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String bit models for superstring

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We extend the model of string as a polymer of string bits to the case of superstring. We mainly concentrate on type II-B superstring, with some discussion of the obstacles presented by not II-B superstring, together with possible strategies for surmounting them. As with previous work on bosonic string we work within the light-cone gauge. The bit model possesses a good deal less symmetry than the continuous string theory. For one thing, the bit model is formulated as a Galilei-invariant theory in $[(D-2)+1]$-dimensional space-time. This means that Poincaré invariance is reduced to the Galilei subgroup in $D-2$ space dimensions. Naturally the supersymmetry present in the bit model is likewise dramatically reduced. Continuous string can rise in the bit models with the formation of infinitely long polymers of string bits. Under the right circumstances (at the critical dimension) these polymers can behave as string moving in D-dimensional space-time enjoying the full $N=2$ Poincaré supersymmetric dynamics of type II-B superstring.

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I. INTRODUCTION

The idea that relativistic string is a composite of point-like entities [1–3] called "string bits" is an appealing alternative to the cumbersome formal apparatus of string field theory. The origins of the idea can be traced to the earliest days of dual models [4] with the attempt, motivated in part by the old parton model of hadrons [5], to understand dual resonance amplitudes as planar "fishnet" Feynman diagrams. After 't Hooft showed that planar diagrams are naturally singled out by the $1/N_c$ expansion [6], the idea was again vigorously explored as a possible link between non-Abelian gauge theory and string theory [7–9]. The attempted linkage failed because, unlike the partons of hadrons (quarks and gluons), the "partons" of string never carry a finite fraction of the string's momentum: string bits are always "wee" partons. From the modern point of view, strings are not hadrons and we advocate that the inevitable weeness of string bits should actually be embraced as a uniquely stringy hallmark [3].

Our main goal in developing string bit models is to devise a truly nonperturbative formulation of string theory. In the earlier work of one of us this idea has been pursued only in light-cone gauge and systematically developed only for bosonic string [10]. Bosonic string (in 26 space-time dimensions) is generally believed to be absolutely unstable, and it is therefore an unfortunate test case for a nonperturbative reformulation. This has not hindered the formal implementation of string bit ideas for this case, since that has so far been limited to a perturbative context. However, there seems little point in attempting nonperturbative studies of bosonic string bit models, other than to confirm that they do not make sense as string theories. We can be much more optimistic in the case of superstring theory which is generally hoped to be a consistent stable theory. Indeed, if a superbit model for superstring can be shown to be a good theory at the nonperturbative level, there is the exciting possibility that many of the conundrums of quantum gravity, such as the consistency of quantum mechanics in the presence of black holes, may be resolved [11,12].

In this paper we present a bit model for superstring, restricting attention for the most part to the type II-B case, which presents the fewest obstacles to a complete treatment. By no means do we claim that our bit model is unique. Universality suggests that the model can be generalized in various ways, and still yield a satisfactory continuum limit. In fact to get the correct string interactions the model *has* to be extended. Producing one or another satisfactory model is useful for studying superstring theory, but we eventually want to restrict the models by some underlying symmetry principles, not by whether they possess a satisfactory continuum limit. Our bit model suggests what some of these principles may be, but it certainly does not give them all.

A dramatic feature of string theory viewed in light-cone gauge is the fact that the longitudinal coordinate $x^-=(t-z)/\sqrt{2}$ is virtually eliminated from the theory. Except for its zero mode, conjugate to P^+, it is solely a function of the transverse coordinates. The string bit idea effectively eliminates even this zero-mode longitudinal degree of freedom, by identifying P^+ with the number of string bits: each bit is free to move around only in the transverse space. The full space-time symmetry group of the string bit dynamics is the Galilei group in $[(D-2)+1]$-dimensional space-time with space coordinates x^k, $k=1,\ldots,D-2$ and time identified with x^+. Each bit has a fixed Newtonian mass m. If M bits can form into long polymers, then mM can be identified in the limit $M\to\infty$ as the string's total P^+. All of this has already been discussed in the simplified context of

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bosonic string [1,3,9,10,13]. To extend the work to superstring, we must decide how the world-sheet spinors are to be fit into the string bit picture. We shall find that they can emerge in the continuous string limit if each bit is in a 256 component supermultiplet of $\mathcal{S}_1\mathcal{G}$, the minimal super-Galilei group [14,15] for eight-dimensional space.

The paper is organized as follows. In Sec. II we review the super-Poincaré algebra in light-cone coordinates and display its super-Galilei subalgebra. Then in Sec. III we devise a suitable discretization of superstring in the light-cone Green-Schwarz formulation. This discretization motivates our proposal for a fully second-quantized superstring bit model. In Sec. IV we present such models, first in $2+1$ dimensions as a warmup, then in $8+1$ dimensions for type II-B superstring. Section V contains our concluding remarks, which include a brief discussion of the open issues we leave for resolution in future work.

II. SUPER-POINCARÉ ALGEBRA IN LIGHT-CONE COORDINATES

We begin by reviewing the D-dimensional super-Poincaré algebra and expressing it in light-cone variables. For simplicity we shall only consider even D. The super-Poincaré generators include a vector P^μ, a rank two antisymmetric tensor $M^{\mu\nu}$, and a Grassmann odd spinor $Q_{\mathcal{A}}$. Greek indices take values from 0 to $D-1$, and capital script indices take values from 1 to $2^{D/2}$, which is the dimension of the spinor representation of the Poincaré group ISO$(D-1,1)$. The algebra satisfied by the generators is given by

$$\begin{aligned}
[P^\mu,P^\nu]&=[Q_{\mathcal{A}},P^\mu]=0,\\
[M^{\mu\nu},P^\rho]&=i(\eta^{\mu\rho}P^\nu-\eta^{\nu\rho}P^\mu),\\
[M^{\mu\nu},M^{\rho\sigma}]&=i(\eta^{\mu\rho}M^{\nu\sigma}+\eta^{\mu\sigma}M^{\rho\nu}\\
&\quad-\eta^{\nu\rho}M^{\mu\sigma}-\eta^{\nu\sigma}M^{\rho\mu}),\\
[M^{\mu\nu},Q_{\mathcal{A}}]&=-\frac{1}{2}(\Sigma^{\mu\nu}\cdot Q)_{\mathcal{A}},\\
\{Q_{\mathcal{A}},Q^\dagger_{\mathcal{B}}\}&=-\frac{1}{\sqrt{2}}(\Gamma\cdot P\Gamma_0)_{\mathcal{A}\mathcal{B}},
\end{aligned} \tag{2.1}$$

where $\eta^{\mu\nu}$ =diag$\{-1,1,\ldots,1\}$, Γ^μ are the Dirac Γ matrices in D dimensions, and $\Sigma^{\mu\nu}=i/2[\Gamma^\mu,\Gamma^\nu]$. Note that the right-hand side of the last equation involves

$$-\Gamma\cdot P\Gamma^0=P^0+P^k\alpha^k,$$

where $\alpha^k\equiv\Gamma^0\Gamma^k, k=1,\ldots,D-1$, are the original Hermitian α matrices introduced by Dirac.

Light-cone coordinates are defined by singling out one of the spatial directions, say x^{D-1}, and letting

$$x^\pm\equiv\frac{1}{\sqrt{2}}(x^0\pm x^{D-1}). \tag{2.2}$$

The role of time is played by x^+, so its conjugate momentum P^- plays the role of the light-cone Hamiltonian. The longitudinal coordinate is x^-, and the transverse coordinates are x^i, with $i=1,\ldots,D-2$. In these coordinates a $[(D-2)+1]$-dimensional super-Galilei algebra emerges as a subalgebra of the full D-dimensional super-Poincaré algebra in the transverse + time directions. Transverse spatial translations are generated by P^i, time translation is generated by P^-, transverse spatial rotations by M^{ij}, and transverse Galilei boosts by M^{+i}. Accordingly, we make the replacements

$$\begin{aligned}
P^-&\to H,\\
M^{ij}&\to J^{ij},\\
M^{+i}&\to K^i.
\end{aligned} \tag{2.3}$$

The part of the super-Galilei subalgebra involving even generators is then given by

$$\begin{aligned}
&[P^i,P^j]=[P^i,H]=[J^{ij},H]=[K^i,K^j]=0,\\
&[J^{ij},P^k]=i(\delta^{ik}P^j-\delta^{jk}P^i),\\
&[K^i,P^j]=-i\delta^{ij}P^+,\\
&[K^i,H]=-iP^i,\\
&[J^{ij},J^{kl}]=i(\delta^{ik}J^{jl}+\delta^{il}J^{kj}-\delta^{jk}J^{il}-\delta^{jl}J^{ki}),\\
&[J^{ij},K^k]=i(\delta^{ik}K^j-\delta^{jk}K^i).
\end{aligned} \tag{2.4}$$

Note that in the above algebra P^+ plays the role of the Newtonian mass. This role will be exploited in constructing the string bit model for discretized light-cone superstring, in which P^+ is the length of a piece of string, and is equal to the total Newtonian mass of all the string bits. The rest of the charges completing the Poincaré algebra do not have a Galilean interpretation, and will not be manifest symmetries in the light-cone gauge.

The supercharge $Q_{\mathcal{A}}$ is a $2^{D/2}$ component SO$(D-1,1)$ spinor. But it decomposes under the transverse SO$(D-2)$ subgroup into two (reducible) $2^{(D-2)/2}$ component spinors playing different roles in the Galilei subalgebra. To display this we choose an appropriate representation for the Γ matrices, convenient for light-cone coordinates. The $2^{D/2}\times 2^{D/2}$ Dirac Γ matrices satisfy the Clifford algebra $\{\Gamma^\mu,\Gamma^\nu\}=-2\eta^{\mu\nu}$. Choose a representation for the Γ matrices such that Γ^0 and Γ^{D-1} are given by

$$\Gamma^0=i\begin{pmatrix}0&-I\\I&0\end{pmatrix},$$
$$\Gamma^{D-1}=i\begin{pmatrix}0&0&\mathbf{1}&0\\0&0&0&-\mathbf{1}\\\mathbf{1}&0&0&0\\0&-\mathbf{1}&0&0\end{pmatrix}. \tag{2.5}$$

where I is the $2^{(D-2)/2}$-dimensional identity matrix, and $\mathbf{1}$ is the $2^{(D-4)/2}$-dimensional identity matrix. This will simplify the superalgebra in light-cone coordinates, singled out by the spatial component $D-1$, since $\alpha^{(D-1)}$ is diagonal:

$$\alpha^{(D-1)}\equiv\Gamma^0\Gamma^{D-1}=\begin{pmatrix}\mathbf{1}&0&0&0\\0&-\mathbf{1}&0&0\\0&0&-\mathbf{1}&0\\0&0&0&\mathbf{1}\end{pmatrix}. \tag{2.6}$$

The choice of representation for the transverse Γ^k, $k=1,\ldots,D-2$ can vary from one dimension to another depending on whether or not one applies Majorana or Weyl constraints (or both). Since we only consider even D, the Weyl constraint may always be imposed. If it is, then convenience dictates a representation for the transverse Γ matrices with the same block form as Γ^{D-1}:

$$\Gamma^k = i\begin{pmatrix} 0 & \gamma^k \\ \gamma^k & 0 \end{pmatrix},$$

where the γ^k are $2^{(D-2)/2}\times 2^{(D-2)/2}$ Hermitian matrices. In such a representation

$$\alpha^k = \begin{pmatrix} \gamma^k & 0 \\ 0 & -\gamma^k \end{pmatrix},$$

and the chirality matrix Γ_{D+1} will be diagonal

$$\Gamma_{D+1} = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

Imposing the Weyl constraint by fixing the chirality of the supercharges to be ± 1 means keeping only the first (last) $2^{(D-2)/2}$ components of $Q_{\mathcal{A}}$. On the other hand, if we want the supercharges to be Hermitian, we must choose the Γ^k to be imaginary (Majorana). Only if $D = 2$ (mod 8) is this possible within the Weyl-friendly representation just described. The Majorana representation is also possible for $D = 4$ (mod 8), but then at least one of the transverse Γ will not have the block form of Γ^{D-1}, so Γ_{D+1} will not be diagonal. For example, in the case $D = 4$, a Majorana representation for the transverse Γ matrices can be taken to be

$$\Gamma^1 = i\begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix}, \quad \Gamma^2 = i\begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}.$$

The Weyl-friendly representation for $D = 4$ would retain the same form for Γ^1 but replace Γ^2 by

$$\Gamma^2 \to i\begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}.$$

The above representation of the Clifford algebra helps us display the Galilei properties of the supercharge $Q_{\mathcal{A}}$. This amounts to describing the embedding $\mathrm{SO}(D-2)\times\mathrm{SO}(1,1) \subset \mathrm{SO}(D-1,1)$ singled out by the light cone. Separate the values of $\mathcal{A}$ into two groups denoted by dotted and undotted capital Latin spinor indices, according to the eigenvalues of the matrix α^{D-1} (2.6), the chirality matrix for SO(1,1):

$$\alpha^{D-1}_{\dot{A}\dot{B}} = -\delta_{\dot{A}\dot{B}},$$
$$\alpha^{D-1}_{AB} = \delta_{AB},$$
$$\alpha^{D-1}_{A\dot{B}} = \alpha^{D-1}_{\dot{A}B} = 0.$$

The dotted and undotted indices each range over $2^{(D-2)/2}$ values (16 for $D = 10$, 2 for $D = 4$). Because the transverse α anticommute with α^{D-1}, it follows that $\boldsymbol{\alpha}_{AB} = \boldsymbol{\alpha}_{\dot{A}\dot{B}} = 0$. The spinor supercharge $Q_{\mathcal{A}}$ then has dotted components $Q_{\dot{A}}$, and undotted components Q_A, transforming (reducibly) as spinors of $\mathrm{SO}(D-2)$. The superalgebra in light-cone coordinates can now be expressed in terms of these spinors. For later convenience we define $R_{\dot{A}} \equiv Q_{\dot{A}}/\sqrt{2}$. In terms of the supercharges Q_A and $R_{\dot{A}}$ the part of the super-Galilei algebra involving odd generators is given by

$$\begin{aligned}
&[P^i, Q_A] = [H, Q_A] = 0, \quad [P^i, R_{\dot{A}}] = [H, R_{\dot{A}}] = 0, \quad \{Q_A, Q_B^\dagger\} = P^+\delta_{AB},\\
&[J^{ij}, Q_A] = -\frac{1}{2}\Sigma^{ij}_{AB}Q_B, \quad [J^{ij}, R_{\dot{A}}] = -\frac{1}{2}\Sigma^{ij}_{\dot{A}\dot{B}}R_{\dot{B}}, \quad \{Q_A, R_{\dot{B}}^\dagger\} = \frac{1}{2}\mathbf{P}\cdot\boldsymbol{\alpha}_{A\dot{B}},\\
&[K^i, Q_A] = 0, \quad [K^i, R_{\dot{A}}] = -\frac{i}{2}\alpha^i_{\dot{A}B}Q_B, \quad \{R_{\dot{A}}, R_{\dot{B}}^\dagger\} = \frac{1}{2}H\delta_{\dot{A}\dot{B}}.
\end{aligned} \tag{2.7}$$

This superalgebra is called $\mathcal{S}_2\mathcal{G}$, where the "2" stands for the two supercharges Q, R. In the Weyl friendly representation described above the spinors $Q_A, R_{\dot{A}}$ each decompose into two inequivalent irreducible spinor representations of $\mathrm{SO}(D-2)$, characterized by opposite values of $\Gamma_{D+1}\alpha^{D-1}$, the chirality matrix for $\mathrm{SO}(D-2)$. To describe this we introduce dotted and undotted lower case Latin indices according to whether this chirality matrix has value -1 or $+1$, respectively:

$$R_{\dot{A}} = \begin{pmatrix} R_{\dot{a}} \\ R_a \end{pmatrix}, \quad Q_A = \begin{pmatrix} Q_a \\ Q_{\dot{a}} \end{pmatrix}.$$

Then the $2^{D/2}$ component supercharge $Q_{\mathcal{A}}$ breaks up in our chosen basis as

$$Q_{\mathcal{A}} = \begin{pmatrix} Q_a \\ \sqrt{2}R_{\dot{b}} \\ \sqrt{2}R_c \\ Q_{\dot{d}} \end{pmatrix}.$$

If the Weyl condition is used to reduce the spinors, which means keeping the top (or bottom) two entries, we simply replace $\alpha^i_{\dot{A}B} \to \gamma^i_{\dot{a}b}$ (or $-\gamma^i_{a\dot{b}}$) and $\Sigma^{ij}_{AB} \to \sigma^{ij}_{ab} \equiv -i[\gamma^i,\gamma^j]_{ab}/2$ (or $\sigma^{ij}_{\dot{a}\dot{b}}$) in (2.7).

The super-Gallilei subalgebra of the super-Poincaré algebra will be relevant in describing the dynamics of superstring bits. In fact it will be the full spacetime symmetry of a field theory of these pointlike constituents of light-cone superstring. The bits are nonrelativistic particles living in the $(D-2)$-dimensional transverse space, with time given by x^+. They do not know about the longitudinal direction x^-, and consequently there is no room for the $M^{-\mu}$ Lorentz generators. However, all information of the longitudinal direction is not lost. When bits form into a long polymer, the conserved bit number operator becomes a candidate for a discretized P^+. In the limit of infinitely long polymers, this "P^+" is effectively continuous and the polymers behave as continuous strings moving in D-dimensional space-time, since x^- is conjugate to P^+. With the formation of infinitely long polymers, the effective dimension of space is increased by one and, at the same time, the Galilean invariance is promoted *in the critical dimension* to a full Poincaré invariance. For the supersymmetric case, it is not immediately obvious how much of the Poincaré superalgebra should be retained in the superbit dynamics. At first glance, one might hope to retain the complete superalgebra displayed in Eq. (2.7). We shall find that this may be too much symmetry for a satisfactory explanation of string, so we should ask how much supersymmetry can be given up while still retaining the full Galilean symmetry. It is clear from Eq. (2.7) that one cannot discard the Q supersymmetries without also discarding the R's. However, it *is* consistent to discard the R supersymmetries while retaining the Q's. This would correspond to the super-Galilei algebra $\mathcal{S}_1\mathcal{G}$ [14,15]. Retaining both dotted and undotted supersymmetries corresponds to the superalgebra $\mathcal{S}_2\mathcal{G}$.

III. DISCRETE SUPERSTRING IN LIGHT-CONE GAUGE

We start with the Green-Schwarz formulation [16,17] of closed superstring theory in light-cone gauge. The bit model is then motivated by first constructing a discretized version of string on the light cone. In the light-cone gauge the world-sheet reparametrization invariance is fixed by choosing $x^+ = \tau$ and choosing σ such that the "+" component of momentum density is constant, $\mathcal{P}^+ = T_0$ with T_0 the string rest tension.

A. II-B

The light-cone world-sheet variables of type II-B superstring theory in $D = 10$ space-time dimensions include, in addition to the coordinates and momenta, the right- and left-moving Majorana-Weyl spinors S^a and $\tilde{S}^a$, transforming in equivalent representations of SO(8), and obeying anticommutation relations

$$\{S^a(\sigma), S^b(\sigma')\} = \delta^{ab}\delta(\sigma - \sigma'),$$
$$\{\tilde{S}^a(\sigma), \tilde{S}^b(\sigma')\} = \delta^{ab}\delta(\sigma - \sigma'). \quad (3.1)$$

Here the indices refer to the undotted indices of a fixed chirality ($\Gamma_{11} = +1$) as described in the previous section, and take the values $1, \ldots, 8$. The light-cone Hamiltonian is given by

$$H = P^- = \frac{1}{2T_0}\int_0^{P^+/T_0} d\sigma[(\mathcal{P}^i)^2 + T_0^2(x^{i\prime})^2 - iT_0 S^a S^{a\prime} + iT_0\tilde{S}^a\tilde{S}^{a\prime}]. \quad (3.2)$$

The indices i, j, k are used for the vector representation of SO(8), and the indices a, b, c, d and $\dot{a}, \dot{b}, \dot{c}, \dot{d}$ are used for the two inequivalent spinor representations of SO(8). The supercharges $Q^a, \tilde{Q}^a, R^{\dot{a}}, \tilde{R}^{\dot{a}}$ generating the $N = 2$ supersymmetry carry both dotted and undotted indices. The undotted ones are essentially the zero modes of the spinor variables:

$$Q^a = \sqrt{T_0}\int_0^{P^+/T_0} d\sigma\, S^a(\sigma),$$
$$\tilde{Q}^a = \sqrt{T_0}\int_0^{P^+/T_0} d\sigma\, \tilde{S}^a(\sigma). \quad (3.3)$$

The dotted components are more complicated bilinears in the spinor and coordinate variables:

$$R^{\dot{a}} = \frac{1}{2\sqrt{T_0}}\int_0^{P^+/T_0} d\sigma\, \gamma^{ib\dot{a}} S^b(\sigma)(\mathcal{P}^i - T_0 x^{\prime i}),$$
$$\tilde{R}^{\dot{a}} = \frac{1}{2\sqrt{T_0}}\int_0^{P^+/T_0} d\sigma\, \gamma^{ib\dot{a}} \tilde{S}^b(\sigma)(\mathcal{P}^i + T_0 x^{\prime i}). \quad (3.4)$$

Consider first how the $N = 2$ superalgebra is realized. It is immediate that all of the Q's and R's anticommute with all the $\tilde{Q}$'s and $\tilde{R}$'s. It follows from (3.1), the canonical commutator of $\mathcal{P}^i$ and x^j, and periodicity of x in σ that

$$\{Q^a, Q^b\} = P^+\delta^{ab},$$
$$\{Q^a, R^{\dot{b}}\} = \frac{1}{2}\mathbf{P}\cdot\gamma^{a\dot{b}}, \quad (3.5)$$

and similarly for the left-moving supercharges $\tilde{Q}, \tilde{R}$. To compute the algebra of the R supercharges we will need the following identities for the SO(8) γ matrices:

$$\gamma^{i\dot{a}c}\gamma^{j\dot{b}c} + (i \leftrightarrow j) = 2\delta^{ij}\delta^{\dot{a}\dot{b}},$$
$$\gamma^{i\dot{a}c}\gamma^{j\dot{a}d} + (i \leftrightarrow j) = 2\delta^{ij}\delta^{cd},$$
$$\gamma^{i\dot{a}c}\gamma^{i\dot{b}d} + (c \leftrightarrow d) = 2\delta^{cd}\delta^{\dot{a}\dot{b}},$$
$$\gamma^{i\dot{a}c}\gamma^{i\dot{b}c} + (\dot{a} \leftrightarrow \dot{b}) = 2\delta^{cd}\delta^{\dot{a}\dot{b}}. \quad (3.6)$$

The top two implement the Clifford algebra, while the bottom two are Fierz identities which follow from the first two by the special triality property of SO(8). We then find that the R supercharges satisfy

$$\{R^{\dot{a}}, R^{\dot{b}}\} = \frac{\delta^{\dot{a}\dot{b}}}{4T_0}\int_0^{P^+/T_0} d\sigma[(\mathcal{P}^i - T_0 x^{i\prime})^2 - 2iT_0 S^a S^{a\prime}] = \delta^{\dot{a}\dot{b}} P_R^-,$$
$$\{\tilde{R}^{\dot{a}}, \tilde{R}^{\dot{b}}\} = \frac{\delta^{\dot{a}\dot{b}}}{4T_0}\int_0^{P^+/T_0} d\sigma[(\mathcal{P}^i + T_0 x^{i\prime})^2 + 2iT_0 \tilde{S}^a \tilde{S}^{a\prime}] = \delta^{\dot{a}\dot{b}} P_L^-, \quad (3.7)$$

where P_R^-, P_L^- are the right- and left-moving parts of the light-cone Hamiltonian respectively, $P^- = P_R^- + P_L^-$.

The above anticommutators show that the right- and left-moving supercharges satisfy independent $N = 1$ $\mathcal{S}_2\mathcal{G}$ algebras, but with *different* Hamiltonians. Thus an $N = 2$ $\mathcal{S}_2\mathcal{G}$ algebra strictly holds *only on the subspace of states satisfying the constraint* $P_R^- = P_L^- (= P^-/2)$. This is just the $L_0 = \tilde{L}_0$ constraint which is indeed required in closed string theory. The first issue we must settle in discretizing the world-sheet coordinate σ is how to treat this constraint. To do this we note that $L_0 - \tilde{L}_0$ is the generator of translations in σ. The states on which it vanishes are precisely those invariant under this translation. When σ is replaced by a discrete label k, the translation becomes discrete: $k \to k+1$. Invariance under this discrete transformation is just a cyclic symmetry requirement on the string wave function:

$$\Psi(x_1\theta_1, x_2\theta_2, \ldots, x_M\theta_M) = \Psi(x_2\theta_2, \ldots, x_M\theta_M, x_1\theta_1), \tag{3.8}$$

where θ_k are the Grassmann odd spinor supercoordinates, defined for type II-B superstring by

$$\theta^a = \frac{1}{\sqrt{2}}(S^a - i\tilde{S}^a). \tag{3.9}$$

In our bit models this symmetry will be an automatic consequence of the identity of string bits and need not be explicitly imposed. Since it is a discrete symmetry, it will not have an infinitesimal interpretation away from the actual continuum limit, so an analog to the constraint $L_0 = \tilde{L}_0$ will not exist in the discretized theory, but will naturally arise in the continuum limit. From this consideration, we see that we need not *and probably should not* require the full $N=2$ supersymmetry in our bit model. The $N=1$ supersymmetry generators $(Q+\tilde{Q})/\sqrt{2}$ and $(R+\tilde{R})/\sqrt{2}$ satisfy the Poincaré superalgebra without constraint, and we might hope to retain this much supersymmetry in the discretized theory.

To set up a model of discrete superstring, we assume that P^+ comes in discrete units m, $P^+ = Mm$ where M is a large integer counting the number of bits in a string. The parameter labeling points on the string thus becomes discrete $\sigma \to km/T_0$, where k is an integer taking the values $1, \ldots, M$. The transverse coordinates are $\mathbf{x}_k$ corresponding to $\mathbf{x}(km/T_0)$ and the conjugate momenta are $\mathbf{p}_k$ corresponding to $m\mathcal{P}(km/T_0)/T_0$. The spinor variables are S_k^a and $\tilde{S}_k^a$ corresponding to $\sqrt{m/T_0}S(km/T_0)$ and $\sqrt{m/T_0}\tilde{S}(km/T_0)$, respectively. The nonvanishing (anti)commutators among these discretely labeled variables are

$$[x_k^i, p_l^j] = i\delta^{ij}\delta_{kl}, \{S_k^a, S_l^b\} = \delta^{ab}\delta_{kl}, \ \{\tilde{S}_k^a, \tilde{S}_l^b\} = \delta^{ab}\delta_{kl}. \tag{3.10}$$

The undotted supercharges should obviously be given by

$$Q^a = \sqrt{m}\sum_{k=1}^{M} S_k^a, \quad \tilde{Q}^a = \sqrt{m}\sum_{k=1}^{M}\tilde{S}_k^a, \tag{3.11}$$

and their algebra is clearly

$$\{Q^a, Q^b\} = mM\delta^{ab}, \{\tilde{Q}^a, \tilde{Q}^b\} = mM\delta^{ab}, \{Q^a, \tilde{Q}^b\} = 0. \tag{3.12}$$

We can also easily guess a discretized form for the R's:

$$\begin{aligned} R^{\dot{a}} &= \frac{1}{2\sqrt{m}}\sum_{k=1}^{M}\gamma^{ib\dot{a}}S_k^b(p_k^i - T_0[x_{k+1}^i - x_k^i]), \\ \tilde{R}^{\dot{a}} &= \frac{1}{2\sqrt{m}}\sum_{k=1}^{M}\gamma^{ib\dot{a}}\tilde{S}_k^b(p_k^i + T_0[x_{k+1}^i - x_k^i]). \end{aligned} \tag{3.13}$$

The anticommutators of $Q, \tilde{Q}$ with $R, \tilde{R}$ are then exactly of the correct form:

$$\begin{aligned} \{Q^a, R^{\dot{b}}\} &= \frac{1}{2}\gamma^{a\dot{b}}\cdot\mathbf{P}, \quad \{\tilde{Q}^a, \tilde{R}^{\dot{b}}\} = \frac{1}{2}\gamma^{a\dot{b}}\cdot\mathbf{P}, \\ \{Q^a, \tilde{R}^{\dot{b}}\} &= \{\tilde{Q}^a, R^{\dot{b}}\} = 0, \end{aligned} \tag{3.14}$$

where $\mathbf{P} = \sum_k \mathbf{p}_k$ is the total transverse momentum carried by the discretized string. However, R fails to anticommute with $\tilde{R}$, breaking the $N=2$ supersymmetry[1]:

$$\{R^{\dot{a}}, \tilde{R}^{\dot{b}}\} = -\frac{iT_0}{4m}\sum_{k=1}^{M}\gamma^{c\dot{a}}\cdot\gamma^{d\dot{b}}S_k^c(\tilde{S}_{k+1}^d + \tilde{S}_{k-1}^d - 2\tilde{S}_k^d).$$

Using the identities of the SO(8) gamma matrices (3.6) we then derive the rest of the superalgebra:

$$\begin{aligned} \{R^{\dot{a}}, \tilde{R}^{\dot{b}}\} + (\dot{a} \leftrightarrow \dot{b}) &= -\frac{iT_0}{2m}\sum_{k=1}^{M}\delta^{\dot{a}\dot{b}}S_k^c(\tilde{S}_{k+1}^c + \tilde{S}_{k-1}^c - 2\tilde{S}_k^c), \\ \{R^{\dot{a}}, R^{\dot{b}}\} &= \frac{1}{4m}\sum_{k=1}^{M}\delta^{\dot{a}\dot{b}}(\mathbf{p}_k - T_0[\mathbf{x}_{k+1} - \mathbf{x}_k])^2 - \frac{iT_0}{2m}\sum_{k=1}^{M}\delta^{\dot{a}\dot{b}}S_k^cS_{k+1}^c, \\ \{\tilde{R}^{\dot{a}}, \tilde{R}^{\dot{b}}\} &= \frac{1}{4m}\sum_{k=1}^{M}\delta^{\dot{a}\dot{b}}(\mathbf{p}_k + T_0[\mathbf{x}_{k+1} - \mathbf{x}_k])^2 + \frac{iT_0}{2m}\sum_{k=1}^{M}\delta^{\dot{a}\dot{b}}\tilde{S}_k^c\tilde{S}_{k+1}^c. \end{aligned} \tag{3.15}$$

[1] Actually it is not hard to modify these definitions so that $\{R, \tilde{R}\} = 0$: simply replace $[x_{k+1}^i - x_k^i]$ by $[x_k^i - x_{k-1}^i]$ in one (*not both*) of the R's. But one would still not get the full $N=2$ algebra because of the constraint problem mentioned earlier. Even worse, we shall see that the resolution of the notorious lattice fermion doubling problem, which is automatic for our choice, would fail for this alternative.

Although we have lost the full $N = 2$ supersymmetry, there remains an $N = 1$ supersymmetry generated by $Q_+ = (Q + \tilde{Q})/\sqrt{2}$ and $R_+ = (R + \tilde{R})/\sqrt{2}$. We easily read off the superalgebra

$$\begin{aligned}
\{Q_+^a, Q_+^b\} &= mM\delta^{ab}, \ \{Q_+^a, R_+^{\dot{b}}\} = \frac{1}{2}\gamma^{a\dot{b}} \cdot \mathbf{P}, \\
\{R_+^{\dot{a}}, R_+^{\dot{b}}\} &= \frac{1}{4m}\sum_{k=1}^{M} \delta^{\dot{a}\dot{b}}(\mathbf{p}_k^2 + T_0^2[\mathbf{x}_{k+1} - \mathbf{x}_k]^2) + \frac{iT_0}{4m}\sum_{k=1}^{M} \delta^{\dot{a}\dot{b}} \tilde{S}_k^c \tilde{S}_{k+1}^c \\
&\quad - \frac{iT_0}{4m}\sum_{k=1}^{M} \delta^{\dot{a}\dot{b}} S_k^c S_{k+1}^c - \frac{iT_0}{4m}\sum_{k=1}^{M} \delta^{\dot{a}\dot{b}} S_k^c(\tilde{S}_{k+1}^c + \tilde{S}_{k-1}^c - 2\tilde{S}_k^c).
\end{aligned} \tag{3.16}$$

The last of these equations gives the Hamiltonian

$$H = \frac{1}{2m}\sum_{k=1}^{M}\left[\mathbf{p}_k^2 + T_0^2(\mathbf{x}_{k+1} - \mathbf{x}_k)^2 - iT_0 S_k^c S_{k+1}^c + iT_0 \tilde{S}_k^c \tilde{S}_{k+1}^c - iT_0 S_k^c(\tilde{S}_{k+1}^c + \tilde{S}_{k-1}^c - 2\tilde{S}_k^c)\right]. \tag{3.17}$$

Note that in the continuum limit the last term is formally subdominant to the others since it involves a second difference. Thus the Green-Schwarz Hamiltonian (3.2) is regained in the continuum. This last term, which arises from the nonzero anticommutator of R with $\tilde{R}$ is in fact extremely valuable. It breaks world-sheet chirality in precisely the way needed (in the manner of Wilson) to remove the annoying fermion doubling problem from the discretized theory. Since the Hamiltonian is a bilinear form in canonical variables, it is easy to confirm this through explicit diagonalization of H. As always with quadratic Hamiltonians this is done by finding eigenoperators under commutation with H. Applying this linear operation to each of the dynamical variables, we find

$$\begin{aligned}
[H, \mathbf{x}_k] &= -i\frac{\mathbf{p}_k}{m}, \quad [H, \mathbf{p}_k] = -i\frac{T_0^2}{m}(\mathbf{x}_{k+1} + \mathbf{x}_{k-1} - 2\mathbf{x}_k), \\
[H, S_k^a] &= \frac{-iT_0}{2m}(S_{k-1}^a - S_{k+1}^a + 2\tilde{S}_k^a - \tilde{S}_{k+1}^a - \tilde{S}_{k-1}^a), \\
[H, \tilde{S}_k^a] &= \frac{+iT_0}{2m}(\tilde{S}_{k-1}^a - \tilde{S}_{k+1}^a + 2S_k^a - S_{k+1}^a - S_{k-1}^a).
\end{aligned} \tag{3.18}$$

To diagonalize these relations we first pass to Fourier modes

$$\begin{aligned}
\mathbf{x}_k &= \frac{1}{\sqrt{M}}\sum_{n=0}^{M-1} \hat{\mathbf{x}}_n e^{-2\pi ink/M}, \quad \mathbf{p} = \frac{1}{\sqrt{M}}\sum_{n=0}^{M-1} \hat{\mathbf{p}}_n e^{-2\pi ink/M} \\
S_k^a &= \frac{1}{\sqrt{M}}\sum_{n=0}^{M-1} \hat{S}_n^a e^{-2\pi ink/M}, \quad \tilde{S}_k^a = \frac{1}{\sqrt{M}}\sum_{n=0}^{M-1} \hat{\tilde{S}}{}_n^a e^{-2\pi ink/M},
\end{aligned} \tag{3.19}$$

with the inverse relations

$$\begin{aligned}
\hat{\mathbf{x}}_n &= \frac{1}{\sqrt{M}}\sum_{k=1}^{M} \mathbf{x}_k e^{+2\pi ink/M}, \quad \hat{\mathbf{p}}_n = \frac{1}{\sqrt{M}}\sum_{k=1}^{M} \mathbf{p}_k e^{+2\pi ink/M}, \\
\hat{S}_n^a &= \frac{1}{\sqrt{M}}\sum_{k=1}^{M} S_k^a e^{+2\pi ink/M}, \quad \hat{\tilde{S}}{}_n^a = \frac{1}{\sqrt{M}}\sum_{k=1}^{M} \tilde{S}_k^a e^{+2\pi ink/M}.
\end{aligned} \tag{3.20}$$

One then finds

$$\begin{aligned}
[H, \hat{\mathbf{x}}_n] &= -i\frac{\hat{\mathbf{p}}_n}{m}, \quad [H, \hat{\mathbf{p}}_n] = 4i\frac{T_0^2}{m}\sin^2\frac{\pi n}{M}\hat{\mathbf{x}}_n, \\
[H, \hat{S}_n^a] &= \frac{-iT_0}{2m}\left(2i\sin\frac{2\pi n}{M}\hat{S}_n^a + 4\sin^2\frac{\pi n}{M}\hat{\tilde{S}}{}_n^a\right), \\
[H, \hat{\tilde{S}}{}_n^a] &= \frac{+iT_0}{2m}\left(2i\sin\frac{2\pi n}{M}\hat{\tilde{S}}{}_n^a + 4\sin^2\frac{\pi n}{M}\hat{S}_n^a\right).
\end{aligned} \tag{3.21}$$

We easily identify the energy lowering operators

$$\begin{aligned}
\mathbf{A}_n &= \frac{1}{\sqrt{2\omega_n}}(\hat{\mathbf{p}}_n - i\omega_n\hat{\mathbf{x}}_n), \\
B_n^a &= \sin\frac{n\pi}{2M}\hat{S}_n^a + i\cos\frac{n\pi}{2M}\hat{\tilde{S}}{}_n^a,
\end{aligned} \tag{3.22}$$

each of which lowers the energy by the amount ω_n/m with $\omega_n \equiv 2T_0\sin(n\pi/M)$. Of course the Hermitian conjugates of these operators are energy raising operators, each of which increases the energy by the same amount.

In the limit $M \to \infty$, with mM fixed, finite energy modes occur for n and $M-n$ finite. These correspond to left- and right-moving modes, respectively, precisely as required for a continuous closed string. The excitation energies for these modes are given by

$$E_n = \frac{2T_0}{m}\sin\frac{n\pi}{M} = \frac{2T_0}{m}\sin\frac{(M-n)\pi}{M}, \tag{3.23}$$

which in the continuum limit with n (or $M-n$) finite approach $2n\pi T_0/P^+$ [or $2(M-n)\pi T_0/P^+$]. Had the $S\tilde{S}$ coupling term been absent there would have been additional lower energy modes with $n - M/2$ infinite.[2]

The ground state of our discretized string is the one annihilated by all of the energy lowering operators. The ground-state energy turns out to be exactly zero. (Implying, of course, the absence of tachyons in the continuum superstring mass spectrum.) The part of $H = H_{xp} + H_{S\tilde{S}}$ involving coordinates and momenta, which just describes a system of harmonic oscillators, applied to the ground state gives half the sum of all the mode excitation energies:

$$H_{xp}|G\rangle = |G\rangle \frac{8}{2m}\sum_{n=1}^{M-1}\omega_n = |G\rangle \frac{8T_0}{m}\sum_{n=1}^{M-1}\sin\frac{n\pi}{M}. \tag{3.24}$$

The "8" appearing here is just the transverse dimension $D-2$ for ten-dimensional space-time. The part of H involving the spinors gives exactly the negative of this, with the "8" in this case being the 8 values of the spinor index a, so

$$H|G\rangle = |G\rangle\left(\frac{8T_0}{m}\sum_{n=1}^{M-1}\sin\frac{n\pi}{M} - \frac{8T_0}{m}\sum_{n=1}^{M-1}\sin\frac{n\pi}{M}\right)$$
$$= 0. \tag{3.25}$$

We can summarize the solution of our discretized superstring model by quoting the Hamiltonian in terms of raising and lowering operators:

$$H = \frac{\mathbf{P}^2}{2mM} + \frac{2T_0}{m}\sum_{n=1}^{M-1}\sin\frac{n\pi}{M}(\mathbf{A}_n^\dagger \cdot \mathbf{A}_n + B_n^{a\dagger}B_n^a), \tag{3.26}$$

where $\mathbf{P}$ is the total momentum. For completeness we also quote the relation of the dynamical variables to raising and lowering operators:

$$\hat{\mathbf{p}}_n = \sqrt{\frac{\omega_n}{2}}(\mathbf{A}_n + \mathbf{A}_{M-n}^\dagger),$$
$$\hat{\mathbf{x}}_n = \frac{i}{\sqrt{2\omega_n}}(\mathbf{A}_n - \mathbf{A}_{M-n}^\dagger),$$
$$\hat{S}_n^a = B_n^a\sin\frac{n\pi}{2M} + B_{M-n}^{a\dagger}\cos\frac{n\pi}{2M},$$
$$\hat{\tilde{S}}_n^a = -i\left(B_n^a\cos\frac{n\pi}{2M} - B_{M-n}^{a\dagger}\sin\frac{n\pi}{2M}\right). \tag{3.27}$$

The discrete II-B superstring model we have presented is the first step toward a string bit model. Its characteristic feature is that it has replaced a closed string by a system of M string bits, which are *ordered around a loop.* The interaction among string bits only exists between nearest neighbors on this loop. Thus it is not quite a standard many body system which would allow interactions between all pairs of particles, and might even include three or more body interactions. It is very well known [3,13] how this peculiar pattern of interactions can arise in a true many body system of particles described by $N_c \times N_c$ matrix creation operators in 't Hooft's $N_c \to \infty$ limit [6]. We shall turn to this in the next section.

A troubling feature of the bit-bit interaction from the string bit point of view is its long-range harmonic form, evident in the Hamiltonian (3.17). However, it is clear that, as with all discretizations, the limit that leads to continuous string should occur for a wide class of interactions, including ones that are short range. Short-range potentials would of course allow a discrete string to dissociate into string bits. All that is necessary to veto dissociation in the superstring continuum limit is that the dissociation energy be of $O(1/m)$ as $M \to \infty$ with mM fixed.

There are many ways we could introduce a short-range nonharmonic dynamics into our model, but it is desirable to retain as much of the supersymmetric structure as possible. One approach is to introduce modifications into $R, \tilde{R}$ and then define the Hamiltonian in terms of these. The simplest possibility is to replace T_0 in (3.13) by a scalar function $\mathcal{V}(|\mathbf{x}_{k+1} - \mathbf{x}_k|)$. This has the virtue of leaving the anticommutator $\{Q_+^a, R_+^{\dot{b}}\}$ of the superalgebra undisturbed. We can also allow a generalization of the spinor structure of the interaction terms in (3.13) compatible with SO(8) invariance and the preservation of $\{Q_+^a, R_+^{\dot{b}}\}$. For definiteness in this paper we shall forego such generalizations and restrict to the following form for R_+:

$$R_+^{\dot{a}} = \frac{1}{2\sqrt{2m}}\sum_{k=1}^{M}\gamma^{b\dot{a}} \cdot [(S_k^b + \tilde{S}_k^b)\mathbf{p}_k$$
$$-(S_k^b - \tilde{S}_k^b)(\mathbf{x}_{k+1} - \mathbf{x}_k)\mathcal{V}(|\mathbf{x}_{k+1} - \mathbf{x}_k|)]. \tag{3.28}$$

Unfortunately, with $\mathcal{V}$ not a constant $\{R_+^{\dot{a}}, R_+^{\dot{b}}\}$ is no longer proportional to $\delta^{\dot{a}\dot{b}}$, so that part of the superalgebra will be lost. In this situation we propose to define the Hamiltonian by the positive SO(8)-invariant bilinear form

$$H \equiv \frac{1}{4}\sum_{\dot{a}=1}^{8}\{R_+^{\dot{a}}, R_+^{\dot{a}}\}. \tag{3.29}$$

[2]The other resolution of the doubling problem (in the manner of Kogut and Susskind) in which these extra modes are accepted as part of the physical spectrum is not satisfactory here because they would include both integer and half integer modes depending on whether M was even or odd. The half integer modes would ruin the superstring interpretation.

This expression automatically commutes with the Q_+^a so the $\mathcal{S}_1\mathcal{G}$ supersymmetry is preserved. Instead of being the square of a Grassmann odd operator, as would be a consequence of $\mathcal{S}_2\mathcal{G}$ supersymmetry, H has the somewhat weaker property of being a sum of squares of eight odd operators. By maintaining this structure we hope to make more likely the recovery of the full Poincaré supersymmetry in the stringy physics. The structure also naturally guarantees that the energy spectrum is bounded from below. For the special case $\mathcal{V} = T_0$, H reduces to the original form. Thus we can assert that a satisfactory free superstring limit will exist provided $\mathcal{V}$ behaves as a nonzero constant as far as low energy collective excitations are concerned.

B. Not II-B

Type II-B superstring studied in the previous section was particularly neat because of the symmetry between left- and right-moving waves on a string. This circumstance allowed a very appealing resolution of the fermion doubling problem, because one can form the SO(8) invariant coupling term $S\tilde{S}$ which raised the energy of the unwanted extra low-lying modes with mode number near $M/2$. When this left-right symmetry is absent, as in the type II-A and heterotic superstring theories, another scheme must be devised to get a satisfactory discretization.

For type II-A superstring the right- and left-moving spinors $S, \tilde{S}$ transform under inequivalent representations of SO(8). Consequently, the coupling term $S\tilde{S}$ is not SO(8) invariant. Therefore one must break the transverse space rotational symmetry in order to get rid of the fermion doubling. In fact, defining canonical spin variables requires a decomposition of the above spin variables with respect to an SU(4)×U(1) subgroup of SO(8),

$$\theta^A = \frac{1}{\sqrt{2}}(S^A + iS^{A+4}), \quad \pi_A = \frac{1}{\sqrt{2}}(S^A - iS^{A+4}), \tag{3.30}$$

and similarly for the left movers. The superscript $A = 1, \ldots, 4$ labels a $\mathbf{4}$ of SU(4), and the subscript A labels a $\bar{\mathbf{4}}$. The decomposition of the representations is

$$\mathbf{8}_s \to \mathbf{4}_{1/2} + \bar{\mathbf{4}}_{-1/2}, \quad \mathbf{8}_c \to \mathbf{4}_{-1/2} + \bar{\mathbf{4}}_{1/2}, \tag{3.31}$$

where $\mathbf{8}_s, \mathbf{8}_c$ are the two inequivalent spinor representations of SO(8). (For a detailed discussion see Chap. 11 of Ref. [17].) Any coupling between the two kinds of spinor would have to break either SU(4) or U(1). One can think of the SU(4)$\sim$SO(6) as the group of rotations in six "internal" dimensions, and the U(1) as the helicity in ordinary four-dimensional space-time. In this view it is preferable to preserve the U(1) symmetry at the discrete level, even at the cost of breaking the SU(4).

For heterotic superstring the situation seems even more complicated, since it has *only* right-moving spinor waves. However, as we shall soon see there may be a more elegant, SO(8) invariant, method to avoid the fermion doubling problem. This method may also be applied to type II-A superstring as an alternative to breaking the SO(8) symmetry.

We start by reminding the reader how the doubling problem arises. Consider the part of the Hamiltonian (3.17) involving only the S spinors,

$$H_S = -iT_0 S_k^a S_{k+1}^a, \tag{3.32}$$

which is all we would have in the heterotic case where $\tilde{S}$ is absent. The Fourier modes $\hat{S}_n$ then satisfy

$$[H_S, \hat{S}_n^a] = \frac{T_0}{m}\sin\frac{2\pi n}{M}\hat{S}_n^a = \frac{2T_0}{m}\sin\frac{\pi n}{M}\cos\frac{\pi n}{M}\hat{S}_n^a. \tag{3.33}$$

We see that the excitation energies are of $O(1/M)$ not only for the desired cases of finite n, $M-n$, but also for finite $n - M/2$. For $n < M/2, \hat{S}_n^a$ raises the energy and is multiplied in its contribution to S_k^a by the time dependent phase $\exp(+iE_nt - 2\pi ink/M)$ with $E_n > 0$. For $M \to \infty$ with finite n this corresponds to a right-moving wave. But in this limit with finite $(M/2) - n$, the unwanted "double mode" excitation is a left-moving wave. Moreover, if M is odd, it acts like a half-integer (antiperiodic) left-moving mode. For $M/2 < n < M$, E_n is negative (the modes are energy lowering operators) and those with finite $M - n$ are right movers for a continuous closed string whereas those with finite $n - M/2$ are left movers. Clearly the Kogut-Susskind resolution of the doubling problem, which is to use the doubled modes as a part of the observable physical modes, would wreck the "heterotic" nature of the model: a continuous closed string would end up with both left- and right-moving spinor modes. Thus the Wilson alternative which worked in the II-B case must somehow be used here.

At the moment, the only way we see to do this is to reintroduce $\tilde{S}$ as an auxiliary field at the discretized level in such a way that it resolves the doubling problem but does not propagate in the continuum limit. Although we shall not try to develop the type II-A and heterotic superbit models in this paper, we illustrate how this might work by examining a spinor model with left-right asymmetry, described by the Hamiltonian

$$\begin{aligned} H_{S\tilde{S}} = &-iT_0 S_k^a S_{k+1}^a + i\eta T_0 \tilde{S}_k^a \tilde{S}_{k+1}^a \\ &-i\xi T_0 S_k^a(\tilde{S}_{k+1}^a + \tilde{S}_{k-1}^a - 2\tilde{S}_k^a). \end{aligned} \tag{3.34}$$

Passing to Fourier modes we have

$$\begin{aligned} [H_{S\tilde{S}}, \hat{S}_n^a] &= \frac{-2iT_0}{m}\sin\frac{n\pi}{M}\left(i\cos\frac{\pi n}{M}\hat{S}_n^a + \xi\sin\frac{\pi n}{M}\hat{\tilde{S}}_n^a\right), \\ [H_{S\tilde{S}}, \hat{\tilde{S}}_n^a] &= \frac{+2iT_0}{m}\sin\frac{n\pi}{M}\left(i\eta\cos\frac{n\pi}{M}\hat{\tilde{S}}_n^a + \xi\sin\frac{n\pi}{M}\hat{S}_n^a\right). \end{aligned} \tag{3.35}$$

We find that $B_n^a = (\hat{S}_n^a + \alpha\hat{\tilde{S}}_n^a)/\sqrt{1+|\alpha|^2}$ is an energy lowering operator provided

$$\alpha = i\left[\frac{1+\eta}{2\xi}\cot\frac{n\pi}{M} + \sqrt{1 + \left(\frac{1+\eta}{2\xi}\right)^2\cot^2\frac{n\pi}{M}}\right],$$

in which case it lowers the energy by an amount

$$E_n = \frac{\omega_n}{m}\left[-\frac{1-\eta}{2}\cos\frac{n\pi}{M} + \sqrt{\xi^2\sin^2\frac{n\pi}{M} + \left(\frac{1+\eta}{2}\right)^2\cos^2\frac{n\pi}{M}}\right]. \qquad (3.36)$$

As long as $\xi \neq 0$ and is real and $\eta > 0$ there are no low energy modes other than the ones for finite n and finite $M-n$, and the doubling problem is avoided. We might as well simplify matters and take $\xi = (1+\eta)/2$. Then

$$\alpha = i\left[\cot\frac{n\pi}{M} + \csc\frac{n\pi}{M}\right]$$

and

$$E_n = \frac{\omega_n}{m}\left[\frac{1+\eta}{2} - \frac{1-\eta}{2}\cos\frac{n\pi}{M}\right]. \qquad (3.37)$$

The energy lowering operators are then simply $B_n^a = \sin\frac{n\pi}{2M}\hat{S}_n^a + i\cos\frac{n\pi}{2M}\hat{\tilde{S}}_n^a$. As $M \to \infty$, the left-moving modes (finite n) have energy $\eta\omega_n/m$ whereas the right movers (finite $M-n$) have energy ω_n/m, the former a factor of η times the latter. As $\eta \to \infty$, the left-moving waves gain infinite energy and would disappear from the spectrum. The discrete theory could have η finite but depend on m in a way that blows up as $m \to 0$.

Extending this trick to type II-A is straightforward. Simply introduce two additional oppositely moving spinor variables, with a Hamiltonian similar to (3.34), except that the new physical spinor is left moving and the new auxiliary spinor is right moving. A type II-A superstring is thus constructed as sort of a combination of right-moving and left-moving heterotic superstrings.

IV. SECOND-QUANTIZED SUPERSTRING BITS

As we saw in the previous section, discrete light-cone superstring seems to be made up of nonrelativistic interacting superparticles carrying spin degrees of freedom and moving in $[(D-2)+1]$-dimensional space-time. If this picture is taken seriously, a superstring is really a composite object, namely, a long closed polymer of infinitesimal string bits. Each of these bits is described by dynamical variables given by its position $\mathbf{x}_k$, its momentum $\mathbf{p}_k$, and spin variables which can be represented in terms of anticommuting Grassmann variables θ_k^a, and their conjugates $\pi_k^a = d/d\theta_k^a$. The possible states of a noninteracting (free) superstring are then given by those of an M-bit polymer, represented by wave functions

$$\Psi(\mathbf{x}_1\theta_1^a, \mathbf{x}_2\theta_2^a, \ldots, \mathbf{x}_M\theta_M^a) \qquad (4.1)$$

subject to the constraint of cyclic symmetry (3.8).

A. $1/N_c$ Expansion and polymers

According to the Hamiltonian for a discrete light-cone free superstring (3.17) each bit interacts only with its nearest neighbors. In order to achieve this nearest neighbor interaction structure in a second-quantized formulation it is necessary to introduce a "color" degree of freedom. The creation operators for superstring bits are then $N_c \times N_c$ matrices transforming in the adjoint representation of $U(N_c)$:

$$\Phi^\dagger(\mathbf{x},\theta)_\alpha^\beta = \sum_{n=0}^{D-2}\frac{1}{n!}\phi^\dagger_{a_1\cdots a_n}(\mathbf{x})_\alpha^\beta\theta^{a_1}\cdots\theta^{a_n}, \qquad (4.2)$$

where the $\phi^\dagger$'s are completely antisymmetric in their spinor indices $a_1\cdots a_n$, and the matrix labels α,β run from 1 to N_c. $\phi^\dagger$ creates a boson or fermion according to whether the number of indices n is even or odd, respectively. The upper limit on n is taken to be $D-2$ because supersymmetry requires the number of components in the spinor θ^a to equal the number of transverse coordinates. This is of course possible only for $D = 4, 6$, and 10. For $D = 10$ (or $D = 4$) there are all together 256 (or 4) components of $\phi^\dagger$, 128 (or 2) bosonic and 128 (or 2) fermionic. The supercreation operator $\Phi^\dagger$ will always be Grassmann even and enjoy commutation relations. The $\phi^\dagger$'s will of course satisfy the graded bracket relations

$$\begin{aligned}[\phi_{a_1\cdots a_n}(\mathbf{x})_\alpha^\beta, \phi^\dagger_{b_1\cdots b_m}(\mathbf{y})_\gamma^\delta]_\pm &\equiv \phi_{a_1\cdots a_n}(\mathbf{x})_\alpha^\beta\phi^\dagger_{b_1\cdots b_m}(\mathbf{y})_\gamma^\delta - (-)^{nm}\phi^\dagger_{b_1\cdots b_m}(\mathbf{y})_\gamma^\delta\phi_{a_1\cdots a_n}(\mathbf{x})_\alpha^\beta \\ &= \delta_{mn}\delta_\alpha^\delta\delta_\gamma^\beta\delta(\mathbf{x}-\mathbf{y})\sum_P(-)^P\delta_{a_1b_{P_1}}\cdots\delta_{a_nb_{P_n}}.\end{aligned} \qquad (4.3)$$

The string bit Fock space is built by acting on the vacuum state $|0\rangle$ with products of the various creation operators, and consists of states transforming in various representations of $U(N_c)$. Singlet states are created by products of traces of matrix products of the matrix creation operators. Each trace creates a closed chain of bits. We identify the discrete free single superstring wave function Ψ with the singlet state $|\Psi\rangle$ in the Fock space of string bits given by

$$|\Psi\rangle = \int\prod_{k=1}^M(d^{D-2}x_k d^{D-2}\theta_k)\mathrm{Tr}[\Phi^\dagger(\mathbf{x}_1\theta_1)\cdots\Phi^\dagger(\mathbf{x}_M\theta_M)]|0\rangle\Psi(\mathbf{x}_1\theta_1,\ldots,\mathbf{x}_M\theta_M). \qquad (4.4)$$

Note that once we agree that our state space is the bit Fock space, *the cyclic symmetry restriction* (3.8) *is an automatic consequence of the identity of string bits and the cyclic property of the trace.* Noninteracting multistring states would contain a product of several such trace structures.

The world-sheet dynamical variables described in the previous section are linear differential operators acting on the single superstring wave function Ψ. On our Fock space we seek to represent these operators as $\mathrm{U}(N_c)$ singlets, i.e., as traces of products of bit creation and annihilation operators. To find the bit Fock space representation of any such dynamical variable Ω, first write down the ket corresponding to $\Omega\Psi$. Then by an integration by parts transfer the differential operator to the creation operators appearing in the trace. Finally, one must identify the function of creation and annihilation operators that reproduces the action of this differential operator. Note that once we have a Fock space representation of an operator, it can act on *any* Fock state, not just singlets. Its action on the single superstring state (4.4) should, however, reproduce the action of the corresponding differential operator on the superstring wave function.

For single body operators like Q^a and the momentum dependent part of $R^{\dot{a}}$, which involve the supercoordinates of only one bit at a time, the Fock space representation is standard. Consider for simplicity only a single component matrix creation operator $a^\dagger(x)^\beta_\alpha$. If we denote the one body differential operator ω_1, its Fock space representation will be given by

$$\Omega_1 = \int dx\, \mathrm{Tr}[a^\dagger(x)\omega_1 a(x)]. \tag{4.5}$$

For two body operators describing *nearest neighbor* interactions, like the coordinate dependent part of $R^{\dot{a}}$, the identification of the Fock space representation is not exact. This is because the second-quantized operators will give interactions between *all pairs* of bits. We are therefore led to an approximate treatment using 't Hooft's $1/N_c$ expansion [6]. To illustrate how this works [13], consider again the simplified case of a single component matrix creation operator $a^\dagger(x)^\beta_\alpha$. Then the sort of two body operator we will need has the structure

$$\Omega_2 = \frac{1}{N_c}\int dx\, dy V(y-x)\mathrm{Tr}[a^\dagger(x)a^\dagger(y)a(y)a(x)]. \tag{4.6}$$

Applying this operator to the singlet Fock state $|M\rangle = \mathrm{Tr}[a^\dagger(x_1)\cdots a^\dagger(x_M)]|0\rangle$, we get, after one contraction,

$$\Omega_2|M\rangle = \frac{1}{N_c}\int dy \sum_k V(y-x_k)\mathrm{Tr}[a^\dagger(x_k)a^\dagger(y)a(y)a^\dagger(x_{k+1})\cdots a^\dagger(x_M)a^\dagger(x_1)\cdots a^\dagger(x_{k-1})]|0\rangle.$$

To continue the evaluation we note that it matters crucially which creation operator the last remaining $a(y)$ contracts against. The contraction with $a^\dagger(x_{k+1})$ produces a factor of $\sum_\alpha \delta^\alpha_\alpha = N_c$ which cancels the $1/N_c$ out front. All other contractions fail to provide a factor of N_c. Thus *in the limit* $N_c \to \infty$,

$$\Omega_2 \mathrm{Tr}[a^\dagger(x_1)\cdots a^\dagger(x_M)]|0\rangle \to \sum_{k=1}^{M} V(x_{k+1}-x_k)\mathrm{Tr}[a^\dagger(x_1)\cdots a^\dagger(x_M)]|0\rangle, \tag{4.7}$$

which is precisely the desired nearest neighbor interaction pattern. The non-nearest neighbor contractions change the trace structure of the state, giving $1/N_c$ times a state with two traces. Thus $1/N_c$ corrections allow a closed polymer chain to rearrange its bonds and transform to two closed polymer chains. In the continuous string limit, this is the origin of string-string interactions. For more details and examples, see Ref. [13].

There is actually some freedom in the choice of the second-quantized two body operator (4.6) which gives in the limit $N_c \to \infty$ a nearest neighbor interaction when acting on singlet states. One can add to Ω_2 terms with nonconsecutive annihilation operators, such as

$$\frac{1}{N_c}\int dx\, dy\, f(x-y) : \mathrm{Tr}[a^\dagger(x)a(y)a^\dagger(y)a(x)] : .$$

This term can be shown to give $1/N_c$ times a state with two traces, and is thus subleading in the limit $N_c \to \infty$. Such modifications will alter the general Fock space properties of Ω_2, but leave unchanged its action on the singlet states in the limit $N_c \to \infty$. In the next two sections we exploit these features of $N_c \to \infty$ to construct second-quantized expressions for the supercharges and Hamiltonian.

B. A superstring bit model in 2 + 1 dimensions

Before developing the (8+1)-dimensional bit model for real ten-dimensional superstring, let us first construct a simpler (2+1)-dimensional model. Long closed polymers in this lower dimensional model will *not* become four-dimensional relativistic strings in the continuum limit, since the full Poincaré algebra is realized only in ten dimensions. But there are two reasons to study this model.

(1) It contains all of the features contained in the higher dimensional model, but with fewer indices. Thus it serves as a pedagogical step toward the higher dimensional model.

(2) We would eventually like to describe strings propagating in four space-time dimensions plus six compactified space dimensions. This might be achieved by such a (2 + 1)-dimensional bit model with additional internal degrees of freedom.

Putting aside for a moment the issue of critical dimension, let us assume that the light-cone Hamiltonian (3.2), or its discrete version (3.17), describes a four-dimensional type II-B superstring. The variables $S, \tilde{S}$ then transform as two-dimensional spinor representations of SO(2). Since SO(3,1) spinors can be either Majorana or Weyl, but not both, S and $\tilde{S}$ are either real two component spinors, or complex one component spinors. For simplicity in matching with the higher dimensional model we shall use the former. The real four-dimensional Majorana representation of SO(3,1) then breaks as follows:

$$\mathbf{4} \to \mathbf{2} + \mathbf{2}.$$

The **2**'s are two-dimensional *reducible* spinor representations of SO(2), and will be labeled by dotted and undotted upper case Latin letters. Recall that lower case Latin indices are reserved for Weyl-restricted spinors, which are inconsistent with Majorana spinors in four dimensions. The two-dimensional representations reduce to the two one-dimensional irreducible representations corresponding to spin $\pm 1/2$ in the plane.

There are two ways to define canonical anticommuting coordinates:

$$\theta^A = \frac{1}{\sqrt{2}}(S^A - i\tilde{S}^A), \quad \pi^A = \frac{1}{\sqrt{2}}(S^A + i\tilde{S}^A), \tag{4.8}$$

or

$$\theta = \frac{1}{\sqrt{2}}(S^1 + iS^2), \quad \pi = \frac{1}{\sqrt{2}}(S^1 - iS^2), \tag{4.9}$$

and similarly for the left movers $\tilde{\theta}, \tilde{\pi}$. The first choice is analogous to the SO(8) preserving formalism (3.9), appropriate for describing type II-B superstring. It defines a pair of two component SO(2) spinors, and is thus termed the "SO(2) formalism." The second choice is analogous to the SU(4)×U(1) formalism (3.30), appropriate for describing both type II-A and II-B superstrings. It defines two complex Grassmann variables and their canonical conjugates, and is thus termed the "U(1) formalism." Note that since SO(2)∼U(1), the two formalisms are equivalent. This is not so in ten dimensions, since SU(4)×U(1)⊂SO(8).

1. SO(2) formalism

From Eq. (4.2) we see that the superstring bit creation operator in the SO(2) formalism is given by

$$\Phi^\dagger = \phi^\dagger + \phi_A^\dagger \theta^A + \frac{1}{2}\phi_{AB}^\dagger \theta^A \theta^B, \tag{4.10}$$

where the indices A, B run from 1 to 2. Consequently there are two bosonic and two fermionic degrees of freedom. Written in terms of the canonical supercoordinates (4.8), the first-quantized supercharges (3.11), (3.13) become

$$\begin{aligned}
Q^A &= \sqrt{\frac{m}{2}}\sum_{k=1}^{M}(\theta_k^A + \pi_k^A), \\
\tilde{Q}^A &= i\sqrt{\frac{m}{2}}\sum_{k=1}^{M}(\theta_k^A - \pi_k^A), \\
R^{\dot{A}} &= \frac{1}{2\sqrt{2m}}\sum_{k=1}^{M}\alpha^{iB\dot{A}}(\theta_k^B + \pi_k^B)(p_k^i - T_0[x_{k+1}^i - x_k^i]), \\
\tilde{R}^{\dot{A}} &= \frac{i}{2\sqrt{2m}}\sum_{k=1}^{M}\alpha^{iB\dot{A}}(\theta_k^B - \pi_k^B)(p_k^i + T_0[x_{k+1}^i - x_k^i]),
\end{aligned} \tag{4.11}$$

where the relevant matrix elements of α^i, as defined in Sec. II are

$$\begin{pmatrix} \alpha^1_{1\dot{1}} & \alpha^1_{1\dot{2}} \\ \alpha^1_{2\dot{1}} & \alpha^1_{2\dot{2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \begin{pmatrix} \alpha^2_{1\dot{1}} & \alpha^2_{1\dot{2}} \\ \alpha^2_{2\dot{1}} & \alpha^2_{2\dot{2}} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{4.12}$$

Recall that even though we have two sets of supercharges, each generating an independent $N = 1$ $\mathcal{S}_2\mathcal{G}$ superalgebra, together they *do not* generate an $N = 2$ superalgebra since R and $\tilde{R}$ fail to anticommute. The combinations $Q + \tilde{Q}$ and $R + \tilde{R}$ satisfy an $N = 1$ $\mathcal{S}_2\mathcal{G}$ superalgebra. Second quantization then follows the steps described in the previous subsection. It is simplest to first find the second-quantized operators associated with θ^A and $\pi^A = d/d\theta^A$. These must satisfy the properties

$$\begin{aligned}
[\Omega_{\theta^A}, \Phi^\dagger(\mathbf{x}\theta)] &= \theta^A \Phi^\dagger(\mathbf{x}\theta), \\
[\Omega_{\pi^A}, \Phi^\dagger(\mathbf{x}\theta)] &= -\frac{d}{d\theta^A}\Phi^\dagger(\mathbf{x}\theta),
\end{aligned} \tag{4.13}$$

where the "−" in the second requirement reflects the fact that a derivative acting on the first-quantized wave function is transferred to the second-quantized ket through an integration by parts. It is easy to confirm the identifications

$$\begin{aligned}
\Omega_{\theta^A} &= \int d\mathbf{x}\, \mathrm{Tr}[\phi^\dagger \phi_A - \phi_{A_1}^\dagger \phi_{AA_1}], \\
\Omega_{\pi^A} &= \int d\mathbf{x}\, \mathrm{Tr}[\phi_A^\dagger \phi - \phi_{AA_1}^\dagger \phi_{A_1}] = \Omega_{\theta^A}^\dagger .
\end{aligned} \tag{4.14}$$

To avoid confusion we will denote the Fock space representations of the supercharges Q and R by the script letters $\mathcal{Q}$ and $\mathcal{R}$. From (4.14) it immediately follows that the Fock space representation of the Q supercharges is given by

$$\begin{aligned}
\mathcal{Q}^A &= \sqrt{\frac{m}{2}}(\Omega_{\theta^A} + \Omega_{\pi^A}) \\
&= \sqrt{\frac{m}{2}}\int d\mathbf{x}\, \mathrm{Tr}[\phi^\dagger \phi_A - \phi_{A_1}^\dagger \phi_{AA_1} + \mathrm{H.c.}], \\
\tilde{\mathcal{Q}}^A &= i\sqrt{\frac{m}{2}}(\Omega_{\theta^A} - \Omega_{\pi^A}) \\
&= i\sqrt{\frac{m}{2}}\int d\mathbf{x}\, \mathrm{Tr}[\phi^\dagger \phi_A - \phi_{A_1}^\dagger \phi_{AA_1} - \mathrm{H.c.}].
\end{aligned} \tag{4.15}$$

These second-quantized supercharges satisfy the same $\mathcal{S}_1\mathcal{G}$ algebra as the first-quantized ones (3.12), with the understanding that the bit number M is replaced by the usual second-quantized number operator:

$$M \to \int d\mathbf{x}\, \mathrm{Tr}\rho(\mathbf{x}), \tag{4.16}$$

where $\rho^{\beta}_{\alpha} \equiv [\phi^{\dagger}\phi + \phi^{\dagger}_{A}\phi_{A} + \frac{1}{2}\phi^{\dagger}_{AB}\phi_{AB}]^{\beta}_{\alpha}$. This is an automatic feature of second-quantized one-body operators, but it can also be confirmed directly from the definitions and (4.3).

The R supercharges contain both one body and two body operators. It is therefore convenient to separate their Fock space representations accordingly:

$$\mathcal{R}^{\dot{A}} = \mathcal{R}^{\dot{A}}_{0} + \mathcal{R}'^{\dot{A}}, \quad \tilde{\mathcal{R}}^{\dot{A}} = \tilde{\mathcal{R}}^{\dot{A}}_{0} + \tilde{\mathcal{R}}'^{\dot{A}}. \tag{4.17}$$

The expressions for $\mathcal{R}_0$ and $\tilde{\mathcal{R}}_0$ are as simple as those for $\mathcal{Q}$ and $\tilde{\mathcal{Q}}$:

$$\mathcal{R}^{\dot{A}}_{0} = \frac{-i}{2\sqrt{2m}} \int d\mathbf{x}\, \alpha^{iB\dot{A}} \mathrm{Tr}[\phi^{\dagger}\partial^{i}\phi_{B} - \phi^{\dagger}_{A_1}\partial^{i}\phi_{BA_1} - \mathrm{H.c.}], \tag{4.18}$$

$$\tilde{\mathcal{R}}^{\dot{A}}_{0} = \frac{1}{2\sqrt{2m}} \int d\mathbf{x}\, \alpha^{iB\dot{A}} \mathrm{Tr}[\phi^{\dagger}\partial^{i}\phi_{B} - \phi^{\dagger}_{A_1}\partial^{i}\phi_{BA_1} + \mathrm{H.c.}].$$

The Fock space representations of the two body operators are less obvious, especially considering the ambiguity alluded to earlier. The simplest choice that succeeds in reproducing the first-quantized free superstring results in the limit $N_c \to \infty$ is given by

$$\begin{aligned} \mathcal{R}'^{\dot{A}} = \frac{-T_0}{2N_c\sqrt{2m}} \int d\mathbf{x}\, d\mathbf{y}\, \boldsymbol{\alpha}^{B\dot{A}} \cdot (\mathbf{y} - \mathbf{x}) \\ \times \mathrm{Tr}[\phi^{\dagger}(\mathbf{x})\rho(\mathbf{y})\phi_{B}(\mathbf{x}) \\ -\phi^{\dagger}_{A_1}(\mathbf{x})\rho(\mathbf{y})\phi_{BA_1}(\mathbf{x}) + \mathrm{H.c.}], \end{aligned} \tag{4.19}$$

$$\begin{aligned} \tilde{\mathcal{R}}'^{\dot{A}} = \frac{iT_0}{2N_c\sqrt{2m}} \int d\mathbf{x}\, d\mathbf{y}\, \boldsymbol{\alpha}^{B\dot{A}} \cdot (\mathbf{y} - \mathbf{x}) \\ \times \mathrm{Tr}[\phi^{\dagger}(\mathbf{x})\rho(\mathbf{y})\phi_{B}(\mathbf{x}) \\ -\phi^{\dagger}_{A_1}(\mathbf{x})\rho(\mathbf{y})\phi_{BA_1}(\mathbf{x}) - \mathrm{H.c.}]. \end{aligned}$$

The $\mathcal{R}$ supercharges then satisfy the following algebra with the $\mathcal{Q}$ supercharges:

$$\begin{aligned} \{\mathcal{Q}^{A}, \tilde{\mathcal{R}}^{\dot{B}}\} &= \{\tilde{\mathcal{Q}}^{A}, \mathcal{R}^{\dot{B}}\} = 0, \\ \{\mathcal{Q}^{A}, \mathcal{R}^{\dot{B}}\} &= \frac{1}{2}\boldsymbol{\alpha}^{A\dot{B}} \cdot \mathbf{P} + \frac{T_0}{2N_c} \int d\mathbf{x}\, d\mathbf{y}\, \boldsymbol{\alpha}^{A\dot{B}} \cdot (\mathbf{x} - \mathbf{y}) :\mathrm{Tr}[\sigma(\mathbf{x})\rho(\mathbf{y})]:, \\ \{\tilde{\mathcal{Q}}^{A}, \tilde{\mathcal{R}}^{\dot{B}}\} &= \frac{1}{2}\boldsymbol{\alpha}^{A\dot{B}} \cdot \mathbf{P} - \frac{T_0}{2N_c} \int d\mathbf{x}\, d\mathbf{y}\, \boldsymbol{\alpha}^{A\dot{B}} \cdot (\mathbf{x} - \mathbf{y}) :\mathrm{Tr}[\sigma(\mathbf{x})\rho(\mathbf{y})]:, \end{aligned} \tag{4.20}$$

where $\sigma^{\beta}_{\alpha} \equiv : [\phi\phi^{\dagger} - \phi_{A}\phi^{\dagger}_{A} + \frac{1}{2}\phi_{AB}\phi^{\dagger}_{AB}]^{\beta}_{\alpha} :$. The integral term on the right-hand side of the last two anticommutators signifies a breakdown of the left- and right-moving $N = 1$ $\mathcal{S}_2\mathcal{G}$ algebras. We expect that acting on a single string state in the limit $N_c \to \infty$ this term will vanish, in order to reproduce the correct first-quantized anticommutators (3.14). It is not immediately obvious from the color structure of the term that this would be so, so we shall verify it explicitly:

$$\begin{aligned} \int d\mathbf{x}\, d\mathbf{y} (\mathbf{x} - \mathbf{y}) \mathrm{Tr} : \sigma(\mathbf{x})\rho(\mathbf{y}) : |\Psi\rangle &= \int d\mathbf{x}\, \mathbf{x} \int d\mathbf{y}\, \mathrm{Tr} : [\sigma(\mathbf{x})\rho(\mathbf{y}) - \sigma(\mathbf{y})\rho(\mathbf{x})] : |\Psi\rangle \\ &\sim \int d\mathbf{x}\, \mathbf{x}\, \mathrm{Tr} \int d\mathbf{y} [\sigma(\mathbf{x})\rho(\mathbf{y}) - \rho(\mathbf{x})\sigma(\mathbf{y})] |\Psi\rangle \\ &= \int d\mathbf{x}\, \mathbf{x}\, \mathrm{Tr} \int d\mathbf{y} [\sigma(\mathbf{x})\sigma(\mathbf{y}) - \rho(\mathbf{x})\rho(\mathbf{y})] |\Psi\rangle \\ &\sim \int d\mathbf{x}\, \mathbf{x}\, \mathrm{Tr}[\sigma(\mathbf{x}) - \rho(\mathbf{x})] |\Psi\rangle \\ &= 0. \end{aligned} \tag{4.21}$$

The second line follows for $N_c \to \infty$ since we have simply discarded subdominant terms. The equality in the third line follows from the fact the $\mathrm{U}(N_c)$ charges given by

$$G^{\beta}_{\alpha} = \int d\mathbf{x} [\sigma(\mathbf{x}) - \rho(\mathbf{x})]^{\beta}_{\alpha} \tag{4.22}$$

annihilate all singlet states. The fourth line again follows for $N_c \to \infty$ by discarding subdominant terms arising from the contractions. The last line follows from the equality of the traces of σ and ρ.

It is immediate from (4.20) that the left+right combinations $\mathcal{Q}_{+} = (\mathcal{Q} + \tilde{\mathcal{Q}})/\sqrt{2}$, $\mathcal{R}_{+} = (\mathcal{R} + \tilde{\mathcal{R}})/\sqrt{2}$ satisfy the correct anticommutation relation

$$\{\mathcal{Q}^{A}_{+}, \mathcal{R}^{\dot{B}}_{+}\} = \frac{1}{2}\boldsymbol{\alpha}^{A\dot{B}} \cdot \mathbf{P}, \tag{4.23}$$

suggesting the possibility that $N = 1$ $\mathcal{S}_2\mathcal{G}$ survives second quantization. Recall from Eq. (3.16) that it was an exact symmetry of the discrete superstring model, or equivalently of the first-quantized superstring bit model. In order for this much supersymmetry to survive second quantization the anticommutator of $\mathcal{R}^{\dot{A}}_{+}$ with itself must have the standard form. This computation yields

$$\{\mathcal{R}_+^{\dot{A}}, \mathcal{R}_+^{\dot{B}}\} = \frac{\delta^{\dot{A}\dot{B}}}{4m}\int d\mathbf{x}\, \mathrm{Tr}[|\nabla\phi|^2 + |\nabla\phi_A|^2 + \tfrac{1}{2}|\nabla\phi_{AB}|^2] + \frac{\delta^{\dot{A}\dot{B}}T_0}{2mN_c}\int d\mathbf{x}\, d\mathbf{y}\Bigg\{\left[1+\frac{T_0}{2}|\mathbf{y}-\mathbf{x}|^2\right]\mathrm{Tr}\phi^\dagger(\mathbf{x})\rho(\mathbf{y})\phi(\mathbf{x})$$
$$+\frac{T_0}{2}|\mathbf{y}-\mathbf{x}|^2\mathrm{Tr}\phi_A^\dagger(\mathbf{x})\rho(\mathbf{y})\phi_A(\mathbf{x}) + \left[-1+\frac{T_0}{2}|\mathbf{y}-\mathbf{x}|^2\right]\mathrm{Tr}\phi_{12}^\dagger(\mathbf{x})\rho(\mathbf{y})\phi_{12}(\mathbf{x})\Bigg\}$$
$$+\frac{\delta^{\dot{A}\dot{B}}T_0}{4mN_c}\int d\mathbf{x}\, d\mathbf{y}\mathrm{Tr}[i\phi^\dagger(\mathbf{y})\phi^\dagger(\mathbf{x})\phi_A(\mathbf{x})\phi_A(\mathbf{y}) + \phi_A^\dagger(\mathbf{y})\phi^\dagger(\mathbf{x})\phi_A(\mathbf{x})\phi(\mathbf{y})$$
$$+i\phi_A^\dagger(\mathbf{y})\phi^\dagger(\mathbf{x})\phi_B(\mathbf{x})\phi_{BA}(\mathbf{y}) + \phi_{AB}^\dagger(\mathbf{y})\phi^\dagger(\mathbf{x})\phi_A(\mathbf{x})\phi_B(\mathbf{y})$$
$$-i\phi^\dagger(\mathbf{y})\phi_A^\dagger(\mathbf{x})\phi_{BA}(\mathbf{x})\phi_B(\mathbf{y}) - \phi_A^\dagger(\mathbf{y})\phi_B^\dagger(\mathbf{x})\phi_{AB}(\mathbf{x})\phi(\mathbf{y})$$
$$-i\phi_A^\dagger(\mathbf{y})\phi_B^\dagger(\mathbf{x})\phi_{CB}(\mathbf{x})\phi_{CA}(\mathbf{y}) - \phi_{AB}^\dagger(\mathbf{y})\phi_C^\dagger(\mathbf{x})\phi_{AC}(\mathbf{x})\phi_B(\mathbf{y}) + \mathrm{H.c.}]$$
$$+[\text{normal-ordered three-body terms}]\ . \qquad (4.24)$$

Some of the three-body terms are not proportional to $\delta^{\dot{A}\dot{B}}$, therefore the $\mathcal{S}_2\mathcal{G}$ algebra is not realized with the seond-quantized supercharges. The terms by which it fails give rise to subleading contributions when acting on single superstring states in the limit $N_c \to \infty$. Consequently the first-quantized supercharges satisfy an $N = 1$ $\mathcal{S}_2\mathcal{G}$ algebra as expected.

As sketched in the previous section, even though we do not have the full $\mathcal{S}_2\mathcal{G}$ superalgebra from which the Hamiltonian is evident, we can still define a Hamiltonian in the following way:

$$H = \sum_{\dot{A}=1}^{2}\{\mathcal{R}_+^{\dot{A}}, \mathcal{R}_+^{\dot{A}}\}. \qquad (4.25)$$

Because of Eq. (4.23) this Hamiltonian possesses an $N = 1$ $\mathcal{S}_1\mathcal{G}$ supersymmetry. It is in fact *one* Fock space representation of the first-quantized Hamiltonian (3.17). As we stated earlier, the Fock space representation of $R_+^{\dot{A}}$, and therefore of the Hamiltonian, is determined only up to terms that give rise to subdominant contributions when acting on the single superstring state $|\Psi\rangle$ in the limit $N_c \to \infty$. One can then try to add such two body terms to $\mathcal{R}_+'^{\dot{A}}$ in the hope of closing the $\mathcal{S}_2\mathcal{G}$ algebra correctly, and ending up with a bit theory possessing the full $N = 1$ $\mathcal{S}_2\mathcal{G}$ supersymmetry. Such extra terms would also change the structure of interactions among different strings, which appear as subleading terms in the $1/N_c$ expansion. From the point of view of critical superstring ($D = 10$) these terms may be necessary to get the correct superstring scattering amplitudes in the continuum limit of the bit model.

C. A string bit model for type II-B superstring

Now that we have warmed up with a $(2 + 1)$-dimensional supersymmetric bit model, let us construct an $(8 + 1)$-dimensional bit model for ten-dimensional type II-B superstring. We shall specify the bit dynamics for type II-B superstring by working out the second-quantized versions of the supersymmetry generators and Hamiltonian. For II-B discrete superstring, the relation of the spinors $S, \tilde{S}$ to the Grassmann variables $\theta, \pi = d/d\theta$ maintains SO(8) covariance:

$$S_k^a = \frac{1}{\sqrt{2}}(\theta_k^a + \pi_k^a),\ \tilde{S}_k^a = \frac{i}{\sqrt{2}}(\theta_k^a - \pi_k^a). \qquad (4.26)$$

Let us first give the second-quantized versions of the undotted supercharges Q^a, $\tilde{Q}^a$, which are examples of one body operators. As with the $(2+1)$-dimensional case we first obtain

$$\Omega_{\theta^a} = \int d\mathbf{x}\sum_{n=0}^{7}\frac{(-)^n}{n!}\mathrm{Tr}\phi_{a_1\cdots a_n}^\dagger\phi_{aa_1\cdots a_n}, \qquad (4.27)$$
$$\Omega_{\pi^a} = \int d\mathbf{x}\sum_{n=0}^{7}\frac{(-)^n}{n!}\mathrm{Tr}\phi_{aa_1\cdots a_n}^\dagger\phi_{a_1\cdots a_n} = \Omega_{\theta^a}^\dagger,$$

from which we find

$$Q^a = \sqrt{\frac{m}{2}}\int d\mathbf{x}\sum_{n=0}^{7}\frac{(-)^n}{n!}\mathrm{Tr}[\phi_{a_1\cdots a_n}^\dagger\phi_{aa_1\cdots a_n} + \mathrm{H.c.}],$$
$$\tilde{Q}^a = i\sqrt{\frac{m}{2}}\int d\mathbf{x}\sum_{n=0}^{7}\frac{(-)^n}{n!}\mathrm{Tr}[\phi_{a_1\cdots a_n}^\dagger\phi_{aa_1\cdots a_n} - \mathrm{H.c.}],$$
$$Q_+^a = \frac{1}{\sqrt{2}}(Q^a + \tilde{Q}^a)$$
$$= \sqrt{\frac{m}{2}}\int d\mathbf{x}\sum_{n=0}^{7}\frac{(-)^n}{n!}\mathrm{Tr}[e^{i\pi/4}\phi_{a_1\cdots a_n}^\dagger\phi_{aa_1\cdots a_n}$$
$$+e^{-i\pi/4}\phi_{aa_1\cdots a_n}^\dagger\phi_{a_1\cdots a_n}]. \qquad (4.28)$$

It is again straightforward to verify that these satisfy the

$\mathcal{S}_1\mathcal{G}$ algebra of their first-quantized counterparts. The number operator is again given by $M = \int d\mathbf{x}\,\mathrm{Tr}\rho(\mathbf{x})$, where the bit density matrix in $8+1$ dimensions is given by

$$\rho(\mathbf{x})_\alpha^\beta = \sum_{n=0}^{8} \frac{1}{n!}[\phi^\dagger_{a_1\cdots a_n}(\mathbf{x})\phi_{a_1\cdots a_n}(\mathbf{x})]^\beta_\alpha. \qquad (4.29)$$

Next we turn to the R supercharges which involve two body operators. As in the $(2+1)$-dimensional case we only present the simplest second-quantized candidates which, in the limit $N_c \to \infty$, produce on single polymer states both R and $\tilde{R}$ given in (3.13). We generalize slightly, replacing T_0 by a general scalar potential $\mathcal{V}(|\mathbf{x}_{k+1} - \mathbf{x}_k|)$. Writing $\mathcal{R} = \mathcal{R}_0 + \mathcal{R}'$ and $\tilde{\mathcal{R}} = \tilde{\mathcal{R}}_0 + \tilde{\mathcal{R}}'$, where the subscript 0 denotes the one body term and prime denotes the two body term, we end up with

$$\mathcal{R}_0^{\dot a} = \frac{-i}{2\sqrt{2m}}\int d\mathbf{x}\,\gamma^{ib\dot a}\sum_{n=0}^{7}\frac{(-)^n}{n!}\mathrm{Tr}[\phi^\dagger_{a_1\cdots a_n}\partial^i\phi_{ba_1\cdots a_n} - \mathrm{H.c.}],$$

$$\tilde{\mathcal{R}}_0^{\dot a} = \frac{1}{2\sqrt{2m}}\int d\mathbf{x}\,\gamma^{ib\dot a}\sum_{n=0}^{7}\frac{(-)^n}{n!}\mathrm{Tr}[\phi^\dagger_{a_1\cdots a_n}\partial^i\phi_{ba_1\cdots a_n} + \mathrm{H.c.}],$$

$$\mathcal{R}'^{\dot a} = \frac{-1}{2N_c\sqrt{2m}}\int d\mathbf{x}\,d\mathbf{y}\sum_{n=0}^{7}\frac{(-)^n}{n!}(\mathbf{y}-\mathbf{x})\cdot\gamma^{b\dot a}\mathcal{V}(|\mathbf{y}-\mathbf{x}|)\mathrm{Tr}[\phi^\dagger_{a_1\cdots a_n}(\mathbf{x})\rho(\mathbf{y})\phi_{ba_1\cdots a_n}(\mathbf{x}) + \mathrm{H.c.}], \qquad (4.30)$$

$$\tilde{\mathcal{R}}'^{\dot a} = \frac{i}{2N_c\sqrt{2m}}\int d\mathbf{x}\,d\mathbf{y}\sum_{n=0}^{7}\frac{(-)^n}{n!}(\mathbf{y}-\mathbf{x})\cdot\gamma^{b\dot a}\mathcal{V}(|\mathbf{y}-\mathbf{x}|)\mathrm{Tr}[\phi^\dagger_{a_1\cdots a_n}(\mathbf{x})\rho(\mathbf{y})\phi_{ba_1\cdots a_n}(\mathbf{x}) - \mathrm{H.c.}].$$

Other terms with nonconsecutive creation operators and with more general spinor structure have not been displayed here, but we expect such terms are needed to get the superstring interactions right. These supercharges again fail to satisfy the $\mathcal{S}_2\mathcal{G}$ algebra. It is still conceivable that with more complicated color routings and spinor structures the full $\mathcal{S}_2\mathcal{G}$ supersymmetry can be restored. But this may not be possible, and we don't think it should necessarily be required at the level of string bits, which should generically exhibit less symmetry than the continuum. At the first-quantized level (equivalent to the second-quantized theory at $N_c \to \infty$) the full $\mathcal{S}_2\mathcal{G}$ superalgebra for Q_+ and R_+ was only present for a constant $\mathcal{V} = T_0$. For nonconstant $\mathcal{V}$ but unchanged spinor structure, we only had the $\mathcal{S}_1\mathcal{G}$ algebra generated by Q_+. For the second-quantized theory at finite N_c our simplest ansatz for $\mathcal{R}'_+$ fails to close the $\mathcal{S}_2\mathcal{G}$ superalgebra because $\{\mathcal{R}'^{\dot a}_+, \mathcal{R}'^{\dot b}_+\}$ is not proportional to[3] $\delta^{\dot a\dot b}$. The offending contributions, however, have a color structure which is subdominant as $N_c \to \infty$. The supersymmetry generated by the $\mathcal{Q}_+$'s remains a symmetry at finite N_c for any $\mathcal{V}$ if the Hamiltonian commutes with $\mathcal{Q}^a_+$, and we shall insist that at least $\mathcal{S}_1\mathcal{G}$ be an exact symmetry of the string bit dynamics. This will automatically hold if we define the Hamiltonian H for the second-quantized theory by (3.29) with second-quantized operators $\mathcal{R}_+$ substituted for the first-quantized R_+. This is because we not only have the $\mathcal{S}_1\mathcal{G}$ superalgebra

$$\{\mathcal{Q}^a_+, \mathcal{Q}^b_+\} = \delta_{ab}m\int d\mathbf{x}\sum_{n=0}^{8}\frac{1}{n!}\mathrm{Tr}\phi^\dagger_{a_1\cdots a_n}\phi_{a_1\cdots a_n} \equiv \delta_{ab}mM, \qquad (4.31)$$

but we also require the $\mathcal{S}_2\mathcal{G}$ anticommutators between $\mathcal{Q}_+$ and $\mathcal{R}_+$:

$$\{\mathcal{Q}^b_+, \mathcal{R}^{\dot a}_+\} = \frac{1}{2}\gamma^{b\dot a}\cdot\int d\mathbf{x}\sum_{n=0}^{8}\frac{1}{n!}\mathrm{Tr}\phi^\dagger_{a_1\cdots a_n}(-i\nabla)\phi_{a_1\cdots a_n} = \frac{1}{2}\gamma^{b\dot a}\cdot\mathbf{P}, \qquad (4.32)$$

where $\mathbf{P}$ is the total momentum of the multibit system. It then follows from our definition of H that $[H, \mathcal{Q}^a_+] = 0$, since all the $\mathcal{R}$'s are translationally invariant, and so commute with $\mathbf{P}$.

We have now presented the ingredients of our proposed string bit model for type II-B superstring. To summarize our results, we recall the steps in the construction of the complete Hamiltonian. First construct $\mathcal{R}^{\dot a}_+ = \mathcal{R}^{\dot a}_{0+} + \mathcal{R}'^{\dot a}_+$ from the expressions listed in (4.30) or from a generalization of them. For example, using the displayed expressions we obtain

[3]For $\mathcal{V} = T_0$, the cross terms $\{\mathcal{R}^{\dot a}_{0+}, \mathcal{R}'^{\dot b}_+\} + (\dot a \leftrightarrow \dot b) \propto \delta^{\dot a\dot b}$. This is not surprising because this operator has a color structure that can survive the limit $N_c \to \infty$, and we already know from the first-quantized theory that $\mathcal{S}_2\mathcal{G}$ holds in that limit when $\mathcal{V} = T_0$.

$$\mathcal{R}_{0+}^{\dot a} = \frac{1}{\sqrt{2}}(\mathcal{R}_0^{\dot a} + \tilde{\mathcal{R}}_0^{\dot a}) = \frac{1}{2\sqrt{2m}}\int d\mathbf{x}\,\gamma^{ib\dot a}\sum_{n=0}^{7}\frac{(-)^n}{n!}\mathrm{Tr}[e^{-i\pi/4}\phi^\dagger_{a_1\cdots a_n}\partial^i\phi_{ba_1\cdots a_n} + \mathrm{H.c.}],$$

$$\mathcal{R}'^{\dot a}_{+} = \frac{1}{\sqrt{2}}(\mathcal{R}'^{\dot a} + \tilde{\mathcal{R}}'^{\dot a}) \qquad (4.33)$$

$$= -\frac{1}{2N_c\sqrt{2m}}\int d\mathbf{x}\,d\mathbf{y}\sum_{n=0}^{7}\frac{(-)^n}{n!}(\mathbf{y}-\mathbf{x})\cdot\gamma^{b\dot a}\mathcal{V}(|\mathbf{y}-\mathbf{x}|)\mathrm{Tr}[e^{-i\pi/4}\phi^\dagger_{a_1\cdots a_n}(\mathbf{x})\rho(\mathbf{y})\phi_{ba_1\cdots a_n}(\mathbf{x}) + \mathrm{H.c.}].$$

Once the $\mathcal{R}_+$'s are pinned down, our proposal for the string bit Hamiltonian will be

$$H_{\text{II-B}} = \frac{1}{4}\sum_{\dot a=1}^{8}\{\mathcal{R}_{0+}^{\dot a} + \mathcal{R}'^{\dot a}_{+}, \mathcal{R}_{0+}^{\dot a} + \mathcal{R}'^{\dot a}_{+}\}$$

$$= \frac{1}{2m}\int d\mathbf{x}\sum_{n=0}^{8}\frac{1}{n!}\mathrm{Tr}|\nabla\phi_{a_1\cdots a_n}|^2 + \frac{1}{2}\{\mathcal{R}_{0+}^{\dot a}, \mathcal{R}'^{\dot a}_{+}\} + \frac{1}{4}\{\mathcal{R}'^{\dot a}_{+}, \mathcal{R}'^{\dot a}_{+}\}, \qquad (4.34)$$

where we have only written out the free part of the string bit Hamiltonian explicitly. The interacting terms are to be worked out using (4.33) or its generalization.

The Hamiltonian (4.34) defines the dynamics we propose for string bits, once we have specified the structure of $\mathcal{R}'_+$. For finite N_c it describes a perfectly well-defined nonrelativistic many-body system. When studied in the limit $N_c \to \infty$, it will, by construction, describe weakly interacting long polymers and the infinitely long ones will have exactly the properties of type II-B free superstrings. Interactions among strings will also be included in (4.34) with strength of order $1/N_c^2$ for the string-string scattering amplitude. Unfortunately, the string interactions arising from the terms displayed in (4.33) do not seem to provide the richness of spinor structure required in the light-cone three-superstring vertex given by Green, Schwarz, and Brink [18]. The basic structure of the correct three-string vertex term in the supercharge is an "overlap" integral of the product of the three string wave functions with an insertion of a complicated seventh order polynomial of the world-sheet spinors at the joining point. Inspection of the $1/N_c$ terms arising from non-nearest neighbor contractions in the action on a polymer state of the terms displayed in (4.33) confirms the basic overlap structure. But these terms can provide only a linear factor of world-sheet spinors at the joining point. Thus it is clear that terms in $\mathcal{R}$ with a more complicated spinor structure will be required.[4] This means that the principles we have so far imposed on our string bit models are not quite strong enough to force the correct dynamics for interacting superstring theory. In Ref. [18] it was shown that requiring the Poincaré superalgebra was sufficient to uniquely determine the three string vertex. Thus, if we could succeed in devising a string bit model with the full $\mathcal{S}_2\mathcal{G}$ supersymmetry at finite N_c *and* a large N_c limit that correctly describes free superstrings, the correct stringy interactions would be virtually guaranteed. So far we have examples which fulfill either of these criteria, but not both. The models given in this paper are constructed to satisfy the second criterion, but they fall short of the first. In Ref. [15] we construct a model possessing the full $\mathcal{S}_2\mathcal{G}$ supersymmetry, but it is unlikely that its large N_c limit describes free superstrings. Lacking a satisfactory model with the full $\mathcal{S}_2\mathcal{G}$ supersymmetry, one should adopt the renormalization group philosophy and allow *all* interactions consistent with $\mathcal{S}_1\mathcal{G}$ symmetry and search for the interesting cases among all possible continuum limits, one of which should be the interacting type II-B superstring theory. The various superstring/bit models and their supersymmetries are summarized in Table I.

[4] If they are restricted to terms with nonconsecutive creation operators, e.g., with the trace structure Tr: $\phi^\dagger\phi\phi^\dagger\phi$:, they will not affect the properties of free strings.

V. AMBIGUITIES AND OPEN ISSUES

In this paper we have made a proposal for the extension of string bit models to superstring, developing most fully the type II-B case while leaving the complete analysis of the not II-B cases for future work. However, we have only made a start on the task of confirming that the proposal reproduces completely all aspects of superstring theory. While we can firmly assert that the $N_c \to \infty$ limit describes free superstrings adequately, the $1/N_c$ corrections which determine the interactions among strings have not yet been well studied. It is transparent from the string bit compositeness that these interactions will be string breaking or joining processes with amplitudes proportional to overlap integrals between initial and final multistring states. We can anticipate also that, as was the case for bosonic string, the interactions can only be fully Poincaré invariant in the critical dimension. However the details, including any modifications required to produce the correct operator insertions at the joining points, have yet to be worked out. These operator insertions are also known to entail contact interactions [19,20] which should of course also be a consequence of our string bit

TABLE I. Superstring models and their supersymmetry (SUSY).

Superstring/bit models			
Model	$\mathcal{V}(\mathbf{x})$	SUSY	Failing (anti)commutators
Covariant type II		$N = 2\ \mathcal{SP}$	None
Light-cone type II, $D = 10$		$N = 2\ \mathcal{SP}$	None
Light-cone type II, $D \neq 10$		$N = 2\ \mathcal{S}_2\mathcal{G}$	$[M^{-i}, M^{-j}] \neq 0$
Discrete light cone		$N = 1\ \mathcal{S}_2\mathcal{G}$	$\{R^{\dot{a}}, \tilde{R}^{\dot{b}}\} \neq 0$
1st quantized super bits	$\mathcal{V} = T_0$	$N = 1\ \mathcal{S}_2\mathcal{G}$	$\{R^{\dot{a}}, \tilde{R}^{\dot{b}}\} \neq 0$
1st quantized super bits	$\mathcal{V}(\mathbf{x})$	$N = 1\ \mathcal{S}_1\mathcal{G}$	$\{R_+^{\dot{a}}, R_+^{\dot{b}}\} \not\propto \delta^{\dot{a}\dot{b}}$
2nd quantized superbits	$\mathcal{V} = T_0$	$N = 1\ \mathcal{S}_1\mathcal{G}$	$\{\mathcal{Q}^a, \mathcal{R}^{\dot{a}}\}, \{\tilde{\mathcal{Q}}^a, \tilde{\mathcal{R}}^{\dot{a}}\}, \{\mathcal{R}_+^{\dot{a}}, \mathcal{R}_+^{\dot{b}}\}$
2nd quantized superbits	$\mathcal{V}(\mathbf{x})$	$N = 1\ \mathcal{S}_1\mathcal{G}$	$\{\mathcal{Q}^a, \mathcal{R}^{\dot{a}}\}, \{\tilde{\mathcal{Q}}^a, \tilde{\mathcal{R}}^{\dot{a}}\}, \{\mathcal{R}_+^{\dot{a}}, \mathcal{R}_+^{\dot{b}}\}$

model. We fully expect that terms must be added to the second-quantized interacting supercharge $\mathcal{R}$ which do not contribute in leading order in the $1/N_c$ expansion. Any monomial such as Tr: $\phi^\dagger\phi\phi^\dagger\phi$:, in which the creation operators are not consecutive is such a term. All of these issues need to be carefully examined in future work.

Assuming that either our proposed Hamiltonian or a suitable modification of it correctly reproduces the interacting superstring theory, there is still the question of uniqueness. Because stringy physics is only a property of infinitely composite string bit polymers, it is natural to expect, in accord with ideas of universality, that there are many microscopic string bit models that yield the same continuum string theory. One aspect of this is our expectation that a wide class of potentials $\mathcal{V}$ will give identical stringy physics to the case $\mathcal{V} = T_0$. The degree of flexibility in the choice of potential still needs to be pinned down. In particular, our conjecture that the potential could even be short range needs to be tested (at the very least numerically). These are all issues that can be addressed at the first-quantized noninteracting (i.e., $N_c \to \infty$) polymer level. But they are also pertinent to the fully interacting (finite N_c) string bit theory. For example, the ambiguities mentioned in the previous paragraph may to some extent be string bit artifacts that can be absorbed into the definition of a small number of macroscopic string parameters and have no further effect on the string interactions.

Finally we say a few words about compactification, a subject we have not yet addressed. A string model of the real world must of course possess precisely four noncompact dimensions, three space plus one time. This means that the corresponding string bit model should have precisely two noncompact spatial dimensions. Accordingly, we must eventually "compactify" six of the eight spatial dimensions of our superbit models. One possibility is, of course, to impose by hand that a six-dimensional subspace is some compact space, be it a toroid, orbifold, or Calabi-Yau manifold. But the string bit picture allows a more dramatic possibility. Polymer formation generically promotes finite internal degrees of freedom on the bit to an effective compact dimension [21]. Indeed the manner in which the world-sheet spinor fields S, $\tilde{S}$ emerge from the 256 component string bit multiplet illustrates this point very nicely. A pair of world-sheet fermion fields can always be "bosonized." The resulting boson world-sheet field then enters the string dynamics in just the way a compactified coordinate would. In particular it would count as part of the D which is required to be 10 for superstring. In this way the string bit model might be properly formulated from the beginning as a $(2+1)$-dimensional super-Galilei invariant theory of string bits, which carry, in addition to the supermultiplet spin labels, a finite number of internal degrees of freedom to play the role of the six compactified dimensions. A successful implementation of this possibility would provide an explicit and concrete realization of 't Hooft's idea [11] that the world is a hologram: That it is fundamentally a system existing in two spatial dimensions, although it gives the appearance of being three dimensional.

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