# Inverse neutrinoless double- $\beta$ decay in gauge theories with CP violation

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We investigate the  $e^-e^- \rightarrow W_i^- W_i^-$  reactions for the various gauge boson production processes in the frame of the standard model with additional right-handed neutrinos and in the left-right symmetric model. The present bounds on the various model parameters are taken into account. The question of the cross section behavior for large energy and the CP violation problems are discussed.

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# I. INTRODUCTION

The nature of the known electron,  $\mu$  and  $\tau$  neutrinos and the existence of the heavier  $(M_{\nu} > M_Z)$  neutrinos are still unresolved problems in particle physics. In the standard model all neutrinos are massless. There is, however, some more or less strongly established evidence which requires massive neutrinos. Problems with (i) the solar (electron) neutrinos, (ii) atmospheric ( $\mu$ ) neutrinos, and (iii) the hot dark matter can be resolved if neutrinos are massive particles. Within the framework of the extended electroweak models neutrinos are usually massive and Majorana-type. At low energies such models can be probed by looking for rare processes, such as the neutrinoless double- $\beta$  decay.

High-energy  $e^-e^-$  collision, a possible option in next linear colliders (NLC), may provide a new test for those  $\Delta L = 2$  interactions via the  $e^-e^- \rightarrow W_i^- W_i^-$  reaction, where  $W_i$  may represent standard model gauge bosons or the additional charged gauge bosons. This inverse neutrinoless double- $\beta$  decay was proposed some time ago [1] and since then has been investigated several times [2]. We have decided to study the process once more for a few reasons. First, we would like to give the numerical values of the total cross sections for the  $e^-e^- \rightarrow W_i^- W_i^$ processes, taking into account the up-to-date bounds for the various model parameters. Second, we would like to study the problem of unitarity and to find the conditions for the correct high-energy behavior of the cross sections in various gauge models. For this purpose it is necessary to consider all model's ingredients [for example, in the left-right (L-R) symmetric model both leftand right-handed double-charged gauge bosons  $\delta_{L,R}^{\pm\pm}$  have to be considered]. The calculated helicity amplitudes for the process give us the opportunity to have a look at the unitarity cancellations in a very precise way. Finally, we investigate the problem of *CP*-symmetry breaking in our process and compare its size for various models.

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In the next section we define the mass Lagrangian and the CP-breaking parameters. In Sec. III we investigate which ingredients of the models are responsible for correct high-energy cross section behavior. The numerical values of total cross sections for model's parameters, which satisfy existing bounds and the size of CP-symmetry breaking, are given in Sec. IV.

#### **II. THE MASS LAGRANGIAN** AND CP VIOLATION

In the class of models which we consider the mass Lagrangian for neutrinos and charged leptons is given by

$$L_{\rm mass} = -\frac{1}{2} (\bar{N}_L^c M_\nu N_R + \bar{N}_R M_\nu^* N_L^c) - (\bar{l}_L M_l l_R + \bar{l}_R M_l^\dagger l_L),$$
  
where (1)

where

$$N_R = \begin{pmatrix} i\gamma^2\nu_L^* \\ \nu_R \end{pmatrix}, \ \ N_L^c = \begin{pmatrix} \nu_L \\ i\gamma^2\nu_R^* \end{pmatrix}$$

are  $(n_L + n_R)$  dimensional row vectors of neutrino fields and  $l_{L(R)}$  are  $n_L$  dimensional charged lepton fields.  $M_l$ and  $M_{\nu}$  are  $n_L \times n_L$  complex and  $(n_L + n_R) \times (n_L + n_R)$ symmetric-complex matrices, respectively. The matrix  $M_{\nu}$  is usually divided into four parts

$$M_{\nu} = \begin{pmatrix} M_L & M_D \\ M_D & M_D \\ M_D^T & M_R \end{pmatrix} {}^{n_R}_{n_R}.$$
(2)

We know, that without changing the physical meaning (all elements of the Lagrangian, except the mass term [Eq. (1)], will not change) some matrices can be made diagonal. In the models without right-handed charged current interaction two matrices, e.g.,  $M_l$  and  $M_R$ , can be chosen diagonal, in the L-R symmetric models only one, e.g.,  $M_l$ . In all kinds of models which we consider the basis of lepton fields can be chosen in such a way that the charged lepton mass matrix  $M_l$  is real and diagonal. Then all the lepton-violating CP phases are present in the neutrino mass matrix  $M_{\nu}$  [Eq. (2)]. The number of CP-breaking phases depends on the model. As an example we consider two kinds of models.

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(1) Standard model with the additional three neutral right-handed singlets (RHS's). In this model  $n_L = n_R = 3$  and, in our basis,

$$M_{\nu} = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} , \qquad (3)$$

where  $M_R = \text{diag}(M_1, M_2, M_3)$  are real numbers and

$$M_D = \begin{pmatrix} a_1 e^{i\alpha_1} & a_2 e^{i\alpha_2} & a_3 e^{i\alpha_3} \\ b_1 e^{i\beta_1} & b_2 e^{i\beta_2} & b_3 e^{i\beta_3} \\ c_1 e^{i\gamma_1} & c_2 e^{i\gamma_2} & c_3 e^{i\gamma_3} \end{pmatrix}.$$
 (4)

The CP symmetry is satisfied if [3]

$$\sin{(lpha_i-lpha_j)}=\sin{(eta_i-eta_j)}=\sin{(\gamma_i-\gamma_j)}=0$$

for i, j = 1, 2, 3 (5)

and six phases  $\alpha_1 - \alpha_2 = \chi_1$ ,  $\alpha_1 - \alpha_3 = \chi_2$ ,  $\beta_1 - \beta_2 = \rho_1$ ,  $\beta_1 - \beta_3 = \rho_2$ ,  $\gamma_1 - \gamma_2 = \eta_1$ , and  $\gamma_1 - \gamma_3 = \eta_2$  break the symmetry.

(2) The left-right symmetric model. We consider two versions of the model. At the beginning we assume that there is the explicit L-R symmetry with all vacuum expectation values real (no spontaneous CP-symmetry breaking) [3]. Then the  $M_L$  and  $M_R$  matrices are proportional,

$$M_L = \text{const} \times M_R , \qquad (6)$$

and  $M_D$  is Hermitian:

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$$M_D = M_D^{\dagger}.\tag{7}$$

There are only six CP-violating phases [3] and we can choose them in the following way:

. .

$$M_R = \begin{pmatrix} M_{11}e^{i\alpha_1} & M_{12} & M_{13} \\ M_{12} & M_{22}e^{i\alpha_2} & M_{23} \\ M_{13} & M_{23} & M_{33}e^{i\alpha_3} \end{pmatrix}$$

and

$$M_D = egin{pmatrix} m_{11} & m_{12}e^{ieta_1} & m_{13}e^{ieta_2} \ m_{12}e^{-ieta_1} & m_{22} & m_{23}e^{ieta_3} \ m_{13}e^{-ieta_2} & m_{23}e^{-ieta_3} & m_{33} \end{pmatrix}.$$

This model is known in the literature as manifest (MLRS) or quasimanifest (QMLRS) *L-R* symmetric one.

We consider also the full version of the *L-R* model where there is no relation between  $M_L$  and  $M_R$  matrices and  $M_D$  is not Hermitian [nonmanifest *L-R* symmetric model (NMLRS)]. In this case there are 18 *CP*-violating phases. The phases of the matrices  $M_L$ ,  $M_R$ , and  $M_D$ , which do not satisfy the *CP*-conserving relations [3],

$$(M_L)_{ij} = \mid (M_L)_{ij} \mid e^{i rac{1}{2} (\delta_i + \delta_j)},$$

$$(M_R)_{ij} = | (M_R)_{ij} | e^{-i\frac{1}{2}(\delta_i + \delta_j)}, \qquad (9)$$

 $\mathbf{and}$ 

(

$$(M_D)_{ij} = \mid (M_D)_{ij} \mid e^{-i\frac{1}{2}(\delta_i - \delta_j)}$$

break the CP symmetry.

The neutrino mass matrix is diagonalized by the complex orthogonal transformation

$$U^{T} M_{\nu} U = \text{diag}[|m_{1}|, |m_{2}|, ..., |m_{n_{L}+n_{R}}|] \equiv M_{\nu d}.$$
(10)

If we denote

$$U = \begin{pmatrix} U_L^* \\ U_R \end{pmatrix}$$

then the mixing matrices  $K_L$  and  $K_R$  in the physical leftand right-charged current interactions [see Eq. (A2) in the Appendix] are given by

$$K_{L,R} = U_{L,R}^{\dagger}.$$
 (11)

There are reversed connections between  $M_{L(R)}, M_D$  and  $M_{\nu d}$ :

$$M_L = K_L^{\dagger} M_{\nu d} K_L^*, \quad M_D = K_L^{\dagger} M_{\nu d} K_R,$$

 $\mathbf{and}$ 

(8)

$$M_R = K_R^T M_{\nu d} K_R$$

which will be useful in the further considerations.

# III. BEHAVIOR OF CROSS SECTIONS FOR HIGH ENERGY

The reduced helicity amplitudes (see the Appendix) have bad high energy behavior. As  $\lim_{s\to\infty} \gamma_{1,2} \sim \frac{\sqrt{s}}{2M_{1,2}}$  the amplitudes with  $|\Delta\lambda| = 1$  proportional to  $\gamma_1$  and the amplitudes with  $\lambda_1 = \lambda_2 = 0$  (proportional to  $\gamma_1\gamma_2$ ) increase with increasing energy. Of course, these divergences cannot appear in the total cross section so there must exist mechanisms which cause their cancellation. As the cancellations of these divergences have influence on the size of the total cross section it is instructive to show how it happens. There are several reasons why the bad high energy behavior does not appear in the helicity amplitudes for  $\sqrt{s} \gg m_a, M_i, a = 1, ..., 6, i = 1, 2$ .

In the L-R symmetric model for  $\Delta \sigma = \pm 1$  in the t and the u channels there are (see the Appendix)

$$R_{t(u)}M_{t(u)}\left(\Delta\sigma = \pm 1, |\Delta\lambda| = 1\right) \longrightarrow \frac{1}{M_i\sqrt{s}\left(1 \mp \cos\Theta\right)} \sum_a (K_{L(R)}^T)_{ma} m_a^2 \left(K_{R(L)}\right)_{an} \tag{13}$$

and

$$R_{t(u)}M_{t(u)}\left(\Delta\sigma = \pm 1, \lambda_1 = \lambda_2 = 0\right) \longrightarrow \frac{1}{2M_iM_j(1 \mp \cos\Theta)} \sum_a (K_{L(R)}^T)_{ma} m_a^2(K_{R(L)})_{an} .$$
(14)

(12)

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To obtain these formulas we use the orthogonality property of the  $K_R$  and  $K_L$  matrices:

$$K_{L(R)}^T K_{R(L)} = 0. (15)$$

For  $\Delta \sigma = 0$  the problem is only with one amplitude for  $\lambda_1 = \lambda_2 = 0$ . Separately, each amplitude in the *u*, *t*, and *s* channels is divergent but when we add them together, then

$$B^{ij}(\sigma_{1},\sigma_{2})\left[R_{t}M_{t}+R_{s}M_{s}\right]+B^{ij}(\sigma_{2},\sigma_{1})R_{u}M_{u}$$

$$\longrightarrow -\frac{\sqrt{2}}{\sqrt{s}M_{i}M_{j}}B^{ij}\left(\pm\frac{1}{2},\pm\frac{1}{2}\right)\left[\frac{1}{2\left(1-\cos^{2}\Theta\right)}\sum_{a}\left(K_{R(L)}^{T}\right)_{ma}m_{a}^{3}\left(K_{R(L)}\right)_{an}\right.$$

$$\left.+\left(M_{\delta_{R,L}}^{2}-i\Gamma_{\delta_{R,L}}M_{\delta_{R,L}}\right)\sum_{a}\left(K_{R,L}^{T}\right)_{ma}m_{a}\left(K_{R,L}\right)_{an}\right].$$
(16)

The crucial thing needed to obtain this high energy behavior is existence of the three s, t, and u channels. Without the  $\delta_{R,L}^{--}$  bosons exchange in the s channel the unitarity would be violated. And it is important that both left  $\delta_{L}^{--}$  and right  $\delta_{R}^{--}$  doubly charged Higgs bosons are present. They give contributions to the different helicity amplitudes (see Appendix).

We can see that in the models with L-R symmetry the appropriate high energy behavior is guaranteed because of the following reasons: (i) the left and right mixing matrices in the charged current are orthogonal [Eq. (15)], and (ii) there exist two doubly charged Higgs bosons  $\delta_{L,R}^{--}$  with proper relations between various couplings.

We can ask now how it is possible, that in the models without L-R symmetry, where only one charged gauge boson  $W^{\pm}$  exists, the unitarity is also satisfied. In this class of models there is no right-handed charged current. Formally this is equivalent with the assumption that the mixing angle  $\xi$  and the mixing matrix  $K_R$  vanish:

$$\xi = 0, \ K_R = 0. \tag{17}$$

There are no doubly charged Higgs bosons so there is no contribution to the amplitude from the s channel:

$$M_s = 0. (18)$$

Only one helicity amplitude in the t and u channels  $M_{t,u}(-\frac{1}{2},-\frac{1}{2},0,0)$  seems to be divergent. But if we look carefully at the high energy behavior we have

$$R_{t(u)}M_{t(u)}\left(-\frac{1}{2},-\frac{1}{2},0,0\right)$$

$$\longrightarrow \frac{1}{\sqrt{2}M_{i}M_{j}}\left[\sqrt{s}\sum_{a}\left(K_{L}^{T}\right)_{ma}m_{a}\left(K_{L}\right)_{an}\right.$$

$$\left.+\frac{1}{\sqrt{s}}\frac{1}{\left(1-\mp\cos\Theta\right)}\sum_{a}\left(K_{L}^{T}\right)_{ma}m_{a}^{3}\left(K_{L}\right)_{an}\right].$$

$$(19)$$

Now from Eq. (12) there is

$$\sum_{a} \left( K_{L}^{T} \right)_{ma} m_{a} (K_{L})_{an} = \left( K_{L}^{T} M_{\nu d} K_{L} \right)_{mn} = \left( M_{L}^{*} \right)_{mn} .$$
(20)

In the RHS models without doubly-charged bosons the

appropriate Yukawa mechanism which generates the mass matrix  $M_L$  is not present, so

$$M_L = 0, (21)$$

and the first term in Eq. (19) which is proportional to  $\sqrt{s}$  disappears, what guarantees the correct high energy behavior of the cross section. In the models where Higgs triplets are present the unitarity is preserved in similar way as in the *L-R* symmetric models.

### **IV. NUMERICAL RESULTS**

First we investigate the total cross sections for production of gauge bosons in  $e^-e^-$  reaction and their dependence on various model parameters. Then we consider the *CP*-symmetry breaking.

### A. Total cross section in L-R model

To calculate the cross section we need to know the values of the model parameters (see [4]): the mixing angle  $\xi$ 

$$\xi \simeq \left(\frac{2k_1k_2}{k_1^2 + k_2^2}\right) \frac{M_{W_1}^2}{M_{W_2}^2}; \tag{22}$$

the masses of the gauge bosons,

$$M_{W_1}^2 \simeq \frac{g^2}{2} \left( k_1^2 + k_2^2 \right), \ M_{W_2}^2 = \frac{g^2}{2} v_R^2;$$
 (23)

the masses of doubly-charged Higgs particles,

$$M_{\delta_L}^2 \simeq \frac{1}{2} v_R^2 , \ M_{\delta_R}^2 \simeq 2 v_R^2.$$
 (24)

In our numerical analysis we take that  $2k_1k_2 \simeq k_1^2 + k_2^2$ , so practically only one parameter, the mass of heavy gauge boson  $M_{W_2}$  is free  $(M_{W_1} \text{ and } g = e/\sin\Theta_W$  are known from the standard model). We do not calculate the decay width for doubly charged Higgs bosons but we put them in the form

$$\Gamma_{\delta_{L,R}^{--}} = \Gamma_{W_1} M_{\delta_{L,R}^{--}} / M_{W_1}.$$
(25)

In addition to the above model parameters the cross sec-

tion depends very strongly on the neutrino masses and mixings. First of all, we can see [Eqs. (15), (A6)–(A8)] that if all neutrinos are massless  $(m_a = 0)$ , then the functions  $R_{t,u,s}$  vanish and the cross section is zero. This fact is very well known. For massless neutrinos there is no way to distinguish between the Dirac and the Majorana cases, lepton number conservation is restored, and our process cannot occur.<sup>1</sup>

From existing terrestrial experiments we know that [5]

$$m_{\nu_{e}} < 5.1 \text{ eV}, \ m_{\nu_{\mu}} < 0.27 \text{ MeV}, \ m_{\nu_{\tau}} < 31 \text{ MeV}$$
 (26)

and the heavy neutrinos, if they exist, must have masses  $M_N > \frac{M_Z}{2}$  [5] or even  $M_N > M_Z$  if additional assumptions about the  $\nu NZ$  coupling are made [6]. There are other laboratory experiments (double- $\beta$  decay and neutrino oscillation in the vacuum) which tell us something about neutrino masses and mixings. Moreover, there are also solar, astrophysical and cosmological observations which can also give some information about masses and mixings; see, e.g., [7]. It follows from all existing observations that the three known neutrinos ( $\nu_e$ ,  $\nu_{\mu}$ ,  $\nu_{\tau}$ ) should have small, almost degenerate masses in the range between 0 and several eV. The other three neutrinos predicted by the *L-R* model must have masses above ~ 100 GeV.

One of the possible choices to assure the mentioned pattern of the neutrino masses is to assume that  $M_D$  is almost a rank 1 matrix and  $M_R$  is almost diagonal with large diagonal elements  $M_i$  (i = 1, 2, 3) which satisfy the constraints [8]

$$\frac{1}{M_1} + \frac{1}{M_2} + \frac{1}{M_3} \simeq 0. \tag{27}$$

On the other hand, the elements of the  $M_R$  matrix are determined by the right-handed vacuum expectation value  $v_R$  and should be bounded by the mass of the heavy gauge boson [Eq. (23)]:

$$M_i \le \frac{2M_{W_2}}{g}.$$
(28)

Let us take the matrices  $M_D$  and  $M_R$  in the form

$$M_D = \begin{pmatrix} 1.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & 1.0 - 10^{-6} \end{pmatrix}$$
(29)

and

$$M_R = \begin{pmatrix} M_1 & 10^{-6} & 10^{-6} \\ 10^{-6} & M_2 & 10^{-6} \\ 10^{-6} & 10^{-6} & M_3 \end{pmatrix}$$

Then one neutrino is massless  $m_1 = 0$ , the other two have very small masses,  $m_2 \simeq 0$ ,  $m_3 \simeq 0$ , and the masses of the heavy neutrinos are given by the diagonal elements of matrix  $M_R$ :

$$m_4 \simeq \mid M_1 \mid, \ m_5 \simeq \mid M_2 \mid, \ m_6 \simeq \mid M_3 \mid$$

For this choice of the neutrino mass matrix the total cross section for various other model parameters is presented on Fig. 1. In the frame of the NMRLS model the present experimental bound on  $M_{W_2}$  is not so high and  $M_{W_2} \ge 600$  GeV is still a reasonable limit [9]. Then the mixing angle  $\xi$  does not suppress so much the helicity amplitudes. Curves depicted by capital A represent the result for  $M_{W_2} = 600$  GeV. In the frame of MLRS or QMLRS models the bound on  $M_{W_2}$  is larger than  $M_{W_2} \ge 1600$  GeV [10] and the cross sections are lower (the line depicted by capital B in Fig. 1). The reasonably high luminosity in the next  $e^-e^-$  collision will not give us the possibility to observe the  $W_1^- W_1^-$  pair production in this case. The cross section depends also on the heavy neutrinos masses. For  $m_4 \sim 100$  GeV (and  $m_5 \sim m_6 \sim 200 \,\, {
m GeV})$  the cross section is small (dashed lines in Fig. 1) and increases with the neutrino mass the solid lines give the cross section for the biggest acceptable values (Eq. (28)). On the figure we can also see the influence of the right-handed resonance  $\delta_R^{--}$ , as for  $M_L \sim 0$  the effect of the left-handed resonance  $\delta_L^{--}$  is not visible (see final remarks in the Appendix).

In the next figures we present the cross sections for production of the light-heavy  $W_1^-W_2^-$  gauge bosons (Fig. 2) and two heavy  $W_2^-W_2^-$  gauge bosons (Fig. 3). The cross sections are higher but of course the thresholds for



FIG. 1. The total cross section for the process  $e^-e^- \rightarrow W_1^-W_1^-$  (*LR* model) as a function of the c.m. energy for various parametrizations of the neutrino mass matrix. Dashed line is for the parametrization given by Eq. (29) with  $M_1 = -100$  GeV. Solid line is for the biggest available  $M_1$  [Eq. (28)]. In all cases  $M_2$  and  $M_3$  are chosen in such a way that Eq. (27) is satisfied.

<sup>&</sup>lt;sup>1</sup>Strictly speaking it is impossible to distinguish between the Dirac and Majorana neutrinos for  $m_a \rightarrow 0$  if there are only left-handed or right-handed current interactions. If both couplings are present (as in the *L-R* symmetric model) the massless Dirac and Majorana neutrinos are still indistinguishable because of the orthogonality of  $K_L$  and  $K_R$  matrices  $(K_{L,R}^T K_{R,L} = 0)$ .



FIG. 2. Process  $e^-e^- \rightarrow W_1^-W_2^-$  (*LR* model) as a function of the c.m. energy for various masses of the heavy neutrinos. Notation is the same as for Fig. 1.

these production processes are too high to hope that the appropriate colliders will be built in the near future.

Figure 4 gives the behavior of the total cross section for two light gauge boson production  $e^-e^- \rightarrow W_1^-W_1^-$  as a function of heavy neutrino masses. We take the following parametrization of the neutrino mass matrix in Eq. (29):

$$M_1 = -M, \ M_2 = M_3 = 2M. \tag{30}$$

The behavior of  $\sigma(M)$  agrees with our previous discussion:  $\sigma \to 0$  for  $M \to 0$ . The cross section increases with the increasing neutrino masses (not shown on the figure) if we go with the mass M above the limit given by Eq.



FIG. 3. Process  $e^-e^- \rightarrow W_2^-W_2^-$  (*LR* model) as a function of the c.m. energy for various masses of the heavy neutrinos. Notation is the same as for Fig. 1.



FIG. 4. Process  $e^-e^- \rightarrow W_1^-W_1^-$  (*LR* model) as a function of the heavy neutrino mass for NLC,  $\sqrt{s} = 500$  GeV (dashed line), and  $\sqrt{s} = 1$  TeV (solid line) with parametrization of the heavy neutrino masses by Eq. (29), with  $M_1 = -M$ ,  $M_2 = M_3 = 2M$ .

(28).

The dependence of the total cross section on the gauge boson mass  $M_{W_2}$ ,  $\sigma(M_{W_2})$ , for various CM energies and different mass matrix parametrizations is depicted in Fig. 5. As was discussed before, the cross section depends strongly on  $M_{W_2}$  and this behavior is not influenced too much by the neutrino matrix parametrization.

#### B. Total cross section in the models with RHS's

We consider the model with three additional righthanded neutrinos which are singlets of the gauge group.



FIG. 5. Process  $e^-e^- \rightarrow W_1^-W_1^-$  (*LR* model) as a function of the heavy gauge boson mass for different energies, (a) for  $\sqrt{s} = 500$  GeV and (b) for  $\sqrt{s} = 1000$  GeV. Again dashed and solid lines are for the neutrino mass paramatrization as in Fig. 1.

At the beginning we assume that the neutrino mass matrix is pure real (CP is conserved) and is the same as before [Eq. (29)]. There are no doubly-charged Higgs bosons so only the t and u channels contribute to the light gauge boson production amplitude  $e^-e^- \rightarrow W_1^-W_1^ (\xi = 0 \text{ and } K_R = 0)$ . Only one helicity amplitude with  $\sigma_1 = \sigma_2 = -1/2$  gives contribution to the total cross section. The appropriate  $K_L$  mixing matrix elements,  $(K_L)_{ei}$  (i = 4, 5, 6), are decreasing functions of the heavy neutrino mass and the same can be observed in the total cross section (Fig. 6). The  $\sigma \left( e^- e^- \to W_1^- W_1^- \right)$  dependence on the c.m. energy is presented in Fig. 7. The cross section is smaller than in the L-R symmetric model and decreases with energy. This is caused by the destructive interferences between heavy neutrinos. It is also very sensitive to the neutrino mass matrix parametrization. If we take, for example,

$$M_D = \begin{pmatrix} 1.0 & 1.0 & 0.9 \\ 1.0 & 1.0 & 0.9 \\ 0.9 & 0.9 & 0.95 \end{pmatrix}, \ M_R = \begin{pmatrix} M_1 & 10 & 20 \\ 10 & M_2 & 10 \\ 20 & 10 & M_3 \end{pmatrix} ,$$
(31)

the cross section is much larger (dashed line in Fig. 7) and its decreasing with the energy is smaller (destructive interferences between heavy neutrinos are absent although such interferences can appear—see the next section), visible only for higher energies (not shown in Fig. 7). For  $\sqrt{s} = 500$  GeV  $\sigma \simeq 0.25 \times 10^{-4}$  fb with parametrization from Eq. (29) and  $\sigma \simeq 0.5 \times 10^{-3}$  fb with parametrization given by Eq. (31). The cross sections which we get are much smaller than that obtained in other papers (see, e.g., Heusch and Minkowski paper in [2]). The reason follows from the way in which the mixing matrix is treated. In our approach the small masses for the known neutrinos are obtained by the see-



FIG. 6. Process  $e^-e^- \rightarrow W_1^-W_1^-$  (RHS model) as a function of heavy neutrino mass for NLC (dashed line) and 1 TeV (solid line) colliders with parametrization of the heavy neutrino masses as in Fig. 4.



FIG. 7. Process  $e^-e^- \rightarrow W_1^-W_1^-$  (RHS model) as a function of the c.m. energy with parametrization of the heavy neutrino masses given by Eqs. (29), (30) with M=100 GeV—solid line and by Eq. (31) with  $M_1 = 100$  GeV,  $M_2, M_3 = 200$  GeV—dashed line.

saw mechanism. We calculate the  $K_L$  matrix by diagonalization of the neutrino mass matrix  $M_{\nu}$  which has strong model foundation and gives realistic spectrum of neutrino masses (three light and three heavy neutrinos). Then the mixing matrix elements have the "seesaw" type

$$(K_L)_{ea} \sim \frac{\langle M_D \rangle}{m_a},$$
 (32)

and are small for large neutrino mass  $m_a$ . The small mixing matrix causes the cross section also to be small. However, such classical "seesaw" mechanism is not the only scenario to explain the small masses of known neutrinos. There are models [11,12] where relation (32) does not work and the mixing matrix elements can be large even for large masses of heavy neutrinos. Then the smallness of the known neutrino mass is guaranteed by the special symmetry arguments which are applicable if number of right-handed neutrinos is larger than 1 (see, e.g., [12]). In such models the mixing matrix elements  $(K_L)_{ea}$  are independent parameters and can be bounded by existing experimental data. The present experimental bounds on  $K_L$  are such that the cross section  $\sigma(e^-e^- \rightarrow W^-W^-)$ can be much larger, e.g.,  $\sigma = 4(64)$  fb for  $\sqrt{s} = 0.5(1)$ TeV (see Heusch and Minkowski paper in [2] and for detailed discussion see also [13]).

#### C. CP-symmetry breaking

There are many phases which can break the CP symmetry [see Eqs. (4),(8)]. We don't study the cross section as a function of all of them. We choose only three phases and we assume that the neutrino mass matrix is given by Eq. (31), where the following changes are made:

$$M_1 \to e^{i\alpha}M, \ M_2 \to 2e^{i\beta}M, \ M_3 \to 3e^{i\gamma}M.$$
 (33)

If all diagonal elements  $M_i$  (i = 1, 2, 3) are real and positive  $(\alpha = \beta = \gamma = 0)$  then the eigenvalues of the neutrino mass matrix are also positive and the *CP* symmetry is conserved if the *CP* parities of heavy neutrinos are the same and equal:

$$\eta_{CP}(N_4) = \eta_{CP}(N_5) = \eta_{CP}(N_6) = +i.$$
(34)

The same happens when all masses  $M_1, M_2, M_3$  are real negative ( $\alpha = \beta = \gamma = \pi$ ). Then CP is also conserved if neutrinos CP parities are negative imaginary:

$$\eta_{CP}(N_4) = \eta_{CP}(N_5) = \eta_{CP}(N_6) = -i.$$
(35)

In both cases above the mixing matrix elements  $(K_{L,R})_{ae}$ , a = 4, 5, 6 are either pure real for  $\alpha = \beta = \gamma = 0$  or pure imaginary if  $\alpha = \beta = \gamma = \pi$ . We have checked that for the  $e^-e^- \rightarrow W_i^-W_j^-$  processes the helicity amplitudes with  $\Delta \sigma = 0$ , proportional to the heavy neutrino masses, are dominant. These helicity amplitudes include either square of the  $K_L$  mixing matrix elements (for  $\sigma_1 = \sigma_2 = -1/2$ ) or square of the  $K_R$  ones (for  $\sigma_1 = \sigma_2 = +1/2$ )—see Eqs. (A6)–(A8). That is why, summing the helicity amplitude over neutrino masses [Eq. (A5)], we get in both cases mentioned constructive contributions from all of them (solid line in Fig. 8).

In the case of mixing CP parities the CP symmetry is also conserved but the destructive interference between contributions from various neutrinos causes that the cross section decreases. In this case some of  $(K_{L,R})_{ae}$ , a =4,5,6 matrix elements are real and some are imaginary.

To obtain these CP-violating effects several  $K_L$  or  $K_R$  matrix elements must interfere in the same helicity amplitude. The structure of the chosen neutrino mass matrix



FIG. 8. Process  $e^-e^- \rightarrow W_1^-W_1^-$  (*LR* model) for  $M_1 = 200, M_2 = 400, M_3 = 600$  GeV [Eq. (32)] and  $M_{W_2} = 1600$  GeV when *CP* parity is conserved in the lepton sector.

causes the  $(K_L)_{ae}$ , a = 4, 5, 6 matrix elements to be of a similar order,

$$|(K_L)_{4e}| \simeq \frac{1}{m_4} > |(K_L)_{5e}| \simeq \frac{1}{m_5} > |(K_L)_{6e}| \simeq \frac{1}{m_6}$$
,  
(36)

but only one suitable element of the  $K_R$  matrix is large:

$$|K_R|_{4e} \sim 1 \gg |(K_R)_{ae}|, \quad a = 5, 6.$$
 (37)

This property causes that only if the  $K_L$  matrix elements [Eq. (36)] contribute to the cross section in the visible way, can the CP breaking be seen. It is just the case for two light gauge bosons  $W_1^-W_1^-$  production where  $\cos^2 \xi$  multiplies the  $K_L$  matrix contributions in  $\sigma_1 = \sigma_2 = -1/2$  helicity amplitude. If the contribu-tion from  $\sigma_1 = \sigma_2 = +1/2$  helicity amplitude (where  $\sin^2 \xi$  is multiplied by  $K_R^T K_R$ ) becomes important, then the CP-symmetry-breaking effect decreases. That is why the effect is visible only outside the  $\delta_R^{--}$  resonance region where only one right-handed mixing matrix  $K_R$  gives an essential contribution [Eq. (37)] and the interference has no importance. It also means that the CPsymmetry breaking is more visible for the larger  $M_{W_2}$ , when  $\sin \xi \to 0$ . For the same reason the *CP*-breaking effect will not be seen in the heavy gauge boson production  $(\cos^2 \xi$  is multiplied by the  $K_R$  matrix contributions) and weakly visible in the light-heavy gauge bosons production  $(\sin \xi \cos \xi \text{ multiplies } K_L^T K_L)$ .

If the *CP* symmetry is violated (phases  $\alpha$ ,  $\beta$ ,  $\gamma \neq 0, \pi$ ), the cross sections lie between two limiting lines in Fig. 8.

The effects of the CP violation in the standard model with the additional right-handed neutrinos are depicted in Fig. 9. For the same energy with the same mass matrix parametrization the effects are almost as large as those in the *L*-*R* symmetric models.



FIG. 9. Process  $e^-e^- \rightarrow W_1^-W_1^-$  (RHS model) for  $M_1 = 200, M_2 = 400, M_3 = 600$  GeV [Eq. (32)] when *CP* parity is conserved in the lepton sector.

In contrast with the case of the CP violation in the quark sector where the effect is small the violation of the CP symmetry in the lepton sector with the Majorana neutrino can be very large. The CP violation phases can change the cross section by even more than one order of the magnitude.

### **V. CONCLUSIONS**

We have calculated the total cross section for various gauge bosons production processes in  $e^-e^-$  scattering. Exact calculations have been done in the frames of two versions of the left-right symmetric model and the standard model with additional right-handed neutrinos. We have checked the high energy behavior for the cross sections, lim  $\sigma(s)$ . Having calculated the helicity amplitudes we have been able to show which ingredients of the models are responsible for the correct high energy limit  $\sigma(s \to \infty) = 0$ . In the left-right symmetric model the unitarity is satisfied because there are s channels with doubly charged Higgs bosons  $\delta^{--}_{L,R}$  and the leftand right-handed mixing matrices are orthogonal. In the standard model with additional RHS's but without Higgs triplets the correct high energy is guaranteed because the fragment of the neutrino mass matrix identically equals zero,  $M_L = 0$ . We have calculated this cross section taking into account existing bounds on the parameters of the considered models. Optimistically, with not very restrictive bounds on the models parameters, we have found that for  $\sqrt{s} = 500$  GeV in the L-R symmetric model  $\sigma \left(e^-e^- \to W_1^- W_1^-\right) \sim 0.01$  fb and for the stan-dard model with a RHS  $\sigma \left(e^-e^- \to W_1^- W_1^-\right) \sim 0.001$  fb. The small values of given cross sections are caused by the small mixing matrix elements  $K_{ae}$  between electron and heavy neutrinos calculated in our seesaw-type models. For nonseesaw models where mixing matrices  $K_{L,R}$ are not calculated and are free parameters bounded only by existing data, the cross sections can be much higher. In the L-R symmetric model the cross sections for

TABLE I. The reduced helicity amplitudes for all polarizations of the  $l_m^- l_n^- \to W_i^- W_j^-$  process  $[c = \cos\Theta \tilde{\eta} = \lambda_1 \Delta \sigma, \ \eta = \Delta \sigma \Delta \lambda, \ \check{\eta} = 2\sigma_1 \lambda_1, \ \hat{\eta} = (\sigma_1 + \sigma_2) \left(\lambda_1 + \lambda_2 
ight)].$ 

$\overline{\lambda_1}$	$\lambda_2$	$\Delta \sigma = \pm 1$	$\Delta \sigma = 0$
		$M_t\left(\Delta\sigma;\lambda_1,\lambda_2 ight)$	
1	1	$[(x- ilde\eta y_1)+rac{ ilde\eta-c}{2}]$	$\frac{1}{\sqrt{2}}(1+\check{\eta}c)$
-1	-1		V2 · · · ·
1	0	$\gamma_2[(1-\eta\beta_2)(x-\eta y_1)+\eta-c]$	$rac{1}{\sqrt{2}}\gamma_2\hat{\eta}\left(1+\hat{\eta}eta_2 ight)$
-1	0		
0	1	$\gamma_1[(1-\etaeta_1)\left(x+\eta y_1 ight)+eta_1-c]$	$rac{1}{\sqrt{2}}\gamma_1\hat{\eta}\left(1+\hat{\eta}eta_1 ight)$
0	-1		¥-
1	-1	$-\frac{1}{\sqrt{2}}$	0
-1	1	· · ·	
0	0	$\gamma_1\gamma_2*$	$-\frac{1}{\sqrt{2}}\gamma_1\gamma_2*$
		$\left[ y_1 \left(eta_2 - eta_1 ight) + x \left(1 - eta_1 eta_2 ight) + eta_1 - c  ight]  ight.$	$\left[1+eta_1\dot{eta_2}-c\left(eta_1+eta_2 ight) ight]$
		$M_{u}\left(\Delta\sigma;\lambda_{1},\lambda_{2} ight)$	
1	1	$ $ $-[(x- ilde\eta y_2)+ frac{ ilde\eta+c}{2}]$ $ $	$\frac{1}{\sqrt{2}}(1-\check{\eta}c)$
-1	-1	_	· · · ·
1	0	$\left  -\gamma_2 [(1+\etaeta_2)\left(x-\eta y_2 ight)+eta_2+c] ight $	$-rac{1}{\sqrt{2}}\gamma_2\hat{\eta}\left(1+\hat{\eta}eta_2 ight)$
-1	0		· -
0	1	$\left  {{\left. { - {\gamma _1}\left[ {\left( {1 + \eta {eta_1}}  ight)\left( {x + \eta {y_2}}  ight) - \eta + c}  ight]} }  ight.  ight $	$-\frac{1}{\sqrt{2}}\gamma_1\hat{\eta}\left(1+\hat{\eta}eta_1 ight)$
0	-1		
1	-1	$-\frac{1}{\sqrt{2}}$	0
-1	1		
0	0	$-\gamma_1\gamma_2*$	$-\frac{1}{\sqrt{2}}\gamma_1\gamma_2*$
		$\left[y_2\left(\beta_1-\beta_2\right)+x\left(1-\beta_1\beta_2\right)+\beta_2+c\right]$	$\left[1+eta_1ecareta_2+c\left(eta_1+eta_2 ight) ight]$
		$M_{s}\left(\Delta\sigma;\lambda_{1},\lambda_{2} ight)$	
1	1	0	$2\sqrt{2}$
-1	-1		
1	0	0	0
-1	0		
0		0	0
<u>U</u> 1	-1	0	0
1 -1	-1	U	U
		0	$-2\sqrt{2}\left(1+\beta_{1}\beta_{2}\right)\approx \alpha_{1}$
U	0	U	$= -2\sqrt{2} (1 + p_1 p_2) \gamma_1 \gamma_2$

production of the non-standard gauge bosons  $e^-e^- \rightarrow W_1^-W_2^-, W_2^-W_2^-$  are larger but then also the thresholds for these processes are higher. For  $\sqrt{s} = 1$  TeV the cross section  $\sigma(e^-e^- \rightarrow W^-(80)W^-(600)) \sim 10$  fb and for  $\sqrt{s} = 1.5$  TeV  $\sigma(e^-e^- \rightarrow W^-(600)W^-(600)) \sim 5$  pb.

We have also checked how the CP-violating parameters influence the total cross section. We have found that  $\sigma_{tot}$  is the biggest if CP is conserved and if all heavy neutrinos CP parities  $\eta_{CP}$  are the same, equal +i or -i. If heavy neutrinos have mixing CP parities (for some  $\eta_{CP} = +i$  and for others  $\eta_{CP} = -i$ ) or if CP is violated then the cross section is smaller. The effect of the CP-symmetry breaking can be very large. This supports the statement that the size of the CP violation can be large if in the lepton sector the Majorana neutrinos are present.

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# APPENDIX: HELICITY AMPLITUDES FOR THE $l_m^- l_n^- \rightarrow W_i W_j$ PROCESS

Helicity amplitudes for two gauge bosons  $W_i^- W_j^-$ (i, j = 1, 2) production in two charged leptons  $l_m^- l_n^-$  scattering processes  $(m, n = e, \mu, \tau)$  are described generally by the Feynman diagrams in the *s*, *t*, and *u* channels (Fig. 10). The vertices for the *t* and *u* channels are determined by the charged current Lagrangian ( $\alpha$  and  $\beta$  are spinor indices)



FIG. 10. The tree level Feynman diagrams which contribute to the  $l_m^- l_n^- \rightarrow W_i^- W_j^-$  process  $(m, n = e, \mu, \tau; i, j = 1, 2)$ . In the *LR* model all three channels are present, while in the RHS model there is no channel *s*.

$$L = \sum_{a=1,\dots,6,b=m,n} \left[ \bar{N}_{a\alpha} \left( \Gamma^{(1)\mu}_{ab} \right)_{\alpha\beta} l_{b\beta} W^{(1)}_{\mu} + \bar{N}_{a\alpha} \left( \Gamma^{(2)\mu}_{ab} \right)_{\alpha\beta} l_{b\beta} W^{(2)}_{\mu} \right], \qquad (A1)$$

where

$$\left(\Gamma_{ab}^{(1)\mu}\right)_{\alpha\beta} = \frac{g}{\sqrt{2}} \{\cos\xi(\gamma^{\mu}P_L)_{\alpha\beta}(K_L)_{ab} \\ -\sin\xi(\gamma^{\mu}P_R)_{\alpha\beta}(K_R)_{ab}\},$$

$$\left(\Gamma_{ab}^{(2)\mu}\right)_{\alpha\beta} = \frac{g}{\sqrt{2}} \{\sin\xi(\gamma^{\mu}P_L)_{\alpha\beta}(K_L)_{ab} \\ +\cos\xi(\gamma^{\mu}P_R)_{\alpha\beta}(K_R)_{ab}\}.$$

$$(A2)$$

For the LR model the mixing angle  $\xi$  and  $6 \times 3$  lepton

TABLE II. High energy behavior of the reduced helicity amplitudes for all polarizations of the  $l_m l_n^- \rightarrow W_i^- W_j^$ process  $[\tilde{\eta} = \lambda_1 \Delta \sigma, \ \tilde{\eta} = 2\sigma_1 \lambda_1, \ \eta = \Delta \sigma \Delta \lambda, \hat{\eta} = (\sigma_1 + \sigma_2)(\lambda_1 + \lambda_2)].$ 

``			
$\frac{\lambda_1}{=}$	$\lambda_2$	$\Delta \sigma = \pm 1$	$\Delta \sigma = 0$
		$M_t\left(\Delta\sigma;\lambda_1,\lambda_2 ight.$	)
1	1	$\frac{1}{2}(1-c)$	$\frac{1}{\sqrt{2}}(1+\check{\eta}c)$
-1	-1	-	V2 · · · /
1	0	$\frac{\sqrt{s}}{2M_2}(1-c)$	$rac{\sqrt{s}}{2\sqrt{2}M_2}\hat{\eta}(1+\hat{\eta}eta_2)$
-1	0		
0	1	$rac{\sqrt{s}}{2M_1}(1-c)$	$\frac{\sqrt{s}}{2\sqrt{2}M_1}\hat{\eta}(1+\hat{\eta}eta_1)$
0	-1	1	
1	-1	$-\frac{1}{\sqrt{2}}$	0
-1	1	· -	
0	0	$\frac{s}{4M_1M_2}(1-c)$	$-\frac{s}{2\sqrt{2}M_1M_2}(1-c)$
		$M_u\left(\Delta\sigma;\lambda_1,\lambda_2 ight)$	)
1	1	$-\frac{1}{2}(1+c)$	$\frac{1}{\sqrt{2}}(1-\check{\eta}c)$
-1	-1	_	
1	0	$-\frac{\sqrt{s}}{2M_2}(1+c)$	$-\frac{\sqrt{s}}{2\sqrt{2}M_{\pi}}\hat{\eta}(1+\hat{\eta}eta_2)$
-1	0	21122	2 2 2 1012
0	1	$-\frac{\sqrt{s}}{2M_1}(1+c)$	$-rac{\sqrt{s}}{2\sqrt{2}M_{\star}}\hat{\eta}(1+\hat{\eta}eta_1)$
0	-1	1	202101
1	-1	$-\frac{1}{\sqrt{2}}$	0
-1	1		
0	0	$-rac{s}{4M_1M_2}(1+c)$	$-\frac{s}{2\sqrt{2}M_1M_2}(1+c)$
	• • • • • • • • • • • • • • • • • • •	$M_{s}\left(\Delta\sigma;\lambda_{1},\lambda_{2} ight)$	)
1	1	0	$2\sqrt{2}$
-1	-1		
1	0	0	0
-1	0		
0	1	0	0
0	-1		
1	-1	0	0
-1	1		
0	0	0	$-rac{\sqrt{2}s}{M_1M_2}$

mixing matrices  $K_{L,R}$  are defined in [4], for example. In the RHS model with  $n_R$  right neutrino singlets  $\xi = 0$ ,  $K_R = 0$  and the  $K_L$  matrix has dimensions  $(3+n_R, 3)$ .

In the model where the doubly charged Higgs bosons

 $\delta^{++}$  exist (e.g., models with the Higgs triplets) the considered process has the *s*-channel diagram too, determined by the  $\delta^{++}W^-W^-$  and the Yukawa Lagrangian couplings. In the *LR* model,

 $L_{HWW} = -\frac{g^2 v_L}{\sqrt{2}} \delta_L^{++} \left( \cos^2 \xi W_1^- W_1^- + \sin^2 \xi W_2^- W_2^- + \sin 2\xi W_1^- W_2^- \right)$  $- \frac{g^2 v_R}{\sqrt{2}} \delta_R^{++} \left( \sin^2 \xi W_1^- W_1^- + \cos^2 \xi W_2^- W_2^- - \sin 2\xi W_1^- W_2^- \right) + \text{H.c.},$ (A3)

$$L_{\text{Yukawa}} = \sum_{a,m,n} \left( \frac{1}{\sqrt{2}v_L} \left[ \delta_L^{++} l_m^T C P_L \left( K_L^T \right)_{ma} m_a (K_L)_{an} l_n \right] + \frac{1}{\sqrt{2}v_R} \left[ \delta_R^{++} l_m^T C P_R \left( K_R^T \right)_{ma} m_a (K_R)_{an} l_n \right] \right) + \text{H.c.}$$

Four Feynman diagrams give contributions to the process  $l_m^- l_n^- \to W_i^- W_j^-$  at the tree level. The differential cross section is given by

$$\frac{d\sigma_{mn}^{ij}(\sigma_1,\sigma_2;\lambda_1,\lambda_2)}{d\cos\Theta} = \frac{x}{16\pi s} \left| M(\sigma_1,\sigma_2;\lambda_1,\lambda_2)_{mn}^{ij} \right|^2, \tag{A4}$$

where  $\sigma_1(\sigma_2)$ ,  $\lambda_1(\lambda_2)$  are the helicities of m(n) fermions and  $W_i(W_j)$  gauge bosons, respectively. The helicity amplitudes can be written in the form ( $\Theta$ ,  $\phi$  are polar angles of  $W_i^-$  in the c.m. frame)

$$M\left(\sigma_{1},\sigma_{2};\lambda_{1},\lambda_{2}\right)_{mn}^{ij} = -\frac{e^{2}}{\sqrt{2}\sin^{2}\Theta_{W}}D_{\Delta\sigma,\ \Delta\lambda}^{J}\left(0,\Theta,\phi\right)$$
$$\times \left\{B^{ij}(\sigma_{1},\sigma_{2})M_{t}(\Delta\sigma;\lambda_{1},\lambda_{2})\left(R_{t}\right)_{mn} + B^{ij}(\sigma_{2},\sigma_{1})M_{u}(\Delta\sigma;\lambda_{1},\lambda_{2})\left(R_{u}\right)_{mn} + B^{ij}(\sigma_{1},\sigma_{2})M_{s}(\Delta\sigma;\lambda_{1},\lambda_{2})\left(R_{s}\right)_{mn}\right\},\tag{A5}$$

where

$$(R_t)_{mn} = \sum_{a} \left( K_{2\sigma_1}^T \right)_{ma} \frac{\left( \frac{m_a}{\sqrt{s}} \right)^{1-|\Delta\sigma|}}{A + x \cos \Theta - \frac{m_a^2}{2s}} (K_{2\sigma_2})_{an}, \tag{A6}$$

$$(R_u)_{mn} = \sum_{a} \left( K_{2\sigma_2}^T \right)_{ma} \frac{\left( \frac{m_a}{\sqrt{s}} \right)^{-1-1}}{A - x \cos \Theta - \frac{m_a^2}{2s}} (K_{2\sigma_1})_{an}, \tag{A7}$$

$$(R_s)_{mn} = \delta_{\sigma_1, 1/2} \frac{1}{1 - \frac{M_{\delta_R}^2}{s} + i \frac{\Gamma_{\delta_R} M_{\delta_R}}{s}} \sum_a \left( K_R^T \right)_{ma} \left( \frac{m_a}{\sqrt{s}} \right) (K_R)_{an} + \delta_{\sigma_1, -1/2} \frac{1}{1 - \frac{M_{\delta_L}^2}{s} + i \frac{\Gamma_{\delta_L} M_{\delta_L}}{s}} \sum_a \left( K_L^T \right)_{ma} \left( \frac{m_a}{\sqrt{s}} \right) (K_L)_{an},$$
(A8)

and

$$\begin{split} K_{\kappa} &= \begin{cases} K_{R} & \text{for } \kappa = +1 \\ K_{L} & \text{for } \kappa = -1 \end{cases}, \\ \Delta \sigma &= \sigma_{1} - \sigma_{2} \; , \; \Delta \lambda = \lambda_{1} - \lambda_{2} \; , \; J = \max(|\Delta \sigma|, |\Delta \lambda|), \\ A &= \frac{M_{W_{i}}^{2} - M_{W_{j}}^{2}}{4s^{2}} - \frac{1 + 4x^{2}}{4}, \\ x &= \frac{k}{\sqrt{s}} \; , \; k = \frac{1}{2\sqrt{s}} \sqrt{s^{2} - 2s(M_{1}^{2} + M_{2}^{2}) + (M_{1}^{2} - M_{2}^{2})^{2}}. \end{split}$$

The factors B are different for various gauge boson productions

$$B^{ij}(\sigma_1, \sigma_2) = \begin{cases} -2\sigma_1(\sin\xi)^{||\Delta\sigma| - 2\sigma_2|}(\cos\xi)^{||\Delta\sigma| - 2\sigma_1|}, & i = 1, j = 2, \\ 4\sigma_1\sigma_2(\sin\xi)^{(1+\sigma_1+\sigma_2)}(\cos\xi)^{(1-\sigma_1-\sigma_2)}, & i = 1, j = 1, \\ (\sin\xi)^{(1-\sigma_1-\sigma_2)}(\cos\xi)^{(1+\sigma_1+\sigma_2)}, & i = 2, j = 2. \end{cases}$$
(A9)

The reduced helicity amplitudes  $M_{t(u,s)}(\Delta\sigma;\lambda_1,\lambda_2)$  are gathered in Table I, where we use the notation

$$y_{1(2)}=rac{E_{1(2)}}{\sqrt{s}}~,~~eta_{1(2)}=rac{k}{E_{1(2)}}~,~~\gamma_{1(2)}=rac{E_{1(2)}}{M_{1(2)}},$$

and

$$E_{1(2)}=rac{s+M_{2(1)}^2-M_{1(2)}^2}{2\sqrt{s}}$$

where  $E_1(E_2)$  and  $M_1(M_2)$  are energies and masses of

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1(2) gauge bosons, respectively. The high energy behavior of the helicity amplitudes are given in Table II. It is worthwhile to notice several interesting properties of the helicity amplitudes [Eq. (A5)]: if  $M_L = 0$  then  $\delta_L^{--}$  does not contribute to the process [Eqs. (12),(A8)]; the  $\delta_L^{--}(\delta_R^{--})$  in the *s* channel contributes only to the amplitude with the electron helicities -1/2(+1/2); although there are no Majorana neutrinos in the *s* channel the amplitude in this channel is also proportional to the neutrino masses; the helicity amplitudes with  $\Delta \sigma = 0$  are proportional to the neutrino masses  $m_a$ .

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