &\quad + 4c_{23}s_{23}c_{13}^2 (c_{12}c_{23} - s_{12}s_{23}s_{13}) \\ &\quad \times (c_{12}s_{23} + s_{12}c_{23}s_{13}) \sin^2\left(\frac{\Delta m_{23}^2 L}{4E}\right). \end{aligned} \tag{3}$$

In (3) the sine-squared factors with large Δm^2 of $\gtrsim 1$ eV^2 are replaced by the average value $\frac{1}{2}$, which can be justified because of the rapid oscillations; the argument of the sine is $\sim$ 10–10^3 (10^4–10^6) for $L = 10$ (10^4) km for $\Delta m^2 = 1 - 100$ eV^2 and $E = 1$ GeV.

We note that there exists the possibility that the atmospheric neutrino anomaly is due to the $\nu_\mu \to \nu_e$ oscillation, a possibility one might not naively expect. It is perfectly consistent with a small event rate in the LSND experiment because the relevant scales of path length and neutrino energy involved in these two experiments are much different. In this case the formula to be used for the oscillation probability is

$$\begin{aligned} P(\nu_\mu \to \nu_e) &= 2c_{12}^2 c_{13}^2 (s_{12}c_{23} + c_{12}s_{23}s_{13})^2 \\ &\quad - 4s_{12}s_{23}c_{13}^2 s_{13} (c_{12}c_{23} - s_{12}s_{23}s_{13}) \\ &\quad \times \sin^2\left(\frac{\Delta m_{23}^2 L}{4E}\right). \end{aligned} \tag{4}$$

Finally the formula for the Bugey experiment takes the form

$$\begin{aligned} 1 - P(\bar{\nu_e} \to \bar{\nu_e}) &= 2c_{12}^2 c_{13}^2 (1 - c_{12}^2 c_{13}^2) \\ &\quad + 4s_{12}^2 c_{13}^2 s_{13}^2 \sin^2\left(\frac{\Delta m_{23}^2 L}{4E}\right), \end{aligned} \tag{5}$$

where the terms with Δm_{12}^2 are averaged as before. It can be justified because the argument of the sine term is of the order of 10–10^3 with $\Delta m^2 =$ 1–100 eV^2, $E = 4$ MeV, and $L = 40$ m, the typical parameters of the Bugey experiment. The second term of (5) may be neglected (as we will do) because the sine-squared factor is $\sim 10^{-2}$ for $\Delta m^2 = 10^{-2}$ eV^2.

We first examine the case that the atmospheric neutrino anomaly is attributed to the $\nu_\mu \to \nu_\tau$ oscillations. As mentioned before our discussion does not distinguish the types (a) and (b) until we address the constraint due to the double β decay.

We demand, for consistency with the gross features of the LSND, the Bugey, and the atmospheric neutrino experiments, the following constraints:

$$c_{12}^2 c_{13}^2 (s_{12}c_{23} + c_{12}s_{23}s_{13})^2 \equiv \epsilon \lesssim 10^{-3}, \tag{6}$$

$$c_{12}^2 c_{13}^2 (s_{12}^2 c_{13}^2 + s_{13}^2) \lesssim \delta = 2.5 \times 10^{-2}, \tag{7}$$

$$(s_{12}c_{23} + c_{12}s_{23}s_{13})^2 (s_{12}s_{23} - c_{12}c_{23}s_{13})^2 \leq 0.1, \tag{8}$$

$$4c_{23}s_{23}c_{13}^2 (c_{12}c_{23} - s_{12}s_{23}s_{13})(c_{12}s_{23} + s_{12}c_{23}s_{13}) \simeq 1. \tag{9}$$

The constraint (6) comes from the LSND experiment. As we have pointed out earlier the rate of oscillation events seems to depend strongly on the fiducial volume cut [13]. We therefore treat ϵ as a small number of less than $\sim 10^{-3}$. We only use the number as a tentative guide when we address the consistency with other experiments. The fate of the solution (B) will be somewhat sensitive to ϵ if it is greater than $\sim 10^{-4}$, but otherwise all the results are insensitive to the value of ϵ.

The equation (7) is due to the bound $1 - P(\bar{\nu}_e \to \bar{\nu}_e) \lesssim 5\%$ obtained in the Bugey experiment [16]. It includes statistical and systematic uncertainties. The remaining two restrictions are from the feature of the atmospheric neutrino data that the zenith-angle dependence is well described by an effective two-flavor-mixing ansatz with $\Delta m^2 \simeq 10^{-2}$ eV^2 and $\sin^2 2\theta \simeq 1$. The constraint (8) arises from a mild requirement that the first term of (3) should be less than 0.2 so as not to disturb the effective two-flavor description. We emphasize that the constraints from the atmospheric neutrino data take the simple forms (8) and (9) because of the mass hierarchy $\Delta m_{12}^2 \approx \Delta m_{13}^2 \gg \Delta m_{23}^2$.

We first notice that from the Bugey constraint (7) $X \equiv c_{12}^2 c_{13}^2$ must satisfy the inequality $X^2 - X + \delta \geq 0$. This inequality is so powerful that it restricts the value of X into the two tiny regions $0 \leq X \leq \delta$ and $1 - \delta \leq X \leq 1$. On the other hand, we must have $c_{13}^2 \simeq 1$ in order to satisfy the requirement (9). Thus we have either $c_{12}^2 \simeq \delta$ or $c_{12}^2 \simeq 1$ corresponding to the small-X and the large-X solutions, respectively. It is also required that $c_{23}s_{23} \simeq \frac{1}{2}$ in order to maximize (9). The small-X solution is then inconsistent with (9). We end up with the unique solution

$$\text{(A)} \qquad s_{12}^2 \lesssim \epsilon, \qquad s_{13}^2 \lesssim \epsilon, \qquad c_{23}^2 \approx s_{23}^2 \simeq \frac{1}{2}, \tag{10}$$

where we have also utilized the LSND constraint (6) to push $s_{12}^2 \simeq \delta$ down to $s_{12}^2 \simeq \epsilon \lesssim 10^{-3}$.

We have explicitly verified that the allowed mixing pattern implied by (10) is physically unique throughout the varying mass hierarchies obtained by the cyclic permutations of 1–3 of (1), as it should be; namely, the light "ν_e" and the almost degenerate strongly mixed heavy "ν_μ" and "ν_τ" for the type (a), and the heavy "ν_e" and the almost degenerate strongly mixed light "ν_μ" and "ν_τ" for the type (b) cases.

We now turn to the case that the atmospheric neutrino anomaly is caused by the $\nu_\mu \to \nu_e$ oscillation. In this case we replace the requirements (8) and (9) by

$$c_{12}^2 c_{13}^2 (s_{12}c_{23} + c_{12}s_{23}s_{13})^2 \leq 0.1 \tag{11}$$

and

$$-4s_{12}s_{23}c_{13}^2 s_{13}(c_{12}c_{23} - s_{12}s_{23}s_{13}) \simeq 1, \tag{12}$$

respectively. By a similar procedure one can show that the consistent solution of the requirements (6), (7), (11), and (12) is uniquely given by

$$\text{(B)} \qquad c_{12}^2 \approx c_{23}^2 \simeq \sqrt{\epsilon}, \qquad c_{13}^2 \approx s_{13}^2 \simeq \frac{1}{2}. \tag{13}$$

The solutions of the other type of mass hierarchies can be obtained in a similar manner and correspond to the redefinition of the mass eigenstates. The allowed mixing pattern is again physically unique: The light "ν_τ" and the almost degenerate strongly mixed heavy "ν_e" and "ν_μ" for the type (a), and the heavy "ν_τ" and the almost degenerate strongly mixed light "ν_e" and "ν_μ" for the type (b) mass hierarchies.

We note that the solutions (A) and (B) are subject to additional constraints from other terrestrial experiments. While the solution (A) solves them automatically nontrivial constraints arise for (B). In particular, the most stringent one comes from the Fermilab E531 experiment [20] for $\Delta m^2 \gtrsim 3$ eV^2 and the ν_μ disappearance experiment by the CDHS group [21] for $\Delta m^2 \lesssim 3$ eV^2. We just translate the result obtained in [18] into the one with the different definition of angles employed in this paper:

$$c_{12}c_{23} \lesssim 0.88 \times 10^{-2} \times \begin{cases} 1 & (1\ \text{eV}^2 \leq \Delta m_{13}^2 \leq 3.3\ \text{eV}^2), \\ \left(\frac{\Delta m_{13}^2}{3.3\ \text{eV}^2}\right)^{-2} & (3.3\ \text{eV}^2 \leq \Delta m_{13}^2 \leq 10\ \text{eV}^2). \end{cases} \tag{14}$$

Thus the solution (B) appears to be consistent with the constraints from these experiments only when the LSND rate is lower by an order of magnitude. We believe that we have to bear in mind such a possibility because the background of oscillation events does not appear to be completely understood [8,13]. Also the bound (14) is the outcome of an approximate treatment done in [18]. An extensive analysis would be required to make a more precise statement as to what confidence level the solution (B) is allowed. Work in this direction is in progress [22]. It is also worth mentioning that the solution (B) can be severely tested [23] at $\Delta m^2 \gtrsim 3$ eV^2 by the ongoing CHORUS and NOMAD experiments [24].

In the case of Majorana neutrinos a further constraint emerges from the nonobservation of the neutrinoless double β decay. The quantity

$$\langle m_{\nu e} \rangle = \sum_{j=1}^{3} \eta_j |U_{ej}|^2 m_j \tag{15}$$

is constrained to be less than ~1 eV by the experiments [15] where $\eta_j = \pm 1$ is the CP phase. Notice that we are working with the representation in which the mixing matrix is real under the assumption of CP invariance.

Generally speaking, the constraint from the double β decay distinguishes between the type (a) and the type (b) mass hierarchies. In the type (a) case there is a chance for cancellation two between nearly degenerate masses, but no chance in the type (b) case because the heavy mass is carried by a unique mass eigenstate.

New features, however, arise in our consistent solutions obtained above. We first discuss the case of the atmospheric neutrino anomaly due to the $\nu_\mu \to \nu_\tau$ oscillation. It can be shown that in the type (a) mass pattern the double-β constraint is automatically satisfied because the heavy masses are always multiplied by small angle factors. On the contrary, the angle factors in front of the unique heavy mass are always of the order of unity in the type (b) mass hierarchy. Therefore, there is no consistent solution of the double β-decay constraint for Majorana neutrinos in the type (b) hierarchy.

In the case of the atmospheric neutrino anomaly due to the $\nu_\mu \to \nu_e$ oscillation, the situation is somewhat different. In the type (a) mass pattern there is a difficulty because almost degenerate heavy masses are multiplied by order 1 coefficients and a tuning, i.e., $s_{12}^2 c_{13}^2 = s_{13}^2$ to better than 0.1, is required for cancellation in addition to the requirement of opposite CP parities. On the contrary, in the type (b) hierarchy, there is no problem with the double-β-decay constraint because the heavy mass is multiplied by small coefficients of the order of $\sqrt{\epsilon}$.

Thus, we have shown in this paper that the neutrino masses and the mixings are strongly constrained by atmospheric and terrestrial experiments under the assumption of DMMN's suggested by the LSND experiments. The constraint is so severe that the mass and the mixing patterns are determined almost uniquely within the uncertainties of the neutrino types and the interpretations of the atmospheric neutrino anomaly. In the case of Majorana neutrinos the additional constraint from the double β decay selects out the unique natural solution in each interpretation.

While the solution (A) is just a collection of independent two-flavor neutrino oscillations, the solution (B) exploits a genuine three-flavor mechanism in which the terms with large and small Δm^2 in the same $\nu_\mu \to \nu_e$ channel are responsible for the LSND events and the atmospheric neutrino anomaly, respectively. Despite the almost degeneracy of "ν_e" and "ν_μ" to $\Delta m^2 \sim 10^{-2}$ eV^2, a large Δm^2 of the order of 1–100 eV^2 can arise through the virtual intermediate state of "ν_τ," thereby rendering the description of LSND-type oscillation events possible [25].

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