Gluon fragmentation into spin-triplet S-wave quarkonium

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The leading color-singlet contribution to the fragmentation function for a gluon to split into spin-triplet S-wave quarkonium is presented. In the case of charmonium, we find that this color-singlet term is always negligible compared to the leading color-octet contribution.

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The dominant mechanism for the production of heavy quarkonium at sufficiently large transverse momentum is parton fragmentation [1]. In Ref. [1], we calculated the fragmentation functions for gluons to split into S-wave quarkonium states to leading order in α_s and to leading order in the nonrelativistic expansion. The fragmentation function for the $^1S_0$ state was given in analytic form. The fragmentation function for the $^3S_1$ state was reduced to a two-dimensional integral, which was evaluated numerically. The integrand was given in the preprint version of Ref. [1], but it was too lengthy to be included in the published version. In this work, we present the detailed formula for this term in the fragmentation function. We also discuss the relative importance of color-singlet and color-octet terms in the fragmentation function.

A rigorous theory of the inclusive production of heavy quarkonium has been developed recently by Bodwin, Braaten, and Lepage [2]. This approach is based on the use of nonrelativistic QCD (NRQCD) to factor the production rate into short-distance coefficients that can be computed in perturbation theory and long-distance factors that are expressed as NRQCD matrix elements. Using this formalism, the fragmentation function for a gluon to split into a quarkonium state X with longitudinal momentum fraction z can be written as

$$D_{g\to X}(z,\mu) = \sum_n d_n(z,\mu)\langle \mathcal{O}_n^X\rangle \ , \qquad (1)$$

where $\mathcal{O}_n^X$ are local four-fermion operators in NRQCD. The short-distance coefficients $d_n(z,\mu)$ are independent of the quarkonium state X. For a fragmentation scale μ of order of the heavy quark mass m_Q, they can be computed using perturbative theory in $\alpha_s(2m_Q)$. The dependence on the quarkonium state X appears in the long-distance factors $\langle \mathcal{O}_n^X\rangle$. The relative magnitude of the various matrix elements for a given state X can be estimated by how they scale with m_Q and with the typical relative velocity v of the heavy quark inside the quarkonium. Thus, the factorization formula (1) for the gluon fragmentation function is a double expansion in α_s and v. To determine the relative importance of the terms in this formula, one should take into account both the scaling in v of the matrix elements and the order in α_s of their coefficients.

We now consider the fragmentation function for a $^3S_1$ state which we denote by V. In the color-singlet model for quarkonium production [3], only a single term in the expansion (1) for the fragmentation function is retained. In the notation of Ref. [2], the matrix element in this term is $\langle \mathcal{O}_1^V(^3S_1)\rangle$. This matrix element is proportional to the probability for the formation of the state V from a pointlike $Q\overline{Q}$ pair in a color-singlet $^3S_1$ state. The leading contribution to its short-distance coefficient $d_1^{(^3S_1)}(z)$ arises from the parton process $g^*\to Q\overline{Q}gg$ and is of order α_s^3. Using the velocity-scaling rules of [2], the matrix element $\langle \mathcal{O}_1^V(^3S_1)\rangle$ is of order v^3, so the contribution to the gluon fragmentation function is of order $\alpha_s^3v^3$. The factorization approach reduces to the color-singlet model in the limit $v\to 0$, since all other matrix elements in the expansion (1) are higher order in v^2.

The charmonium and bottomonium systems are probably not sufficiently nonrelativistic that matrix elements that are suppressed by powers of v^2 can be completely neglected. In the case of a $^3S_1$ state, there is one matrix element that is suppressed by a single power of v^2 relative to $\langle \mathcal{O}_1^V(^3S_1)\rangle$, but it also has a short-distance coefficient $d(z)$ of order α_s^3. There are several "color-octet" matrix elements that are suppressed by two powers of v^2, including $\langle \mathcal{O}_8^V(^1S_0)\rangle$, $\langle \mathcal{O}_8^V(^3S_1)\rangle$, and $\langle \mathcal{O}_8^V(^3P_J)\rangle$. Of particular importance in gluon fragmentation is the matrix element $\langle \mathcal{O}_8^V(^3S_1)\rangle$, because it has a short-distance coefficient of order α_s due to the parton process $g^*\to Q\overline{Q}$. All other matrix elements that are suppressed by v^4 have short-distance coefficients of order α_s^2 or higher. The matrix element $\langle \mathcal{O}_8^V(^3S_1)\rangle$ is proportional to the probability for the formation of the state V (plus other particles) from a pointlike $Q\overline{Q}$ pair in a color-octet $^3S_1$ state. The corresponding contribution to the fragmentation function is of order α_sv^7, compared to $\alpha_s^3v^3$ for the leading color-singlet term. The two fewer powers of α_s can compensate for the suppression of the matrix element by v^4. Keeping only these two terms, the gluon fragmentation function for the $^3S_1$ state at the initial scale $2m_Q$ can be written as

$$D_{g\to V}(z,2m_Q)=d_1^{(^3S_1)}(z,2m_Q)\langle\mathcal{O}_1^V(^3S_1)\rangle +d_8^{(^3S_1)}(z,2m_Q)\langle\mathcal{O}_8^V(^3S_1)\rangle\,. \quad (2)$$

To leading order in α_s, the coefficient of the color-singlet matrix element in (2) can be deduced from the Feynman amplitude for $g^*\to Q\overline{Q}gg$, where the $Q\overline{Q}$ pair is produced in a color-singlet $^3S_1$ state with vanishing relative momentum. The square of the amplitude for this process can be extracted from a calculation of the matrix element for $e^+e^-\to\psi gg$ [4]. There are several typographical errors that must be corrected in Eq. (5) of Ref. [4]. In the sixth term on the right side, the factor $(1-\mu^2-x_2^2)$ should be $(1-\mu^2-x_2)^2$. In the last term, $(1-\mu-x_1)^2$ should be $(1-\mu^2-x_1)^2$, $(1-\mu-x_2)^2$ should be $(1-\mu^2-x_2)^2$, $(K_-\cdot L_-)^2$ should be $(K_-\cdot L_-)$, and the overall sign of this last term should be changed from minus to plus. Having made these corrections, one can reproduce previous results for the energy distribution for $\gamma^*\to\psi gg$ [5]. The fragmentation function for the $^3S_1$ state can be calculated by the same method used for the $^1S_0$ state in Ref. [1]. The calculation is rather involved and we present only the final result:

$$d_1^{(^3S_1)}(z,2m_Q)=\frac{5}{5184\pi m_Q^3}\alpha_s(2m_Q)^3\int_0^z dr\int_{(r+z^2)/2z}^{(1+r)/2}dy\,\frac{1}{(1-y)^2(y-r)^2(y^2-r)^2}\times\sum_{i=0}^{2}z^i\left(f_i(r,y)\;+\;g_i(r,y)\frac{1+r-2y}{2(y-r)\sqrt{y^2-r}}\ln\frac{y-r+\sqrt{y^2-r}}{y-r-\sqrt{y^2-r}}\right)\,. \quad (3)$$

The integration variables are $r=4m_Q^2/s$ and $y=p\cdot q/s$, where p and q are the four-momenta of the quarkonium and the fragmenting gluon and $s=q^2$. The functions f_i and g_i are

$$f_0(r,y)=r^2(1+r)(3+12r+13r^2)\;-\;16r^2(1+r)(1+3r)y-2r(3-9r-21r^2+7r^3)y^2\;+\;8r(4+3r+3r^2)y^3 -4r(9-3r-4r^2)y^4-16(1+3r+3r^2)y^5\;+\;8(6+7r)y^6\;-\;32y^7\,, \quad (4)$$

$$f_1(r,y)=-2r(1+5r+19r^2+7r^3)y\;+\;96r^2(1+r)y^2\;+\;8(1-5r-22r^2-2r^3)y^3 +16r(7+3r)y^4\;-\;8(5+7r)y^5\;+\;32y^6\,, \quad (5)$$

$$f_2(r,y)=r(1+5r+19r^2+7r^3)\;-\;48r^2(1+r)y\;-\;4(1-5r-22r^2-2r^3)y^2 -8r(7+3r)y^3\;+\;4(5+7r)y^4\;-\;16y^5\,, \quad (6)$$

$$g_0(r,y)=r^3(1-r)(3+24r+13r^2)\;-\;4r^3(7-3r-12r^2)y\;-\;2r^3(17+22r-7r^2)y^2 +4r^2(13+5r-6r^2)y^3\;-\;8r(1+2r+5r^2+2r^3)y^4\;-\;8r(3-11r-6r^2)y^5+8(1-2r-5r^2)y^6\,, \quad (7)$$

$$g_1(r,y)=-2r^2(1+r)(1-r)(1+7r)y\;+\;8r^2(1+3r)(1-4r)y^2+4r(1+10r+57r^2+4r^3)y^3 -8r(1+29r+6r^2)y^4\;-\;8(1-8r-5r^2)y^5, \quad (8)$$

$$g_2(r,y)=r^2(1+r)(1-r)(1+7r)\;-\;4r^2(1+3r)(1-4r)y-2r(1+10r+57r^2+4r^3)y^2 +4r(1+29r+6r^2)y^3\;+\;4(1-8r-5r^2)y^4. \quad (9)$$

The integrals over r and y in (3) must be evaluated numerically to obtain the fragmentation function at the energy scale $\mu=2m_Q$. The fragmentation function at higher energy scales μ is then obtained by Altarelli-Parisi evolution.

The short-distance coefficient of the color-octet matrix element in (2) can be calculated to leading order in α_s from the Feynman amplitude for $g^*\to Q\overline{Q}$. The result is [6]

$$d_8^{(^3S_1)}(z,2m_Q)=\frac{\pi\alpha_s(2m_Q)}{24m_Q^3}\delta(1-z)\,. \quad (10)$$

The radiative correction of order α_s^2 has also been calculated recently by Ma [7].

The relative importance of the color-singlet and color-octet contributions to the fragmentation functions can be determined by integrating the initial fragmentation function (2) over z to get the fragmentation probability at the scale $2m_Q$:

$$\int_0^1 dz\,D_{g\to V}(z,2m_Q) =(8.28\times10^{-4})\frac{\alpha_2^3(2m_Q)}{m_Q^3}\langle\mathcal{O}_1^V(^3S_1)\rangle +(1.31\times10^{-1})\frac{\alpha_s(2m_Q)}{m_Q^3}\langle\mathcal{O}_8^V(^3S_1)\rangle\,. \quad (11)$$

The value of the color-singlet matrix element $\langle\mathcal{O}_1^V(^3S_1)\rangle$ can be determined from the electronic width of the vector meson state. In the case of the charmonium states J/ψ and ψ', the matrix elements $\langle\mathcal{O}_1^V(^3S_1)\rangle$ are approximately 0.73 GeV^3 and 0.11 GeV^3, respectively. The most reliable determinations of the color-octet matrix elements come from recent data on prompt charmonium production from the Collider Detector at Fermilab (CDF) at the Tevatron. The color-octet contributions are necessary to explain the magnitude of the cross section for prompt

J/ψ and ψ' production at large transverse momentum, and they also explain the shape of the transverse momentum distribution [8]. The values of the matrix elements $\langle \mathcal{O}_8^V(^3S_1)\rangle$ that are obtained by fitting the observed cross sections are approximately 1.5×10^{-2} GeV^3 for J/ψ and 4.3×10^{-3} GeV^3 for ψ' [9]. Using the value $\alpha_s(2m_c) = 0.26$, we find that the two terms in the fragmentation probability (11) are approximately 3.2×10^{-6} and 1.5×10^{-4} in the case of J/ψ and 4.7×10^{-7} and 4.3×10^{-5} in the case of ψ'. The color-octet term is larger by about a factor of 50 for the J/ψ and larger by about a factor of 100 for the ψ'. We conclude that the color-singlet term is always negligible compared to the color-octet term for charmonium.

In the case of bottomonium, the color-singlet term in the gluon fragmentation function is probably also negligible. Because of the running of the coupling constant between the scales $2m_c$ and $2m_b$, the suppression of the color-singlet term by α_s^2 decreases its relative importance by about a factor of 2. On the other hand, the suppression of the color-octet term by v^4 decreases its relative importance by about a factor of 10. The net effect is that the color-singlet term is still likely to be more than an order of magnitude smaller than the color-octet term.

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