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Nontrivial asymptotically nonfree gauge theories and dynamical unification of couplings

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Evidence for the nontriviality of asymptotically nonfree (ANF) Yang-Mills theories is found on the basis of optimized perturbation theory. It is argued that these theories with matter couplings can be made nontrivial by means of the reduction of couplings, leading to the idea of the dynamical unification of couplings (DUC). The second-order reduction of couplings in the ANF SU(3)-gauged Higgs-Yukawa theory, which is assumed to be nontrivial here, is carried out to motivate independent investigations on its nontriviality and DUC.

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Asymptotically free (AF) theories [1] do not suffer from the problem of triviality [2]. It is widely believed that the question of triviality cannot be addressed within the framework of perturbation theory, and so far there is no real indication for the existence of a nontrivial four-dimensional theory that is not AF. It is, however, tempting to think that if an infrared-free theory has an ultraviolet fixed point which is small, perturbation theory might be intact even near the fixed point and hence could be applicable to the triviality problem.

About ten years ago, Sakakibara, Stevenson, and I [3] considered perturbation theory near a fixed point. We formulated the problem as the problem of the renormalization scheme (RS) dependence, because at any finite order in perturbation theory even the existence of a positive zero of the β function depends on the RS. We performed our investigation on the basis of optimized perturbation theory (OPT) [4], which, as is well known, yields RS-invariant perturbative approximations and has already experienced certain successes in perturbative QCD [5] and also in QED [6]. We found that one needs a perturbative calculation of a physical quantity of at least third order in order to be able to apply our method. Investigating concrete field theory examples, we argued that under certain circumstances perturbative analyses based on OPT near a fixed point could be believable.

Recently, using the third-order QCD corrections of the e^+e^- cross sections [7], Mattingly and Stevenson [8] applied OPT and concluded that AF QCD has an infrared-stable fixed point. Although the assumption on the existence of an infrared fixed point in AF QCD has no logical inconsistency, it is not clear at all how much the fixed point found in a perturbative approach can describe the physics in the infrared regime, because the nonperturbative effects play the essential role in understanding the low energy physics of AF QCD. In the ultraviolet regime, on the other hand, the nonperturbative nature may be neglected in describing the basic part of the physics of AF QCD.

One of the main assumptions of this paper, which is partly motivated by this fact, is that this is true even in asymptotically nonfree (ANF) Yang-Mills theories. Of course, this is a very strong assumption but there is neither internal inconsistency of this assumption nor known facts against it (at least to my knowledge [9]). Moreover, as will be seen, the investigation based on OPT indicates that ANF Yang-Mills theories could have an ultraviolet fixed point so that they could be well-defined, interacting theories in the ultraviolet limit.

The existence of an ultraviolet fixed point in the Yang-Mills theory fits indeed with the idea of a "walking technicolor gauge coupling" [10]. Here I would like to emphasize its relation to unification of couplings that does not follow from a symmetry principle, because such a unification scheme has become desirable for the following reasons. (1) Most grand unified theories (GUT's) become ANF if one attempts to obtain a realistic fermion mass matrix by introducing additional Higgs fields, and (2) it has been found [11] that various supersymmetric GUT's with gauge-Yukawa unification can predict the top-bottom mass hierarchy correctly, but the theoretical possibility of unifying the gauge and Yukawa couplings from a symmetry principle within the framework of field theory is extremely limited and mostly unrealistic.

The gauge-Yukawa unified models, proposed in Ref. [11], are constructed on the basis of the principle of reduction of couplings [12, 13]. Though there are certain successes of these models, the reduction principle is associated with no intuitive, physical meaning. Dynamical unification of couplings (DUC), which I propose in this paper, is based on the assumption that Yang-Mills theories with matter couplings can have an ultraviolet fixed point and hence are well defined in the ultraviolet limit if a reduction of couplings is appropriately carried out. Therefore, DUC gives a reduction of couplings a simple, theoretical meaning at least. This is speculation at this moment, and nontriviality of realistic gauge-Yukawa unified models has to be verified on a case by case analysis of course. However, because of the complexity of the realistic models, they are not appropriate to be considered if one only wants to test the idea of DUC. In the second part of this paper, I will consider a simplified toy model, the SU(3)-gauged Higgs-Yukawa model, and carry out the reduction program in second order to motivate independent investigations on its nontriviality.

We begin by recalling the basic result obtained in Ref. [3]. Consider a physical quantity $\mathcal{R}(p_k,\mu,\alpha(\mu)/\pi)$ in a massless renormalizable theory, where p_k stand for the physical external momenta, μ is the renormalization scale, and $\alpha(\mu)$ is the renormalized coupling. In the nth order of perturbation theory, $\mathcal{R}$ can be written as

$$\mathcal{R}^{(n)}(p_k,\mu,a) = \gamma a \left[1+\sum_{i=1}^{n-1} r_i(p_k,\mu)\, a^i\right] , \qquad a \equiv \alpha/\pi ,$$

while the β function takes the form

$$\beta^{(n)}(a) = -a^2 \sum_{i=0}^{n-1} b_i a^i .$$

The coefficients b_i's ($i \geq 2$) are RS dependent, and along with μ they can uniquely parametrize the RS dependence. Therefore, $\mathcal{R}$, being a physical quantity, has to satisfy $\mu\partial\mathcal{R}/\partial\mu + \beta\partial\mathcal{R}/\partial a = 0$ and also $\partial\mathcal{R}/\partial b_i = 0$ ($i \geq 2$), which we altogether symbolically denote by $d\mathcal{R}/d(\mathrm{RS}) = 0$. Then the essence of OPT is to demand the optimization condition [4]

$$\left.\frac{d\mathcal{R}^{(n)}}{d\,(\mathrm{RS})}\right|_{\mathrm{RS=OPT\,RS}} = 0 , \qquad (1)$$

and to fix from this an optimized RS for a given physical quantity. Note that in perturbation theory one has only $d\mathcal{R}^{(n)}/d(\mathrm{RS}) = O(a^{n+1})$. In Ref. [3], we assumed that OPT makes sense even near a fixed point and found that the fixed point a^*_{OPT} in third order can be obtained from

$$0 = \frac{7b_0}{4b_1} + a^*_{\mathrm{OPT}} + \frac{3b_0}{b_1}\rho_2\,(a^*_{\mathrm{OPT}})^2 \quad \text{for third order}, \qquad (2)$$

$$0 = \frac{83b_0}{52b_1} + a^*_{\mathrm{OPT}} + \frac{12}{13}\left(\frac{b_0}{b_1}\rho_2 + \frac{b_1}{4b_0}\right)(a^*_{\mathrm{OPT}})^2 + \frac{64b_0}{13b_1}\rho_3\,(a^*_{\mathrm{OPT}})^3 \quad \text{for fourth order}, \qquad (3)$$

where

$$\rho_2 = r_2 + \frac{b_2}{b_0} - \left(r_1 + \frac{b_1}{2b_0}\right)^2 , \quad \rho_3 = r_3 + \frac{b_3}{2b_0} - r_1\left(\frac{b_2}{b_0} + 3r_2 - 2r_1^2 - \frac{b_1 r_1}{2b_0}\right) \qquad (4)$$

are the RS-independent quantity for a given $\mathcal{R}$. From Eqs. (2) and (3), one sees that the more negative the ρ's are, the more likely is the existence of a positive a^*_{OPT}.

Before I come to the non-Abelian case, I would like to discuss QED with many flavors in fourth-order. The result will be compared with that in third order to check the reliability of OPT. Using the fourth-order calculations together with the lower-order results [14], one can extend our third-order analysis [3] to the next order. As we did there, I consider

$$\mathcal{R}(-p^2/\mu^2,a) = -2\frac{d}{d\,\ln(-p^2/\mu^2)}\ln[1+\Pi(-p^2/\mu^2)] = \frac{2}{3}fa\left(1+\sum_{i=1} r_i a^i\right)$$

as a RS-invariant quantity, where $\Pi(-p^2/\mu^2,a)$ is the photon self-energy. From Ref. [14] I obtain the coefficients of the β function,

$$b_0 = -\frac{2}{3}f , \quad b_1 = -\frac{1}{2}f , \quad b_2 = \frac{1}{16}f + \frac{11}{72}f^2 ,$$
$$b_3 = \frac{23}{64}f - \left[\frac{95}{432} - \frac{13}{18}\zeta(3)\right]f^2 + \frac{77}{1944}f^3$$

(f is the number of flavors), and also [15]

$$r_1(-p^2/\mu^2=1) = \frac{3}{4} - \frac{5}{9}f , \quad r_2(-p^2/\mu^2=1) = -\frac{3}{32} - \left[\frac{47}{16} - 2\zeta(3)\right]f + \frac{25}{81}f^2 ,$$
$$r_3(-p^2/\mu^2=1) = -\frac{69}{128} + \left[\frac{11}{72} + \frac{121}{24}\zeta(3) - \frac{15}{2}\zeta(5)\right]f + \left[\frac{16829}{2592} - \frac{29}{16}\zeta(3)\right]f^2 - \frac{125}{729}f^3 ,$$

where $\zeta(3) = 1.202057\ldots$, $\zeta(5) = 1.036928\ldots$, and the quantities above are defined in the $\overline{\mathrm{MS}}$ scheme. Inserting them into ρ's defined in Eq. (4), I obtain

$$\rho_2 = -\frac{93}{64} - \left[\frac{23}{12} - 2\zeta(3)\right]f \simeq -1.453125 + 0.48745f , \qquad (5)$$

$$\rho_3 = \frac{135}{256} + \left[\frac{301}{64} - \frac{15}{2}\zeta(5)\right]f + \left[\frac{9}{4} - \frac{3}{2}\zeta(3)\right]f^2$$
$$\simeq 0.527344 - 3.07384f + 0.44691f^2 . \qquad (6)$$

One sees that ρ_3 is positive for $f \geq 7$ so that indeed there is no positive zero of Eq. (3) and hence no indication for the existence of an ultraviolet fixed point for $f \geq 7$ at fourth order. For $f \leq 6$, one has a positive zero; $\alpha^*_{\rm OPT}(=a^*_{\rm OPT}\pi) \simeq$3.04, 2.42, 2.08, 1.85, 1.68, 1.55 for $f = 1,2,3,4,5,6$, respectively. But they are too large that one cannot trust the result. So our third-order conclusion [3] remains unchanged at fourth order, supporting the reliability of the analysis based on OPT.

What follows is a slight generalization of the analysis of Ref. [8] in QCD, but with completely different physics and its applications in mind. The β-function coefficients of the first three orders in the $\overline{\rm MS}$ scheme can be found in Ref. [16]:

$$b_0 = \frac{11}{6}C_A - \frac{2}{3}T_F f\ , \quad b_1 = \frac{17}{12}C_A^2 - \left(\frac{5}{6}C_A + \frac{1}{2}C_F\right) \times T_F f\ , \tag{7}$$

$$b_2 = \frac{2857}{1728}C_A^3 + \left(-\frac{1415}{864}C_A^2 + \frac{79}{432}C_A T_F f - \frac{205}{288}C_A C_F + \frac{11}{72}C_F T f + \frac{1}{16}C_F^2\right) T_F f\ , \tag{8}$$

where C_A, C_F, and T_F are the usual group theoretic coefficients. Asymptotic nonfreedom requires that $f > 11C_A/4T_F$, and I concentrate only on such cases from the reason given before. I will below calculate ρ_2 in ANF SU(2), SU(3), SU(5), and SO(10) gauge theories with f Dirac fermions in the fundamental representation [$C_A = N, C_F = (N^2-1)/2N, T_F = 1/2$ for SU(N) and $C_A = 8$, $C_F = 9/2$, $T_F = 1$ for SO(10)]. To this end, I use the third-order corrections to (A) $\sigma_{\rm tot}(e^+e^- \to \text{hadrons})$ [7] and (B) the Gross-Llewellyn Smith sum rule for deep inelastic neutrino-nucleon scattering [17].

(A) $\sigma_{\rm tot}(e^+e^- \to \text{hadrons})$. The first quantity is the so-called R ratio $R(s/\mu^2, a(\mu)) = d_R \sum_f Q_f^2[1 + \mathcal{R}(s/\mu^2, a)]$, which is defined by $\sigma_{\rm tot}(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ in the e^+e^- annihilation, where s is the center of mass energy, d_R is the dimension of the quark representation, and Q_f stands for the electric charge of the f quark. Since it is unlikely that the real electric charge of the quark is related to the existence of a fixed point in a non-Abelian gauge theory, I instead use the fermion number and assume that $Q_f = 1$ for all the fermions. Under this assumption, I recall the third-order result of Ref. [7]:

$$\mathcal{R}^{(3)} = \frac{3}{4}C_F\, a\,(1 + r_1 a + r_2 a^2)\ , \tag{9}$$

where

$$r_1(s/\mu^2=1) = \left[\frac{41}{8} - \frac{11}{3}\zeta(3)\right] C_A - \frac{1}{8}C_F + \left[-\frac{11}{6} + \frac{4}{3}\zeta(3)\right] T_F f\ , \tag{10}$$

$$\begin{aligned} r_2(s/\mu^2=1) = &\left[\frac{90445}{2592} - \frac{2737}{108}\zeta(3) - \frac{121}{432}\pi^2\right] C_A^2 - \left[\frac{127}{48} + \frac{143}{12}\zeta(3)\right] C_A C_F - \frac{23}{32}C_F^2 \\ &+55\left[-\frac{1}{18}C_A + \frac{1}{3}C_F\right]\zeta(5)C_A + \left[\frac{302}{81} - \frac{76}{27}\zeta(3) - \frac{1}{27}\pi^2\right] T_F^2 f^2 \\ &+\left[\frac{11}{144} - \frac{1}{6}\zeta(3)\right]\frac{d^{abc}d^{abc}}{C_F d_R} f + \left[\left(-\frac{1940}{81} + \frac{448}{27}\zeta(3) + \frac{10}{9}\zeta(5) + \frac{11}{54}\pi^2\right) C_A \right. \\ &\left. +\left(-\frac{29}{48} + \frac{19}{3}\zeta(3) - \frac{20}{3}\zeta(5)\right) C_F\right] T_F f\ . \end{aligned} \tag{11}$$

Using these three- and four-loop results, one can now computes ρ_2 defined in Eq. (4):

$$\begin{aligned} \rho_2 &\simeq [-4.2140 + 0.03224f + 0.05455f^2 - 8.12\times10^{-4}f^3 - 1.53\times10^{-4}f^4][1 - f/11]^{-2} \quad [\text{for SU(2)}] \\ &\simeq [-8.4102 - 0.50203f + 0.10845f^2 - 2.066\times10^{-3}f^3 - 6.78\times10^{-5}f^4][1 - 2f/33]^{-2} \quad [\text{for SU(3)}] \\ &\simeq [-21.9066 - 1.30140f + 0.13594f^2 - 1.71\times10^{-3}f^3 - 2.44\times10^{-5}f^4][1 - 2f/55]^{-2} \quad [\text{for SU(5)}] \\ &\simeq [-47.8948 - 1.6234f + 0.2569f^2 - 1.680\times10^{-3}f^3 - 1.53\times10^{-4}f^4][1 - f/22]^{-2} \quad [\text{for SO(10)}]\ , \end{aligned}$$

where $d^{abc}d^{abc} = 0$ for SU(2) and SO(10), and 40/3 and 504/5 for SU(3) and SU(5), respectively, have been used. Then I investigate whether Eq. (2) has a positive solution if $f \geq 12, 17, 28, 23$ for SU(2), SU(3), SU(5), SO(10). The result is shown in Table I. As one can see from Table I, $\alpha^*_{\rm OPT}$ for some cases is small so that one may trust the results.

TABLE I. The third-order fixed points ($\alpha^*_{\rm OPT} = a^*_{\rm OPT}\pi$) from the R ratio.

	SU(2)			SU(3)			SU(5)			SO(10)	
f	$(b_0/b_1)\rho_2$	$\alpha^*_{\rm OPT}$	f	$(b_0/b_1)\rho_2$	$\alpha^*_{\rm OPT}$	f	$(b_0/b_1)\rho_2$	$\alpha^*_{\rm OPT}$	f	$(b_0/b_1)\rho_2$	$\alpha^*_{\rm OPT}$
12	−3.317	0.494	17	−18.197	0.096	28	−54.289	0.033	23	−35.281	0.049
13	−1.912	0.856	18	−5.689	0.294	30	−10.073	0.167	26	−8.999	0.183
14	−1.815	0.960	19	−3.794	0.441	32	−6.583	0.259	28	−7.670	0.222
15	−2.014	0.940	20	−3.365	0.516	34	−6.100	0.296	30	−7.852	0.230

(B) The Gross-Llewellyn Smith sum rule. This sum rule says that the first moment of the isospin singlet structure function for the hadronic matrix element which describes deep inelastic processes is 6 at the parton model level: $\int_0^1 dx(F_3^{\bar{\nu}p} + F_3^{\nu p})(x, Q^2/\mu^2, a) = 6\,[\,1 + \mathcal{R}(Q^2/\mu^2, a)\,]$, where x is one of the scaling variables in the processes. The third-order QCD correction has been computed by Larin and Vermaseren [17]:

$$\mathcal{R}^{(3)} = \frac{3}{4} C_F\, a\,(\,1 + r_1 a + r_2 a^2\,)\;, \tag{12}$$

$$r_1(Q^2/\mu^2 = 1) = \frac{23}{12} C_A - \frac{7}{8} C_F - \frac{1}{3} f\;, \tag{13}$$

$$\begin{aligned} r_2(Q^2/\mu^2 = 1) = {} & \left[\frac{5437}{648} - \frac{55}{18}\zeta(5)\right] C_A^2 - \left[\frac{1241}{432} - \frac{11}{9}\zeta(3)\right] C_A C_F + \frac{1}{32} C_F^2 \\ & + \left[\left(-\frac{3535}{1296} - \frac{1}{2}\zeta(3) + \frac{5}{9}\zeta(5)\right) C_A + \left(\frac{133}{864} + \frac{5}{18}\zeta(3)\right) C_F \right. \\ & \left. + \left(\frac{11}{144} - \frac{1}{6}\zeta(3)\right) \frac{d^{abc}d^{abc}}{C_F N_C}\right] f + \frac{115}{648} f^2\;. \end{aligned} \tag{14}$$

As in the case (A), I insert the r_1 and r_2 into ρ_2 in Eq. (4) to obtain

$$\begin{aligned} \rho_2 & \simeq [\,6.8068 - 3.90512 f + 0.57496 f^2 - 3.157 \times 10^{-2} f^3 + 5.48 \times 10^{-4} f^4\,][\,1 - f/11\,]^{-2} \quad [\text{for SU(2)}] \\ & \simeq [\,16.5809 - 6.45245 f + 0.630222 f^2 - 2.2537 \times 10^{-2} f^3 + 2.44 \times 10^{-4} f^4\,][\,1 - 2f/33\,]^{-2} \quad [\text{for SU(3)}] \\ & \simeq [\,47.7897 - 11.25861 f + 0.658477 f^2 - 1.3979 \times 10^{-2} f^3 + 8.77 \times 10^{-5} f^4\,][\,1 - 2f/55\,]^{-2} \quad [\text{for SU(5)}] \\ & \simeq [\,132.1687 - 23.63008 f + 1.340446 f^2 - 2.8751 \times 10^{-2} f^3 + 1.37 \times 10^{-4} f^4\,][\,1 - f/22\,]^{-2} \quad [\text{for SO(10)}]\;. \end{aligned}$$

The values of ρ_2 and $\alpha^*_{\rm OPT}$ for some different $f(> 11C_A/4T_F)$ are shown in Table II.

Note that even in the ideal situation the fixed point values are RS dependent, and they are process dependent in OPT. What should not depend on them are its existence and the value of the anomalous dimensions at the fixed point (critical exponents). Therefore, results (A) and (B) found above are surprisingly similar in the sense that the size of the fixed point values is similar in both cases so that in both cases one could believe the OPT results which indicate the existence of ultraviolet fixed points in ANF Yang-Mills theories.

Triviality of gauged Higgs-Yukawa systems is widely expected, unless they are completely asymptotically free. A rigorous treatment of the asymptotic behavior of the theory with more than one coupling is given in Ref. [12]. It was found [13] that by imposing a certain relation among the gauge, Higgs, and Yukawa couplings which are consistent with perturbative renormalizability, it is possible to make the SU(3)-gauged Higgs-Yukawa system completely asymptotically free and hence nontrivial [18]. This renormalization-group-invariant relation among couplings is a consequence of the reduction of couplings [12].

Inspired by the possibility that ANF Yang-Mills theories may be nontrivial under certain circumstances and by the fact that gauged Higgs-Yukawa systems can be made asymptotically free by means of the reduction of couplings, one may be naturally led to the idea that even ANF gauged Higgs-Yukawa systems are nontrivial if the reduction of couplings is appropriately carried out. One then would achieve a dynamical unification of couplings in a theory, because these couplings are forced in a dynamically consistent fashion to be related to each other in order for the theory to remain well defined and interacting in the ultraviolet limit.

OPT for systems with more than one couplings does not exist yet, because there is no known systematic way to control the propagation of the RS dependence of lower orders to higher orders. But it is clear that once the reduction of couplings is applied to a system with many couplings so that the reduced system contains only one independent coupling, one can employ all the facilities of OPT. Unfortunately, third-order calculations in gauged Higgs-Yukawa systems do not exist yet. Here I would like to present the result of the two-loop reduction in the ANF SU(3)-gauged Higgs-Yukawa theory to motivate corresponding higher-order calculations.

Let me first mention a few words about the reduction of couplings, and consider a massless, renormaliz-

TABLE II. The third-order fixed points ($\alpha^*_{\rm OPT} = a^*_{\rm OPT}\pi$) from the Gross-Llewellyn Smith sum rule.

	SU(2)			SU(3)			SU(5)			SO(10)	
f	$(b_0/b_1)\rho_2$	$\alpha^*_{\rm OPT}$	f	$(b_0/b_1)\rho_2$	$\alpha^*_{\rm OPT}$	f	$(b_0/b_1)\rho_2$	$\alpha^*_{\rm OPT}$	f	$(b_0/b_1)\rho_2$	$\alpha^*_{\rm OPT}$
12	−2.896	0.568	17	−17.196	0.100	28	−52.488	0.034	23	−38.544	0.046
13	−1.279	1.133	18	−4.681	0.339	30	−8.261	0.193	26	−10.708	0.161
14	−1.063	1.333	19	−2.766	0.558	32	−4.725	0.331	28	−8.599	0.204
15	−1.104	1.314	20	−2.296	0.684	34	−4.148	0.390	30	−8.107	0.225

able gauge theory based on a simple gauge group with N other couplings, where the gauge coupling is denoted by α, and the others by α_i , $i = 1, \ldots, N$. The complete reduction of couplings [12] is equivalent to demanding that α_i be written as a power series of α, i.e., $\alpha_i = \sum_{n=0}^{\infty} \eta_i^{(n)} (\alpha/\pi)^n \alpha$, $i = 1, \ldots, N$. As a consequence, the reduced system contains only α as the independent coupling. It was shown [12] that the power series is consistent with perturbative renormalizability only if the reduction equations

$$\beta_\alpha(\alpha, \alpha_i(\alpha)) \frac{d\alpha_i(\alpha)}{d\alpha} = \beta_i(\alpha, \alpha_i(\alpha)) \tag{15}$$

are satisfied, where β_α stands for the β function of α, and β_i for that of α_i. The uniqueness of the power series solution can be decided at the one-loop level, and the η's can be computed order by order in perturbation theory [12].

TABLE III. The expansion coefficients for the reduction of couplings in the SU(3)-gauged Higgs-Yukawa theory.

n_d	$\eta_t^{(0)}$	$\eta_t^{(1)}/\pi$	$\eta_h^{(0)}$	$\eta_h^{(1)}/\pi$
9	2	3.294	1.283	2.586
10	2.296	4.356	1.533	3.592

The gauged Higgs-Yukawa model I consider below can be obtained from the standard model by switching off the SU(2) and U(1) gauge couplings, dropping all leptons, and allowing n_d families of quarks. I also assume that only one of the (up-type) Yukawa couplings is nonvanishing; the simplified system contains only the SU(3) gauge coupling α, the Yukawa coupling α_t, and the Higgs self-coupling α_h. Here I am interested in the case for $n_d > 8$, and recall the β functions [21]

$$\begin{aligned}
\frac{\beta_3}{\pi} &= a^2 \left[-\frac{11}{2} + \frac{2}{3} n_d + \left(\frac{19}{6} n_d - \frac{51}{4} \right) a - \frac{1}{4} a_t + \cdots \right] , \\
\frac{\beta_t}{\pi} &= a_t \left[\frac{9}{4} a_t - 4a + \frac{9}{2} a a_t - \frac{3}{4} a_h a_t - \frac{3}{2} a_t^2 + \left(\frac{10}{9} n_d - \frac{101}{6} \right) a^2 + \frac{3}{16} a_h^2 + \cdots \right] , \\
\frac{\beta_h}{\pi} &= 3a_h^2 + 3a_h a_t - 3a_t^2 - 4a a_t^2 - \frac{3}{16} a_h a_t^2 + \frac{15}{4} a_t^3 - \frac{39}{8} a_h^3 + 5a a_h a_t - \frac{9}{2} a_h^2 a_t + \cdots ,
\end{aligned} \tag{16}$$

where $a_i = \alpha_i/\pi$. It can be shown that the power series solution of the reduction equations (15) with $i = t, h$, i.e.,

$$\alpha_i = \sum_{n=0}^{\infty} \eta_i^{(n)} \left(\frac{\alpha}{\pi} \right)^n \alpha , \quad i = t, h , \tag{17}$$

exists uniquely to all orders in perturbation theory so that the original system with three independent couplings can uniquely be reduced to a system with only one independent coupling, α. The first- and second-order coefficients can be computed by solving Eq. (15) with the second-order β functions (16), and the results are given in Table III. The reduced system has only one β function:

$$\frac{\beta}{\pi} = a^2 \left[-\frac{11}{2} + \frac{2}{3} n_d + \left(-\frac{151}{12} + \frac{169}{54} n_d \right) a + O(a^2) \right] .$$

The fact that the first two coefficients of β for $n_d \geq 9$ are positive (as they are in the previous cases) does not mean anything about a fixed point within the framework of OPT; one needs a complete third-order calculation to obtain ρ_2 and then to solve Eq. (2). If it is negative and large, there will be a small, positive a^*_{OPT}.

Unification of the gauge couplings in ANF extensions of the standard model were previously considered in Ref. [19]. In contrast with the present idea, it was assumed there that the gauge couplings asymptotically diverge so that if one requires the couplings to become strong simultaneously at a certain energy scale, one can predict their low energy values [20]. There are many papers based on this idea, but none of them discusses nontriviality of ANF unified gauge models and its possible relation to unification of couplings. Obviously, there will be many applications of the idea of DUC in constructing realistic unified gauge models, and it is therefore most desirable to justify the assumptions leading to the idea of DUC independently in different approaches.

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