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Electroweak penguin diagrams and two-body B decays

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We discuss the role of electroweak penguin diagrams in B decays to two light pseudoscalar mesons. We confirm that the extraction of the weak phase α through the isospin analysis involving $B \to \pi\pi$ decays is largely unaffected by such operators. However, the methods proposed to obtain weak and strong phases by relating $B \to \pi\pi$, $B \to \pi K$, and $B \to K\bar{K}$ decays through flavor SU(3) will be invalidated if electroweak penguin diagrams are large. We show that, although the introduction of electroweak penguin contributions introduces no new amplitudes of flavor SU(3), there are a number of ways to experimentally measure the size of such effects. Finally, using SU(3) amplitude relations we present a new way of measuring the weak angle γ which holds even in the presence of electroweak penguin diagrams.

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I. INTRODUCTION

The B system is the ideal place to measure the phases of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The weak phases α, β, and γ can be measured in numerous ways through asymmetries and rate measurements of various B decays [1]. Ultimately it will be possible to verify the relation $\alpha = \pi - \beta - \gamma$, predicted within the standard model.

The conventional method for obtaining the angle α is through the measurement of the time-dependent rate asymmetry between the process $B^0 \to \pi^+\pi^-$ and its CP conjugate. This assumes that the decay is dominated by one weak amplitude—the tree diagram. However, there is also a penguin contribution to the decay, which has a different weak phase than the tree diagram. This introduces a theoretical uncertainty into the extraction of α. Fortunately, this uncertainty can be removed by the use of isospin [2]. The two final-state pions can be in a state with $I = 2$ or $I = 0$. But the penguin diagram, which is mediated by gluon exchange, contributes only to the $I = 0$ final state. Thus, by isolating the $I = 2$ component, one can isolate the tree contribution, thereby removing the uncertainty due to the penguin diagrams. This can be done through the use of an isospin triangle relation among the amplitudes for $B^+ \to \pi^+\pi^-$, $B^0 \to \pi^+\pi^-$, and $B^0 \to \pi^0\pi^0$. By measuring the rates for these processes, as well as their CP-conjugate counterparts, it is possible to isolate the $I = 2$ component and obtain α with no theoretical uncertainty. The crucial factor in this method is that the $I = 2$ amplitude is pure tree and hence has a well-defined CKM phase.

Recently, it was proposed that the phases of the CKM matrix could be determined through the measurement of various decay rates of B mesons to pairs of light pseudoscalars [3–5]. This was based on two assumptions: (i) a flavor SU(3) symmetry [6–8] relating $B \to \pi\pi$, $B \to \pi K$, and $B \to K\bar{K}$ decays, and (ii) the neglect of exchange- and annihilation-type diagrams, which are expected to be small for dynamical reasons. For example, it was suggested that the weak phase γ [equal to $\arg(V_{ub}^*)$ in the Wolfenstein parametrization [9]], could be found by measuring rates for the decays $B^+ \to \pi^0 K^+$, $B^+ \to \pi^+ K^0$, $B^+ \to \pi^+\pi^0$, and their charge-conjugate processes [4]. The πK final states have both $I = 1/2$ and $I = 3/2$ components. The key observation is that the gluon-mediated penguin diagram contributes only to the $I = 1/2$ final state. Thus, a linear combination of the $B^+ \to \pi^0 K^+$ and $B^+ \to \pi^+ K^0$ amplitudes, corresponding to $I = 3/2$ in the πK system, could be related via flavor SU(3) to the purely $I = 2$ amplitude in $B^+ \to \pi^+\pi^0$, permitting the construction of an amplitude triangle. The difference in the phase of the $B^+ \to \pi^+\pi^0$ side and that of the corresponding triangle for B^- decays was found to be 2γ. Taking SU(3) breaking into account, the analysis is unchanged, except that one must include a factor f_K/f_π in relating $B \to \pi\pi$ decays to the $B \to \pi K$ decays [10]. The weak phase γ can also be extracted in an *independent* way, along with the CKM phase α and all the strong final-state phases, by measuring the rates for another set of seven decays, along with the rates for the charge-conjugate decays [5]. [SU(3)-breaking effects are discussed in [10].] This method also relies on the

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SU(3) relation between the $I = 3/2\,\pi K$ amplitude and the $I = 2\,\pi\pi$ amplitude.

The crucial ingredient in the above analyses is that the penguin diagram is mediated by gluon exchange. However, there are also electroweak contributions to the processes $b \to sq\bar{q}$ and $b \to dq\bar{q}$, consisting of γ and Z penguin and box diagrams [11]. (From here on, we generically refer to all of these as "electroweak penguin diagrams.") Since none of the electroweak gauge bosons is an isosinglet, these diagrams can affect the above arguments. For the $B \to \pi\pi$ isospin analysis, the result is that the $I = 2$ state will no longer have a well-defined weak CKM phase. For the $B \to \pi\pi/\pi K$ analyses, in the presence of electroweak penguin diagrams there are no longer triangle relations among the $B \to \pi K$ and $B \to \pi\pi$ amplitudes. Theoretical estimates [12] have indicated that electroweak penguin diagrams are expected to be relatively unimportant for $\pi\pi$. However, they are expected to play a significant role in the πK case, introducing considerable uncertainties in the extraction of γ as described above.

The purpose of the present paper is to examine the role of electroweak penguin diagrams in all $B \to PP$ decays, where P denotes a light pseudoscalar meson. We wish to address the following questions: (1) To what B decays do electroweak penguin diagrams contribute? (2) Can one obtain information on their magnitude directly from the data? (3) Can one extract weak CKM phases in the presence of electroweak penguin diagrams?

We answer the first question by including the electroweak penguin contributions in a general graphical description of all $B \to PP$ amplitudes, which was shown to be a useful representation of flavor SU(3) amplitudes [3].

The second question is answered in the affirmative. An explicit calculation of electroweak penguin diagrams [13] suggests that they could dominate in decays of the form $B_s \to (\phi \text{ or } \eta) + (\pi \text{ or } \rho)$. We find that there are additional measurements which are indirectly sensitive to such contributions.

As to the third question, we find that it is indeed possible to obtain information about the CKM angle γ, even in the presence of electroweak penguin diagrams. While the method proposed makes use of a considerably larger number of measurements than the original simple set proposed in [3–5], there is no difficulty *in principle* in obtaining the necessary information from experiment alone. Whether these measurements are feasible *in practice* in the near term is another story, which we shall address as well. The four amplitudes for different charge states in $B \to \pi K$ decays satisfy a quadrangle relation dictated entirely by isospin. When sides are chosen in an appropriate order, we find that one diagonal of the quadrangle is related to the rate for $B_s \to \pi^0\eta$, so that (up to discrete ambiguities) the quadrangle is of well-defined shape. The difference between the other diagonal and the corresponding quantity for charge-conjugate processes, when combined with the rate for $B^+ \to \pi^+\pi^0$, provides information on $\sin\gamma$.

We discuss general aspects of electroweak penguin diagrams in Sec. II, with particular emphasis on estimates of the size of such effects. In Sec. III we examine the electroweak penguin contributions to $B \to PP$ decays. The quadrangle for $B \to \pi K$ decays is treated in Sec. IV. Experimental prospects are noted in Sec. V, while Sec. VI summarizes.

II. ELECTROWEAK PENGUIN DIAGRAMS: GENERAL CONSIDERATIONS

Our approach is intended to rely to the greatest possible degree on model-independent flavor SU(3) arguments rather than on specific *a priori* calculations of amplitudes. Thus the estimates to be presented in the present section are intended primarily to indicate which contributions should be retained in an SU(3) analysis, and which may be safely neglected.

A. How big are electroweak penguin diagrams?

The standard penguin diagram involves a charge-preserving, flavor-changing transition of a heavy quark to a lighter one by means of a loop diagram involving a virtual W and quarks, and emission of one or more gluons. The penguin diagrams involving $\bar{b} \to \bar{d}$ transitions change isospin by 1/2 unit, while $\bar{b} \to \bar{s}$ transitions leave isospin invariant.

Penguin diagrams in which the $\bar{b} \to \bar{q}$ system is coupled to other quarks through the photon or Z (or through box diagrams involving W's) instead of through gluons have more complicated isospin properties. There will be contributions in which the additional quark pair is isoscalar (as in the conventional penguin graphs), but others in which it is isovector.

The importance of electroweak penguin (EWP) diagrams was realized in the calculation of the parameter ϵ'/ϵ describing direct CP violation in $K_L \to \pi\pi$ [14]. That parameter requires an imaginary part of the ratio A_2/A_0, where the subscript denotes the isospin $I_{\pi\pi}$ of the $\pi\pi$ system. The EWP can provide an $I_{\pi\pi} = 2$ contribution, whereas the conventional penguin diagram cannot. The numerical importance of the EWP diagram involving Z exchange is enhanced by a factor of m_t^2/M_Z^2 [15].

A similar circumstance was realized by Deshpande and He [12] to apply to two cases: (a) An isospin triangle for $B \to \pi\pi$ decays, while continuing to hold, receives small contributions from electroweak penguin diagrams. This can in principle affect the analysis proposed in [2] for extracting the weak phase α. (b) The validity of the SU(3) triangle proposed in [3–5], involving the comparison of $B \to \pi\pi$ and $B \to \pi K$ decays, is also affected.

The dominant electroweak penguin contribution arises from Z exchange. There are two such diagrams, shown in Fig. 1. The distinction between the two is that the diagram of Fig. 1(a) is color allowed, while that of Fig. 1(b) is color suppressed. We refer to these as $P_{\rm EW}$ and $P_{\rm EW}^C$, respectively. Thus EWP effects will be most important when the $P_{\rm EW}$ diagram is involved, that is, when there is a nonstrange neutral particle in the final state, such as π^0, η, ρ^0, or ϕ. All-charged final states will be less affected

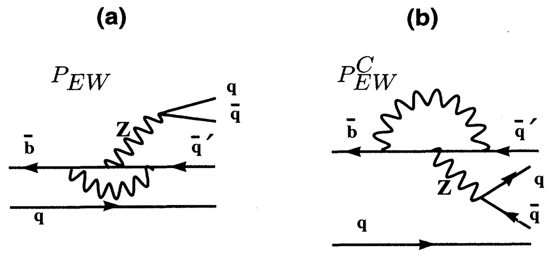

FIG. 1. (a) Color-allowed Z-penguin diagram, (b) color-suppressed Z-penguin diagram.

by the presence of electroweak penguin diagrams, since in this case only the $P^C_{\rm EW}$ diagram can arise. EWP diagrams which involve the annihilation of the quarks in the initial B meson are suppressed by a factor of $f_B/m_B \approx 5\%$. As we will see from the hierarchy of diagrams discussed in the next section, this means that we will always be able to ignore annihilation-type EWP diagrams.

The ratio of a $P_{\rm EW}$ electroweak penguin to a gluonic penguin contribution P in b quark decays contains a factor of $\alpha_2/\alpha_s \approx (1/30)/0.2 \approx 1/6$, where we have evaluated both couplings at m_b. The electroweak penguin diagram for Z exchange contains a factor of $(m_t/M_Z)^2 \approx 4$ in contrast with a logarithm $\ln(m_t^2/m_c^2) \approx 9$ in the gluonic penguin diagram. Thus the overall electroweak penguin diagram's amplitude should be $\sim 10\%$ that of the gluonic penguin, modulo group-theoretic factors. This is in qualitative accord with the result of [11,12].

A more quantitative calculation of the ratio $P_{\rm EW}/P$ will necessarily involve hadronic physics. In particular, the matrix elements for $P_{\rm EW}$ and P are almost certainly different, since the two diagrams clearly have different dynamical structures. Such model-dependent calculations are fraught with uncertainties [16]. (For example, although it might be argued that factorization applies to the $P_{\rm EW}$ diagram, it is considerably more doubtful for P.) Thus theoretical calculations of $P_{\rm EW}/P$ [11,12] should be viewed with a certain amount of skepticism. Still, the magnitude of this ratio is very important. As we will see in the following sections, the methods presented in [3–5] for the extraction of weak and strong phases will be invalidated if EWP's are too large, say $P_{\rm EW}/P \gtrsim 20\%$. For these reasons it is important to try to obtain information about electroweak penguin diagrams from the data.

B. Diagrams and hierarchies

There are, of course, other diagrams which contribute to $B \to PP$ decays, and it is equally important to estimate the size of electroweak penguin diagrams relative to these other contributions.

Excluding electroweak penguin diagrams, there are six distinct diagrams which contribute to B decays: (1) a (color-favored) "tree" amplitude T, T'; (2) a "color-suppressed" amplitude C, C'; (3) a "penguin" amplitude P, P'; (4) an "exchange" amplitude E, E'; (5) an "annihilation" amplitude A, A'; (6) a "penguin annihilation" amplitude PA, PA'. (We refer the reader to Ref. [3] or [10] for a more complete discussion of the diagrams.) For T, C, E, and A, the unprimed and primed amplitudes contribute to the decays $\bar b \to \bar u u \bar d$ and $\bar b \to \bar u u \bar s$, respectively, and the primed amplitudes are related to their unprimed counterparts by a factor of $|V_{us}/V_{ud}| \simeq \lambda = 0.22$. For P and PA the unprimed and primed amplitudes contribute to the decays $\bar b \to \bar d$ and $\bar b \to \bar s$, respectively. In this case, the primed amplitudes are actually *larger* than the unprimed amplitudes by a factor of $|V_{ts}/V_{td}|$, which is of order $1/\lambda$.

In Ref. [10] we estimated the relative sizes of these diagrams in $B \to PP$ decays. Here we include electroweak penguin diagrams, justifying our estimates of their magnitudes after presenting the expected hierarchies.

(1) $\bar b \to \bar u u \bar d$ and $\bar b \to \bar d$ transitions. The dominant diagram is T. Relative to the dominant contribution, we expect

$$\begin{aligned} 1 &: |T| \ , \\ O(\lambda) &: |C|, |P| \ , \\ O(\lambda^2) &: |E|, |A|, |P_{\rm EW}| \ , \\ O(\lambda^3) &: |PA|, |P'^C_{\rm EW}| \ . \end{aligned} \tag{1}$$

(2) $\bar b \to \bar u u \bar s$ and $\bar b \to \bar s$ transitions. Here the dominant diagram is P'. Relative to this, we estimate

$$\begin{aligned} 1 &: |P'| \ , \\ O(\lambda) &: |T'|, |P'_{\rm EW}| \ , \\ O(\lambda^2) &: |C'|, |PA'|, |P'^C_{\rm EW}| \ , \\ O(\lambda^3) &: |E'|, |A'| \ . \end{aligned} \tag{2}$$

The use of the parameter $\lambda = 0.22$ here is unrelated to CKM matrix elements—it is simply used as a measure of the approximate relative sizes of the various contributions. For instance, $|C/T| \sim \lambda$ is due to color suppression, while E and A are suppressed relative to T by the factor $f_B/m_B \approx 0.05 \sim \lambda^2$. Similarly, $PA/P \sim f_B/m_B$. Although it is fairly certain that P' dominates the second class of decays, the value of the ratio $|T'/P'|$ is less clear. Our value of λ for this ratio is probably a reasonable estimate. Finally as discussed in Ref. [10], we expect the SU(3) corrections to a diagram to be roughly 20% ($\sim \lambda$) of that particular diagram. We shall discuss SU(3)-breaking effects in the cases of several specific processes of interest in Secs. III and IV.

Note that both of the above hierarchies are educated guesses—it is important not to take them too literally. Since λ is not that small a number, a modest enhancement or suppression (due to hadronic matrix elements, for example) can turn an effect of $O(\lambda^n)$ into an effect of $O(\lambda^{n\pm1})$. Ultimately experiment will tell us exactly how large the various diagrams are.

Some combination of the decays $B^0 \to \pi^+\pi^-$ and $B^0 \to K^+\pi^-$ has been observed [17]. The most likely branching ratios for these two modes are both about 10^{-5} (though all that can be conclusively said is that their sum is about 2×10^{-5}). One then concludes that the T and P' amplitudes are about the same size. In this case, the estimated hierarchies in Eqs. (1) and (2) can be combined.

The above estimated hierarchies can be used to judge how large electroweak penguin effects should be. Our naive estimate of $P_{\rm EW}/P$ was $\sim 10\%$. Allowing for some variation in either direction, we have $P_{\rm EW}/P \sim O(\lambda) - O(\lambda^2)$. Thus, for $\bar b \to \bar u u \bar d/\bar b \to \bar d$ decays, EWP's are at most $O(\lambda^2)$ of the dominant T contribution. For this reason it is unlikely that electroweak penguin diagrams will significantly affect $B \to \pi\pi$ decays. On the other hand, for $\bar b \to \bar u u \bar s/\bar b \to \bar s$ decays, EWP contributions can be as much as $O(\lambda)$ of the dominant P' diagrams, which is why they may be important in $B \to \pi K$ decays.

As discussed in the previous section the color-suppressed electroweak penguin diagram $P_{\rm EW}^C$ should be smaller than its color-allowed counterpart $P_{\rm EW}$ by approximately a factor of λ. Thus this contribution is probably completely negligible in $\bar b \to \bar u u \bar d/\bar b \to \bar d$ decays, and is at most a 5% effect in $\bar b \to \bar u u \bar s/\bar b \to \bar s$ decays relative to the dominant P' contribution.

III. $B \to PP$ DECAYS

A. Decomposition in terms of SU(3) amplitudes

We review briefly the SU(3) discussion of [3]. The weak Hamiltonian operators associated with the transitions $\bar b \to \bar q u \bar u$ and $\bar b \to \bar q$ ($q = d$ or s) transform as a $\mathbf{3}^*$, $\mathbf{6}$, or $\mathbf{15}^*$ of SU(3). These combine with the triplet light quark in the B meson and couple to a symmetric product of two octets (the pseudoscalar mesons) in the final state, leading to decays characterized by one singlet, three octets, and one 27-plet amplitude. Separate amplitudes apply to the cases of strangeness-preserving and strangeness-changing transitions. The diagrams T–PA are a useful representation of flavor SU(3) amplitudes. Although there are six types of diagram (excluding electroweak penguin diagrams), they only appear in five linear combinations in $B \to PP$ decays, in accord with the group theory result.

The inclusion of electroweak penguin diagrams does not affect this picture. The ratio of transitions $\bar b \to \bar q u \bar u$, $\bar b \to \bar q d \bar d$, and $\bar b \to \bar q s \bar s$ is altered, but the $\bar b \to \bar q d \bar d$ and $\bar b \to \bar q s \bar s$ terms remain equal. [This is obvious for the γ and Z penguin diagrams. For the box diagrams, this equality is ensured by the Glashow-Iliopoulos-Maiani (GIM) mechanism. There are contributions from the boxes which break this equality, but they are much suppressed relative to the dominant term.] The weak Hamiltonian thus continues to contain terms transforming as a $\mathbf{3}^*$, $\mathbf{6}$, or $\mathbf{15}^*$ of SU(3), but in different proportions. Thus, even if one includes electroweak penguin graphs, there must continue to be five independent amplitudes describing $\Delta S = 0$ decays and five other amplitudes describing $|\Delta S| = 1$ decays. However, some of the correspondence between $\Delta S = 0$ and $|\Delta S| = 1$ decays present in the previous description will be altered. In this section we extend the decomposition of $B \to PP$ decays in terms of the diagrams T–PA to include the electroweak penguin diagrams of Fig. 1. In this way we see explicitly how $B \to \pi\pi$ and $B \to \pi K$ decays are affected by electroweak penguin diagrams.

In [3] it was argued that the diagrams E, A, and PA (and their primed counterparts) are negligible since they are suppressed by a factor of $f_B/m_B = O(\lambda^2)$ and hence are unlikely to be important in many cases. However, there are processes such as $B^0 \to \pi^0\pi^0$, $B^+ \to K^+\bar K^0$, and $B_s \to \pi^0 \bar K^0$ which are dominated by the $O(\lambda)$ terms C and/or P. In these cases diagrams suppressed by $O(\lambda^2)$ with respect to the dominant T contributions, such as E, A, and $P_{\rm EW}$, can cause a significant change in the rate. There are situations, which we will soon discuss, when one cannot neglect such seemingly small diagrams. These are precisely the cases where EWP's are important.

We continue to use the approximation of ignoring E, A, and PA-type diagrams when considering electroweak penguin effects as long as their effects are $O(\lambda^2)$ with respect to the dominant contribution to a process. Annihilation-type electroweak penguin amplitudes will always be subdominant by at least $O(\lambda^2)$ in all the processes we will consider and hence we can ignore them. In $\Delta S = 0$ decays (Table I), the E contribution to $B^0 \to \pi^+\pi^-$ can really be ignored; we include it only to make evident the isospin triangle relation among the $B^+ \to \pi^+\pi^-$, $B^0 \to \pi^+\pi^-$, and $B^0 \to \pi^0\pi^0$ amplitudes. Similarly, in $|\Delta S = 1|$ decays (Table II), the C' contribution should really be dropped, since it is expected to be of the same order as the PA' diagram, which has been neglected. Nevertheless, we continue to keep track of the C' contribution in such decays since it is related to the non-negligible C diagram in $\Delta S = 0$ decays. (Obviously our results should not, and do not, depend on keeping or ignoring the C' contribution.)

The distinction between the gluonic penguin P and the electroweak penguin diagram $P_{\rm EW}$ is the coupling to the light quarks. In P, the quarks u, d, and s have equal couplings to the gluon. In $P_{\rm EW}$, however, the u and d/s quarks are treated differently. Schematically, we can represent the couplings of the strong and electroweak penguin diagrams as follows:

$$P : u\bar u + d\bar d + s\bar s \ ,$$
$$P_{\rm EW}, P_{\rm EW}^C : \frac{2}{3} u\bar u - \frac{1}{3}(d\bar d + s\bar s) \ . \qquad (3)$$

TABLE I. Decomposition of $B \to PP$ amplitudes for $\Delta C = \Delta S = 0$ transitions in terms of graphical contributions of Refs. [3,10] and Fig. 1. For completeness we include color-suppressed $P_{\rm EW}^C$ contributions even when they are estimated to be negligible.

Final state	T, C, P contributions	Electroweak penguin diagrams
$B^+ \to \pi^+\pi^0$	$-(T+C)/\sqrt{2}$	$-[P_{\rm EW} + P_{\rm EW}^C]/\sqrt{2}$
$K^+\bar K^0$	$P + A$	$-\frac{1}{3}P_{\rm EW}^C$
$B^0 \to \pi^+\pi^-$	$-(T+P+E)$	$-\frac{2}{3}P_{\rm EW}^C$
$\pi^0\pi^0$	$-(C-P-E)/\sqrt{2}$	$-[P_{\rm EW} + \frac{1}{3}P_{\rm EW}^C]/\sqrt{2}$
$K^0\bar K^0$	P	$-\frac{1}{3}P_{\rm EW}^C$
$B_s \to \pi^+K^-$	$-(T+P)$	$-\frac{2}{3}P_{\rm EW}^C$
$\pi^0\bar K^0$	$-(C-P)/\sqrt{2}$	$-[P_{\rm EW} + \frac{1}{3}P_{\rm EW}^C]/\sqrt{2}$

TABLE II. Decomposition of $B \to PP$ amplitudes for $\Delta C = 0, |\Delta S| = 1$ transitions in terms of graphical contributions of Refs. [3,10] and Fig. 1. For completeness we include C' and the color-suppressed $P'^{C}_{\rm EW}$ contributions even though they are estimated to be negligible.

Final state	P', T', C' contributions	Electroweak penguin diagrams
$B^+ \to \pi^+ K^0$	P'	$-\frac{1}{3}P'^{C}_{\rm EW}$
$\pi^0 K^+$	$-(P'+T'+C')/\sqrt{2}$	$-[P'_{\rm EW} + \frac{2}{3}P'^{C}_{\rm EW}]/\sqrt{2}$
$B^0 \to \pi^- K^+$	$-(P'+T')$	$-\frac{2}{3}P'^{C}_{\rm EW}$
$\pi^0 K^0$	$(P'-C')/\sqrt{2}$	$-[P'_{\rm EW} + \frac{1}{3}P'^{C}_{\rm EW}]/\sqrt{2}$
$B_s \to K^+ K^-$	$-(P'+T')$	$-\frac{2}{3}P'^{C}_{\rm EW}$
$K^0 \bar{K}^0$	P'	$-\frac{1}{3}P'^{C}_{\rm EW}$

Here the electroweak penguin diagram is taken to couple to the charge of the quarks as it would if it arose purely from photon exchange; any residual effect may be subsumed under the much larger gluonic penguin contribution.

In Tables I and II we present the decomposition of the 13 $B \to PP$ decays in terms of the various diagrams, for $P = \pi$ or K. We warn the reader that non-negligible SU(3)-breaking corrections can lead to differences in certain decays that appear equal in the above tables. For example, according to Table II, $B^+ \to \pi^+ K^0$ and $B_s \to K^0 \bar{K}^0$ will have the same rate. However, SU(3)-breaking effects introduce a rate difference here. We refer the reader to Ref. [10] for more details. We shall, however, correctly include SU(3)-breaking effects when discussing specific examples in the following sections. Note that the inclusion of electroweak penguin diagrams leads to the following replacement in the corresponding tables of Ref. [3]: $P \to P - \frac{1}{3}P^{C}_{\rm EW}, C \to C + P_{\rm EW}, T \to T + P^{C}_{\rm EW}$.

B. Effects on CP analyses

There are several interesting aspects of Tables I and II worth mentioning.

(1) $B \to \pi\pi$ decays. Consider the $B \to \pi\pi$ decays in Table I. The decay $B^+ \to \pi^+\pi^0$, which is purely $I = 2$, has an electroweak penguin component. If our estimated hierarchy is accurate, this component should be between $O(\lambda^2)$ and $O(\lambda^3)$ of the dominant T contribution. This is in agreement with Deshpande and He [12], who find that $|A_{\rm EWP}/A_T| \approx 1.6\%|V_{td}/V_{ub}|$ for this decay. In other words, the EWP contribution to $B^+ \to \pi^+\pi^0$ is very small. It is even smaller in the decay $B^0 \to \pi^+\pi^-$, since only the color-suppressed EWP can contribute here. On the other hand, electroweak penguin diagrams can be more significant in $B^0 \to \pi^0\pi^0$ decays, since this decay suffers color suppression.

The size of EWP's is relevant to the extraction of α via the analysis proposed in [2]. Let us study this effect in detail. This analysis requires measuring the (time-integrated) rates of $B^+ \to \pi^+\pi^0$, $B^0 \to \pi^+\pi^-$, $B^0 \to \pi^0\pi^0$, and their CP-conjugate counterparts, and

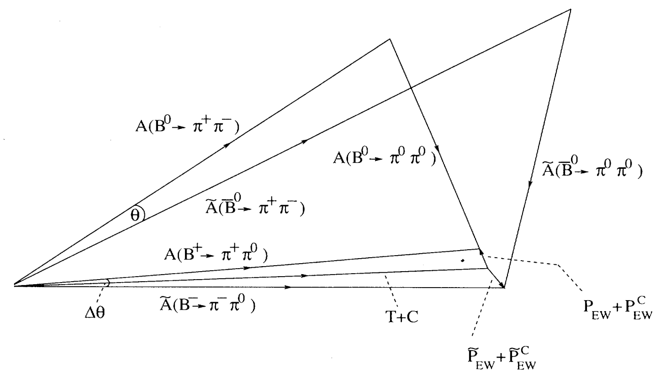

FIG. 2. Isospin analysis of $B \to \pi\pi$ decays with the inclusion of electroweak penguin diagrams. The amplitudes $\tilde{A}(\bar{B} \to \pi\pi)$ are defined as $\exp(2i\gamma)A(\bar{B} \to \pi\pi)$, with similar definitions for $\tilde{P}_{\rm EW}$ and $\tilde{P}^{C}_{\rm EW}$.

observing the time-dependence of $B^0(t) \to \pi^+\pi^-$. The amplitudes of these six processes form two triangles, as shown in Fig. 2, in which the CP-conjugate amplitudes have been rotated by a common phase $\tilde{A}(\bar{B} \to \pi\pi) \equiv \exp(2i\gamma)A(\bar{B} \to \pi\pi)$ (and similarly for $\tilde{P}_{\rm EW}$ and $\tilde{P}^{C}_{\rm EW}$). The CKM phase α is measured from the time-dependent rate of $B^0(t) \to \pi^+\pi^-$, which involves a term

$$\left|\frac{\tilde{A}(\bar{B}^0 \to \pi^+\pi^-)}{A(B^0 \to \pi^+\pi^-)}\right| \sin(2\alpha + \theta)\sin(\Delta m t) \ , \qquad (4)$$

where Δm is the neutral B mass difference. The angle θ is measured as shown in Fig. 2.

The effect of the EWP amplitudes on determining θ and correspondingly fixing α is rather clearly represented by the small vectors at the right bottom corner of the Fig. 2. These terms, given by $P_{\rm EW} + P^{C}_{\rm EW}$ and its CP conjugate, have unknown phases relative to the $T + C$ term which dominates $A(B^+ \to \pi^+\pi^0)$ and its charge conjugate. This leads to a very small uncertainty in the relative orientation of the two triangles. [In the limit of neglecting EWP amplitudes, one would have $\tilde{A}(B^- \to \pi^-\pi^0) = A(B^+ \to \pi^+\pi^0)$.] The uncertainty in measuring θ, and consequently in determining α, is given by

$$\Delta\alpha \approx \frac{1}{2}\Delta\theta \leq \left|\frac{P_{\rm EW} + P^{C}_{\rm EW}}{T + C}\right| \ . \qquad (5)$$

We therefore conclude that the effects of EWP amplitudes on the measurement of α are at most of order λ^2 and are negligible.

Since a different conclusion has been claimed in [12,18], let us clarify the apparent disagreement. The authors of [12,18] have only shown that the error in determining α from the rate of $B^0(t) \to \pi^+\pi^-$ is large. This is dominantly the effect of the gluonic penguin diagram, as already noted in [19]. They have not separated the effect of EWP amplitudes. Figure 2 shows clearly how small this effect is.

(2) $B \to \pi K$ decays. We now turn to the $B \to \pi K$ decays in Table II. In the absence of electroweak penguin diagrams, one can write two triangle relations involving amplitudes in both the $\Delta S = 0$ and $|\Delta S| = 1$ sectors:

$$\sqrt{2}A(B^0 \to \pi^0 K^0) + A(B^0 \to \pi^- K^+) = \lambda\sqrt{2}A(B^+ \to \pi^+\pi^0)$$
$$-(C' - P') - (P' + T') = -\lambda(T + C) \ , \quad (6)$$
$$\sqrt{2}A(B^+ \to \pi^0 K^+) + A(B^+ \to \pi^+ K^0) = \lambda\sqrt{2}A(B^+ \to \pi^+\pi^0)$$
$$-(T' + C' + P') + (P') = -\lambda(T + C) \ . \quad (7)$$

SU(3) breaking can be taken into account by including a factor of f_K/f_π on the right-hand side [10]. In Eq. (7) above, SU(3) relates the $I = 3/2\,\pi K$ amplitude to the $I = 2\,\pi\pi$ amplitude. By measuring the three rates involved in the triangle relation, as well as their CP conjugates, the weak CKM angle $\gamma = \arg(V_{ub}^*)$, which is the weak phase of $A(B^+ \to \pi^+\pi^0)$, can be extracted [4]. By using both Eqs. (6) and (7), strong final-state phases and the sizes of the different diagrams can also be extracted [5].

When electroweak penguin diagrams are included, however, these two triangle relations no longer hold. For example, the left-hand side of Eq. (7) is now equal to

$$-[T' + C' + P'_{\rm EW} + P'^{C}_{\rm EW}] \ , \quad (8)$$

while the right-hand side is

$$-\lambda[T + C + P_{\rm EW} + P^{C}_{\rm EW}] \ . \quad (9)$$

Despite their similarity, these two expressions are not equal since the relation between nonpenguin contributions ($T'/T = C'/C = \lambda$) does not hold for the electroweak penguin diagrams: $|P'_{\rm EW}/P_{\rm EW}| = |V_{ts}/V_{td}| \sim 1/\lambda$.

From our previous discussion, we estimate that $|P'_{\rm EW}/T'|$ may be as much as ~ 1. Eventually, it will be up to experiment to determine the size of electroweak penguin diagrams. However, in a realistic scenario, with hierarchies such as those discussed in Sec. II B, EWP's lead to large uncertainties in the extraction of weak CKM angles and strong phases through the analyses of Refs. [4,5]. In Sec. IV we extend the SU(3) triangle analysis of Ref. [4] to a quadrangle relation, using more decay rate measurements to exhibit a new way of measuring the weak angle γ which holds even in the presence of electroweak penguins.

C. Experimental signals

As discussed above, the fate of the analyses of Refs. [4,5] for extracting weak CKM phase information depends crucially on the size of electroweak penguins. Rather than relying on theoretical calculations, which inevitably have uncertainties due to hadronic matrix elements, it would be preferable to obtain this information from experiment.

Electroweak penguins are expected to dominate decays of the form $B_s \to (\phi \text{ or } \eta) + (\pi \text{ or } \rho)$ [13]. This is easy to understand in terms of diagrams:

$$A[B_s \to (\phi \text{ or } \eta) + (\pi \text{ or } \rho)] \sim -C' + E' - P'_{\rm EW} \ . \quad (10)$$

We have already argued that the E' diagram is small, so, from Eq. (2) and the discussion following it, we see that the dominant contribution is $P'_{\rm EW}$.

Unfortunately, even though these decays are dominated by electroweak penguins, their branching ratios are all small, less than $\sim 10^{-6}$. Furthermore, they all involve the decays of B_s mesons, which are not as accessible experimentally. This leads to the obvious question: are there signals for electroweak penguin diagrams which involve decays of $B^\pm$ or B^0 mesons, and which have large branching ratios? Indeed there are. Consider the decays $B^+ \to \pi^0 K^+$ and $B^0 \to \pi^- K^+$. From Table II, we have

$$\sqrt{2}A(B^+ \to \pi^0 K^+) \simeq -[T' + P' + P'_{\rm EW}],$$
$$A(B^0 \to \pi^- K^+) \simeq -[T' + P'] \ , \quad (11)$$

where we have dropped the (much smaller) terms C' and $P'^{C}_{\rm EW}$. Both of these decays should have branching ratios of $\sim 10^{-5}$ as a result of the dominant P' contribution. A difference in the ratio $2B(B^+ \to \pi^0 K^+)/B(B^0 \to \pi^- K^+)$ from 1 would most likely signify the presence of electroweak penguin diagrams (although it could also indicate the importance of the omitted C' term, or other terms such as A' which we expect to be even smaller). Though indirect, this is very likely to be the first experimental test of such effects. Similarly, the most likely source of a difference in the ratio $2B(B^0 \to \pi^0 K^0)/B^+ \to \pi^+ K^0$ from 1 will be the contribution of electroweak penguin diagrams.

IV. AMPLITUDE QUADRANGLES

A. SU(3)-invariant analysis for $B \to \pi K$

The decays $B \to \pi K$ involve a weak Hamiltonian with both $I = 0$ and $I = 1$ terms. The $I = 0$ piece can lead only to a πK final state with $I = 1/2$, while the $I = 1$ piece can lead to both $I = 1/2$ and $I = 3/2$ final states. Thus there are two decay amplitudes leading to $I_{\pi K} = 1/2$ and one leading to $I_{\pi K} = 3/2$. Since there are four amplitudes for $B \to \pi K$ decays, they satisfy a quadrangle, which we may write as [20,21]

$$A(B^+ \to \pi^+ K^0) + \sqrt{2}A(B^+ \to \pi^0 K^+) = \sqrt{2}A(B^0 \to \pi^0 K^0) + A(B^0 \to \pi^- K^+) = A_{3/2} \ . \quad (12)$$

With the phase conventions adopted in [3], the quadrangle has the shape shown in Fig. 3, with two short diagonals. These diagonals are

$$D_1 = -[T' + C' + P'_{\rm EW} + P'^{C}_{\rm EW}] \ ,$$
$$D_2 = -C' - P'_{\rm EW} - A' \ . \quad (13)$$

The first of these diagonals, D_1, is just the amplitude $A_{3/2}$. The key point is that $A(B_s \to \pi^0\eta) = -[C' + P'_{\rm EW} - E']/\sqrt{3}$, for an octet η. Thus, ignoring the very small E' and A' diagrams, the second diagonal D_2 is in

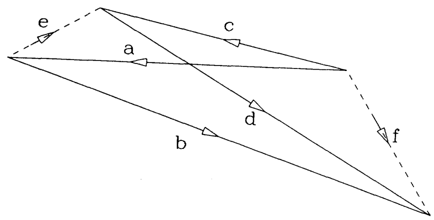

FIG. 3. Amplitude quadrangle for $B \to \pi K$ decays. (a) $A(B^+ \to \pi^+ K^0)$; (b) $\sqrt{2}A(B^+ \to \pi^0 K^+)$; (c) $\sqrt{2}A(B^0 \to \pi^0 K^0)$; (d) $A(B^0 \to \pi^- K^+)$; (e) the diagonal $D_2 = \sqrt{3}A(B_s \to \pi^0\eta)$; (f) the diagonal $D_1 = A_{3/2}$ corresponding to the $I = 3/2$ amplitude.

fact equal to $\sqrt{3}A(B_s \to \pi^0\eta)$. Therefore the shape of the quadrangle is uniquely determined, up to possible discrete ambiguities. The case of octet-singlet mixtures in the η simply requires us to replace the $\sqrt{3}$ by the appropriate coefficient [22], since one can show that the singlet piece of η does not contribute appreciably here.

The quadrangle has been written in such a way as to illustrate the fact, noted in Refs. [3–5], that the $B^+ \to \pi^+ K^0$ amplitude receives only penguin contributions in the absence of $O(f_B/m_B)$ corrections. The weak phases of both gluonic and electroweak $\bar{b} \to \bar{s}$ penguins, which are dominated by a top quark in the loop, are expected to be π. We have oriented the quadrangle to subtract out the corresponding strong phase.

The $I = 3/2$ amplitude is composed of two parts:

$$A_{3/2} = |A^T_{\pi K}|e^{i\gamma}e^{i\tilde{\delta}_T} - |A^{\rm EWP}_{\pi K}|e^{i\tilde{\delta}_{\rm EWP}} , \qquad (14)$$

where we have explicitly exhibited electroweak and final-state phases, and the tildes denote differences with respect to the strong phase shift in the $B^+ \to \pi^+ K^0$ amplitude. The corresponding charge-conjugate quadrangle has one diagonal equal to

$$\bar{A}_{3/2} = |A^T_{\pi K}|e^{-i\gamma}e^{i\tilde{\delta}_T} - |A^{\rm EWP}_{\pi K}|e^{i\tilde{\delta}_{\rm EWP}} , \qquad (15)$$

so that one can take the difference to eliminate the electroweak penguin contribution:

$$A_{3/2} - \bar{A}_{3/2} = |A^T_{\pi K}| 2i \sin\gamma e^{i\tilde{\delta}_T} . \qquad (16)$$

In diagrammatic language, the quantity $|A^T_{\pi K}|$ is just $|T' + C'|$. But this can be related to the $I = 2$ $\pi\pi$ amplitude in order to obtain $\sin\gamma$. Specifically, if we neglect electroweak penguin effects in $B^+ \to \pi^+\pi^0$ (a good approximation, as noted in Sec. III B), we find that

$$|A^T_{\pi K}| = \lambda(f_K/f_\pi)\sqrt{2}|A(B^+ \to \pi^+\pi^0)| . \qquad (17)$$

Thus we can extract not only $\sin\gamma$, but also a strong phase shift difference $\tilde{\delta}_T$, by comparing (16) and (17). Of course, if such a strong phase shift difference exists, the B and $\bar{B}$ quadrangles will necessarily have different shapes, and CP violation in the B system will already have been demonstrated.

We should remark that the quadrangle construction for $B \to \pi K$ decays introduced in [20] and refined in [21] assumed the presence of a single weak phase in the amplitude $A_{3/2}$, and no longer is valid in the presence of electroweak penguin diagrams.

B. SU(3)-breaking effects in $B \to \pi K$

The analysis presented above relies on the equality of two small amplitudes—the diagonal D_2 of the πK quadrangle and the decay amplitude $\sqrt{3}A(B_2 \to \pi^0\eta)$. Thus one might worry that small effects, which we have ignored up to now, might break this equality. We address this question here.

First, we have ignored E' and A' diagrams in equating these two amplitudes. This should not cause any problems. We expect that $P'_{\rm EW}$ is roughly of the same size as T'. But E' and A' are suppressed by $f_B/m_B \approx 5\%$ relative to T'. Thus their neglect introduces at most a small error into our analysis.

The second possibility involves SU(3) breaking. The effects of SU(3) breaking in two-body decays of B mesons have been analyzed by us in more detail in a longer paper [10]. The largest terms in the present case involve the effect of SU(3) breaking on the dominant gluonic penguin term (P') in $B \to K\pi$. These terms are of the same strength in all the $B \to K\pi$ amplitudes illustrated in Fig. 3, and hence cancel in the construction of the two diagonals. The next most important term involves SU(3) breaking in the ratio of the $|\Delta S| = 1$ and $\Delta S = 0$ nonpenguin amplitudes. However, this is expected to be well approximated by the ratio f_K/f_π [10] (see also [3,24]), as in Eq. (17). The critical term turns out to be the effect of SU(3) breaking on the electroweak penguin diagram. Specifically, the $B_s \to \pi^0\eta$ decay involves a spectator s quark, whereas the spectator quark in the $B \to \pi K$ decays is u or d. Thus the SU(3) breaking corresponds here to a difference in the form factors for the two types of decays. Although we expect SU(3)-breaking effects to be typically of order 25% (i.e., the difference between f_π and f_K), here they are expected to be smaller, since the mass ratio m_η/m_K is much closer to unity than is m_K/m_π. Still, this SU(3) breaking does introduce some theoretical uncertainty into this method for obtaining γ.

C. The processes $B \to \pi K^*$ and $B \to \rho K$

We have carried out a similar analysis for the decays $B \to \pi K^*$. Clearly it is still possible to write an amplitude quadrangle for these processes; the question is simply the interpretation of the diagonals.

There are more SU(3) amplitudes in $B \to PV$ decays since the final-state particles do not belong to the same octet. Nevertheless, one can still use a graphical analysis in the spirit of Ref. [3]—there are just more diagrams. For example, instead of one T diagram, there are two (T_P and T_V), corresponding to the cases where the spectator quark hadronizes into the P or V meson in the final state.

Carrying out such a graphical analysis, we find that the diagonals of the πK^* quadrangle are

$$D_1^* = -[T_P' + C_P' + P_{\mathrm{EW},V}' + P_{\mathrm{EW},V}'^C] ,$$
$$D_2^* = -C_P' - P_{\mathrm{EW},V}' , \tag{18}$$

where the subscripts P and V represent the spectator quark hadronizing into the π and K^*, respectively. (In the above we have ignored annihilation-type contributions.) Remarkably, the diagonal D_2^* [labeled by (e) in Fig. 3] corresponds to $\sqrt{2}A(B_s \to \pi^0\phi)$. Again, the shape of the quadrangle can be specified by experimental measurements. The other diagonal D_1^* contains both an electroweak penguin piece (which we can eliminate in the manner noted in Sec. IV A above), and a nonpenguin piece $-(T_P' + C_P')$. This latter piece is closely related to the amplitude for the decay $B^+ \to \pi^0\rho^+$:

$$\sqrt{2}A(B^+ \to \pi^0\rho^+) = -[T_P + C_P - P_P + P_V + P_{\mathrm{EW},V}] . \tag{19}$$

If the penguin diagrams are unimportant in this decay, or if the two types of penguin contributions P_P and P_V cancel (the EWP is expected to be quite small here), the analysis can be carried through exactly as in Sec. IV A. In this case, the precision on the measurement of γ is roughly of order $|(P_P - P_V)/(T_P + C_P)|$.

Another quadrangle relation holds for the amplitudes of $B \to \rho K$. They are obtained from the amplitudes of $B \to \pi K^*$ by replacing $T_P', C_P', P_{\mathrm{EW},V}'$, etc., by $T_V', C_V', P_{\mathrm{EW},P}'$, etc. Here one of the diagonals of the quadrangle is given by $\sqrt{3}A(B_s \to \eta\rho^0)$. The other diagonal [obtained from D_1^* by substituting $P \leftrightarrow V$ in (18)] contains $-(T_V' + C_V')$ and an electroweak penguin term. When the latter is eliminated as in Sec. IV A, the remaining $-(T_V' + C_V')$ term is approximately equal to $\sqrt{2}A(B^+ \to \pi^+\rho^0)$.

V. DATA: STATUS AND PROSPECTS

The measurements proposed here are not all easy. The $B \to \pi K$ decays should be characterized by branching ratios of order 10^{-5} for charged pions and about half that for neutral pions if the $B \to \pi^-K^+$ decay really has been observed at the 10^{-5} level [17] and if the gluonic penguin amplitude is dominant. The amplitudes in Fig. 3 are drawn to scale using the calculations of Ref. [12], neglecting strong final-state phase differences, and assuming $\gamma = \pi/2$. The effects of electroweak penguin diagrams can be seen not only in the rotation of the phase of $A_{3/2}$ from its nonpenguin value, but in substantial differences in the lengths of the sides of the quadrangle. It may well be that electroweak penguin effects make their first appearance in such rate differences, as mentioned at the end of Sec. III.

The $B_s \to \pi^0\eta$ decay will be very difficult to measure. The calculations of Ref. [12] indicate a branching ratio of a couple of parts in 10^7. One has to distinguish a B_s from a $\bar{B}_s$. In order to observe the $\pi^0\eta$ decay at a hadron machine, where the displaced vertex of the B_s would seem to be a prerequisite, one would have to observe the η in a mode involving charged particles.

Somewhat more hope is offered in the corresponding $B \to \pi K^*$ case, if we can trust the very small branching ratio for $B_s \to \pi^0\phi$ of a couple of parts in 10^8 predicted in Ref. [13]. (See also [25].) The corresponding electroweak penguin effects [characterizing the diagonal (e) in Fig. 3] are expected to be smaller here, whereas it is quite likely that the basic $B \to \pi K^*$ decays can be observed soon.

The possibility of degeneracies in lengths of the sides of the quadrangles can lead to a large amplification of errors in the amplitudes (e) when used to predict the length of side (f). For example, imagine that (e) were really zero and (a) = (c), (b) = (d). The length of (f) then would be indeterminate. On the other hand, if the diagonal (e) of the quadrangle is sufficiently small, the quadrangle reduces to two nearly degenerate triangles in which the effects of electroweak penguin diagrams are negligible. In this case, the second diagonal is given to a good approximation by $\sqrt{2}A(B^+ \to \pi^0\rho^+)$ [assuming some cancellation between the P_P and P_V terms of Eq. (19)], and the relative phase between this amplitude and its charge conjugate measures 2γ. Indeed, the very small value of $\mathcal{B}(B_s \to \pi^0\phi)$ calculated in Ref. [13] suggests that this may be happening for the decays $B \to \pi K^*$.

VI. CONCLUSIONS

We have found the following results.

(a) Electroweak penguin diagrams (EWP's) are not expected to substantially affect the discussion in Ref. [2] regarding $B \to \pi\pi$ decays.

(b) EWP's *are* more likely to be important in the comparisons [3–5] of $B \to \pi K$ and $B \to \pi\pi$ decays, though such conclusions are dependent on the evaluation of hadronic matrix elements of operators.

(c) EWP's do not introduce new amplitudes of flavor SU(3), so that one cannot detect their presence merely by modification of flavor-SU(3) amplitude relations.

(d) A deviation of the rate ratios $2\Gamma(B^+ \to \pi^0K^+)/\Gamma(B^0 \to \pi^-K^+)$ and $2\Gamma(B^0 \to \pi^0K^0)/\Gamma(B^+ \to \pi^+K^0)$ from unity indicates the presence of EWP's or the importance of color-suppressed or annihilation contributions above anticipated levels. Since the corresponding branching ratios are expected to be $\sim 10^{-5}$, these are likely to be the first (indirect) experimental signals of EWP's. Electroweak penguin diagrams are expected to dominate decays of the form $B_s \to (\phi \text{ or } \eta) + (\pi \text{ or } \rho)$ [13], but the branching ratios for these processes are expected to be significantly smaller.

(e) A quadrangle analysis has been presented for such decays as $B \to \pi K$, $B \to \pi K^*$, and $B \to \rho K$. One diagonal of the quadrangle is related to the amplitude for a physical process such as $B_s \to \pi^0\eta$ or $B_s \to \pi^0\phi$, so that one can perform a construction to obtain the other diagonal. From the magnitude and phase of this amplitude, one can obtain $\sin\gamma$, where $\gamma \equiv \arg(V_{ub}^*)$. A related construction [23] makes use of a triangle relation satisfied by the amplitudes for $B^+ \to (\pi^0K^+, \pi^0K^+, \eta K^+)$.

(f) The $B \to \pi K^*$ processes hold out hope for a small electroweak penguin contribution, if the $B_s \to \pi^0\phi$ branching ratio is as small as cited in Ref. [13]. In such a case, the quadrangle will degenerate into two nearly identical triangles, so that the original analysis of Ref. [4], suitably modified to take account of the presence of one vector and one pseudoscalar meson, may be more trustworthy. We have presented the ingredients of such an analysis in Sec. IV C.

[*Note added.* After this paper was submitted for publication a work appeared [23] in which the amplitude for $B^+ \to \eta_8 K^+$ is employed instead of our quadrangle construction in order to specify the phase between the amplitudes for $B^+ \to \pi^+ K^0$ and $B^+ \to \pi^0 K^+$. Here η_8 denotes an octet member. In our phase convention, $\sqrt{6}A(B^+ \to \eta_8 K^+) = -[T' + C' - P' + P'_{\rm EW} + \frac{4}{3}P^{C\prime}_{\rm EW}]$, so that

$$\sqrt{2}A(B^+ \to \pi^0 K^+) + 2A(B^+ \to \pi^+ K^0) = \sqrt{6}A(B^+ \to \eta_8 K^+) \ . \quad (20)$$

This relation, which follows immediately from Table II of Ref. [3], is not affected by EWP diagrams since these diagrams introduce no new SU(3) amplitudes. One then finds that the amplitude $\sqrt{6}A(B^+ \to \eta_8 K^+) - \sqrt{6}A(B^- \to \eta_8 K^-)$ connecting the vertices of the two triangles is just Eq. (16), and the subsequent analysis proceeds as above. Since the physical η is an octet-singlet mixture with mixing angle of around 20° [22], $A(B^+ \to \eta_8 K^+)$ must be extracted from measurements of $B^+ \to \eta K^+$ and $B^+ \to \eta' K^+$ with the help of an assumption about relative phases.]

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