Duality symmetric quantization of superstrings

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A general covariant quantization of a superparticle, Green-Schwarz superstring, and a supermembrane with manifest supersymmetry and duality symmetry is proposed. This quantization provides a natural-quantum mechanical description of curved BPS-type backgrounds related to the ultrashort supersymmetry multiplets. Half-size commuting and anticommuting Killing spinors admitted by such backgrounds in quantum theory become truncated κ-symmetry ghosts. The symmetry of Killing spinors under dualities transfers to the symmetry of the spectrum of states. A GS superstring in the generalized semi-light-cone gauge can be quantized consistently in the background of ten-dimensional supersymmetric gravitational waves. Upon compactification they become supersymmetric electrically charged black holes, either massive or massless. However, the generalized light-cone gauge breaks S duality. We propose a new family of gauges, which we call black hole gauges. These gauges are suitable for quantization both in flat Minkowski space and in the black hole background, and they are duality symmetric. As an example, a manifestly S-duality symmetric black hole gauge is constructed in terms of the axion-dilaton-electric-magnetic black hole hair. We also suggest the U-duality covariant class of gauges for type II superstrings.

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I. INTRODUCTION

For many years one of the main goals in the quantization of superstrings, superparticles, and supermembranes was to perform the quantization-preserving manifest ten-dimensional supersymmetry and Poincaré invariance. This proved to be an extremely complicated problem; its formal solution involved operations with an infinite number of ghosts for ghosts. It may happen that eventually we will find a simple way of working with this formalism. However, it is not inconceivable that the requirement of ten-dimensional Poincaré invariance is excessively strong, since it prevents us from studying string theory in the self-consistent gravitational background created by string excitations.

Indeed, the main idea behind the standard approach was to use perturbation theory near the flat Poincaré-invariant background. However, recently it was conjectured that among the string eigenstates there are extreme black holes. These black holes in some respects behave as ordinary elementary particles. A consistent quantization of string theory should describe such states as well. Meanwhile, black holes certainly cannot be represented as small perturbations of the flat Minkowski space. Some eigenstates of the string theory, which look like extreme black holes in $d = 4$, can be considered as gravitational waves in $d = 10$. However, these gravitational waves are not the usual plane waves obtained by solving equations for small perturbations in the linear approximation (which could be associated with gravitons), but rather exact solutions of the full nonlinear gravitational equations.

These solutions have many interesting properties. First of all, the bosonic configurations have unbroken supersymmetries related to the existence of the Killing spinors. This leads to a supersymmetric nonrenormalization theorem [1] in gravitational theories. This theorem implies that all perturbative quantum gravity corrections to the on-shell classical action vanish in these backgrounds. This is very unusual, since previously the only background with such a property was the trivial flat Minkowski space. Also, these solutions, which are called Bogomolny-Prasad-Sommerfield (BPS) states, saturate the supersymmetric positivity bound

$$M = |Z| \; . \tag{1}$$

They have some (nonperturbative) duality symmetries, which relate to each other, e.g., electric and magnetic black holes. There are strong indications that understanding of such states will be essential for the investigation of nonperturbative properties of string theory.

One may try to find a generally covariant string quantization procedure which will preserve such nonperturbative symmetries as duality invariance. This will be the main goal which we will try to pursue in the present paper. If successful, such a program may provide a maximally symmetric quantization compatible with the nonperturbative structure of the theory. Simultaneously, it may give us an adequate quantum-mechanical description of extreme black holes, gravitational waves, and other BPS states in terms of string theory.

Investigation of the BPS states it the key feature of recent activities in string theory. However, these states have a dual status. To obtain the corresponding solutions one is using purely classical concepts of space and time and classical fields, including the metric of extreme black holes or fundamental strings and membranes. On the other hand, one is attributing a certain quantum-

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mechanical meaning to these states. One would like to get a coherent description of these "geometry states" from the point of view of string theory. As a step towards understanding the quantum-mechanical meaning of the states describing various geometries, we suggest finding the place of the BPS states in the quantization of Siegel's κ symmetries.

It is known that the gauge κ symmetries on the world line of the superparticle, on the world sheet of the Green-Schwarz (GS) superstring, and in the world volume of the supermembrane share the following important property: the spinorial parameter of the gauge transformation has to be somehow broken into two parts. Only one-half of the spinor has to take part in the transformations, the second part has to be thrown out to comply with unitarity principle. This implies that the BPS-type geometries with one-half of supersymmetry unbroken may help us to perform the quantization of κ-symmetric objects. In addition, the procedure of quantization via such geometries may help us to reveal the hidden symmetries of classical supergravities [2] in the spectra of the quantized string.

Another important feature of κ symmetries is that for the classical (not quantized) superstring to be invariant under the κ symmetry in a nontrivial background requires the background to satisfy classical equations of motion. Thus even the classical κ-symmetric string lives only in configurations which solve equations of motions, in particular, in the soliton-type configurations. We will find that for the quantized string the constraints on the background are even stronger.

Thus we propose to implement the structure of BPS states and the hidden symmetries of classical supergravities [2] into the quantization of the gauge symmetries of the superparticle, of the Green-Schwarz superstring and of the supermembrane. Upon quantization one may expect that only the S, T, and U dualities [3–6] will survive. We would also like to reveal via quantization the spectrum of the black hole multiplets with states classified by USp(4)×SU(2) and the value of the left-moving charge for the heterotic string, and by USp(8)×SU(2) for a type-II superstring.

Our approach will be very closely related to the method of quantization in the generalized semi-light-cone gauge suggested some time ago by the author and by Morozov [7, 8]. Semi-light-cone gauge is the gauge where the two-dimensional metric is in the conformal gauge and the spinor in the light-cone gauge $\gamma^+\theta = 0$. Generalized semi-light-cone gauge has a more general constraint on the spinor in terms of a null vector $n_\mu \gamma^\mu \theta = 0$, $n^2 = 0$. Also an alternating set of gauges with two null vectors $n^2 = 0, m^2 = 0$, $2mn = 1$ was used.

The formalism of Ref. [7, 8] was further developed by Grisaru, Nishino, and Zanon [9] and by Candiello, Lechner, and Tonin [10] for the heterotic σ model in a curved background. The most recent results are described in [10] where also the reference to previous work can be found. Both groups have found the constraints on possible backgrounds where the heterotic string can be quantized consistently in the generalized light-cone gauge. It remained unnoticed, however, that their constraints are satisfied, in particular, by the background of extreme electric black holes. We describe the backgrounds in which the already existing quantization is valid and pay special attention to the unusual properties of massless black holes.

The generalized light-cone gauge, in which the heterotic GS string was already quantized, breaks S duality since the above-mentioned constraints on Killing spinors are not satisfied by magnetic black holes. The basic reason for this is the fact that the Killing spinors of electric black holes are constrained by the null condition whereas in magnetic case the constraint is chiral.

In this paper we will introduce a more general class of gauges, which will allow us to perform a consistent string quantization without an infinite number of ghosts in an arbitrary half-supersymmetric background. In particular, quantization can be performed in backgrounds including all known types of extreme electric and magnetic black holes and gravitational waves which could be obtained by the black hole uplifting to $d = 10$. We will be able to go from an electric black hole background to a magnetic one simply by changing a gauge.

Note that we will be able to use these gauges even in Minkowski space, i.e., even in the absence of black holes. Thus one can use our method of quantization even in the limit when the background becomes trivial. However, this method becomes especially adequate for investigation of physical processes in the nontrivial backgrounds corresponding to the eigenstates of the string theory, in particular when the background is given by massive and/or massless black holes. The saturation of the supersymmetric positivity bound in the limit

$$M = |Z| \to 0 \tag{2}$$

to the massless states will be included in our analysis.

The paper is organized as follows. In Secs. I–III we formulate the general covariant quantization scheme for κ-symmetric theories in arbitrary half-supersymmetric backgrounds. In Secs. IV–IX we apply the new quantization rules mostly in case of the four-dimensional black hole backgrounds.

In Sec. II we introduce a new principle of a general covariant quantization of the κ-symmetric objects in arbitrary half-supersymmetric curved backgrounds. We also identify the background in which heterotic GS superstring in generalized light-cone gauge is known to be quantized consistently, in compactified theory, with the extreme electrically charged black holes. By passing, we describe the unusual singularities of massless supersymmetric black holes. In Sec. III we formulate the general duality covariant constraint on the ghosts, which provides a truncation of infinite reducibility of κ symmetry. The constraint is given in terms of the zero mode of the supercharge of the background. The central charge of the background is used for algebraic constraint on spinors. Sec. IV displays the algebra of supercharges describing the BPS states. In Sec. V we present duality covariant gauges for the heterotic string. Sec. VI is devoted to the procedure of quantization of κ-symmetric objects in flat backgrounds, for which the gauge-fixing condition is defined in the limit of the vanishing background. In Sec. VII we describe the path integral for the Green-Schwarz heterotic superstring in duality-symmetric gauges. Section VIII contains a detailed description of the S-duality covariant class of gauges in terms of the axion-dilaton black hole hair. In particular, we show that the spinorial part of the black hole gauge behaves as a modular form of the weight $\left(\frac{1}{4}, -\frac{1}{4}\right)$ for the left-handed part and as the modular form of the weight $\left(-\frac{1}{4}, \frac{1}{4}\right)$ for the right-handed part under S-duality transformations. This is the condition under which the partition function on the torus is duality invariant. In Sec. IX the U-duality covariant class of gauges is described.

In Appendix A we present some details on supersymmetric gravitational waves in which the GS string can be quantized consistently in the generalized light-cone gauge. We also display the four-dimensional electrically charged black holes related to these waves. In Appendix B we discuss *massless* four-dimensional multi-black holes related to ten-dimensional supersymmetric waves.

II. GENERAL COVARIANT QUANTIZATION OF κ SYMMETRY

Consider the Green-Schwarz superstring (as well as other objects as superparticle and supermembrane which have local κ symmetry) coupled to a most general on-shell background[1] whose bosonic part has one-half of unbroken supersymmetries. This means that the supersymmetric variation of gravitino[2] vanishes,

$$\delta_{\rm SUSY}\Psi_\mu = \hat{\nabla}_\mu \epsilon_k = 0, \tag{3}$$

for some nonvanishing values of the supersymmetry parameter ϵ_k which is called Killing spinor. For the supermembrane we are looking for the Killing spinors of 11-dimensional supergravity. For type-II string theory the background gravitino is given by two $d = 10$ Majorana-Weyl spinors (of opposite chirality for IIA and same chirality for IIB string) and by one Majorana-Weyl spinor for the heterotic string and for the superparticle. In all cases the background defines the split of the full spinor ϵ into two parts, which we will call Killing spinor ϵ_k and anti-Killing spinor $\epsilon_{\bar{k}}$:

$$\epsilon = \epsilon_k + \epsilon_{\bar{k}} \ . \tag{4}$$

In terms of this split the concept of one-half of unbroken supersymmetry means that for a given background only the Killing spinor is the nonvanishing covariantly constant spinor, and its dimension is equal to one-half of the dimension of the full spinor, or, equivalently, the dimension of the Killing spinor equals that of the anti-Killing one:

$$\hat{\nabla}_\mu \epsilon_k = 0, \quad \epsilon_k \neq 0 \ , \tag{5}$$

and

$$\hat{\nabla}_\mu \epsilon_{\bar{k}} = 0, \quad \epsilon_{\bar{k}} = 0 \ . \tag{6}$$

In addition,

$$\dim\,[\epsilon_k] = \dim\,[\epsilon_{\bar{k}}] \ . \tag{7}$$

Typically, for the opposite sign of electric and magnetic charges of the background the role of the Killing and anti-Killing spinors is reversed:

$$q \to -q, \quad p \to -p, \quad \epsilon_k \to \epsilon_{\bar{k}}, \quad \epsilon_{\bar{k}} \to \epsilon_k \ , \tag{8}$$

i.e., in the "antibackground," characterized by the opposite charges, the anti-Killing spinor is the nonvanishing covariantly constant spinor.[3]

We will treat separately the backgrounds with all supersymmetries unbroken (like flat space or Robinson-Bertotti-type geometries), where

$$\hat{\nabla}_\mu \epsilon_k = 0, \quad \epsilon_k \neq 0 \ , \tag{9}$$

and

$$\hat{\nabla}_\mu \epsilon_{\bar{k}} = 0, \quad \epsilon_{\bar{k}} \neq 0 \ . \tag{10}$$

For the purpose of quantization in flat background or in any other maximally supersymmetric background (9) and (10) the role of the half-supersymmetric background (5) and (6) is to motivate the choice of the gauge fixing of κ-symmetric theories even when the curved background is already absent: there will be still some special trace of it in the system, like magnetization in the absence of a magnetic field. In particular, even in the flat background we will basically use the black hole hair which carries the duality property of the theory and represents the property of the curved space at infinity.

The Killing–anti-Killing split of the full spinor in half-supersymmetric backgrounds (5) and (6) is described as a specific algebraic relation of the type

$$\begin{aligned} \epsilon_k &= \chi_k \epsilon \ , \\ \epsilon_{\bar{k}} &= \chi_{\bar{k}} \epsilon \ , \end{aligned} \tag{11}$$

where the projectors χ_k and $\chi_{\bar{k}}$ have the properties

$$\begin{aligned} \chi_k \chi_{\bar{k}} &= 0 \ , \\ \chi_k \chi_k &= \chi_k \ , \\ \chi_{\bar{k}} \chi_{\bar{k}} &= \chi_{\bar{k}} \ . \end{aligned} \tag{12}$$

The covariantly constant spinors may or may not depend on space-time coordinates (depending on the configuration and on the frame) but in all cases the algebraic relation of the type shown above is valid for the constant part of the spinors which they approach at infinity (for asymptotically flat space-times).

The basic problem in quantization of κ symmetry for the superparticle, for the Green-Schwarz string theory

[1] Under the on-shell background we mean the background satisfying the classical field equations, which however may have to be corrected to avoid κ-symmetry anomalies.

[2] For configurations solving field equations and admitting supercovariantly constant spinors defined in Eq. (3) the supersymmetry variations of dilatino and gluino with the Killing spinor parameter ϵ_k vanishes, as will be explained later.

[3] In presence of SL$(2, Z)$ symmetry, as we will see later, transformation (8) is a part of SL$(2, Z)$.

and for the supermembrane is the following. The gauge symmetry starts with the classical fields of the action, but after it is fixed by using the first generation ghosts, the ghost system also requires a gauge fixing, etc. The origin of the problem is in the fact that the generator of the gauge symmetry of the first-generation ghosts in these theories is nilpotent on shell [7].

A procedure to truncate this infinite set of gauge symmetries was suggested in [7] on the basis of Batalin-Vilkovisky [11] quantization method. We have proposed to use some algebraic constraint on the mth generation of the ghosts of κ symmetry, which makes the dimension of the truncated ghost equal to one-half of the nontruncated one. The untruncated ghost is a gauge field which requires a next generation of ghosts, whereas the truncated one does not require a gauge fixing, or, to be more precise, it does nor require the next generation of the propagating ghosts. The truncation was presented in Eq. (22) of [7] in the form

$$\sigma^a{}_\alpha C^\alpha{}_{(m)} = 0, \quad a = 1, \ldots, 8, \quad \alpha = 1, \ldots, 16,$$
$$C^\alpha{}_{(m)} = \tilde{\sigma}^\alpha{}_a C^\alpha{}_{(m)}, \quad \sigma^a{}_\alpha \tilde{\sigma}^\alpha{}_b = 0. \qquad (13)$$

The issues of gauge independence as well as independence on the truncation procedure were clarified in this paper. It is clear now that despite many years of existence of this formal quantization in terms of *arbitrary* orthogonal projectors $\sigma, \tilde{\sigma}$ we were lacking many interesting examples of such projectors which we know now. Moreover, as we will see later in various examples, duality symmetries are the symmetries which rotate these projectors, or, in other words, make all of them possible. The algebraic constraint in our new quantization will come out from the algebraic constraints which the Killing spinors of the half-supersymmetric backgrounds satisfy. From this point of view there will be no preference to any constraint: they will appear on equal footing in the quantized string.

At the time when the quantization [7] was performed the set of algebraic constraints which was available was not very rich. In addition to the standard light-cone condition $\gamma^+\theta = (\gamma^0 + \gamma^9)\theta = 0$ we have introduced a generalized light-cone condition, in which the constraint on spinors was realized in terms of two null vectors:

$$n_\mu n^\mu = m_\mu m^\mu = 0 \ , \quad m_\mu n^\mu = \frac{1}{2} \ . \qquad (14)$$

In particular, we have imposed the algebraic constraint on the first-generation ghosts $C_{(1)}$ in the form

$$\not{m} \not{n} \, C_{(1)} = 0 \ , \quad C_{(1)} = \not{n} \not{m} C_{(1)} \ . \qquad (15)$$

When constraint of this type is imposed, the theory can be quantized as an irreducible theory with one generation of ghosts of κ symmetry. The gauge, in which the two-dimensional metric was considered in the conformal gauge and the the fermionic coordinate of the GS string θ in the light-cone gauge, was called the semilight cone gauge. When the fermionic variable θ was constrained in terms of two-null vectors, as explained above, this gauge was called the generalized semi-light-cone gauge.

We have also considered a less restricted differential, nonalgebraic gauge for θ in which besides the standard Faddeev-Popov (FP) ghosts also the propagating Nielsen-Kallosh (NK) ghosts (related to Nakanishi-Lathrup fields) had to be taken into account. The role of these ghosts in Becchi-Rouet-Stora-Tyutin (BRST) quantization was clarified in [12]. In this gauge the space-time supersymmetry is realized linearly, as different from the one with the algebraic constraint.

The partition function for the GS heterotic superstring was constructed in [7, 8]. It was shown to be independent (at least formally) on the choice of the truncation condition on the ghosts and on the choice of the gauge condition on the fermionic variable θ. In particular, in this way one proves the independence on the directions n, m in the choice of constraints. The contribution of the second-class constraints was taken in the form in which it was derived for the first time in the series of papers by Fradkin and collaborators [13].

If one makes a special choice of the vectors n_μ, m_μ one can recover the standard light-cone gauge. This corresponds to the choice $\not{n} = \gamma^0 + \gamma^9$. However, for arbitrary choice of the vectors there is no need to pick up the direction 9, it could be any directions in the nine-dimensional space $1, 2, 3, 4, 5, 6, 7, 8, 9$. Using the modern language one can summarize this presentation by the statement that our generalized light-cone gauge has a T-duality symmetry, SO(6) part of it, whereas the standard light-cone gauge $(\gamma^0 + \gamma^9)\theta = 0$ breaks T duality. A remarkable thing about the proof [7, 8] of the independence of the physical states on the choice of the direction n, m is that it suggests a proof of the T duality of the states which arise in the quantization of the string.

Comparing our old truncation condition (15) with the properties of Killing spinors of the half-supersymmetric backgrounds in general, given in Eqs. (11) and (12), we see that we have used one particular example of the general projectors. Our projectors in (15) obviously satisfy the relations

$$(\not{m} \not{n})(\not{n} \not{m}) = 0 \ , \quad (\not{m} \not{n})^2 = (\not{m} \not{n}) \ , \quad (\not{n} \not{m})^2 = (\not{n} \not{m}) \ . \qquad (16)$$

Therefore, the generalized (m, n) light-cone-type truncation of fermionic symmetry, which was used in [7, 8], is associated with the Killing spinors of the backgrounds for which

$$\chi_k(\mathrm{GLC}) = \not{n} \not{m} \ , \quad \chi_{\bar{k}}(\mathrm{GLC}) = \not{m} \not{n} \ . \qquad (17)$$

The heterotic GS superstring σ model was constructed in [9]. It was discovered there that the quantization of the heterotic string in generalized (m, n) light-cone gauge is consistent only when the background is constrained in a specific way, the constraint being stronger than the requirement that the background satisfies classical equations of motion. The most recent quantization of the GS heterotic σ model was performed in [10]. Both groups have studied the issues of κ anomalies.

We would like to reformulate here the constraint on the background as given in [9, 10] in a form which is suitable for the generalization to most general BPS states. These states correspond to backgrounds of the superstring, which have fermionic isometries related to Killing spinors of dimension equal to one-half of the full spinor. To classify those isometries we will introduce the following definitions.

Supersymmetric gravitational waves are the supergeometries whose bosonic part admits a supercovariant Killing spinor and a null Killing vector.

Supersymmetric gravitational waves of electric type are the supergeometries whose bosonic part admits a supercovariant Killing spinor satisfying the null constraint $\not{n}\epsilon_k = 0$ where n is a null Killing vector.

Supersymmetric pp waves are the special set of supersymmetric gravitational waves of electric type whose bosonic part admits a covariantly constant null Killing vector.

Supersymmetric gravitational waves of magnetic type are the supergeometries whose bosonic part admits a supercovariant Killing spinor satisfying a chiral constraint $(1-\Gamma^5)\epsilon_k = 0$, where $1-\Gamma^5$ is a chiral projector in any SO(4) subspace of the full SO(1,9) tangent space of the supergeometry. They also admit at least one null Killing vector.

Supersymmetric gravitational waves of electromagnetic type are the supergeometries whose bosonic part admits at least one null Killing vector and the supercovariant Killing spinor satisfies the constraint which is neither null nor chiral.

The constraints on the backgrounds in which the heterotic string can be quantized consistently in the generalized light-cone gauge were presented in [9, 10]. It remained unnoticed that these constraints require the background to correspond to electric BPS states. In more precise form our analysis shows the following.

The heterotic GS string can be consistently quantized in (m,n) light-cone gauge in the background of ten-dimensional supersymmetric gravitational waves of electric type or in any compactified form of it. In particular when the supersymmetric wave is reduced to four-dimensional theory, one gets the most general electrically charged extreme black-hole-type solutions of heterotic string.

Indeed, the background has to admit algebraically constrained covariantly constant spinors to comply with the requirements of truncation of gauge symmetry. When the algebraic constraint on the ghost is $\not{n}C_{(1)} = 0$, we are looking for the most general configurations which admit covariantly constant spinors satisfying this constraint. Since the corresponding vector n_μ is null, we are simultaneously looking for geometries which admit a null Killing vector. Indeed the constant null vector of the flat background will become covariantly constant in the curved background. This brings us to the backgrounds which have one covariantly constant null vector: to supersymmetric *pp* waves [14] which we have called supersymmetric spin waves (SSW's). The metric is that of Brinkmann, and other fields are adjusted for supersymmetry. The configuration may depend on $u, x^1, \ldots, x^8$ but has to be independent on v. If both null vectors are used in the alternative-type gauges one can relax covariant constancy of both vectors and look only for the supersymmetric backgrounds which admit two null Killing vectors. Those depend only on transverse coordinates $x^1, \ldots, x^8$ but have to be independent on both light-cone coordinates u, v. These configurations include fundamental strings [15], generalized fundamental strings [16], etc. These configurations also fall into the definitions of gravitational waves, since they admit a null Killing vector. The chiral null models of Horowitz and Tseytlin [17] also admit a supercovariantly constant Killing spinor satisfying the null constraint and belong to the class of gravitational waves of electric type. We present the relevant details on these configurations and their relation to the most general supersymmetric electrically charged multi-black-hole-type solutions in Appendices A and B. As an example we describe here the spherically symmetric electrically charged black holes in which the heterotic string is quantized consistently. They have the following four-dimensional canonical metric:

$$ds^2 = e^{2\phi} g^{-2} dt^2 - e^{-2\phi} g^2 (d\vec{x})^2 \ , \tag{18}$$

where the four-dimensional dilaton field is given by

$$e^{-2\phi} = \frac{1}{g^2}\left(1 + \frac{4mG_N}{r} + \frac{4g^2(N_L-1)}{r^2}\right)^{\frac{1}{2}} \ . \tag{19}$$

In Eq. (18) G_N is the Newton constant and N_L is the contribution from the left-moving oscillators. The Bogomolny bound in notation of [18] (for $G_N = 2$) states that the mass of the black hole is equal to the central charge of the graviton multiplet, which in turn is defined by the right-moving electric charge as well as by the combination of the left-moving charge and N_L:

$$m^2 = |Z|^2 = \frac{\vec{Q}_R}{8g^2} = \frac{g^2}{8}\left(\frac{\vec{Q}_L^2}{g^4} + 2N_L - 2\right) \ . \tag{20}$$

The relation of this solution to supersymmetric ten-dimensional gravitational waves and to four-dimensional black holes [18] is explained in Appendix A. The black hole solution (18) interpolates nicely between the $a=1$ and $a=\sqrt{3}$ heterotic string supersymmetric electrically charged black holes. Indeed, for $a=1$ solutions the left-moving charge Q_L is vanishing and therefore the dilaton is given by the harmonic function

$$e^{-2\phi} = \frac{1}{g^2}\left(1 + \frac{8m}{r} + \frac{16m^2}{r^2}\right)^{\frac{1}{2}} = \frac{1}{g^2}\left(1 + \frac{4m}{r}\right) \ . \tag{21}$$

For $a=\sqrt{3}$ we have $N_L = 1$. For this solution the dilaton is given by the square root of the harmonic function:

$$e^{-2\phi} = \frac{1}{g^2}\left(1 + \frac{8m}{r}\right)^{\frac{1}{2}} \ . \tag{22}$$

Note, however, that the general solution (18) and (19) corresponds not to black holes with arbitrary dilaton coupling a, but to more generic dimensionally reduced supersymmetric gravitational waves. It has been noticed by Behrndt [19] that there exists a massless black hole configuration in (18) and (19). Indeed, the two-parameter solution with $m = |Z| = 0$ and $N_L = 0$ has the form

(18), where the canonical four-dimensional metric is

$$ds^2 = \left(1 - \frac{4g^2}{r^2}\right)^{-\frac{1}{2}} dt^2 - \left(1 - \frac{4g^2}{r^2}\right)^{\frac{1}{2}} d\vec{x}^2 , \quad (23)$$

and the four-dimensional dilaton is

$$e^{-2\phi} = \frac{1}{g^2}\left(\frac{r^2 - 4g^2}{r^2}\right)^{\frac{1}{2}} , \quad e^{-2\phi}(r \to \infty) \equiv \frac{1}{g^2} . \quad (24)$$

One of the striking properties of massless dilaton black holes is the appearance of a new type of singularity. Massive extreme black holes have the singularity and the horizon both situated at $r = 0$ (the only nonsingular solution is the pure magnetic $a = 1$ massive extreme black hole in stringy frame). Massless states are getting an additional singularity at $r \neq 0$. The position of the singularity is related to the string coupling constant.

The electric solution is singular at $r = 2g$ (and at $r = 0$). At $r \to 2g$ the dilaton blows up. As different from massive electrically charged black holes, which near singularity $r = 0$ have small gauge coupling, the massless electrically charged black holes have infinite coupling near the singularity $r = 2g$. The singularity at $r = 0$ and the fact that the string coupling becomes small are irrelevant for massless electric black holes,

$$(e^{2\phi})^{\text{el}}_{r\to 2g} \to \infty . \quad (25)$$

The magnetic massless solution[4] has the form

$$ds^2 = \left(1 - \frac{4}{g^2 r^2}\right)^{-\frac{1}{2}} dt^2 - \left(1 - \frac{4}{g^2 r^2}\right)^{\frac{1}{2}} d\vec{x}^2 \quad (26)$$

and the four-dimensional dilaton of the magnetic solution is:

$$e^{2\phi} = g^2 \left(1 - \frac{4}{g^2 r^2}\right)^{-\frac{1}{2}} , \quad e^{2\phi}(r \to \infty) \equiv g^2 . \quad (27)$$

This solution is singular at $r = \frac{2}{g}$ (and at $r = 0$). Here again we have the picture quite opposite to the usual properties of massive magnetically charged dilaton black holes. Near the singularity $r = \frac{2}{g}$ the string coupling vanishes. Indeed, in this case

$$(e^{2\phi})^{\text{magn}}_{r\to 2/g} \to 0 . \quad (28)$$

It is particularly important that even in the limit of the vanishing ADM mass of the black hole considered above the configuration still has unbroken supersymmetry and the Killing spinor satisfies the same constraint as for the black holes with the nonvanishing mass.

The multi-black-hole solutions generalizing those in Eq. (18) as well as the most general known to us stationary supersymmetric solutions can be found in Appendix A. The massless black holes, some of their properties, including singularities, as well as more general massless black hole and multi-black-hole solutions are presented in Appendix B.

All those configurations have one-half of unbroken supersymmetry and therefore the heterotic string can be quantized consistently in these backgrounds.

III. NEW TRUNCATION OF κ SYMMETRY

The backgrounds with half of supersymmetries unbroken, which were intensively studied in the recent years, see, e.g., [20, 21] for a review, offer a much more general class of truncation of κ symmetries. To be more explicit, we may use any BPS background to get the orthogonal projectors needed for truncation of κ symmetry and defined in Eqs. (13) in old form and in Eq. (12) in a form related to the Killing–anti-Killing split in Eqs. (5), (6), and (11). Moreover, all solutions of the Killing equations (3) for 11- and 10-dimensional supergravities which are not known yet and still wait to be discovered are already included in quantization.

Consistent quantization of truncated κ symmetry is possible in the backgrounds with one-half of unbroken supersymmetry. The integrability condition for the existence of Killing spinors of the bosonic part of the background is the consistency condition for the quantization. This defines the curved superspace in which quantized κ-symmetric objects exist.

The reason why the geometries with one-half of supersymmetries unbroken (BPS-states) are singled out is related to the fact that the dimension of the truncated κ-symmetry ghost has to be one-half of the untruncated one to preserve unitarity of the quantization and the correct counting of the physical degrees of freedom.

The most general truncation of κ symmetry can be achieved in terms of the most general algebraic constraint, which the Killing spinors of the half-supersymmetric backgrounds satisfy. Our goal is not to use any specific background for this purpose, but the most general one which may define the Killing–anti-Killing split of the spinor. The key role in our quantization of the κ-symmetric systems belongs to the supercharge of the background.

The supercharge of the gravitational supersymmetric theory was defined by Teitelboim [22] in asymptotically flat spaces as the surface integral in terms of the gravitino Ψ_μ field of the configuration, solving the field equations

$$\mathcal{Q} = \oint_{\partial\Sigma} d\Sigma_{\mu\nu}\gamma^{\mu\nu\lambda}\Psi_\lambda . \quad (29)$$

The surface over which the integration has to be performed depends on the choice of configuration. In all cases it is the same surface the integration over which defines the Arnowitt-Deser-Misner (ADM) mass of a given system or the ADM mass per unit area (length). The on-shell backgrounds with one-half of supersymmetry unbroken in bosonic sectors have the vanishing supersym-

[4] These solutions do not fit into the dimensionally reduced supersymmetric gravitational waves of electric type, for which the quantization performed in [9, 10] can be applied directly. However, it will be shown later that for supersymmetric waves of magnetic type there exists a more general gauge condition, in which the quantization can be performed.

metry variation of the gravitino, when the parameters are Killing spinors, as defined in Eqs. (3), (5), and (6):

$$\mathcal{Q}_k = \oint_{\partial\Sigma} d\Sigma_{\mu\nu}\gamma^{\mu\nu\lambda}\hat{\nabla}_\lambda \epsilon_k = 0 \; . \tag{30}$$

Let us stress that κ symmetry of the classical action is preserved only in the on-shell superbackground. This means that the bosonic part of the background in absence of fermions in the solutions has to solve classical equations of motion. For the heterotic string this means that the vanishing of supersymmetry variations of gravitino Ψ_μ is a sufficient condition for the vanishing of the supersymmetry variations of dilatino λ and gluino χ. To prove it one can use the Nester construction in the form used in [23]. Thus we start with

$$\delta_{\epsilon_k}\Psi_\mu = 0 \Longrightarrow N^{\mu\nu} = \bar{\epsilon}_k\,\gamma^{\mu\nu\lambda}\hat{\nabla}_\lambda\epsilon_k = 0 \; . \tag{31}$$

Using Eq. (3.14) from [23] we get

$$\hat{\nabla}_\mu N^{\mu 0} = (\delta_{\epsilon_k}\lambda)^\dagger(\delta_{\epsilon_k}\lambda) + (\delta_{\epsilon_k}\chi)^\dagger(\delta_{\epsilon_k}\chi) + \text{field eqs.} = 0 \; . \tag{32}$$

Since the field equations have to be satisfied for the background of the superstring even before quantization, we conclude that the existence of a supercovariantly constant spinor (30) is necessary and sufficient condition for the bosonic background to have half of supersymmetries unbroken. The full background corresponding to such bosonic backgrounds has fermionic isometries of dimension equal to one-half of the dimension of the full fermionic part of the superspace.

For anti-Killing spinors the supercharge is not vanishing. For the black hole multiplets it defines the so-called superhair of the black hole:

$$\mathcal{S}_{\text{superhair}} \equiv \mathcal{Q}_{\bar{k}} = \oint_{\partial\Sigma} d\Sigma_{\mu\nu}\gamma^{\mu\nu\lambda}\hat{\nabla}_\lambda\epsilon_{\bar{k}} \; . \tag{33}$$

The concept of the *superhair* was defined for the first time for extreme Reissner-Nordström black holes in [24] and studied more recently in the context of more general extreme black holes in [25].

We postulate the new truncation of infinite reducibility of κ symmetry by requiring some odd (even) generation κ-symmetry ghost to be a commuting $m = 2n+1$ (anticommuting $m = 2n$) fermionic zero mode of the *zero supercharge condition*[5]

$$\mathcal{S}_{\text{ghost}} = \oint_{\partial\Sigma} d\Sigma_{\mu\nu}\gamma^{\mu\nu\lambda}\hat{\nabla}_\lambda C_{(m)} = 0 \; . \tag{34}$$

In other words, we require the parameter of the κ-symmetry transformation to be a Killing spinor of the geometries associated with the states which saturate the supersymmetric positivity bound and the BPS bound. This gives a perfect and universal accomplishment of the goal: to truncate the infinite reducibility of any κ-symmetric theory. The integration in (34) has to be performed over the suitably defined surface.

[5]The fermionic coordinate of the string θ is included in this set as $p = 0$ case.

For example, in the context of the ten-dimensional heterotic string toroidally compactified to four dimensions, the relevant surface defining the ADM mass of the four-dimensional black holes is the two-sphere at spatial infinity times the internal space $\partial\Sigma = S^2_\infty \times T^6$. However, for each particular class of problems the choice of a surface in the definition of the supercharge depends on the class of configurations which are interesting in specific problems. For example, supersymmetric fundamental string [15] provides a supercharge per unit of length, and the surface $\partial\Sigma$ is an eight-dimensional spacelike surface. One may expect that for the supermembrane the surface will be the same as the one defining the ten-dimensional ADM mass of the black holes. Various surfaces for the quantization of κ-symmetric objects in various phases still have to be identified. In particular, for various supersymmetric p-branes and black holes in diverse dimensions there will be all kind of relevant surfaces. A nontrivial situation may occur when the mass of the black holes vanishes due to the shrinking of the corresponding area, as shown by Strominger for type-II B string theory [26]. However even in this limit the background still provides a Killing spinor suitable for quantization.

The definition of the supercharge (as well as the definition of the ADM mass) does not violate general covariance, it is just the way to describe the gravitating systems with special behavior at infinity.

Upon integration the truncation condition acquires a form of the algebraic constraint on the Killing spinor in the half-supersymmetric bosonic background of the form

$$\mathcal{S}_{\text{ghost}} = \chi_{\bar{k}} C_{(m)} = 0 \; , \quad C_{(m)} = \chi_k C_{(m)} \; . \tag{35}$$

The wonderful property of the half-supersymmetric backgrounds is that they admit both commuting and anticommuting Killing spinors.Therefore we may use this algebraic condition either on the anticommuting variables θ in the classical action corresponding to the unitary gauge with non-propagating κ-symmetry ghosts, or on any generation of the commuting-anticommuting ghosts, since their statistics alternates.

Thus we propose to truncate the infinite reducibility of κ symmetry identifying the fermionic ghosts with the asymptotic value of the Killing spinor of the bosonic part of the background. The constraint (34) is the most general constraint which allows the truncation and consistent quantization in a given background. Simultaneously it restricts the backgrounds by requiring them to admit supercovariantly constant spinors of the dimension $\frac{1}{2}$ of the dimension of the original spinor.

Thus what remains is to find the most general background for each theory (superparticle in arbitrary dimensions, GS type-II superstring, the heterotic string, and the supermembrane) which has one half of unbroken supersymmetry. This would supply us with the most general algebraic constraint for the truncation of κ symmetry in each of the above-mentioned theories.

To us the best known example of such kind is the list of all metrics admitting supercovariantly constant spinors in $N = 2$ supergravity (and more recently in $N = 4$), performed by Tod [27]. In $N = 2$ supergravity interacting with $N = 2$ matter Tod has listed all metrics and has found all supercovariantly constant spinors. The reason for this was partially related to the fact that $N = 2$ supersymmetry with two Majorana spinors, or one Dirac spinor is suitable for the use of the highly developed Newman-Penrose formalism with commuting Dirac spinors.

In most of the other cases related to $N = 4$ supergravity with matter, or in $N = 8$ supergravity in compactified theories, or directly in $4 \geq d \geq 11$ there is a rapidly growing amount of information about bosonic configurations with one-half of unbroken supersymmetries. Those configurations are related by dualities, by dimensional reduction, and/or uplifting. Examples include extreme black holes, fundamental strings, p-branes, pp-waves, dual strings, and dual waves. However, at present we do not have all solutions of integrability conditions for the existence of Killing spinors in higher dimensional supersymmetries.

IV. EXTREME BLACK HOLE SUPERSPACE AND THE SUPERCHARGE ALGEBRA

We are interested in the quantization of the κ-symmetric superstring the background superspace with the following properties: it is an on-shell superspace (in the first approximation prior to the the study of κ-symmetry anomalies). However, only special on-shell superspaces are allowed: half of fermionic directions are isometries. This means that the system of coordinates exists in which the configuration is independent on half of fermionic coordinates of the superspace. This corresponds to the fact that the bosonic part of the background admits supercovariantly constant Killing spinors. The supercharge which forms the Clifford algebra, defining the ultra-short supermultiplet of string excitations is built in terms of anti-Killing spinor whose dimension is the same as that of the Killing spinor. Such backgrounds allow the general covariant truncation of infinite reducible κ symmetry and consistent quantization of the superstring.

Most of our attention here will be directed to the extreme four-dimensional black hole supermultiplets and their spectra. Therefore we will describe the algebra of the supercharges $\mathcal{Q}$ representing the ten-dimensional Majorana-Weyl or Majorana spinors as $d = 4, N = 4$ or $d = 4, N = 8$ spinors. However, the strategy for quantization of generic κ-symmetric objects remains the same if one is interested in the spectrum of higher-dimensional configurations.

The algebra which the supercharges $\mathcal{Q}$ satisfy in backgrounds with half of supersymmetries unbroken is most conveniently described for massive states at rest with $M = |Z|$ in terms of a $2N$-component spinors[6] [28]. In doublet form they are given by

$$Q^a_\alpha = \begin{pmatrix} Q^i_\alpha \\ Q^{*\alpha i} \end{pmatrix}, \quad Q^a_\alpha = Q^i_\alpha \quad \text{for a} = 1, \ldots, N,$$

$$Q^a_\alpha = Q^{*\alpha i} = \epsilon^{\alpha\beta} Q^{*i}_\beta \quad \text{for a} = N+1, \ldots, 2N. \tag{36}$$

These spinors satisfy a symplectic reality condition

$$Q^{*a}_\alpha = \epsilon^{\alpha\beta} \Omega_{ab} Q^b_\beta \tag{37}$$

with

$$\Omega^{ab} = -\Omega_{ab} = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \tag{38}$$

$$\{Q^a_\alpha, Q^b_\beta\} = \epsilon_{\alpha\beta} \begin{pmatrix} Z & |Z|I \\ -|Z|I & Z^* \end{pmatrix} \equiv \epsilon_{\alpha\beta}\, \mathcal{Z}^{ab} . \tag{39}$$

The $2N \times 2N$ matrix $\mathcal{Z}^{ab}$ is written in terms of $N \times N$ numerical antisymmetric complex matrix Z^{ij} and $|Z|\delta^{ij}$. The numbers Z^{ij} are the eigenvalues of the central charge operators in a given supermultiplet. For the purpose of quantization we need to consider the special BPS case when

$$-ZZ^* = ZZ^\dagger = \delta^{il}|Z|^2 . \tag{40}$$

If we would have a massive multiplet without central charges the algebra generating the states would be

$$\{Q^a_\alpha, Q^b_\beta\} = \epsilon_{\alpha\beta} \begin{pmatrix} 0 & M \\ -M & 0 \end{pmatrix} \equiv \epsilon_{\alpha\beta}\, \Omega^{ab} M . \tag{41}$$

The physical massive states of N-extended supersymmetry without central charges are classified according to USp($2N$)$\times$SU(2). When the extreme constraint is relaxed, i.e., $M \neq |Z|$, the algebra is

$$\{Q^a_\alpha, Q^b_\beta\} = \epsilon_{\alpha\beta} \begin{pmatrix} Z & MI \\ -MI & Z^* \end{pmatrix} \equiv \epsilon_{\alpha\beta}\, \mathcal{P}^{ab} . \tag{42}$$

The USp($2N$) symmetry of Eq. (41) is broken in presence of central charges Z^{ij}. The effect of Z^{ij} is to reduce the U(N) symmetry of supersymmetry algebra to subgroups of U(N) which leave the complex skew-symmetric numerical matrix Z^{ij} invariant. Therefore one has to classify the orbits of the twofold antisymmetric representation $[N]_2$ of U(N). From all possibilities to have central charges for the purpose of quantization we are interested only in one: in *special critical orbit with USp(N) invariance.* In this case only the central charge matrix $\mathcal{Z}^{ab}$ has the properties required to fix κ symmetry: the rank of this matrix[7] equals N whereas the dimension is $2N$.

Thus we would like to use only the backgrounds with the supercharge satisfying the algebra (39). The condition of exactly one-half of unbroken supersymmetry requires all nonvanishing eigenvalues of $||Z||$ coincide. Under this condition exactly half of generators $\mathcal{Q}_k$ drop from the algebra, and USp($2N$) symmetry is broken down to

[6]The notation of Ref. [28] is used in this section.

[7]The rank of the matrix is the maximal size of its invertible square minor.

USp(N). Let us stress that the extremality condition $M = |Z|$ means that in the absence of central charges, there are no massive states. This is actually well known from black hole theory: supersymmetric extreme black holes require the presence of central charges [24, 29].

Any complex antisymmetric matrix Z^{ij} can be brought to the normal form using some U(N) rotation. For example, for $N = 4$ and for $N = 8$ respectively we get

$$\tilde{Z}_{ij} = i\sigma_2 \begin{pmatrix} |Z| & 0 \\ 0 & |Z| \end{pmatrix} ,$$

$$\tilde{Z}_{ij} = i\sigma_2 \begin{pmatrix} |Z| & 0 & 0 & 0 \\ 0 & |Z| & 0 & 0 \\ 0 & 0 & |Z| & 0 \\ 0 & 0 & 0 & |Z| \end{pmatrix} , \tag{43}$$

where $|Z|$ is a non-negative real number. When the central charge matrix is in the normal form, one can perform a symplectic transformation over $Q^a_\alpha \to T^a{}_b Q^a_\alpha = S^a_\alpha$ with some numerical matrix T [28]. As a result, we get for supersymmetry generators in the "electric black hole" basis,

$$\{S^a_\alpha, S^b_\beta\} = \epsilon_{\alpha\beta}\, \tilde{Z}^{ab} = \begin{pmatrix} 0 & |Z|(1+\sigma_3) \\ -|Z|(1+\sigma_3) & 0 \end{pmatrix} . \tag{44}$$

Now it is easy to define the projectors which separate the vanishing supercharge $\mathcal{Q}_{\bar{k}}$ from the anti-Killing one $\mathcal{Q}_k$:

$$\mathcal{Q}_k = \frac{1}{\sqrt{2}}[1-(-1)^{a+1}]S^a_\alpha \ , \quad \mathcal{Q}_{\bar{k}} = \frac{1}{\sqrt{2}}[1+(-1)^{a+1}]S^a_\alpha \ . \tag{45}$$

This means that in our basis the even in a components of the symplectic spinor S^a_α commute, whereas the odd components generate the spectrum. We introduce the notation

$$(\mathcal{Q}_k)^m_\alpha = \frac{1}{\sqrt{2}}[1-(-1)^{a+1}]S^a_\alpha, \quad m \equiv 2a = 2,4,\ldots,2N, \tag{46}$$

$$(\mathcal{Q}_{\bar{k}})^{\hat{m}}_\alpha = \frac{1}{\sqrt{2}}[1-(-1)^{a+1}]S^a_\alpha \ , \qquad \hat{m} \equiv 2a+1 = 1,3,\ldots,2N-1. \tag{47}$$

Each of the N-component Killing and anti-Killing spinors $(\mathcal{Q}_k)^m_\alpha$ and $(\mathcal{Q}_{\bar{k}})^{\hat{m}}_\alpha$ satisfies the symplectic reality condition which does not mix them:

$$(\mathcal{Q}_k)^{*m}_\alpha = \epsilon^{\alpha\beta}\Omega_{mn}(\mathcal{Q}_k)^n_\beta \ , \quad (\mathcal{Q}_{\bar{k}})^{*\hat{m}}_\alpha = \epsilon^{\alpha\beta}\Omega_{\hat{m}\hat{n}}(\mathcal{Q}_k)^{\hat{n}}_\beta \ . \tag{48}$$

In doublet form they are given by the N-component spinors

$$(\mathcal{Q}_k)^m_\alpha = \begin{pmatrix} (\mathcal{Q}_k)^p_\alpha \\ (\mathcal{Q}_k)^{*\alpha p} \end{pmatrix} ,$$

$$(\mathcal{Q}_{\bar{k}})^{\hat{m}}_\alpha = \begin{pmatrix} (\mathcal{Q}_{\bar{k}})^{\hat{p}}_\alpha \\ (\mathcal{Q}_{\bar{k}})^{*\alpha\hat{p}} \end{pmatrix} , \quad p,\hat{p} = 1,\ldots,\frac{N}{2} \ . \tag{49}$$

In normal basis we may rewrite the supercharge algebra (39) as

$$\{(\mathcal{Q}_k)^m_\alpha, (\mathcal{Q}_k)^n_\beta\} = 0 \ , \tag{50}$$

$$\{(\mathcal{Q}_{\bar{k}})^{\hat{m}}_\alpha, (\mathcal{Q}_{\bar{k}})^{\hat{n}}_\beta\} = |Z|\ \epsilon_{\alpha\beta}\Omega^{\hat{m}\hat{n}} \ , \tag{51}$$

$$\{(\mathcal{Q}_k)^m_\alpha, (\mathcal{Q}_{\bar{k}})^{\hat{n}}_\beta\} = 0 \ . \tag{52}$$

Thus, our extreme black hole basis for extended supersymmetry in a normal form represents two USp(N) doublets instead of the one original USp($2N$) doublet Q^a_α. Only one of those doublets (anti-Killing one) forms the Clifford algebra, the second one (Killing one) anticommutes with both of them. The Clifford vacuum Ω is annihilated by $(\mathcal{Q}_{\bar{k}})^{*\alpha\hat{p}}$ as well as by $(\mathcal{Q}_k)^{\alpha p}$. It has to be doubled since the CPT conjugation adds the states where the role of Killing and anti-Killing part of the spinors is reversed. This double degeneracy of the Clifford vacuum shows that black holes with opposite sign of central charges behave as particle-antiparticle to each other. The spectrum of states generated by the action of $(\mathcal{Q}_{\bar{k}})^{*\alpha\hat{p}}$ on the vacuum is described as follows. One set of states comes from

$$\Omega_{\bar{k}} \ , \quad (\mathcal{Q}_{\bar{k}})^{*\hat{p}}_\alpha \Omega_{\bar{k}} \ , \ldots, \quad (\mathcal{Q}_{\bar{k}})^{*\hat{p}_1}_\alpha \cdots (\mathcal{Q}_{\bar{k}})^{*\hat{p}_{N/2}}_\alpha \Omega_{\bar{k}} \ . \tag{53}$$

The Clifford vacuum state is a bosonic black hole with positive value of the central charge, other states in this chain are black hole superpartners. The set of CPT conjugate states is based on the analogous chain which starts with the Clifford vacuum which is a black hole with the opposite sign of the central charge. The states are classified by the representations of USp(N)×SU(2) group.

The generators of both groups are constructed as bilinear combinations of supercharges, where either the spinorial space-time indices or the internal ones are contracted. In particular, the generator of the USp(N) transformations which labels the states of the given spin is

$$s^{\hat{m}\hat{n}} = -\frac{i}{2\sqrt{|Z|}}\{(\mathcal{Q}_{\bar{k}})^{\hat{m}}_\alpha, (\mathcal{Q}_{\bar{k}})^{\hat{n}}_\beta\}\epsilon^{\alpha\beta} \ , \quad \hat{m},\hat{n} = 1,\ldots,N \ . \tag{54}$$

It generates the algebra of USp(N),

$$[s^{\hat{m}\hat{n}},\ s^{\hat{k}\hat{l}}] = \Omega^{\hat{m}\hat{k}}s^{\hat{n}\hat{l}} + \cdots . \tag{55}$$

Since we are going to use for gauge fixing the split of the spinor defined by the supercharge of the background, one may expect that eventually the algebra generating the full set of the BPS states will appear via Noether charges of the quantized string. We will show that in the normal form (52) the algebra will be associated with the light-cone gauge and electrically charged black holes. The same algebra in general will be shown to represent the most general extreme black holes saturating the BPS bound.

V. DUALITY-COVARIANT GAUGES FOR THE HETEROTIC STRING

We would like to consider the quantization of the heterotic string in the four-dimensional background of extreme black holes. To be able to accommodate the black hole hair using the standard fields of the Green-Schwarz string, which include the ten-dimensional Majorana-Weyl spinor depending on the world-sheet coordinates $z, \bar{z}$, we will use the form of the constraints which Killing spinors satisfy in this background, adapting them to the ten-dimensional form. We may choose a commuting (anticommuting) κ-symmetry ghost of some generation, which is a ten-dimensional spinor, to satisfy the constraint

$$\chi_{\bar{k}}\, C(z,\bar{z}) = \frac{1-\Gamma}{2} C(z,\bar{z}) = 0 \ . \tag{56}$$

The numerical Hermitian matrix Γ is defined by the properties of the Killing spinors at asymptotic infinity of the target space. In our case it is defined by the central charges of the background:

$$\Gamma = \frac{Z}{|Z|}\ , \quad Z^2 = |Z|^2 \ . \tag{57}$$

We may use our constraint on the Killing spinor in the form which correspond to that given by Harvey and Liu [23] and Sen [30] in their presentation of the form of the Bogomolny bound.

$$\Gamma = \frac{i(\bar{\lambda}_0 - \lambda_0)}{2|Z|}\gamma^0 \sum_{a=4}^{a=9} \gamma^a (Q_a + i\gamma_5 P_a) = \Gamma^\dagger\ , \quad \Gamma^2 = 1\ , \tag{58}$$

where

$$\lambda_0 = a_0 + ie^{-2\phi_0} \tag{59}$$

is the value of the dilaton-axion complex scalar at infinity, far away from the black hole. Six electric Q_a and six magnetic P_a charges of the black hole satisfy the conditions

$$\gamma^{[ab]} Q_a P_b = 0 \tag{60}$$

and

$$-e^{-4\phi_0} g^{00} g^{ab} (Q_a Q_b + P_a P_b) = |Z|^2 \ . \tag{61}$$

The 12 charges Q_a, P_a can be defined also via 28-dimensional charges $(q_{\hat{a}})_{\rm el}$, $(q_{\hat{a}})_{\rm mag}$ introduced by Sen [30]:

$$Q_a = E_a{}^{\hat{a}} (q_{\hat{a}})_{\rm el}\ , \quad P_a = E_a{}^{\hat{a}} (q_{\hat{a}})_{\rm mag} \ . \tag{62}$$

The matrices $E_a{}^{\hat{a}}$ are defined by the nonvanishing expectations values of the scalars of the four-dimensional theory (or, equivalently, by the geometry of the compactified dimensions). Condition (60) was not spelled explicitly in [23, 30]. For us this condition is of great importance: in this form it reflects the critical orbit with USp(N) symmetry, discussed in Sec. IV. At the technical level, without (60) we would not be able to get the required projectors in the presence of both electric and magnetic charges.

The Killing spinor can be represented in the form

$$C(z,\bar{z}) = \chi_k\, C(z,\bar{z}) = \frac{1+\Gamma}{2} C(z,\bar{z}) \ . \tag{63}$$

Our new projectors $\chi_{\bar{k}}$, χ_k indeed satisfy all the requirements (12), since

$$\left(\frac{1+\Gamma}{2}\right)\left(\frac{1-\Gamma}{2}\right) = 0\ , \tag{64}$$

$$\left(\frac{1+\Gamma}{2}\right)^2 = \left(\frac{1+\Gamma}{2}\right)\ , \tag{65}$$

$$\left(\frac{1-\Gamma}{2}\right)^2 = \left(\frac{1-\Gamma}{2}\right) \ . \tag{66}$$

Under S- and T-duality transformations the central charges are covariant. The corresponding covariant transformation of spinors makes our constraint on κ-symmetry ghost duality covariant. Before discussing the details of the covariant gauge fixing, let us break both S and T duality of the new class of gauges and reconstruct the familiar class of gauges: the light-cone one and the generalized light-cone gauge.

Example 1: light-cone gauge. Our first example is a pure electric U(1) dilaton black hole. We choose the hair

$$Q_9 = e^{2\phi_0}|Z|\ , \quad Q_4 = \cdots = Q_8 = P_4 = \cdots = P_9 = 0 \ . \tag{67}$$

The black hole projector becomes a light-cone projector

$$\chi_{\bar{k}} = \left(\frac{1-\Gamma}{2}\right) = \left(\frac{1-\gamma^0\gamma^9}{2}\right)\ ,$$
$$\chi_k = \left(\frac{1+\Gamma}{2}\right) = \left(\frac{1+\gamma^0\gamma^9}{2}\right) \ . \tag{68}$$

Thus in terms of the ten-dimensional Majorana-Weyl spinor the electric solutions admit a Killing spinor which satisfies the light-cone constraint

$$(\gamma^- \gamma^+)\epsilon_k = (\gamma^- \gamma^+) C(z,\bar{z}) = 0\ , \tag{69}$$

and the anti-Killing spinor satisfies equation

$$(\gamma^+ \gamma^-)\epsilon_{\bar{k}} = 0 \ . \tag{70}$$

If we would choose a negative value of the electric charge

$$Q_9 = -e^{2\phi_0}|Z|\ , \quad Q_4 = \cdots = Q_8 = P_4 = \cdots = P_9 = 0\ , \tag{71}$$

the constraint on the ghost would become the one for the anti-Killing spinor in the previous choice,

$$(\gamma^+ \gamma^-) C(z,\bar{z}) = 0 \ . \tag{72}$$

Thus in this example the electric black hole hair breaks the ten-dimensional Lorentz symmetry SO(1.9) down to SO(1.1)×SO(8). We can consider the limit of our gauge-fixing function (56) when the central charge vanishes, $|Z| \to 0$. This limit exists and has the same form $(\gamma^+\gamma^-)C(z,\bar{z}) = 0$. Thus using the light-cone gauge one can either consider the massive four-dimensional black hole states or massless four-dimensional states.

Example 2: generalized light-cone gauge. Let us choose

$$Q_a = l_a e^{2\phi_0}|Z|\ , \quad P_4 = \cdots = P_9 = 0 \ . \tag{73}$$

The constraint on the ghosts depends now on the arbitrary six-dimensional vector l_a satisfying the constraint $l^2 = 1$:

$$(1 - \gamma^0 \gamma^a l_a) C(z, \bar{z}) = 0 \; . \tag{74}$$

This is a special choice of our generalized light-cone gauge when the vector $n_\mu = (1, 0, 0, 0, l_a)$, and $n^2 = 0$ due to the fact that $l^2 = 1$.

Example 3: magnetic gauge. The quantization of superstring theory as well as of heterotic string theory was performed only in light-cone or generalized light-cone gauge. Therefore it was widely believed that the elementary excitations of string can be associated only with electrically charged black holes. However we may change the gauge now. Let us first consider the simplest magnetic U(1) dilaton black hole:

$$P_4 = e^{2\phi_0} |Z| \; , \;\; Q_4 = \cdots = Q_9 = Q_5 = \cdots = P_9 = 0 \; . \tag{75}$$

Now the Killing spinor and the κ-symmetry ghost in terms of the ten-dimensional Majorana-Weyl spinor are constrained to be chiral in the four-dimensional Euclidean subspace, $\Gamma^5 = \gamma^1 \gamma^2 \gamma^3 \gamma^4$,

$$(1 + \Gamma^5) \, \epsilon_k = (1 + \Gamma^5) \, C(z, \bar{z}) = 0 \; , \tag{76}$$

and the anti-Killing spinor is antichiral:

$$(1 - \Gamma^5) \epsilon_{\bar{k}} = 0 \; . \tag{77}$$

Again, by changing the sign of the magnetic charge we have the antichiral ghost. Such a split breaks the ten-dimensional Lorentz symmetry to SO(1.5)×SO(4). This algebraic constraint has not been used before for the gauge fixing of the κ symmetry.

Example 4: generalized magnetic gauge. One can choose a more general magnetic charge with SO(6) symmetry as

$$P_a = l_a e^{2\phi_0} |Z| \; . \tag{78}$$

The ghost will satisfy the condition

$$(1 + \gamma^1 \gamma^2 \gamma^3 \gamma^a l_a) \, C(z, \bar{z}) = 0 \; . \tag{79}$$

Example 5: electric-magnetic-axion-dilaton U(1) black hole. We choose

$$P_4^2 + Q_4^2 = e^{4\phi_0} |Z|^2$$

$$Q_5 = \ldots = Q_9 = P_5 = \cdots = P_9 = 0 \; . \tag{80}$$

The Killing spinor satisfies the constraint

$$\left(1 - \frac{\gamma^0 \gamma^4 e^{-2\phi_0} (Q_4 + \gamma_5 P_4)}{|Z|}\right) \, C(z, \bar{z}) = 0 \; . \tag{81}$$

This gauge was also never used before for the quantization of the heterotic string.

In dealing with central charges of supersymmetry algebra related to supersymmetric extreme black holes it is more convenient to use the chiral basis as described in Sec. IV. This basis is associated with the symplectic spinors which in a clear way shows how the black hole multiplets form the representations of USp(N)×SU(2) algebra. In this basis the truncation condition takes the form

$$\frac{\mathcal{Z}}{|Z|} \, C(z, \bar{z}) = 0 \; , \tag{82}$$

where the numerical symplectic matrix $\mathcal{Z}$ is defined in Eq. (39), and the ghost forms a symplectic spinor. In a more detailed form the constraint is

$$\begin{pmatrix} \frac{Z^{ij}}{|Z|} & \mathrm{I} \\ -\mathrm{I} & \frac{Z^{*ij}}{|Z|} \end{pmatrix} \begin{pmatrix} C^j_\alpha \\ C^{*\alpha j} \end{pmatrix} = \begin{pmatrix} \frac{Z^{ij}}{|Z|} C^j_\alpha + C^{*\alpha i} \\ -C^i_\alpha + \frac{Z^{*ij}}{|Z|} C^{*\alpha i} \end{pmatrix} = 0 \; . \tag{83}$$

The advantage of using symplectic form of the constraint is that, e.g., the second line on the right-hand side of Eq. (83) can be obtained from the first one by multiplication on Z^{ki}. The black hole basis for supersymmetry which was used in [29] is very close to the one which is used there. In particular, the SU(4) matrices α, β were used instead of six matrices γ^a. Thus, if we know the antisymmetric matrix Z^{ij}, we can build the symplectic matrix $\mathcal{Z}$ and have a symplectic spinor gauge fixing in the form (82). For example, pure electric solution with $P = 0$ and positive electric charge $Z^{ij} = \alpha^3_{ij} Q/\sqrt{2}$, and with

$$Z^{ij} = \alpha^3_{ij} |Z| = i\sigma_2 \begin{pmatrix} |Z| & 0 & 0 & 0 \\ 0 & |Z| & 0 & 0 \\ 0 & 0 & |Z| & 0 \\ 0 & 0 & 0 & |Z| \end{pmatrix}$$

$$= |Z| \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \tag{84}$$

provides the following symplectic 8×8 matrix for Eq. (82):

$$\left(\frac{\mathcal{Z}}{|Z|}\right)_{\mathrm{el}} = \begin{pmatrix} i\sigma_2 \, I & I \\ -I & i\sigma_2 \, I \end{pmatrix} \; , \tag{85}$$

where I is the unit 4×4 matrix. If we would consider a pure magnetic solution we would get $Z^{ij} = \beta^3_{ij} P/\sqrt{2}$,

$$Z^{ij} = \beta^3_{ij} |Z| = i\sigma_2 \begin{pmatrix} -|Z| & 0 & 0 & 0 \\ 0 & -|Z| & 0 & 0 \\ 0 & 0 & |Z| & 0 \\ 0 & 0 & 0 & |Z| \end{pmatrix}$$

$$= |Z| \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} , \tag{86}$$

and the corresponding symplectic 8×8 matrix would be

$$\left(\frac{\mathcal{Z}}{|Z|}\right)_{\mathrm{mag}} = \begin{pmatrix} i\sigma_2 \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} & \mathrm{I} \\ -\mathrm{I} & i\sigma_2 \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{pmatrix} . \tag{87}$$

Thus indeed we see that the electric solution corresponds to the central charge matrix in the normal form, with all eigenvalues equal, whereas the magnetic solutions presents the central charge in the form related to the normal one by some U(4) transformation.

We have used in this section the ten-dimensional form of the constraints on κ symmetry to be able to display the relations between known gauge-fixing conditions and the electric black holes (or supersymmetric gravitational waves in ten-dimensional context). We have also given examples of the magnetic and mixed electromagnetic gauges which were never used for quantization before. We have also shown that the chiral four-dimensional spinors, especially in symplectic form, are very convenient way to display dualities via the transformations of the central charges.

VI. CENTRAL CHARGES IN THE FLAT SPACE LIMIT

When the string is quantized in the BPS background, the central charges are those of the background. To perform the quantization in the flat space we may consider different possibilities. We may simply use the central charge matrix of the background to gauge-fix the fermions as in Eq. (82) even when the background is not there anymore. The second possibility is to consider the limit of central charges going to zero,

$$\lim_{\mathcal{Z}\to 0} \left(\frac{\mathcal{Z}}{|Z|}\right) C \equiv \chi_{\bar{k}} C = 0 . \tag{88}$$

Such a limit exists, we have shown examples in the previous section. In particular when the background is pure electric or pure magnetic, the constraint has the same for as the limit when the central charge goes to zero. We may however take the following attitude. In the curved background with the central charges the vacuum expectation value of the string variable (or zero mode, $z,\bar{z}$-independent value) $\Pi_z{}^{\mu}(z,\bar{z})$ is such as to reproduce the central charge in the supersymmetry algebra. The ten-dimensional string variable

$$\Pi_{\bar{z}}{}^{\mu}(z,\bar{z}) = \partial_{\bar{z}} x^{\mu} - \bar{\theta}\gamma^{\mu}\partial_{\bar{z}}\theta , \quad \mu = 0,\dots,9 , \tag{89}$$

consists of the four-dimensional part $\Pi_{\bar{z}}{}^{\hat{\mu}}(z,\bar{z})$, $\hat{\mu} = 0,\dots,3$, and of the six-dimensional part $\Pi_{\bar{z}}{}^{a}(z,\bar{z})$, $a = 4,\dots,9$. The extreme black hole background with specific values of the central charge enforces a nonvanishing value of the vacuum expectation of the string variables Π of the form:

$$\langle(\gamma_a C)_{\alpha\beta ij}\Pi_{\bar{z}}{}^{a}(z,\bar{z})\rangle = \begin{pmatrix} Z^{ij}\epsilon_{\alpha\beta} & 0 \\ 0 & Z^{*ij}\epsilon_{\dot{\alpha}\dot{\beta}} \end{pmatrix} . \tag{90}$$

Here $\mathcal{C}$ is the charge conjugation matrix. The four-dimensional part we take directly in the rest frame be

$$\langle(\gamma_{\hat{\mu}} C)_{\alpha\beta ij}\Pi_{\bar{z}}{}^{\hat{\mu}}(z,\bar{z})\rangle = \begin{pmatrix} 0 & |Z|I \\ -|Z|I & 0 \end{pmatrix} , \tag{91}$$

where I is the unit matrix. Thus we define the string black hole state as the state with vanishing ten-dimensional mass of the state, however the four-dimensional mass is not vanishing since the state has nonvanishing central charge. Indeed we have

$$\langle(\gamma_{\mu}\mathcal{C}\ \Pi_{\bar{z}}{}^{\mu})\rangle\,\langle(\gamma_{\mu}\mathcal{C}\ \Pi_{\bar{z}}{}^{\mu})^{\dagger}\rangle = m_{10}^2 = 0 ; \tag{92}$$

however

$$\langle(\Pi_z{}^{a}\gamma_a\mathcal{C})\rangle\,\langle(\Pi_z{}^{a}\gamma_a\mathcal{C})^{\dagger}\rangle = -|Z|^2 I ,$$

$$\langle(\gamma_{\hat{\mu}}\mathcal{C}\ \Pi_z{}^{\hat{\mu}})\rangle\,\langle(\gamma_{\hat{\mu}}\mathcal{C}\ \Pi_z{}^{\hat{\mu}})^{\dagger}\rangle = |Z|^2 I . \tag{93}$$

In this way we have reproduced the property of the centron BPS multiplet discussed in [31] that the BPS state corresponds to a massless ten-dimensional state but to a massive four-dimensional one.

To summarize, the string variable $\Pi_z{}^{\mu}(z,\bar{z})$, $\mu = 0,\dots,9$ gets a nonvanishing $(z,\bar{z})$-independent numerical value defined by the central charge of the background:

$$\langle(\gamma_{\mu} C)_{\alpha\beta ij}\Pi_{\bar{z}}{}^{\mu}(z,\bar{z})\rangle = \begin{pmatrix} Z^{ij} & |Z|I \\ -|Z|I & Z^{*ij} \end{pmatrix} \equiv \mathcal{Z} . \tag{94}$$

Thus even if the BPS background which supplies this matrix is absent we may attribute the numerical values of the central charge matrix to the vacuum expectation of the string momenta. This again gives us a constraint on κ-symmetry ghost in the form (82).

The construction above suggests the following idea. One can rewrite the classical GS action for the heterotic string by using the string variables in the form

$$x^{\hat{\mu}},\ \theta^{i}_{\alpha},\ (\theta^{i}_{\alpha})^{*},\ x^{ij},\ x^{*ij},$$

$$\hat{\mu} = 0,1,2,3 , \quad i,j = 1,2,3,4 , \quad \alpha = 1,2 . \tag{95}$$

The antisymmetric matrices $x^{ij} = ||x||$ which are the new bosonic coordinates of the compactified string, are defined as

$$(\gamma_a C)_{\alpha\beta ij} x^a(z,\bar z) = \left(x^{ij}(z,\bar z)\epsilon_{\alpha\beta} 00 \quad x^{*ij}(z,\bar z)\epsilon_{\dot\alpha\dot\beta} \right) \equiv X(z,\bar z) \ , \tag{96}$$

and have the property

$$||x||||x^\dagger|| = \left(\sum_4^9 x^a x_a\right) I \ . \tag{97}$$

In terms of these variables the classical Green-Schwarz action for the heterotic string has the global $(z,\bar z)$-independent U(4) symmetry under which the fermionic string variables as well as bosonic variables X transform. The symmetry is best expressed in terms of the symplectic transformations in the form given in Sec. III. The symplectic spinor is rotated as

$$\theta \to T\theta \ , \quad X \to TXT^T \ , \quad T = \begin{pmatrix} U & 0 \\ 0 & U^* \end{pmatrix} \ . \tag{98}$$

When the string action is considered in these variables, the generation of central charges matrix becomes a very natural step. This may lead to a reformulation of the $N=4, N=8$ supersymmetry in a basis with central charge-type coordinates. In particular, it was suggested in [32] to consider a $d=4$ superspace with additional coordinates for describing extended supergravities with hidden supersymmetries. The full set of coordinates (for $N=8$) is

$$x^{\hat\mu},\ \theta^I,\ \bar\theta^I,\ t_{IJ},\ \bar t^{IJ} \ , \quad \hat\mu = 0,1,2,3 \ , \quad I,J = 1,\dots,8 \ . \tag{99}$$

The bosonic coordinates t_{IJ}, $\bar t^{IJ}$ correspond to Cartan antisymmetric tensors x^{ij}, y_{ij} , $i,j=1,\dots,8$ which gives the explicit form of E_7 generators. We suggested to introduce the new vielbein forms E_{ij}, $\bar E^{ij}$ in addition to the usual ones. Those forms in curved superspace at $\theta=\bar\theta=0$ are defined by the scalar field matrix of Cremmer and Julia [2]

$$\begin{pmatrix} U^{IJ}{}_{ij} & \bar V^{IJij} \\ V_{IJij} & \bar U_{IJ}{}^{ij} \end{pmatrix} \ . \tag{100}$$

One can expect that the development of this direction will lead to the better understanding of the role of central charges in supersymmetric theories. In particular, the crucial property of all nonlinear manifestly supersymmetric on-shell invariants of $N=8$ supergravity is their E_7 symmetry and the fact that they are build as the integrals over the full superspace [32, 33].

The nonrenormalization theorem for extreme black holes which was presented in [1, 29] had only one crucial requirement: fermionic isometries, which make the superfields covariantly independent on some fermionic coordinates. It seems to become clear now that if the extreme black holes with manifest E_7 symmetry[8] will be discovered as solutions of $N=8$ supergravity, they will represent the U-duality symmetry of the spectra of quantized states of the superstring theory. In addition, they will (i) form the most general background of the compactified to $d=4$ type-II GS string theory in which the consistent truncation and quantization of κ symmetry is possible and (ii) these black holes will be subject to supersymmetric non-renormalization theorem of the type described in [1, 29].

VII. GS SUPERSTRING PATH INTEGRAL IN BLACK HOLE GAUGES

For the time being, before the reinterpretation of string variables responsible for accommodation of central charges is performed, we will study the quantization of the GS heterotic string in the old variables $(x^\mu, \theta_\alpha,\ \mu=0,1,\dots,9,\ \alpha=1,\dots,16)$, but in duality-covariant gauges.

The gauge-fixed path integral in semi-light-cone gauge $\gamma^+\theta=0$, $g_{\alpha\beta}=\rho g^m_{\alpha\beta}$, where $g^m_{\alpha\beta}$ is some background metric, was presented in Eq. (2.2) of [8] in the form[9]

$$\sum_{\text{topologies}} e^{-4\pi\chi\phi_0} \int_{\text{moduli}} \int Dx^\mu D\theta D\psi Db Dc (\text{Det } u_{\bar z})^{-4} \times \exp\left(- \int d^2z (\partial_z x^\mu \partial_{\bar z} x^\mu + \bar\theta\gamma^- \Pi_{\bar z}{}^+ \partial_z \theta + L'_{\text{gauge}}(\psi) + b\bar\partial c + \bar b \partial \bar c) \right) \ , \tag{101}$$

where $u_{\bar z} = \partial_{\bar z} x^+ = \Pi_{\bar z}{}^+$. In a generalized semi-light-cone gauge $\chi_k(\text{glc}) = (\not p \not n)$ the only difference would come in the θ term in the action which will read

$$\theta^T U_{\bar z} \partial_z \theta \tag{102}$$

with

$$U_{\bar z}(z,\bar z) = \chi_k{}^T C \not\Pi_{\bar z}, \chi_k \ , \tag{103}$$

and the local measure of integration is $(\text{Det } u_{\bar z})^{-4}$, which is equal to the inverse square root of the determinant of the maximum square invertible minor of the matrix $||U||$.

We may rewrite this path integral now in the form

[8]Under manifest E_7 symmetry we mean the following. When the black hole hair (the values of the scalar field at infinity and the electric and magnetic charges of the black hole) undergoes the global E_7 rotation, the total solution, as a function of three-dimensional space, will rotate according to E_7. This property was demonstrated for manifestly SL$(2,R)$ symmetric black holes in $N=4$ supergravity [34]. In all cases the classical symmetry groups are broken down in quantum theory to subgroups with integer parameters only.

[9]The semi-light-cone quantization of the heterotic GS string was performed by Carlip [35] in a slightly different form.

where the Killing–anti-Killing split of the spinor is realized in terms of the most general possible central charge in USp(4) critical orbit. Thus we consider the generic "black hole gauge"

$$\chi_{\bar{k}}\theta = 0 \ , \quad \theta = \theta_k \ , \tag{104}$$

which means that only the Killing part of the anticommuting spinor θ propagate. In this case we have the same path integral as explained for the generalized light-cone gauge, however with *any possible choice of the projector* $\chi_{\bar{k}}$ (see examples in Sec. IV) and not only the light-cone one. Thus the path integral is

$$\sum_{\text{topologies}} e^{-4\pi\chi\phi_0} \int_{\text{moduli}} \int Dx^\mu D\theta D\psi Db Dc (\text{Det } u_{\bar{z}})^{-4} \times \exp\left(-\int d^2z[\partial_z x^\mu \partial_{\bar{z}} x^\mu + \theta^T({\chi_k}^T C \not{\Pi}_{\bar{z}}\, \chi_k)\partial_z\theta + L'_{\text{gauge}}(\psi) + L_{\text{rep ghosts}}]\right) . \tag{105}$$

It was explained in [8] that the local measure of integration provides at least formally the independence of the theory of the way in which local fermionic gauge symmetry was fixed. This means that the part of the path integral given by

$$\int Dx^\mu D\theta D\psi Db Dc (\text{Det } u_{\bar{z}})^{-4} \times \exp\left(-\int d^2z[\partial_z x^\mu \partial_{\bar{z}} x^\mu + \theta^T({\chi_k}^T C \not{\Pi}_{\bar{z}}\, \chi_k)\partial_z\theta + L'_{\text{gauge}}(\psi) + L_{\text{rep ghosts}}]\right) \tag{106}$$

is invariant under the change of the gauge conditions. This gauge is a unitary gauge for the fermionic symmetry (all fermionic ghosts are not propagating). The local measure of integration is exactly the contribution of the second class constraints as predicted in [13,11]. The class of gauges which were considered before and the transformations from one to another did not involve any changes of the vacuum expectation value of the dilaton which controls the loop expansion. Therefore for this class of gauges the fact the that integral in (106) is invariant by construction is sufficient to claim that the total path integral including the loop integrations

$$\sum_{\text{topologies}} e^{-4\pi\chi\phi_0} \int_{\text{moduli}} \tag{107}$$

is gauge invariant. Now we are considering more general class of gauges which are related by S-duality transformations from one gauge to another. The partition function on the torus where $e^{-4\pi\chi\phi_0} = 1$ is now duality invariant by construction. However if we are interested in partition functions for different topologies, we have to take into account that S-duality will act on the string coupling constant as

$$(e^{-2\phi_0})' = \left(c(a_0 + ie^{-2\phi_0}) + d\right)^{-1} \times \left(c(a_0 - ie^{-2\phi_0}) + d\right)^{-1} e^{-2\phi_0} \ , \tag{108}$$

where a_0 is the value of the axion field at infinity and c, d are some integers. The part of the path integral given in Eq. (106) is invariant. However each term in the path integral in (105) with the nonvanishing Euler number χ transforms when we use the full SL$(2, Z)$ transformation to change a constraint on the spinor[10] and on the string coupling constant. Thus when duality transformation includes the dilaton whose vacuum expectation value plays the role of the string coupling constant, each term in the Green-Schwarz path integrals given in Eq. (105) is covariant rather than invariant: the change of the gauge has to be followed by the corresponding change of the coupling constant. There is a puzzling resemblance here with the observation about S duality due to Witten [36]. He found that the partition function on a general four-manifold is not modular invariant but transforms as a modular form of the weight depending on the topology of the manifold. Using the fact that our partition function consists of the invariant part given in Eq. (106) and using the SL$(2, Z)$ transformation (108) one can see that for each topology our partition function transforms as a modular form of the weight depending on the topology of the manifold.

The expansion in topologies makes the understanding of S duality more complicated and perhaps the better way to proceed is to use the first quantization, suggested above only to get the elementary string states, which are duality invariant, according to Eq. (106). The next step would be to construct the BRST operator for the first quantized theory. The resulting second quantized theory may have better way of realizing S duality and may give us a possibility to avoid the loop expansion in the form (107).

Therefore for the moment we will concentrate on the part of the path integral given in Eq. (106) which has a clear behavior under the change of the gauge conditions including the S-duality type. For example in the pure magnetic gauge where $\chi_k = \frac{1}{\sqrt{2}}(1 - \Gamma^5)$ the fermionic part of the action is

$$\frac{1}{2}\bar{\theta}(1 - \Gamma^5)\not{\Pi}_{\bar{z}}\,(1 - \Gamma^5)\partial_z\theta = \frac{1}{2}\bar{\theta}(1 - \Gamma^5)\left(\gamma_0 {\Pi_{\bar{z}}}^0 + \sum_{a=5}^{a=9}\gamma_a {\Pi_{\bar{z}}}^a\right)\partial_z\theta \ . \tag{109}$$

Thus the kinetic term of the fermionic variables depends

[10] We will consider in detail the S-duality covariant gauges in Sec. VIII.

on SO(1,5) vector Π^0, Π^a, whereas the spinor is chiral in SO(4). The local measure for the magnetic solution is

$$\left[\mathrm{Det}\left(-(\Pi_{\bar z}{}^0)^2 + \sum_{a=5}^{a=9}(\Pi_{\bar z}{}^a)^2\right)\right]^{-2} . \tag{110}$$

There are different ways to proceed with this action. In [8] we performed some change of variables after which we got the SU(4) form of the path integral. The analogous change of variables can be performed starting with any gauge of the type (104). This result is

$$\int Dx^\mu D\eta_{\bar z}{}^i D\theta_i DbDc \exp\left[-\int d^2z\left(\partial_z x^\mu \partial_{\bar z} x^\mu + \sum_{i=1}^{i=4}\eta_{\bar z}{}^i\partial_z\theta_i + L'_{\text{gauge}}(\psi) + L_{\text{rep ghosts}}\right)\right] . \tag{111}$$

This form of the GS superstring path integral shows in a clear way that the idea that the elementary string excitations have to be associated only with electric black holes is based on the light-cone gauge quantization. In the generic class of gauges the elementary string excitations cannot be qualified as electric black holes: they are given by generic black holes. We have presented the string partition function in the form in which there is no dependence left on the choice of the constraint on the spinor. Therefore there is no difference whether we have started with electric-type constraint $\gamma^+\theta = 0$ or magnetic-type constraint $(1+\Gamma^5)\theta = 0$. One can claim on the basis of this construction that the elementary string excitations is invariant under duality transformations as the soliton configurations solving the classical equations of motions and as the Bogomolny bound.

Another way to proceed with the path integral in the semi-light-cone gauge was suggested in [37]. These authors rescaled the eight-dimensional Killing spinors θ without breaking it into two SU(4) spinors. These procedure seems to be more suitable for dealing with anomalies. At this stage we would prefer to postpone the issue of anomalies and just work out the black hole class of gauges which contains the light-cone gauge as a subclass. We hope that the situation with conformal and gauge fermionic symmetry anomalies will be studied later along the lines of [35, 8, 37].

One more comment about the black hole gauges is in order. If we would choose the light-cone gauge also for the bosonic variables of the string, i.e.,

$$x^+ = \tau P^+ , \quad \partial_{\bar z} x^+ = P^+ , \tag{112}$$

we would have to identify the variable P^+ with the mass of the black hole. Indeed, in our picture the origin of the light-cone gauge is traced back to the central charges. However, they appear via the zero modes of the string momenta. Therefore

$$\langle \Pi_{\bar z}{}^0 + \Pi_{\bar z}{}^9\rangle = P^+ = 2|Z| = 2M , \tag{113}$$

where M is the mass of the black hole.

Taking into account that P^+ plays such an important role in the Green-Schwarz string field theory based on the light-cone first quantization one may hope that our picture may lead to string field theory describing the duality symmetric interactions of extreme black hole multiplets.

Manifestly supersymmetric black-hole gauge. Manifestly supersymmetric gauge was suggested in [7] with the purpose to keep linear realization of supersymmetry. The algebraic constraint was imposed on the first generation of fermionic ghosts, whereas the gauge for the θ variable was chosen to contain a derivative. We may present now the gauge-fixed action for manifestly supersymmetric black hole gauges. The constraint on the first generation of ghosts is defined by the central charge matrix. We will use it in the form $\chi_{\bar k} C(z,\bar z) = 0$. The gauge-fixed action according to Eqs. (38) from [7] and Eq. (3.4) of [8] is

$$\mathcal{L}_{\rm cl} + \bar\pi_{\bar z}\chi_{\bar k}\partial_z\theta + (\partial_z \bar C_{\bar z})\chi_{\bar k}\not{\Pi}_{\bar z}\,\chi_k C_z , \tag{114}$$

where the propagating FP ghosts $\bar C_{\bar z}$, C_z are commuting whereas the propagating NK ghosts $\bar\pi_{\bar z}$ are anticommuting. By performing some change of variables and by adjusting the local measure of integration one can prove that the physical states are independent of the choice of the fermionic gauge fixing.

VIII. S-DUALITY SYMMETRIC FAMILY OF BLACK HOLE GAUGES

The basic new feature of the quantization which we propose is the use of the algebraic constraint on Killing spinor of the BPS background. The simple and universal form of the constraint is given in Eq. (82). When the background undergoes any duality transformation, the Killing spinor and the algebraic constraint on Killing spinor transform in a way which reflects *the symmetry under dualities of equations of motion including fermions.*

In this section we would like to study the new quantization for the special case of the axion-dilaton black holes in manifestly S-duality symmetric form [34]. For this configuration we know exactly how background (in our example the superspace, whose bosonic part consists of axion-dilaton black holes) transforms under S duality and what happens with the constraint on Killing spinor. After the detailed analysis of this configuration we will reformulate in the next section the information available about the S-duality-covariant gauges to the form suitable for the generalization to the U-duality symmetric gauges.

The axion-dilaton family of black holes [34] has a feature which justifies the word family. We consider the solution in a form in which it is characterized by some generic values of the the axion-dilaton field and electric and magnetic charges. After the S-duality transformation the solution keeps the same functional form. It is important that one considers the generic values of the black hole hair and not the exceptional cases such as pure magnetic or pure electric solutions, for example. In such special cases, as it was demonstrated in [38,39], one starts with pure electric solutions and after S-duality transformation one gets a new solution which is characterized by the electric as well as magnetic charge and by some value of the axion field, which was not present in the original pure electric dilaton black hole. However, after the manifest S-duality symmetric form of the solutions is found one does not generate new solutions by performing an additional S-duality transformation, they are all there, in the family. For simplicity we will consider a U(1) axion-dilaton black hole [34] with only one vector field, which has both magnetic and electric charge.[11]

The solution has the form

$$\begin{aligned} ds^2 &= e^{2\mathcal{U}}dt^2 - e^{-2\mathcal{U}}d\vec{x}^2, \\ e^{-2\mathcal{U}}(\vec{x}) &= i[\mathcal{H}_2(\vec{x})\,\overline{\mathcal{H}}_1(\vec{x})] - \mathcal{H}_1(\vec{x})\,\overline{\mathcal{H}}_2(\vec{x})\ , \\ A^t(\vec{x}) &= (Q+iP)\mathcal{H}_2(\vec{x}) + \text{c.c.}, \\ \lambda(\vec{x}) &= a(\vec{x}) + ie^{-2\phi(\vec{x})} = \frac{\mathcal{H}_1(\vec{x})}{\mathcal{H}_2(\vec{x})}\ , \\ \tilde{A}^t(\vec{x}) &= -(Q+iP)\mathcal{H}_1(\vec{x}) - \text{c.c.}\ , \end{aligned} \tag{115}$$

where $\mathcal{H}_1(\vec{x}), \mathcal{H}_2(\vec{x})$ are two complex harmonic functions (for simplicity we are considering a one-black-hole solution)

$$\begin{aligned} \mathcal{H}_1(\vec{x}) &= \frac{e^{\phi_0}}{\sqrt{2}}\left(\lambda_0 + \frac{\lambda_0 M + \overline{\lambda}_0\Upsilon}{|\vec{x}|}\right) \equiv v_1 + \frac{\mathcal{M}_1}{|\vec{x}|}\ , \\ \mathcal{H}_2(\vec{x}) &= \frac{e^{\phi_0}}{\sqrt{2}}\left(1 + \frac{M+\Upsilon}{|\vec{x}|}\right) \equiv v_2 + \frac{\mathcal{M}_2}{|\vec{x}|} \end{aligned} \tag{116}$$

where $\lambda_0 \equiv \lim_{|\vec{x}|\to\infty}\lambda(\vec{x})$.

The S-duality transformation on the half supersymmetric bosonic background is given by the fractional transformation on the complex scalar $\lambda(x)$:

$$\lambda'(x) = \frac{a\lambda(x)+b}{c\lambda(x)+d}\ . \tag{117}$$

Here the SL(2, Z) matrix Λ is

$$\Lambda = \begin{vmatrix} a & b \\ c & d \end{vmatrix}\ ,\quad \Lambda^{-1} = \begin{vmatrix} d & -b \\ -c & a \end{vmatrix}\ ,\quad \det\Lambda = 1\ , \tag{118}$$

where a, b, c, d are some real integers.

We will present a useful form of this solution where only the SL(2, Z) doublets enter. This will be helpful for clarifying the transformation property of the gauge-fixing condition under the S duality. This form will be also suggestive for the U duality. Consider the harmonic matrix

$$\begin{aligned} &\partial_i\partial_i V(x) = 0\ , \\ &V(x) = \begin{pmatrix} \mathcal{H}_2(\vec{x}) & -\mathcal{H}_1(\vec{x}) \\ \overline{\mathcal{H}}_2(\vec{x}) & -\overline{\mathcal{H}}_1(\vec{x}) \end{pmatrix}\ , \\ &\det V(x) = ie^{-2\mathcal{U}(x)}\ . \end{aligned} \tag{119}$$

Under S duality this matrix transforms as

$$V'(x) = h\,V(x)\Lambda^{-1}\ , \tag{120}$$

where

$$h = \begin{pmatrix} U & 0 \\ 0 & U^* \end{pmatrix},\quad U = \frac{|S_0|}{S_0}\ ,\quad S_0 \equiv c\lambda_0 + d\ . \tag{121}$$

This transformation is known to be a compensating U(1) transformation which supports the choice of the local U(1) gauge fixing under the SL(2, Z) transformation.

We can also use two harmonic doublets

$$\begin{aligned} (\,\mathcal{H}_2\,, -\mathcal{H}_1\,)' &= \frac{|S_0|}{S_0}(\,\mathcal{H}_2\,, -\mathcal{H}_1\,)\,\Lambda^{-1}\ , \\ \begin{pmatrix} \mathcal{H}_1 \\ \mathcal{H}_2 \end{pmatrix}' &= \frac{|S_0|}{S_0}\,\Lambda\begin{pmatrix} \mathcal{H}_1 \\ \mathcal{H}_2 \end{pmatrix}\ . \end{aligned} \tag{122}$$

The vector fields are also organized in doublets. We have a harmonic doublet potential

$$\partial_i\partial_i\mathcal{A} = 0\ ,\quad \mathcal{A} = \left[\,A^t(x)\,, \tilde{A}^t(x)\,\right]\ . \tag{123}$$

Under S duality it transforms as

$$\mathcal{A}' = \mathcal{A}\,\Lambda^{-1}\ . \tag{124}$$

The vector field strength also can be presented in the doublet form

$$\begin{aligned} \mathcal{F} &\equiv \left(\,F_{\text{tr}}\,, -i\tilde{F}_{\text{tr}}\,\right) = \frac{1}{|\vec{x}|^2}(\,q\,,\tilde{p}\,)\ , \\ {}^*\mathcal{F} &\equiv \begin{pmatrix} {}^*\tilde{F}_{\text{tr}} \\ -i{}^*F_{\text{tr}} \end{pmatrix} = \frac{1}{|\vec{x}|^2}\begin{pmatrix} \tilde{q} \\ p \end{pmatrix}\ . \end{aligned} \tag{125}$$

Those two doublets transform as follows under S duality:

$$\mathcal{F}' = \mathcal{F}\,\Lambda^{-1}\ ,\quad {}^*\mathcal{F}' = \Lambda\,{}^*\mathcal{F}\ . \tag{126}$$

Our matrix V consists of the value of this matrix at infinity when $|\vec{x}| \to \infty$ and of the $\frac{1}{|\vec{x}|}$ part of this matrix:

$$\begin{aligned} V(x) &= v + \frac{1}{|\vec{x}|}\mathcal{M}\ , \\ v &\equiv \begin{pmatrix} v_2 & -v_1 \\ \bar{v}_2 & -\bar{v}_1 \end{pmatrix}, \\ \mathcal{M} &\equiv \begin{pmatrix} \mathcal{M}_2 & -\mathcal{M}_1 \\ \overline{\mathcal{M}}_2 & -\overline{\mathcal{M}}_1 \end{pmatrix}. \end{aligned} \tag{127}$$

The matrices v and $\mathcal{M}$ transform as

$$v' = h\,v\Lambda^{-1}\ ,\quad \mathcal{M} = h\,\mathcal{M}\Lambda^{-1}\ . \tag{128}$$

[11] In this section we are using the notation of [34] unless otherwise specified.

Consider the following product of doublets:

$$(v\,{}^*\mathcal{F})' = h(v\,{}^*\mathcal{F}) \ . \tag{129}$$

Such products transform only in terms of the compensating h transformation. They will be useful for the form of the gauge fixing which will transform under SL$(2,Z)$ only in terms of the h matrix. Now we can build the combination suitable for exhibiting the S-duality-covariant form of the superstring κ-symmetry gauge fixing.

The Killing spinor admitted by the axion-dilaton black holes was found by Ortín in [40]:

$$\epsilon_I(\vec{x}) = e^{\frac{1}{2}\mathcal{U}(\vec{x})}\left(\frac{\mathcal{H}_2(\vec{x})}{\overline{\mathcal{H}}_2(\vec{x})}\right)^{\frac{1}{4}}\epsilon_{I(0)} \ , \tag{130}$$

where $\epsilon_{I(0)}$ is the value of the chiral part of the Killing spinor at infinity, satisfying an algebraic constraint which halves the spinor. We will study this constraint both in the doublet form as well as in the form in which only the h part of the symmetry is relevant.

The doublet form of the constraint is

$$\chi_{\bar{k}}\epsilon = 0 \Longrightarrow \begin{pmatrix} i\mathcal{M}_1 \\ i\mathcal{M}_2 \end{pmatrix}\epsilon_{I(0)} - \begin{pmatrix} \tilde{q} \\ p \end{pmatrix}_{IJ}\gamma^0\epsilon^J_{(0)} = 0 \ . \tag{131}$$

Under S-duality transformations the chiral part of the Killing spinor transforms as

$$[\epsilon_I(\vec{x})]' = e^{\frac{1}{2}\mathcal{U}(\vec{x})}\left[\left(\frac{\mathcal{H}_2(\vec{x})}{\overline{\mathcal{H}}_2(\vec{x})}\right)^{\frac{1}{4}}\right]'(\epsilon_{I(0)})'$$
$$= e^{\frac{i}{2}\arg S}\epsilon_I(\vec{x}) \ , \tag{132}$$

where

$$S(\vec{x}) \equiv c\lambda(\vec{x}) + d = c\,\frac{\mathcal{H}_1(\vec{x})}{\mathcal{H}_2(\vec{x})} + d \ . \tag{133}$$

Under S duality the constant part of Killing spinors transforms in terms of the asymptotic value of $S(\vec{x})$ at $|\vec{x}| \to \infty$ which is equal to $S_0 = c\lambda_0 + d$:

$$(\epsilon_{I(0)})' = e^{\frac{i}{2}\arg S_0}\epsilon_{I(0)} \ , \quad ({\epsilon_{(0)}}^J)' = e^{-\frac{i}{2}\arg S_0}{\epsilon_{(0)}}^J \ . \tag{134}$$

Taking into account the transformation of doublets above we find that the Killing spinor constraint in the doublet form transforms as

$$\left[\begin{pmatrix} i\mathcal{M}_1 \\ i\mathcal{M}_2 \end{pmatrix}\epsilon_{I(0)} - \begin{pmatrix} \tilde{q} \\ p \end{pmatrix}_{IJ}\gamma^0\epsilon^J_{(0)}\right]'$$
$$= e^{-\frac{i}{2}\arg S_0}\begin{vmatrix} a & b \\ c & d \end{vmatrix}\left[\begin{pmatrix} i\mathcal{M}_1 \\ i\mathcal{M}_2 \end{pmatrix}\epsilon_{I(0)} - \begin{pmatrix} \tilde{q} \\ p \end{pmatrix}_{IJ}\gamma^0\epsilon^J_{(0)}\right]$$
$$= 0 \ . \tag{135}$$

An alternative form of the Killing constraint can be obtained by multiplying Eq. (131) by the doublet $(\bar{v}_2\ , -\bar{v}_1)$ as suggested by Eq. (129). Indeed we may simplify things by using the fact that

$$(\bar{v}_2\ , -\bar{v}_1) \times \begin{pmatrix} i\mathcal{M}_1 \\ i\mathcal{M}_2 \end{pmatrix} = -M \ ,$$
$$(\bar{v}_2\ , -\bar{v}_1) \times \begin{pmatrix} \tilde{q} \\ p \end{pmatrix} = \frac{1}{\sqrt{2}}(Q + iP) \ . \tag{136}$$

The Killing spinor constraint (131) after multiplication by $(\bar{v}_2\ , -\bar{v}_1)$ becomes

$$M\epsilon_{I(0)} + \frac{1}{\sqrt{2}}(Q + iP)_{IJ}\gamma^0\epsilon^J_{(0)} = 0 \ . \tag{137}$$

In this form it transforms as

$$\left(M\epsilon_{I(0)} + \frac{1}{\sqrt{2}}(Q + iP)_{IJ}\gamma^0\epsilon^J_{(0)}\right)'$$
$$= e^{i\arg S_0}\left(M\epsilon_{I(0)} + \frac{1}{\sqrt{2}}(Q + iP)_{IJ}\gamma^0\epsilon^J_{(0)}\right) = 0 \ , \tag{138}$$

and we have taken into account that $(Q + iP)' = e^{\frac{i}{2}\arg S_0}(Q + iP)$.

We considered a solution with only one vector field, i.e., our choice of $(Q + iP)_{IJ}$ was $\alpha^3_{IJ}[(Q + iP)]$ where α^3_{IJ} is one of the SU(4) matrices α, β. In a more general situation we would have $(Q + iP)_{IJ} = \alpha^n_{IJ}(Q + iP)_n + \beta^{\tilde{n}}(P + iQ)_{\tilde{n}}$ where $n, \tilde{n} = 1, 2, 3$. In six-dimensionally covariant form[12] we would have the constraint defined in the notation of Sec. IV:

$$\chi_{\bar{k}}\epsilon = 0 \Longrightarrow \epsilon - \gamma^0\gamma^a(Q_a + i\gamma^5 P_a)\epsilon = 0, \quad a = 4, \ldots, 9. \tag{139}$$

Under S duality the constraint in this form transforms as

$$\left(\epsilon - \gamma^0\gamma^a(Q_a + i\gamma^5 P_a)\epsilon\right)'$$
$$= e^{-\frac{i\gamma^5}{2}\arg S_0}\left(\epsilon - \gamma^0\gamma^a(Q_a + i\gamma^5 P_a)\epsilon\right) = 0 \ . \tag{140}$$

A nice form of this transformation comes out if we use symplectic notation for the constraint on symplectic Killing spinor C as given in the form

$$\left(\frac{\mathcal{Z}}{|Z|}C\right)' = h_{\rm sp}\left(\frac{\mathcal{Z}}{|Z|}C\right) = 0 \ , \quad |Z|' = |Z| \ , \tag{141}$$

where

$$h_{\rm sp} = \begin{pmatrix} U^{1/2} & 0 \\ 0 & (U^*)^{1/2} \end{pmatrix},$$
$$U^{1/2} = (S_0)^{\frac{1}{4}}(\bar{S}_0)^{-\frac{1}{4}} = e^{\frac{i}{2}\arg S_0} \ ,$$
$$S_0 \equiv c\lambda_0 + d \ . \tag{142}$$

It is important to stress that that if in the flat background we have chosen a specific gauge-fixing condition (truncation) and afterwards have decided to put

[12] There is a factor of $\sqrt{2}$ difference in the definition of charges used in [34], [23].

the quantized string in the background, we could do it only under condition that the algebraic constraint on the ghost defines the algebraic constraint on the Killing spinor of the half-supersymmetric background. For example the string quantized in the light-cone gauge can be placed in the electric black hole background, but the string quantized in the magnetic gauge can be placed only in the magnetic background. The string quantized in the electromagnetic gauge can be placed in the electromagnetic black hole background. That is why we call those gauges black hole gauges.

An additional useful way to summarize this section is to use Witten's definition of the modular forms of the weight (u,v) [36] if the expression transforms as

$$F' = (c\lambda_0 + d)^u \, (c\bar{\lambda}_0 + d)^v F \; . \tag{143}$$

Our new black hole gauge conditions are modular forms of the weight $\left(\pm\frac{1}{4}, \mp\frac{1}{4}\right)$ for the left-handed (upper sign) or right-handed (lower sign) parts of spinorial gauge conditions, see Eq. (142).

IX. U-DUALITY-COVARIANT CLASS OF GAUGES

Hidden symmetries of supergravities, restricted to the subgroups with integer parameters to provide the black hole charge quantization, are realized in extreme black holes solutions. In this respect *extreme black holes are as good for realizing dualities as are quarks for realizing the fundamental representation of* $SU(3)$. In type-II strings or in the supermembrane case we may look for a general class of duality-symmetric gauges although we do not know yet a black hole solution with one-half of unbroken supersymmetry and E_7 [or $SL(8,R)$] covariant black hole hair. However, one can use the information available about the hidden symmetry of $N = 8$ supergravity [2]. We may just use the fact that it is known how the hidden symmetries act on fields, in particular, on spinors. In symmetric gauge $N = 8$ supergravity has an SO(8) symmetric form. The scalar matrix V in this form satisfies the condition

$$V = V^\dagger \; . \tag{144}$$

This condition is provided by the SU(8) gauge transformation. The theory can be formulated in the inhomogeneous coordinates of $\frac{E_7}{SU(8)}$. In these coordinates $Y_{AB,CD}$, $A,\ldots,D = 1,\ldots,8$ are the scalar fields of the theory. If we would know the most general black hole solutions they would probably give us those scalar fields as some functions of the space coordinates $\vec{x}$, and the metric, the vector fields, etc., would be adjusted to the scalar matrix expectation value (values at infinity) as well as scalar charges, which will form the $\frac{1}{|\vec{x}|}$ part of this matrix. Thus as different from the previously studied case of S duality we do not have the solution available which exhibits this hidden symmetry, i.e., we do not know yet the configuration which solves classical equations of motion of $N = 8$ supergravity in a form in which the solution has manifest U duality. However the general structure of the theory suggests that the algebraic constraint on the Killing spinor will undergo a transformation generalizing that of S duality, which is described above.

Let us first reformulate the information about the S duality described above in the form suitable for the generalization to U duality.

The action of U(1,1) [complex version of $SL(2,R)$] on spinors of $N = 4$ supergravity interacting with matter with the local U(4) gauge symmetry fixed was known since 1977 [41]. The scalars $Y(x)$ are related to the axion-dilaton field $\lambda(x)$ described above. For some arbitrary parameter ζ (which is equal to 1 in the absence of matter) the scalar matrix may be chosen in the symmetric gauge $V = V^\dagger$:

$$V\left[Y(x)\right] = \begin{pmatrix} \frac{1}{\sqrt{1-Y\bar{Y}}} & \frac{Y}{\sqrt{1-Y\bar{Y}}} \\ \frac{\bar{Y}}{\sqrt{1-Y\bar{Y}}} & \frac{1}{\sqrt{1-Y\bar{Y}}} \end{pmatrix} \; , \tag{145}$$

were $Y = \zeta^{\frac{1}{2}} \left(\frac{1+i\lambda(x)}{2}\right)$.

When the scalars are subject to fractional transformation of U(1,1),

$$Y = \frac{aY' + \bar{c}}{cY' + a} \; , \quad |a|^2 - |c|^2 = 1 \; , \tag{146}$$

the gravitino has to transform as

$$\Psi_\mu = \exp\left(\frac{i\zeta}{4}\gamma_5 \theta(Y', \bar{Y}')\right)(\Psi_\mu)' \; , \tag{147}$$

where

$$\exp[i\theta(Y', \bar{Y}')] = \frac{cY' + \bar{a}}{\bar{c}\bar{Y}' + a} \; . \tag{148}$$

The Killing spinor, being defined as a zero mode of the equations

$$(\delta_{\mathrm{SUSY}}\Psi_\mu) = \exp\left(\frac{i\zeta}{4}\gamma_5\theta(Y', \bar{Y}')\right)(\delta_{\mathrm{SUSY}}\Psi_\mu)' = 0 \; , \tag{149}$$

obviously has to transform under duality transformation SU(1,1) as

$$(\hat{\nabla}_\mu)\epsilon_k = \exp\left(\frac{i\zeta}{4}\gamma_5\theta(Y', \bar{Y}')\right) \left((\hat{\nabla}_\mu)\epsilon_k\right)' = 0 \; . \tag{150}$$

This is all we need to show that our choice of the gauge fixing defined by the Killing spinors of the background is duality covariant. Available black hole solutions gave us examples of such rotations of the Killing spinors. In particular, the Killing spinor admitted by the axion-dilaton black holes in symplectic form is

$$\begin{pmatrix} \epsilon(\vec{x}) \\ \epsilon^*(\vec{x}) \end{pmatrix} = e^{\frac{1}{2}\mathcal{U}(\vec{x})} \begin{pmatrix} \left(\frac{\mathcal{H}_2(\vec{x})}{\overline{\mathcal{H}}_2(\vec{x})}\right)^{\frac{1}{4}} & 0 \\ 0 & \left(\frac{\overline{\mathcal{H}}_2(\vec{x})}{\mathcal{H}_2(\vec{x})}\right)^{\frac{1}{4}} \end{pmatrix} \begin{pmatrix} \epsilon \\ \epsilon^* \end{pmatrix}_{(0)} , \tag{151}$$

where ϵ_0 is the constant x-independent part of the Killing spinor (its value at $|\vec{x}| \to \infty$).

We would like to stress that the compensating U(1) transformation acting on the Killing spinors of the background is neither global nor local: it is *rigid*. Indeed, the $h_{\rm sp}$ matrix in our example is

$$\begin{pmatrix} \epsilon(\vec{x}) \\ \epsilon^*(\vec{x}) \end{pmatrix}' = \begin{pmatrix} \left(\frac{c\lambda(\vec{x})+d}{c\bar{\lambda}(\vec{x})+d}\right)^{\frac{1}{4}} & 0 \\ 0 & \left(\frac{c\bar{\lambda}(\vec{x})+d}{c\lambda(\vec{x})+d}\right)^{\frac{1}{4}} \end{pmatrix} \begin{pmatrix} \epsilon(\vec{x}) \\ \epsilon^*(\vec{x}) \end{pmatrix}, \tag{152}$$

where $\lambda(x) = \frac{\mathcal{H}_1}{\mathcal{H}_2}$ and $\mathcal{H}_{1,2}(x)$ are complex harmonic functions defined in Eq. (116). The role of rigid symmetries in connection with the hypermultiplet action in the black hole background was discussed before in [42].

Taking into account the fact that the choice of the gauge condition for the heterotic string has not required any specific knowledge of available black holes but only the properties of $N = 4$ supergravity under S duality (including the spinors) we may proceed with the duality symmetric gauges for type-II string and supermembrane.

We may choose a scalar matrix in a symmetric gauge $V = V^\dagger$ as suggested in [2]. This condition is provided by the special choice of the local SU(8) gauge fixing:

$$V = \begin{pmatrix} \frac{1}{\sqrt{1-Y\bar{Y}}} & \frac{Y}{\sqrt{1-Y\bar{Y}}} \\ \frac{\bar{Y}}{\sqrt{1-Y\bar{Y}}} & \frac{1}{\sqrt{1-Y\bar{Y}}} \end{pmatrix}. \tag{153}$$

Each entry of this matrix is defined by the matrix $Y_{AB,CD}$, $A,\dots,D = 1,\dots,8$, i.e., by the inhomogeneous coordinates of $\frac{E_7}{SU(8)}$.

$E_{7(+7)}$ acts on the coordinates $Y_{AB,CD}$ by fractional transformation

$$Y' = \frac{B+YD}{A+YC}. \tag{154}$$

The action of U duality on the scalar matrix V is

$$V(Y)\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} U(Y) & 0 \\ 0 & U(Y)^* \end{pmatrix} V(Y'), \tag{155}$$

where

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \tag{156}$$

is a constant matrix of $E_{7(+7)}$, and A, B, C, D are 28×28 constant matrices defined in [2]. The local SU(8) transformation U is determined by the condition that after the action of $E_{7(+7)}$ the matrix $V(Y') = V(Y')^\dagger$ remains unitary.

The local SU(8) compensating transformation which keeps the theory in the same symmetric gauge after the duality transformation is defined by

$$\begin{pmatrix} U(Y) & 0 \\ 0 & U(Y)^* \end{pmatrix} = V(Y)\begin{pmatrix} A & B \\ C & D \end{pmatrix} V^{-1}\left(\frac{B+YD}{A+YC}\right). \tag{157}$$

In this form the matrix U still acts on 28-dimensional representation. In the form in which it acts on a single spinor in an eight-dimensional representation with $U(Y)_{\rm sp}$ it gives an explicit transformation of the gauge condition under U duality.

Thus we have shown that given a half-supersymmetric background of $d = 11$ or $d = 10, N = 2$ supersymmetry, the Killing spinor of this background is covariant under the action of U duality on spinors [via compensating SU(8)]. Therefore if the gauge-fixing condition on spinors of type-II superstring or supermembrane in a half-supersymmetric background is chosen in a manifestly duality covariant form (34), this gauge fixing will transform into an equivalent one related to the original one by the action of $E_{7(+7)}$ (or of some subgroup of it) on gravitino and therefore on the Killing spinor.

In absence of the background we may still use the fact that the algebraic constraint on Killing spinors at infinity is defined in terms of the $E_{7(+7)}$ black hole hair. The algebraic constraint will transform under duality in terms of the global SU(8) compensating transformation

$$\begin{pmatrix} U(Y_0) & 0 \\ 0 & U(Y_0)^* \end{pmatrix} = V(Y_0)\begin{pmatrix} A & B \\ C & D \end{pmatrix} V^{-1}\left(\frac{B+Y_0D}{A+Y_0C}\right), \tag{158}$$

where $(Y_{AB,CD})_0$ is the value of the scalar fields at infinity, or the vacuum expectation value of the scalar field $\langle Y_{AB,CD}\rangle$. The gauge-fixing condition will transform according to the transformation property of the spinors. And again, in symplectic form we have

$$\left(\frac{\mathcal{Z}}{|Z|}C\right)' = \begin{pmatrix} U(Y_0)_{\rm sp} & 0 \\ 0 & U(Y_0)^*_{\rm sp} \end{pmatrix} \left(\frac{\mathcal{Z}}{|Z|}C\right) = 0, \qquad |Z|' = |Z|. \tag{159}$$

Thus the manifest U duality is the property of the general covariant gauge fixing in terms of the supercharge of the background and presented in Eq. (34). This gauge fixing incorporates all possible half-supersymmetric backgrounds of the theory.

X. CONCLUSION

The main result of this work is the gauge fixing of manifestly supersymmetric string theory in a form in which duality symmetry is also manifest. The difference with the known quantization methods is in the fact that string is placed in the curved BPS background which admits a Killing spinor of the half-size of the supersymmetry parameter. The gauge-fixing condition uses the supercharge of the background, see Eq. (34), and therefore general covariance is not broken in the quantization. The algebraic constraint on the ghosts which allows to truncate κ symmetry has a simple form in terms of the symplectic central charge matrix of the background defined in Eq. (39) and is given in Eq. (82). Under duality transformation of the background this constraint transforms as shown in Eq. (135) in doublet form, in Eq. (138) in the contracted form, and in Eq. (141) in symplectic form for S duality and in Eq. (159) in the symplectic form for U duality. The partition function on a torus is by construction gauge independent, and therefore it is duality symmetric at least at the formal level, before anomalies are taken into account. The absence of anomalies of κ symmetry in the heterotic string background requires the possibility to perform the embedding of the spin connection with torsion into the non-Abelian gauge group [10]. Such embedding was already studied for different black holes and their uplifted versions. It was found that it does not work equally well for all configurations, depending on whether the holonomy group is a subgroup of the gauge group of the heterotic string or not [14, 43]. Thus we expect that the future study of the anomalies of the GS string in duality covariant gauges will help us to understand better the quantum aspects of this theory which were not yet covered in this paper: here we have only shown how to generalize the light-cone gauge to the most general possible gauges where the κ-symmetry is truncated in terms of the central charges of the BPS backgrounds.

We consider the quantization in the absence of the curved background as a limiting procedure when curvature goes to zero. This means that we can use the central charge of the background even in the flat space to fix the fermionic symmetry. The central charge can be considered as the vacuum expectation value of the string momenta, see Eq. (94). It was explained in [31] that the saturation of the four-dimensional BPS bound $m_4 = |Z|$ is the condition that the ten-dimensional configuration is massless, since $m_{10}^2 = m_4^2 - |Z|^2 = 0$. The massless ten-dimensional state may correspond also to the massless four-dimensional state. The form of the gauge-fixing fermionic symmetry (39), which we are using has a well-defined limit when $|Z| \to 0$ which allows us to perform the quantization both in the massless black hole background (23) as well as in the flat background.

Our main conclusion is the following. The nature of infinite reducible κ symmetry, which is the gauge symmetry of the manifestly supersymmetric versions of the string theory, provides a clear request for the existence of the geometries associated with the states saturating the supersymmetric positivity bound: extreme black holes, strings, membranes, *pp* waves. Those geometry states play a special role in gravitational theories. What was considered previously as a misfortune of infinite reducibility of κ symmetry becomes its enormous advantage: the consistent truncation can be performed in the BPS-type backgrounds with one-half of unbroken supersymmetry, which describe the duality-invariant geometries. We have found duality-symmetric gauges for fixing κ symmetry. In those gauges the elementary excitations of the supersymmetric string described by the first quantized partition function on the torus are duality invariant. The second quantization of this theory may lead to a better understanding of a quantized string and black holes.

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APPENDIX A: SUPERSYMMETRIC WAVES AND BLACK HOLES

It was observed some time ago that the uplifted extreme electrically charged black holes become supersymmetric gravitational waves with the Killing spinor satisfying the light-cone constraint [42]. The most general known to us half-supersymmetric backgrounds of the heterotic string which admits a Killing spinor satisfying the null condition $n_\mu \gamma^\mu \epsilon = 0$ is given by a bosonic configurations which also admits a null Killing vector. We call such configurations supersymmetric gravitational waves (in the context of string theory they were called chiral null models [17]). Some of them (Brinkmann's plane fronted waves) admit the Killing vector which is covariantly constant [14]. They were called supersymmetric string waves (SSW's). The configurations related to the SSW by T duality are also supersymmetric and were called generalized fundamental string solutions or dual waves [16]. The bosonic part of the ten-dimensional background for the heterotic string which is defined by the integrability condition for the truncation of κ symmetry is given by such solutions.[13] Here we would like to consider the half-supersymmetric SO(8)-symmetric solutions build from ten functions of transverse x^i, $i = 1, \dots 8$. This metric admits two null Killing vectors n, m, being independent on u and v coordinates. Indeed, the gauge fixing in the generalized (m, n) light-cone gauge in the presence of the background naturally requires the background to admit two null vectors in addition to a half-size Killing spinor.

The metric is

$$ds^2 = 2e^{2\hat{\phi}} du(dv + A_\mu dx^\mu) - \sum_1^8 dx^i dx^i,$$
$$A_v = 0 \ . \tag{A1}$$

The two-form field is

$$B = 2e^{2\hat{\phi}} du \wedge (dv + A_\mu dx^\mu) \ . \tag{A2}$$

The ten-dimensional dilaton $e^{-2\hat{\phi}}$ and the u component of the field A_μ are harmonic functions in the eight-dimensional flat space,

$$\sum_1^8 \partial_i \partial_i e^{-2\hat{\phi}} = 0 \ , \quad \sum_1^8 \partial_i \partial_i A_u = 0 \ . \tag{A3}$$

The eight transverse functions A_i satisfy the equations

[13] More general ten-dimensional solutions may still be discovered and at the moment of this writing there is no information available about the most general half-supersymmetric backgrounds for the heterotic string. However, on the basis of the $N = 2, d = 4$ investigations of Tod [27] one may expect that all such backgrounds may be listed.

$$\sum_1^8 \partial_i(\partial_{[j}A_{i]}) = 0 \ . \tag{A4}$$

For some of the solutions (not all of them) it is known how to proceed with the spin embedding to cancel α' corrections coming from anomalies [14, 16].

It is instructive to present here also the form of these solutions in terms of the four-dimensional geometry, when the functions describing the ten-dimensional waves depend only on x^1, x^2, x^3. Dimensional reduction of supersymmetric gravitational waves was performed in [44]. The stationary metric in the canonical frame is

$$ds^2 = e^{2\phi}\left(dt + \sum_1^3 A_i dx^i\right)^2 - e^{-2\phi}\sum_1^3 dx^i dx^i \ , \tag{A5}$$

where the four-dimensional dilaton is given by

$$e^{-2\phi} = \left(e^{-2\hat\phi}A_u - \sum_4^8 (A_i)^2\right)^{\frac{1}{2}} \ . \tag{A6}$$

The other fields can be also deduced from the ten-dimensional configuration and are presented explicitly in [44]. If we would take a special subclass of dimensionally reduced gravitational waves (A1) with

$$A_1 = A_2 = A_3 = 0 \ , \tag{A7}$$

we will get the supersymmetric black holes with metric

$$ds^2 = e^{2\phi}dt^2 - e^{-2\phi}\sum_1^3 dx^i dx^i \ , \tag{A8}$$

where the four-dimensional dilaton is defined in Eq. (A6). The functions defining the solutions are taken in the form [44]

$$\begin{aligned} e^{-2\hat\phi} &= 1 + \sum_1^s \frac{2\tilde m_k}{r_k} \ , \\ A_u &= 1 + \sum_1^s \frac{2\hat m_k}{r_k} \ , \\ A_i &= \sum_1^s \frac{2(q_k)_i}{r_k} \ , \\ i &= 4, \ldots, 8. \end{aligned} \tag{A9}$$

This is a multi-black-hole solution with S black holes and $r_k \equiv |\vec x - \vec x_k|$. The four-dimensional dilaton is given by

$$e^{-2\phi} = \left\{ \left(1 + \sum_1^s \frac{2\tilde m_k}{r_k}\right)\left(1 + \sum_1^s \frac{2\hat m_l}{r_l}\right) - \left[\sum_1^s \frac{2(q_k)_i}{r_k}\right]^2 \right\}^{\frac{1}{2}} \ . \tag{A10}$$

If we would take one-black-hole solutions with all functions $A_i = 0$ we would reproduce the supersymmetric electrically charged two-parameter black hole solutions of Sen [30] for $g = 1$. The canonical metric is given in Eq. (A8), where the dilaton is given by

$$e^{-2\phi} = \left(1 + \frac{2(\tilde m + \hat m)}{r} + \frac{4\tilde m \hat m}{r^2}\right)^{\frac{1}{2}} \ . \tag{A11}$$

One can rescale this solution by introducing $\langle e^{-2\phi}\rangle = e^{-2\phi_0} = \frac{1}{g^2}$. This allows us to bring our dimensionally reduced wave solution to the form of Eq. (19). Two independent parameters in the wave solutions are related to those in (19) as follows:

$$\tilde m + \hat m = 2mG_N \ , \quad \tilde m \hat m = g^2(N_L - 1) \ . \tag{A12}$$

The first parameter is the mass of the black hole, which in our case is $m = \frac{\tilde m + \hat m}{2G_N}$. The second parameter is related to the left-handed charge of the black hole and to the parameter $m_0^2 = 4\tilde m \hat m$ introduced by Sen [30]. Thus if one would wish to generalize Sen's solutions to the multi-black-hole case using the duality rotations from Kerr's four-dimensional black holes, this would be very difficult. However, a duality between ten-dimensional supersymmetric waves and four-dimensional black holes established in [42] helps to get the most general in this class multi-black hole solutions defined in Eqs. (A8) and (A10).

Now we may conclude that the general electrically charged black-hole-type solutions (A5) indeed form the background in which the heterotic string in Green-Schwarz form is known to be quantized consistently. The uplifted geometry admits the Killing spinor and two null vectors required by our choice of the gauge condition. In dimensionally reduced form this geometry includes all known electrically charged supersymmetric black holes of the heterotic string theory saturating the BPS bound.

APPENDIX B: SUPERSYMMETRIC MASSLESS MULTI-BLACK HOLES

The massless black hole configuration found by Behrndt [19] by dimensional reduction of the T-self-dual supersymmetric wave solutions is given by $\tilde m = -\hat m$, see Eqs. (A8), (A11), and (A12). In this appendix we will describe a rather nontrivial space-time structure of this solution and find a more general set of massless black holes.

Note that for the solutions obtained by Sen [30] the black hole mass m and the left-handed charge Q_L have the following parametrization: $m^2 = \frac{m_0^2}{16}\cosh^2\alpha$ and $(Q_L)^2 = \frac{g^2}{2} m_0^2 \sinh^2\alpha$. Therefore the point where the mass of the black hole is zero seems to require also the left-handed charge to vanish, i.e., the solution becomes trivial. However, if one starts with ten-dimensional supersymmetric gravitational waves (A1), the parameters $\tilde m$ and $\hat m$ are independent, and there is no obvious reason not to consider the configuration $\tilde m + \hat m = 0$. (The configurations with $\tilde m + \hat m < 0$, which would correspond to a negative ADM mass, would violate supersymmetric positivity bound.) Bearing in mind that massless black holes are not quite usual solutions of four-dimensional gravity interacting with matter, one may try to study these configurations in more detail.

As we have already mentioned in Sec. II, the massless electric black hole (23) has a singularity at $r = 2g$, in addition to the singularity ar $r = 0$. Meanwhile, the massless magnetic black hole (26) has a singularity at $r =$

$\frac{2}{g}$. The relevant question to ask is whether the singularity of the metric of the electric black hole at $r = 2g$ and of the magnetic one at $r = \frac{2}{g}$ is a true singularity, or it can be removed by the change of coordinate system. To answer this question we calculated the curvature scalar for these solutions in canonical Einstein frame. In the electric case the curvature scalar has a singularity at $r = 2g$:

$$R^{\mathrm{el}}_{\mathrm{can}} = \frac{4g^2(2g^2 + r^2)}{r(r^2 - 4g^2)^{\frac{5}{2}}} \ . \qquad \text{(B1)}$$

In the magnetic case the canonical curvature is

$$R^{\mathrm{magn}}_{\mathrm{can}} = \frac{4g(2 + g^2r^2)}{r(g^2r^2 - 4)^{\frac{5}{2}}} \ . \qquad \text{(B2)}$$

For completeness of the picture we will check that the new singularity is present also in stringy frame. For electric solution in stringy frame the metric is

$$ds^2 = \left(1 - \frac{4g^2}{r^2}\right)^{-1} dt^2 - d\vec{x}^2 \ , \qquad \text{(B3)}$$

and the curvature is

$$R^{\mathrm{el}}_{\mathrm{str}} = \frac{-8g^2(8g^2 + r^2)}{r^2(r^2 - 4g^2)^2} \ . \qquad \text{(B4)}$$

The magnetic massless solutions in stringy frame has the metric

$$ds^2 = dt^2 - \left(1 - \frac{4}{g^2r^2}\right) d\vec{x}^2 \qquad \text{(B5)}$$

and the curvature scalar is

$$R^{\mathrm{magn}}_{\mathrm{str}} = \frac{16g^2(2 + g^2r^2)}{(r^2g^2 - 4)^3} \ . \qquad \text{(B6)}$$

Thus the singularity at $r = 2g$ ($r = 2/g$) does not vanish when we change from electric to magnetic solutions or change from canonical to stringy frame. For comparison, we present here the pure magnetic $a = 1$ massive black hole in stringy frame

$$ds^2 = dt^2 - \left(1 + \frac{2m}{r}^2\right) d\vec{x}^2 \ . \qquad \text{(B7)}$$

The curvature is completely nonsingular

$$R^{\mathrm{magn}}_{\mathrm{str}}(a = 1) = \frac{8m^2}{(2m + r)^4} \ . \qquad \text{(B8)}$$

Nothing like that happens with the massless configuration, the singularity is present and since it is related to string coupling, it somehow reflects the presence of a string.

One can find a solution describing a more general family of four-dimensional black holes with a vanishing ADM mass. For this purpose we may use the fact that in gravitational wave solutions in $d = 10$ one can use more general harmonic functions. For one-black-hole case one may take

$$\begin{aligned} e^{-2\hat{\phi}} &= e^{-2\hat{\phi}_0} + \frac{2\tilde{m}}{r} \ , \\ A_u &= (A_u)_0 + \frac{2\hat{m}}{r} \ , \\ A_i &= (A_i)_0 + \frac{2(q)_i}{r} \ , \\ i &= 4, \ldots, 9. \end{aligned} \qquad \text{(B9)}$$

The four-dimensional dilaton is now given by

$$e^{-2\phi} = \left\{ \left(e^{-2\hat{\phi}_0} + \frac{2\tilde{m}_k}{r}\right)\left((A_u)_0 + \frac{2\hat{m}_k}{r}\right) - \left[(A_i)_0 + \frac{2(q_k)_i}{r}\right]^2 \right\}^{\frac{1}{2}} \ . \qquad \text{(B10)}$$

Obviously there are many ways to make the $\frac{1}{r}$ term in this expression equal to zero, and to make the ADM mass of the four-dimensional black holes vanishing. The condition on the parameters of the harmonic functions which provides the massless black holes is

$$e^{-2\hat{\phi}_0}\hat{m} + (A_u)_0\tilde{m} - 2\sum_{i=4}^{9}(A_i)_0 q_i = 0 \ . \qquad \text{(B11)}$$

The main difference with the previous massless case comes from the nonvanishing asymptotic value of the ten-dimensional component of the metric $g_{ui} = (A_i)_0$. This modification cannot be removed by simple rescaling of coordinates. Thus generalizing higher-dimensional configurations one can find more massless four-dimensional black hole-type solutions. Different choices of harmonic functions in $d = 10$ solutions describe different geometries of the six-dimensional space. The massless multi-black holes are also available. We may choose the following harmonic functions in the supersymmetric waves (160):

$$\begin{aligned} e^{-2\hat{\phi}} &= e^{-2\hat{\phi}_0} + \sum_1^s \frac{2\tilde{m}_k}{r_k} \ , \\ A_u &= (A_u)_0 + \sum_1^s \frac{2\hat{m}_k}{r_k} \ , \\ A_i &= (A_i)_0 + \sum_1^s \frac{2(q_k)_i}{r_k} \ , \\ i &= 4, \ldots, 9. \end{aligned} \qquad \text{(B12)}$$

The total configuration may consist of many massless black holes, the condition that each black hole in the configuration is massless requires

$$e^{-2\hat{\phi}_0}\hat{m}_k + (A_u)_0\tilde{m}_k - 2\sum_{i=4}^{9}(A_i)_0(q_k)_i = 0 \ . \qquad \text{(B13)}$$

The existence of massless black hole solutions presents a new challenge. Some time ago the very possibility of black holes being massless would seem unthinkable. One could expect that in the limit $m \to 0$ gravitational field disappears, and space becomes exactly flat. Now we have a vector multiplet which acts a a source of gravity. This leads to existence of a large family of states which have nontrivial geometric properties even when their ADM mass vanishes. One should note that it is somewhat misleading to call these states "massless black holes." First of all, they are massless in the sense of their ADM mass, however the configuration has a rest frame. Also, gravitational attraction becomes increasingly strong near usual black holes. Meanwhile, in our case massless black holes are in equilibrium with each other, and they gravitationally *repel* usual test particles which come to their vicinity. In this sense they behave like white holes rather than black ones [45].

Note also that these solutions appear in a situation where we have massless charged vector fields. In such a situation, just like in the theory of confinement in QCD, nonperturbative effects may completely change the nature of charged black hole solutions. In particular, one may study whether nonperturbative effects may lead to confinement or condensation of electrically and/or magnetically charged massless black holes. Therefore physical interpretation of massless black hole solutions and of their possible role in string theory requires further investigation.

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