Spontaneous breaking of vector symmetries and the nondecoupling light Higgs particle

Hanqing Zheng*
Paul Scherrer Institute, 5232 Villigen PSI, Switzerland
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Using a four fermion interaction Lagrangian, we demonstrate that the spontaneous breaking of vector symmetries requires the existence of a light (comparing with the heavy fermion mass) scalar particle, and the low energy effective theory (the σ model) obtained after integrating out heavy fermion degrees of freedom is asymptotically a renormalizable one. When applying the idea to the electroweak symmetry breaking sector of the standard model, the Higgs particle's mass is of the order of the electroweak scale.

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The spontaneous breaking of vector symmetries (SBVS) is an interesting subject in quantum field theory and also in particle physics, as long as it continues to be a possible characteristic of nature. In a previous paper [1] we have made an attempt in considering the breaking of the electroweak symmetry as a consequence of the SBVS between fermions with heavy bare masses. The motivation of such a consideration is to break the electroweak symmetry dynamically but with least influence on the low energy physics [1]. We modeled SBVS by a low energy effective Higgs-Yukawa interaction and, after integrating out the heavy fermion fields in the mean field approximation of the Higgs particle, estimated the low energy residual effects of these heavy fermions. It is shown that the heavy fermion fields are essentially decoupling at low energies, except that they can generate massless Goldstone excitations to be absorbed by the weak gauge fields. The low energy effective theory (the standard model) should therefore be weakly interacting — a picture different from the technicolor models.

Nothing can be said about the mass of the composite Higgs particle, within the context of the effective Higgs-Yukawa model in Ref. [1]. A question then arises: Can it be as heavy as the heavy fermion mass? From general physical consideration we know that this should not be the case. Because if the Higgs particle's mass is heavy enough, the remaining fields must be in a strongly interacting system because of the well known tree-level unitarity argument [2] — a result contrary to our motivation and the general expectation from the decoupling phenomena. The aim of this paper is to resolve this Higgs mass ambiguity, using a model of four fermion interactions with the dynamically generated SBVS. The SBVS through four fermion interactions was first analyzed by Preskill and Weinberg [3] to study the possible violation of the "persistent mass condition." For a four fermion interaction with a global vector symmetry, there are primarily two scales, the cutoff scale Λ and the bare fermion mass $M (M < \Lambda)$, and in addition, the interaction strength is characterized by a dimensionless coupling constant, G. Preskill and Weinberg have shown that, for a given cutoff Λ and a sufficiently large G, there exists a critical value M_c. When the fermion mass is below this critical point, $M < M_c$, the vector (isospin) symmetry is spontaneously broken down. As a consequence, there exist massless particles composed of massive constituents leading to a violation of the persistent mass condition. When M exceeds M_c, the system is in a symmetry phase and the decoupling phenomenon occurs. The symmetry breaking is of second order and characterized by a new scale m (the fermion mass splitting) obtained after some fine tuning.

For our purpose, the four fermion interaction Lagrangian can be written as[1]

$$\mathcal{L} = \bar{\Psi}^i \left(i\partial\!\!\!/ - M\right) \Psi^i - \frac{G}{N_c\Lambda^2}[(\bar{\Psi}^i\rho_3\Psi^i)^2 + (\bar{\Psi}^i\rho_1\vec{\tau}\Psi^i)^2] , \tag{1}$$

where $\Psi = (\psi_1, \psi_2)^T$ and $\psi_{1,2}$ are SU(2) isospin doublets. The index i refers to the "color" degree of freedom and runs over 1 to N_c. We assume N_c is large in the following. τ_i are generators of the SU(2) isospin group, and ρ_i are Pauli matrices of the "parity doublet" space (i.e., space between ψ_1 and ψ_2). The Lagrangian equation (1) is invariant under the following SU(2)$\times$SU(2) rotations:

$$\Psi \to e^{i\vec{\alpha}\cdot\vec{\tau} + i\rho_2\vec{\beta}\cdot\vec{\tau}}\Psi . \tag{2}$$

To match the electroweak physics one of the SU(2) global symmetries will be gauged as SU(2)$_W$ [of course, the local U(1)$_Y$ should also be introduced]. The other "custodial" SU(2) symmetry remains as a global one and can be broken explicitly but slightly. The latter constraint comes from the experimental value of the ρ parameter. Since these are already discussed in Ref. [1]

*Electronic address: hanqing.zheng@psi.ch

[1]One can add more terms and couplings; see for example Ref. [1]. The present Lagrangian is the minimal one suitable for the discussion in the present paper.

and are not very relevant to the topic in the present paper we will no longer discuss them but just work with the effective Lagrangian equation (1). To study the spontaneous symmetry breaking we look at the gap equation and search for a solution of the fermion mass matrix of the type $m = m_s + \rho_3 m_3$. In the large N_c limit we obtain

$$m_s = M \ , \tag{3}$$

$$m_3 = \frac{iG}{\Lambda^2}\int^{\Lambda} \frac{d^4p}{(2\pi)^4}\mathrm{tr}(\rho_3 S_F) \ , \tag{4}$$

where

$$S_F = \begin{pmatrix} S_F^1 & \\ & S_F^2 \end{pmatrix} \ . \tag{5}$$

The above equation (4) can be written in a simple form. To define

$$f(m) = \frac{m}{\Lambda^2}\int^{\Lambda} \frac{q_E^2 dq_E^2}{q_E^2 + m^2} \ , \tag{6}$$

we have

$$m_1 - m_2 = \frac{G}{\pi^2}[f(m_1) - f(m_2)] \ , \tag{7}$$

where $m_{1,2} = M \pm m_3$. For small but nonvanishing $m_1 - m_2$ the above equation can be further approximated as

$$\frac{\pi^2}{G} \simeq f'(M) + \tfrac{1}{6} m_3^2 f'''(M) \ . \tag{8}$$

For small values of M (or M_c),[2] $f'(M)$ is a decreasing function of M and $f'''(M)$ is negative. Therefore we observe from the above formula that when π^2/G is smaller than unity there exists the critical value M_c, $\pi^2/G = f'(M_c)$. When M is less than M_c there exists a nonvanishing solution of m_3,

$$m_3 = \sqrt{6M_c(M_c - M)} \ , \tag{9}$$

which holds in the $m_3 << M$ (or $M \to M_c$) limit. Once M exceeds the critical value M_c there is only the trivial solution $m_3 = 0$ in the above equation (8).

Up to now we have said little more than the result obtained in Ref. [3] except that in our case the vector group is SU(2)×SU(2) which spontaneously breaks down to SU(2) and therefore there are 3 Goldstone bosons. The appearance of these massless Goldstone excitations implies that the decoupling of the heavy fermions exists in a nontrivial manner because of the existence of the phase transition.

To understand more about the dynamics of SBVS it is necessary to solve the Lagrangian equation (1) in the large N_c limit. For our purpose it is appropriate to discuss the following two point functions:

$$\Pi_P(q^2) \equiv i\int d^4x e^{iqx}\langle|T\{\bar\Psi^i(x)\rho_1\Psi^j(x)\bar\Psi^j(0)\rho_1\Psi^i(0)\}|\rangle \ , \tag{10}$$

$$\Pi_S(q^2) \equiv i\int d^4x e^{iqx}\langle|T\{\bar\Psi(x)\rho_3\Psi(x)\bar\Psi(0)\rho_3\Psi(0)\}|\rangle \ , \tag{11}$$

$$\begin{aligned}\Pi_M^\mu(q^2) &\equiv i\int d^4x e^{iqx}\langle|T\{\bar\Psi^i(x)\rho_2\gamma^\mu\Psi^j(x)\bar\Psi^j(0)\rho_1\Psi^i(0)\}|\rangle \\ &\equiv iq^\mu \Pi_M(q^2) \ .\end{aligned} \tag{12}$$

In above equations i, j denote isospin indices and we dropped out the color indices for simplicity. These two point functions are obtainable by summing up fermion bubble chains. We use the functions with overbars to denote the one-loop contribution to the two point functions. Direct calculation leads to

$$\Pi_P(q^2) = \frac{\overline{\Pi}_P(q^2)}{1 - G/\Lambda^2\overline{\Pi}_P(q^2)} \ , \tag{13}$$

$$\Pi_S(q^2) = \frac{\overline{\Pi}_S(q^2)}{1 - G/\Lambda^2\overline{\Pi}_S(q^2)} \ , \tag{14}$$

and

$$\Pi_M^\mu(q) = \frac{\overline{\Pi}_M^\mu(q)}{1 - G/\Lambda^2\overline{\Pi}_P(q^2)} \ , \tag{15}$$

where $\overline{\Pi}_P$ and $\overline{\Pi}_S$ are quadratically divergent and $\overline{\Pi}_M^\mu$ only contains logarithmic divergence (the latter one is linearly divergent in the NJL model). One must be careful in dealing with the quadratic divergence in order to avoid the dependence on the choice of the internal momentum flow. The standard method to overcome this difficulty is to calculate firstly the imaginary part of the two point functions using the Cutkosky rule and then use dispersion relations to evaluate the full amplitudes. The dispersion integrals are usually divergent and need subtractions. To deal with $\overline{\Pi}_P$ it is useful to rewrite it as $\overline{\Pi}_P(q^2) = \overline{\Pi}_P(0) + q^2\overline{\Pi}_P'(q^2)$. The function $\overline{\Pi}_P'(q^2)$ [which coincides with $(d/dq^2)\overline{\Pi}_P(q^2)$ at the origin] now only contains logarithmic divergence, the quadratic cutoff dependent term is already absorbed into $\overline{\Pi}_P(0)$. In order to have a Goldstone pole in the function $\Pi_P(q^2)$ we read off the self-consistency condition for SBVS which should be equivalent to the gap equation,

$$1 - G/\Lambda^2\overline{\Pi}_P(0) = 0 \ . \tag{16}$$

[2]In the present case, the gap equation is more sensitive than in the Nambu–Jona-Lasinio (NJL) model to the higher order terms in the $1/\Lambda$ expansion. Unambiguous results can only be obtained when keeping M/Λ small, since these subleading terms are regularization scheme dependent.

The mixed function Π_M can then be written as

$$\Pi_M = \frac{\overline{\Pi}_M(q^2)}{-G/\Lambda^2 q^2 \overline{\Pi}'_P(q^2)} , \tag{17}$$

while[3]

$$\frac{1}{\pi}\mathrm{Im}\overline{\Pi}_M(t) = \frac{m_1 - m_2}{4\pi^2}\left(1 - \frac{(m_1+m_2)^2}{t}\right)^{3/2}\left(1 - \frac{(m_1-m_2)^2}{t}\right)^{1/2}$$
$$= (m_1 - m_2)\frac{1}{\pi}\mathrm{Im}\overline{\Pi}'_P(t) . \tag{18}$$

The use of unsubtracted dispersion relations (with a truncated integrand at $4\Lambda^2$) as proposed in Ref. [4] immediately leads to $\overline{\Pi}_M/\overline{\Pi}'_P \equiv$ const. This is not an accident, as can be proven using the equal time anticommutation relation of the quark fields and the current conservation condition,[4]

$$\Pi^\mu_M \equiv \frac{2iq^\mu}{q^2}\langle|\overline{\Psi}\rho_3\Psi|\rangle . \tag{19}$$

Therefore from Eq. (17) we obtain

$$\langle|\overline{\Psi}\rho_3\Psi|\rangle = -\frac{N_c}{2G}\Lambda^2(m_1 - m_2) . \tag{20}$$

For the two point function $\overline{\Pi}_S(q^2)$, one can write $\overline{\Pi}_S(q^2) = \overline{\Pi}_P(q^2) + \delta\overline{\Pi}_S(q^2)$. Again the quadratic divergence is absorbed into $\overline{\Pi}_P(0)$ and $\delta\overline{\Pi}_S(q^2)$ only contains logarithmic divergence. The fine tuning is isolated in Eq. (16) or in the gap equation. The Higgs particle's mass is obtained by looking for the pole position of the scalar two point function. We read off from Eq. (14) that

$$m_H^2 = -\delta\overline{\Pi}_S(m_H^2)/\overline{\Pi}'_P(m_H^2) . \tag{21}$$

In the symmetry phase (i.e., $m_1 = m_2$) Π_S is identical to Π_P as a result of the global symmetry. Since these Green functions are continuous in m_3, $\delta\overline{\Pi}_S \sim (m_1 - m_2)^2$ and therefore m_H^2 is a small quantity. Approximately we have $m_H^2 = -\delta\overline{\Pi}_S(0)/\overline{\Pi}'_P(0)$. Simple calculation yields[5]

$$m_H = 2m_3 . \tag{22}$$

This result indicates that the scalar particle's mass is small, i.e., at the symmetry breaking scale (comparing with the fermion mass scale and the cutoff parameter). Especially it has nothing to do with the whole fermion mass, rather it is only related to the dynamically generated part of the fermion mass. In the NJL model for chiral symmetry breaking it happens to be that the two masses coincide. It is worth pointing out that the light mass of the scalar particle composed of heavy fermion fields is a consequence of the symmetry and be model independent, at least in a system with second order phase transition. This property is not shared by other possible composite particles. For example one could add another four fermion interaction term in the Lagrangian equation (1) with vector-vector couplings and the mass of the vector resonance, if it exists, is $\sim \Lambda$ and can be large.

To discuss the electroweak physics, comparing with the expression of the decay constant of the Goldstone field [1], $f_\pi^2 = \frac{N_c}{2\pi^2}m_3^2\ln(\Lambda^2/M^2)$, we obtain

$$m_H = \frac{2\pi v}{\sqrt{N_c\ln(\Lambda/M)}} . \tag{23}$$

Taking for example $\Lambda/M \simeq 10$ we may obtain the upper bound of the Higgs particle's mass and taking $\Lambda \sim 10^{18}$ GeV and $M \sim 10^3$ GeV the lower bound may be estimated. We have

$$260/\sqrt{N_c}\ \mathrm{GeV} \le m_H \le 1000/\sqrt{N_c}\ \mathrm{GeV} , \tag{24}$$

This result is compatible with the present experimental lower bound and also lies within the range detectable by future hadron colliders. Since the Higgs particle's mass is lighter than 1 TeV, i.e., the scale signaling the strong interaction in the electroweak symmetry breaking sector, SBVS induced electroweak symmetry breaking is "weak," and the symmetry can be realized linearly in the Higgs sector. Moreover the low energy effective theory is *renormalizable*: all the nonrenormalizable terms are screened by the heavy fermion mass [1] (m^2/M^2 suppressed). This is different from the technicolor interaction (in which the spontaneously broken symmetry is the chiral symmetry) induced electroweak symmetry breaking.

The correct low energy theory, after integrating out the heavy fermion fields, should therefore be the effective $O(p^4)$ Lagrangian for Goldstone fields obtained in Ref. [1] plus the standard electroweak interaction Lagrangian of

[3] $\frac{1}{\pi}\mathrm{Im}\overline{\Pi}_P(t) = \frac{1}{4\pi^2}[t - (m_1+m_2)^2]\sqrt{(1 - \frac{(m_1+m_2)^2}{t})(1 - \frac{(m_1-m_2)^2}{t})}$, $\frac{1}{\pi}\mathrm{Im}\overline{\Pi}_S(t) = \frac{1}{2}\{\frac{1}{4\pi^2}(t - 4m_1^2)\sqrt{1 - \frac{4m_1^2}{t}} + (m_1 \to m_2)\}$. At the critical point of the phase transition ($m_1 = m_2$) these two functions are equal. This is of course the consequence of the symmetry.

[4] Similar results were obtained in the NJL model in Ref. [5] where different regularization schemes are used.

[5] This expression receives $O[1/\ln(\Lambda/M)]$ corrections which cannot be determined unambiguously [6], although it is practically unimportant in the present case. In particular, adding more four fermion interaction terms with higher derivatives in the effective Lagrangian as pointed out in the first paper of Ref. [6] will not lead m_H to be proportional to M rather than m_3.

the Higgs field.[6] It is also helpful to not integrate out fermion fields completely but first down to an arbitrary scale μ to study the heavy fermion contributions to the running coupling constants of the composite Higgs field. We have

$$\lambda_0(\mu) = \frac{N_c}{8\pi^2} \ln\left(\frac{\Lambda^2 + M^2}{\mu^2 + M^2}\right) , \tag{25}$$

$$Z_H(\mu) = \frac{N_c}{4\pi^2} \ln\left(\frac{\Lambda^2 + M^2}{\mu^2 + M^2}\right) , \tag{26}$$

$$m_H^2(\mu) = \frac{N_c}{2\pi^2}\left\{\frac{\pi^2}{G}\Lambda^2 - \Lambda^2 + \mu^2 + 3M^2 \ln\left(\frac{\Lambda^2 + M^2}{\mu^2 + M^2}\right)\right\} . \tag{27}$$

We see from the above expressions that only the high frequency modes ($\mu > M$) contribute to the wave function renormalization constant (Z_H) and the bare coupling constant of ϕ^4 self interactions (λ_0). The low frequency modes only contribute to the fine tuning of the Higgs mass.

Once we have introduced the matter field (quarks and leptons) couplings in the same way as in the SM we can set up the complete equivalence between the SM and our model of SBVS, Eq. (1), [7] in the $m/M << 1$ limit, even at the energy scale E much larger than the electroweak scale as long as $E << M$, within the constraints on the Higgs particle's mass. It is interesting to note that our model shares many low energy properties of the top-color model [7], although we have a very different physical motivation from the very beginning. Our result implies that the Higgs particle's mass is naturally of the order of the electroweak scale which, if confirmed by future experiments, may therefore not necessarily be considered as a support to the top-color model.

Before concluding, we would like to stress that it is also appealing to study the property of heavy vector fermions in the phenomenology aspects. As has been pointed out in Ref. [8], the inclusion of the heavy chiral fermions may violate the stability of the SM vacuum. According to the analysis on the one-loop effective potential, the Higgs mass is therefore forced to become heavy by the appearance of heavy chiral fermions. If this is regarded as unnatural in the sense of perturbative vacuum stability, heavy vector fermions may be the only reasonable candidates in searching for new matter constituents of nature.

To conclude, we start from a nonperturbative four fermion interaction with spontaneously broken symmetry between heavy fermions in vector representations of the symmetry group and derive an asymptotically renormalizable low energy effective theory, with a light scalar particle. This remarkable property of the decoupling-nondecoupling phase transition phenomena of SBVS, we believe, is model independent. To what extent our model will be of realistic importance when applying to, for example, electroweak physics may depend on whether or how can it be read off from a more fundamental theory since there are restrictions on SBVS [9], if one respects to the gauge interactions as the first principle. However, it is very interesting to point out that our model is close in spirit to the "composite Higgs model" proposed by Georgi and Kaplan [10] in which the $SU(2)_W$ gauge group is vectorlike and is spontaneously broken by a confining gauge interaction due to vacuum misalignment. Since the present model, as a low energy effective theory, only deals with the low energy symmetries, we expect that it is helpful to us in understanding the general issue of SBVS, in a model independent way.

Finally, it is worth emphasizing that there may exist the possibility that it is vague to say the Higgs particle is "composite" or "elementary," since the two cases may practically be indistinguishable, as shown by the above example.

Note added. After this paper was completed, the author became aware of the recent work by Maekawa [11] in which ideas similar to Ref. [1] and the present paper were discussed.

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[6]The light Higgs particle brings new nondecoupling effects. For example if we further integrate it out (if it is allowed, i.e., not to be as light as $\sim 2M_W$), at tree level, there is an additional contribution to L_1, $\delta L_1 = v^2/8m_H^2$, which is the same as the standard model one. The claim made in [1] on the difference between the two L_1 terms obtained in the present model and in the standard model (SM) is therefore incorrect.

[7]There is no difficulty to reproduce the CKM matrix, and even multi-Higgs models in an extended version of four fermion interactions.

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