Particle path-integral approach to the study of Dirac spin-$\frac{1}{2}$ field systems

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(Received 17 February 1995)

A novel approach to the study of fermionic systems in d-dimensional Euclidean spacetime is presented according to which an original, field-theoretical form of description is converted into a particle-based language. An important aspect of the advocated procedure is that it employs a spacetime resolution scale which does not have to serve, at the same time, as an ultraviolet cutoff for matter field fluctuations. At the particle level of description, such fluctuations are independently regularized by a scale associated with a "proper-time" parameter. A key feature of our representation for fermionic systems is its purely geometrical content. In particular, Polyakov's spin factor, which enters the path integral description of spin-$\frac{1}{2}$ entities, emerges very naturally in the course of passing from the field-theoretical to the particle-based language. The applications considered in this paper pertain to evaluations of the Dirac determinant. In the presence of a coupling to an external gauge field, such computations lead to effective-action terms. Both Maxwell and topological terms are retrieved in two, three, and four spacetime dimensions.

PACS number(s): 11.10.Ef, 11.10.Kk, 11.15.Tk

I. INTRODUCTION

The quantum-mechanical description of elementary particles propagating in Minkowski spacetime runs into an immediate obstacle: A localized "position" operator $\hat{X}_\mu$, which corresponds to the set of spacetime coordinates x_μ, does not have a well-defined meaning. Field theory bypasses this problem via second quantization and yields a particle picture through the Fock space construction. On the other hand, the Wick rotation which inevitably enters field-theoretical calculations renders the aforementioned impasse irrelevant, since no topological ambiguities appear in the course of defining a coordinate operator in *Euclidean spacetime.* There is no *a priori* reason, then, why the principles and methodologies of first quantization cannot be pursued within the Euclidean spacetime setting.

It so happens that the quantum propagation of particlelike entities in Euclidean spacetime has been discussed by Polyakov, who has furnished the relevant path-integral rules for its description [1,2]. An important aspect of the whole scheme is that the parameter τ, which serves to label dynamical development along a given path, has pure mathematical meaning and cannot be identified with physical time as in the case of the ordinary quantum-mechanical path integral. Reparametrization transformations of the form

$$\tau \to \tau'(\tau) , \qquad \dot{\tau}'(\tau) \geq 0 ,$$

where the overdot signifies differentiation with respect to τ, serve to impose a gauge-type invariance requirement on the theory and, as such, they should be appropriately accommodated by the integration measure. As for the phase assigned to each path, it does possess a direct *geometrical* meaning and serves to incorporate quantities such as the length, curvature, etc., of each particular path.

A suitable choice of gauge leads to the following expression for the probability amplitude associated with a free pointlike entity that has no additional intrinsic degrees of freedom (e.g., spin):

$$F(x,y) = \int_0^\infty dT\, e^{-\mu_0 T} \int_{\substack{x(0)=x \\ x(T)=y}} [dx(\tau)]\, \delta\big[\dot{x}(\tau)^2 - 1\big] \ . \tag{1}$$

We shall refer to the parameter $\tau(\in [0,T])$ entering the above expression as "proper time."

The geometrical accommodation of spin involves additional notions the nature of which need not concern us for the moment. Suffice it to state that, for closed-path propagation in $\mathbb{R}^d$, the quantum description of spin-1/2 particles requires the injection into the functional integral of the so-called spin factor $\Phi(C)$, given by [1]

$$\Phi(C) = \mathcal{P} \exp\left\{ \frac{1}{8} \int_0^T d\tau\, \omega_{\mu\nu}[x(\tau)]\, [\gamma_\mu, \gamma_\nu] \right\} , \tag{2}$$

where C denotes the closed path $x(0) = x(T)$, $\mathcal{P}$ stands for path ordering, and

$$\omega_{\mu\nu}[x(\tau)] = \frac{1}{2}\, [\dot{x}_\mu(\tau)\ddot{x}_\nu(\tau) - \ddot{x}_\mu(\tau)\dot{x}_\nu(\tau)] , \quad \dot{x}^2 = 1 ,$$

is the orientation tensor of the local perpendicular plane.

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What basically serves as a source of inspiration for the formulation of the above scheme is the statistical study of models such as the Ising, Heisenberg, etc., in terms of geometrical configurations appropriate for each system. Such configurations facilitate an interpretation according to which the underlying Euclidean space can be alternatively viewed as spacetime. The natural next step is to look for direct connections between path integrals for pointlike entities and field theory. In fact, the idea of a quantum description of the Klein-Gordon and Dirac particles in terms of paths, labeled by proper time, was put forth by Feynman [3] in his original paper on the subject. Since then, the connection between quantum field theory and particle path-integral representations has been discussed by several authors (see, e.g., [4] and references cited therein).

In addition, the dynamics of relativistic spinning particles has been described by means of (first-quantized) superparticle path integrals [5–8], using additional Grassmann variables. More recently, it was shown that such world line (superparticle) approaches can be used for calculating (multiloop) QCD amplitudes very effectively [9–11].

A quantitative exploitation of pointlike representations was also conducted by Brandt, Neri, and Zwanziger [12] in connection with the quantum treatment of magnetic-charge-carrying particles. The present authors' recent interest in particle-based representations of field systems can be found in Refs. [13–16].

In this paper we pursue the connection between quantum field theories and Euclidean path integrals with exclusive reference to spin-1/2 fields. With an eye towards gaining computational advantages at the nonperturbative level, we adopt the point of view that the initial casting of the field system should have a built-in resolution scale. We shall then proceed, in Sec. II, to *translate* the quantum version of the field system into path-integral form via a series of mathematically well-defined steps. In this way, we shall be able to identify the field-theoretical origin of the geometrical features which enter the Euclidean path integrals.

A characteristic aspect of our approach is that the path-integral expressions inherit, explicitly or implicitly, the spacetime resolution scale from the field system. Sending the resolution scale to zero (in the sense that it is taken to be much smaller than the observational scale) leads to interesting physical realizations. A relevant analysis will be carried out in Sec. III, with special emphasis being placed on the spin factor.

The fact that our regularized casting of the field system takes place within a continuous spacetime bakground, as opposed to having introduced a lattice set of points, encourages the thought that no continuum property has been lost during the translation into path-integral language. We verify this property by performing specific calculations in Secs. IV and V, concerning logarithms of the Dirac determinant. In this way, we shall (re)produce both conventional and topological terms entering effective actions. Our conclusions are formulated in Sec. VI, while some technical details are provided in the Appendix.

II. TRANSCRIPTION OF SPINORIAL FIELD SYSTEMS INTO A PARTICLE PATH-INTEGRAL REPRESENTATION

Our immediate goal is to accomplish the translation of a given quantum field theoretical system, formulated in Euclidean spacetime, into a particle-based, i.e., quantum-mechanical, language. An approach which realizes this objective at a discrete level of description will be carried out in the present section.

On the quantum-mechanical side, the relevant representation is furnished by a discrete set of coordinate states $\{|n\alpha\rangle\}$ which obey the orthonormality and completeness conditions

$$\langle n\alpha|n'\alpha\rangle = \frac{1}{\alpha^d}\,\delta_{nn'}\ , \qquad \sum_n \alpha^d|n\alpha\rangle\langle n\alpha| = 1\ . \tag{3}$$

On the field-theoretical side, the discrete formulation is encoded in the action functional. Thus, for a free Dirac field, one writes

$$S^D = -\sum_n \alpha^d\bar{\psi}(n\alpha)\left(\gamma_\mu\partial_\mu^D + m^D\right)\psi(n\alpha)\ . \tag{4}$$

The precise definition of the differential operator ∂_μ^D specifies the discretization scheme one whishes to adopt. Irrespectively of any particular choice, the functional integral, which furnishes the quantum content of the theory, utilizes the well-defined Berezin integration measure [17] $d\psi(n\alpha)$ and $d\bar{\psi}(n\alpha)$ over the Grassmannian variables $\psi(n\alpha)$ and $\bar{\psi}(n\alpha)$. For example, one obtains for the partition function

$$\begin{aligned} Z_\alpha &= \int\prod_n\,[d\psi(n\alpha)][d\bar{\psi}(n\alpha)]\exp\left(-S^D\right) \\ &= \det\left(\gamma_\mu\partial_\mu^D + m^D\right)\ . \end{aligned} \tag{5}$$

The calculation of the determinant, or, better, of $\mathrm{Tr}\left[\ln(\gamma_\mu\partial_\mu^D + m^D)\right]$ in some convenient representation, must now take into account the precise definition of ∂_μ^D. Given the well-known doubling problem in lattice theories, various successful strategies have been developed which make use of concepts such as that of a hopping parameter [18], staggered fermions [19], the SLAC derivative [20], etc. Invariably, the above determinant refers to a sparse matrix.

Our purpose, on the other hand, is not to define the field-theoretical system on the lattice *per se* but to translate it into a particle-based language. Not only that, but we also want to eventually arrive at path-integral expressions which can be "read" as describing the propagation of pointlike entities in *continuous* Euclidean spacetime. To this end, we shall employ a scheme within which discretization emerges as averaging over cells rather than as a replacement of the continuum by a lattice of points. In particular, the derivative operator is defined through its action on the spinor field at the site $n\alpha$ as [16]

$$\begin{aligned} \left(\gamma_\mu\partial_\mu^D + m^D\right)\psi(n\alpha) = -&\int d^dy\,[(\gamma_\mu\partial_\mu - m)f(|y|)] \\ &\times\psi(n\alpha + y)\ , \end{aligned} \tag{6}$$

where $f(|y|)$ is a distribution of fast decrease which practically falls to zero whithin the discretization range α. Moreover, $f(|y|)$ has the property $f(|y|) \stackrel{\alpha\to 0}{\longrightarrow} \delta(y)$, which is satisfied, e.g., by the Gaussian distribution.

According to (6), the derivative of the spinor field assigned to site $n\alpha$ is given by an appropriately weighted average of regular derivatives within a volume of the order α^d surrounding the site. More generally, we might view our discrete action S^D as resulting, by virtue of averaging within cells of volume α^d, from the nonlocal Dirac action

$$S^{\rm NL} = \int d^dx \int d^dy\ \bar{\psi}(x)\left[(\gamma_\mu\partial_\mu - m)f(|y|)\right]\psi(x+y)\ . \tag{7}$$

Observe that the nonlocality in the above expression is totally under control as the standard Dirac theory is recovered in the limit $\alpha \to 0$.

Our field-theoretical characterization of the derivative operator $\hat{\partial}_\mu$ according to the nonlocal version (7) immediately points to its quantum-mechanical content. In coordinate representation, we write

$$\langle x|\hat{\partial}_\mu|x'\rangle = \frac{\partial}{\partial x_\mu} f(|x-x'|)\ . \tag{8}$$

Of more immediate interest is the representation of ∂_μ^D in the basis offered by the discrete set of coordinates with orthonormality and completeness relations given by (3). We shall make direct use of this representation in connection with the computation of $\mathrm{Tr}[\ln(\gamma_\mu\partial_\mu^D + m^D)]$. As a first step in this direction, let us use Schwinger's proper-time representation [21] to write

$$\ln Z_\alpha \stackrel{c\to 0}{=} -\mathrm{Tr}\int_c^\infty \frac{dT}{T}\ e^{-T\left(\gamma_\mu\partial_\mu^D + m^D\right)}\ , \tag{9}$$

where c makes its appearance as an *a priori* independent length scale. Evaluating the trace in the representation offered by the complete and orthonormal set of discrete coordinate states $\{|n\alpha\rangle\}$ we obtain, by standard methods,

$$-\ln Z_{\alpha,c} = \mathrm{tr}\int_c^\infty \frac{dT}{T}\ e^{-Tm} \lim_{N\to\infty}\left(\prod_{i=0}^{N}\sum_{n_i}\alpha^d\right)\frac{\delta_{n^0,n^N}}{\alpha^d}\prod_{i=1}^{N}\langle n^{i-1}\alpha\,|\left(1-\epsilon\gamma\cdot\partial^D\right)|\,n^i\alpha\rangle\ . \tag{10}$$

We have picked up, it will be noticed, yet one more length scale, namely, $\epsilon \equiv T/N$. In (10), tr stands for the trace over Dirac indices.

The emerging picture is the following: Each given closed path in $\mathbb{R}^d$, of total length T, involved in the evaluation of the trace, has been divided into N intervals. Upon each division we have inserted a complete set of discrete coordinate states. Accordingly, the index i labels *independent* discretizations of $\mathbb{R}^d$. We are now faced with the task of specifying the matrix element $\langle n^{i-1}\alpha\,|\partial_\mu^D|\,n^i\alpha\rangle$. We determine

$$\langle n^{i-1}\alpha\,|\partial_\mu^D|\,n^i\alpha\rangle = \frac{1}{\alpha}\frac{\partial}{\partial n_\mu^{i-1}}\int_{-\pi/\alpha}^{\pi/\alpha}\frac{d^dp_i}{(2\pi)^d}\ \tilde{f}(p^{i\,2}\alpha^2)\,e^{-ip^i\cdot(n^i-n^{i-1})\alpha}\ , \tag{11}$$

where $\tilde{f}(p^2\alpha^2)$ denotes the Fourier transform of $f(|y|)$. Given that $\tilde{f}(p^2\alpha^2)$ falls off to zero within the first Brillouin zone, we conclude that

$$\langle n^{i-1}\alpha\,|\partial_\mu^D|\,n^i\alpha\rangle = \frac{1}{\alpha}\frac{\partial}{\partial n_\mu^{i-1}}\ f\left(|n^i - n^{i-1}|\right)\ . \tag{12}$$

The independent discretization of each copy of $\mathbb{R}^d$, per division of the (closed) path, means that the distance $|n^i - n^{i-1}|$ takes on continuous values. We visualize the lattice of points as being able to "slide" continuously in $\mathbb{R}^d$; i.e., the lattice arrangement can shift from $\mathbb{R}^d$ slice to $\mathbb{R}^d$ slice during the motion along the path. Returning to the free energy, we find, after some straightforward manipulations,

$$-\ln Z_{\alpha,\epsilon,c} = \mathrm{tr}\int_c^\infty \frac{dT}{T}\ e^{-Tm}\left(\prod_{i=0}^{N}\sum_{n_i}\alpha^d\right)\frac{\delta_{n^0,n^N}}{\alpha^d}\prod_{i=1}^{N}\left[1-\frac{\alpha}{\epsilon}\left(n^i-n^{i-1}\right)_\mu\gamma_\mu\right]\prod_{i=1}^{N}\frac{1}{\alpha^d}F\left(|n^i-n^{i-1}|\right)\ , \tag{13}$$

where we have introduced

$$F\left(|n^i-n^{i-1}|\right) \equiv -\frac{(\epsilon/\alpha)^2\alpha^d}{|n^i-n^{i-1}|}\frac{\partial}{\partial\,|n^i-n^{i-1}|}\ f\left(|n^i-n^{i-1}|\right)\ . \tag{14}$$

A convenient choice for $F\left(\left|n^i - n^{i-1}\right|\right)$, consistent with the requirements that have been already spelled out, is

$$F\left(\left|n^i - n^{i-1}\right|\right) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}\left|n^i - n^{i-1}\right|^2} . \tag{15}$$

The above distribution will play a key role throughout our analysis. It furnishes a relative weight which relates the lattice structures at i and $i-1$, given the fact that they can "slide" with respect to each other. Its significance for our scheme is directly attributed to the presence of "proper time" as an additional parameter.[1]

In principle, any other established lattice scheme could serve the purpose, provided an appropriate dependence on "proper time" is devised. For example, in the Wilson case [18], one might conceive of employing statistical averages over hopping parameters K_i. Our present discrete casting of the fermionic system seems to have a natural, built-in capacity to average over repositionings of the lattice structure as one moves in the "proper-time" direction. Another issue which calls for special attention concerns the various length scales entering (10). According to our construction, α accounts for localization in spacetime. We may associate it with "particle size." The scale ϵ sets the pace by which successive spacetime slices enter the splitting of a given (closed) path. They both correspond to spacetime resolution scales and one is well advised to send them to zero (in the local limit) at unit ratio, i.e., $\alpha, \epsilon \to 0$ with $(\alpha/\epsilon) = 1$. Turning our attention to c, we observe that it sets a lower bound on the length of the closed trajectories entering the calculation of the trace. Obviously, $c \gg \epsilon$, so that it should be the last scale to be sent to zero. As it turns out, c will assume the role of an ultraviolet cutoff for matter field fluctuations, once the spacetime resolution scale has been safely set equal to zero.

Our next task is to move towards continuous expressions. On one hand, $\alpha \to 0$ entails the transitions

$$n\alpha \to x\ , \qquad \left(\prod_{i=1}^{N}\sum_{n_i}\alpha^d\right) \to \int \prod_{i=1}^{N} d^d x_i\ , \qquad \frac{\delta_{n^0,n^N}}{\alpha^d} \to \delta(x^0 - x^N)\ .$$

On the other hand, $\epsilon \to 0$ generates path-related quantities:

$$\int \prod_{i=1}^{N} dx_i \to \int_{\tau\in[0,T]} [dx(\tau)]\ , \qquad \sum_{i=1}^{N}\left(x^{i-1} - x^i\right)_\mu \to \int_0^T d\tau\, \dot{x}_\mu(\tau)\ .$$

Effecting the above "changes" in the expression for $\ln Z$, we write, after taking into consideration (15),

$$-\ln Z_{\substack{c\to 0\\ \epsilon\to 0}} = \mathrm{tr}\int_c^\infty \frac{dT}{T}\, e^{-Tm}\mathcal{N}\int_{x(0)=x(\tau)}[dx(\tau)]\, e^{-T/2\epsilon}\, \mathcal{P}\exp\left[\frac{1}{\epsilon}\int_0^T d\tau\, \gamma_\mu \dot{x}_\mu(\tau)\right]\exp\left[-\frac{1}{2\epsilon}\int_0^T d\tau \dot{x}(\tau)^2\right]\ , \tag{16}$$

where $\mathcal{N}$ is a residual factor which comes from the definition of $F\left(\left|n^i - n^{i-1}\right|\right)$; it is equal to $(2\pi)^{-Nd/2}\epsilon^{-Nd}$. The above expression should be simply viewed as a continuous version of (13) and not as the result of taking the limits $\alpha, \epsilon \to 0$ (at unit ratio). In fact, the resolution scale ϵ enters (16) explicitly and this matter will occupy our attention in the next section. From here on, however, we shall skip explicit reference to the $\epsilon \to 0$ and/or $c \to 0$ limits.

We close the present section by extending our analysis to the case where an external Abelian gauge field coupling is also included in the field-theoretical model. Our discrete action reads

$$S^D = -\alpha^d \sum_{n,\mu} \bar{\psi}(n\alpha)\left(\gamma_\mu D_\mu^D + m^D\right)\psi(n\alpha)\ . \tag{17}$$

The covariant derivate operator D_μ^D is defined by its action on the spinor field at site $n\alpha$ as

$$\left(\gamma_\mu D_\mu^D + m^D\right)\psi(n\alpha) = -\int d^d y\, \left(\gamma_\mu \partial_\mu - m\right) f(|y|)\, U\left(L_{n\alpha, n\alpha+y}\right)\, \psi(n\alpha + y)\ , \tag{18}$$

where

$$U\left(L_{x,x+y}\right) \equiv \exp\left[-ig\int_{L_{x,x+y}} A_\mu(z) dz_\mu\right] \tag{19}$$

[1] On the other hand, no visible advantage is offered by our discretization procedure with respect to pure field-theoretical analyses, where the lattice structure remains static.

and $L_{x,x+y}$ denotes the (arbitrary) path from x to $x+y$ in $\mathbb{R}^d$. Going through the same steps as before, we write

$$-\ln Z_A = \mathrm{Tr} \int_c^\infty \frac{dT}{T}\, e^{-T(\gamma_\mu D_\mu^D + m^D)} \,. \tag{20}$$

We shall refer to the above quantity as the (Euclidean) effective action. The calculation of the trace will be conducted, as before, in the representation offered by the complete set of discrete coordinate states. The key relation is

$$\langle n^{i-1}\alpha \,|\,(1-\epsilon\gamma_\mu D_\mu^D)\,|\, n^i\alpha\rangle \approx \frac{1}{\alpha^d}\, U(L_{n^{i-1}\alpha,n^i\alpha}) \left[1 - \frac{\alpha}{\epsilon}\gamma_\mu (n^i - n^{i-1})_\mu\right] F\left(|n^i - n^{i-1}|\right) \,. \tag{21}$$

Standard manipulations lead to the following discrete expression for the effective action (containing all the length scales):

$$\begin{aligned}-\ln Z_A = \;& \mathrm{tr} \int_c^\infty \frac{dT}{T}\, e^{-Tm} \left(\prod_{i=0}^N \sum_{n_i} \alpha^d\right) \frac{\delta_{n^0,n^N}}{\alpha^d} \prod_{i=1}^N \left[1 - \frac{\alpha}{\epsilon}\left(n^i - n^{i-1}\right)_\mu \gamma_\mu\right] \\ & \times \prod_{i=1}^N \frac{1}{\alpha^d} F\left(|n^i - n^{i-1}|\right) \exp\left[-ig \sum_{i=1}^N \alpha (n^i - n^{i-1})_\mu A_\mu(n^i\alpha)\right] \,.\end{aligned} \tag{22}$$

Finally, the continuous expression corresponding to (16) reads

$$\begin{aligned}-\ln Z_A = \;& \mathrm{tr} \int_c^\infty \frac{dT}{T}\, e^{-Tm} \mathcal{N} \int_{x(0)=x(T)} [dx(\tau)]\, e^{-T/2\epsilon}\, \mathcal{P} \exp\left[\frac{1}{\epsilon}\int_0^T d\tau\, \gamma_\mu \dot{x}_\mu(\tau)\right] \\ & \times \exp\left[-\frac{1}{2\epsilon}\int_0^T d\tau\, \dot{x}(\tau)^2\right] \exp\left\{ig \int_0^T d\tau\, \dot{x}_\mu(\tau) A_\mu[x(\tau)]\right\} \,.\end{aligned} \tag{23}$$

III. EXTRACTION OF THE SPIN FACTOR

The path-integral expressions (16) and (23) arrived at in the previous section still retain the spacetime resolution scale ϵ. Our intention, on the other hand, was to attain a form for these expressions pertaining to the resolution being much smaller than the observation scale, i.e., being consistent with sending ϵ to zero. One possible way of eliminating ϵ, which bypasses direct confrontation of the trace over Dirac matrices, is to make use of the identities

$$\mathcal{N} \exp\left[-\frac{1}{2\epsilon}\int_0^T d\tau\, \dot{x}(\tau)^2\right] = \int [dp(\tau)] \exp\left[i \int_0^T d\tau\, p_\mu(\tau)\, \dot{x}_\mu(\tau)\right] \exp\left[-\frac{\epsilon}{2}\int_0^T d\tau\, p(\tau)^2\right] \,, \tag{24}$$

$$\begin{aligned}\mathcal{P} \exp\left[\frac{1}{\epsilon}\int_0^T d\tau\, \gamma_\mu\, \dot{x}_\mu(\tau)\right] \exp\left[i\int_0^T d\tau\, p_\mu(\tau)\, \dot{x}_\mu(\tau)\right] = \;& \mathcal{P} \exp\left[-\frac{i}{\epsilon}\int_0^T d\tau\, \gamma_\mu \frac{\delta}{\delta p_\mu(\tau)}\right] \\ & \times \exp\left[i \int_0^T d\tau\, p_\mu(\tau)\, \dot{x}_\mu(\tau)\right] \,,\end{aligned} \tag{25}$$

and

$$\begin{aligned}\mathcal{P} \exp\left[\frac{i}{\epsilon}\int_0^T d\tau\, \gamma_\mu \frac{\delta}{\delta p_\mu(\tau)}\right] \exp\left[-\frac{\epsilon}{2}\int_0^T d\tau\, p(\tau)^2\right] = \;& \mathcal{P} \exp\left(-i \int_0^T d\tau\, \gamma\cdot p\right) \\ & \times \exp\left(-\frac{\epsilon}{2}\int_0^T d\tau\, p^2\right) e^{T/2\epsilon} \,.\end{aligned} \tag{26}$$

Upon inserting the above identities into, say, (23) and integrating by parts, we can safely set ϵ to zero. We then obtain

$$\begin{aligned}\ln Z_A = -\mathrm{tr} \int_c^\infty \frac{dT}{T}\, e^{-Tm} & \int_{x(0)=x(T)} [dx(\tau)] \int [dp(\tau)] \exp\left(i \int_0^T d\tau\, \dot{x}\cdot p\right) \\ & \times \mathcal{P} \exp\left(-i \int_0^T d\tau\, \gamma\cdot p\right) \exp\left(ig \int_0^T d\tau\, \dot{x}\cdot A\right) \,.\end{aligned} \tag{27}$$

An alternative derivation of the above result, within the framework of the present approach, has been furnished in Ref. [16]. A Green function version of (27) has been obtained by Migdal [22] through a formal procedure applied on the local Dirac operator.

A different way to deal with the explicit dependence on ϵ in (16) and (23) is via the direct confrontation of the Dirac trace. To this end, we introduce a set of anticommuting variables $\psi_\mu(\tau)$, $\mu = 1,\ldots,d$, defined on a closed path parametrized by $\tau \in [0,T]$, imposing antiperiodic boundary conditions. Upon taking into account that the correlator $\langle\psi_\mu(\tau)\psi_\nu(\tau')\rangle$ with respect to the action $\frac{1}{4}\int_0^T d\tau\psi_\mu\dot{\psi}_\mu$ is simply $\frac{1}{2}\delta_{\mu\nu}\,\mathrm{sgn}(\tau-\tau')$, we determine

$$\begin{aligned}\int_{\mathrm{AP}}[d\vec{\psi}]\exp\left(\frac{1}{4}\int_0^T d\tau\,\psi_\mu\dot{\psi}_\mu\right)\mathcal{P}\exp\left(\frac{1}{\epsilon}\int_0^T d\tau\,\psi_\mu\dot{x}_\mu\right)\\ =\mathcal{P}\exp\left\{\frac{1}{\epsilon}\int_0^T d\tau\,\dot{x}_\mu(\tau)\frac{\delta}{\delta\xi_\mu(\tau)}\right\}\left\{\exp\left[\frac{1}{2}\int_0^T d\tau_1\int_0^T d\tau_2\,\xi_\mu(\tau_1)\,\mathrm{sgn}(\tau_1-\tau_2)\,\xi_\mu(\tau_1)\right]\right\}_{\xi=0}\\ =1+\frac{1}{\epsilon^2}\int_0^T d\tau_1\int_{\tau_1}^T d\tau_2\,\dot{x}_{\mu_1}(\tau_1)\,\dot{x}_{\mu_2}(\tau_2)\,\delta_{\mu_1\mu_2}+\frac{1}{\epsilon^4}\int_0^T d\tau_1\cdots\int_{\tau_3}^T d\tau_4\,\dot{x}_{\mu_1}(\tau_1)\cdots\dot{x}_{\mu_4}(\tau_4)\\ \times(\delta_{\mu_1\mu_2}\delta_{\mu_3\mu_4}-\delta_{\mu_1\mu_3}\delta_{\mu_2\mu_4}+\delta_{\mu_1\mu_4}\delta_{\mu_2\mu_3})+\cdots\,,\end{aligned}\tag{28}$$

where the path integral on the left-hand side (LHS) is normalized to unity for $\dot{x}=0$. For the case of even dimensions, we deduce, from the above expression,

$$\mathrm{tr}\mathcal{P}\exp\left(\frac{1}{\epsilon}\int_0^T d\tau\,\gamma\cdot\dot{x}\right)=\mathrm{tr}\mathbf{1}\int_{AP}[d\vec{\psi}]\exp\left(\frac{1}{4}\int_0^T d\tau\,\psi_\mu\dot{\psi}_\mu\right)\mathcal{P}\exp\left(\frac{1}{\epsilon}\int_0^T d\tau\,\psi_\mu\dot{x}_\mu\right)\,.\tag{29}$$

We shall now carry out an independent evaluation of the RHS of (29) via the following procedure: For a given closed curve C we introduce the orthonormal basis $\{\vec{e},\vec{n}_i\}$, $i=1,\ldots,d-1$, where $\vec{e}$ is a unit tangential vector [with $e_\mu(\tau)=\dot{x}_\mu(\tau)$] and $\vec{n}_i$ are mutually orthogonal unit vectors spanning the (hyper)plane perpendicular to the curve. Expanding $\vec{\psi}$ within this basis according to

$$\vec{\psi}=2\left(\phi_0\vec{e}-\phi_i\vec{n}_i\right)\,,\tag{30}$$

the integration over the Grassmannian modes (ϕ_0,ϕ_i) can be carried out explicitly. Now, motion along the curve generates the rates

$$\begin{aligned}\dot{\vec{e}}(\tau)&=B_i(\tau)\vec{n}_i(\tau)\,,\\ \dot{\vec{n}}_i(\tau)&=C_{ij}(\tau)\vec{n}_j(\tau)-B_i(\tau)\vec{e}(\tau)\,.\end{aligned}\tag{31}$$

By a suitable choice of coordinates, namely, $\vec{\psi}\cdot\vec{e}=\psi_\mu\dot{x}_\mu=\text{const.}$, the integral on the rhs of (29) will involve only the perpendicular modes. We thereby obtain

$$\begin{aligned}\int_{\mathrm{AP}}[d\vec{\psi}]\exp\left(\frac{1}{4}\int_0^T d\tau\,\psi_\mu\dot{\psi}_\mu\right)\mathcal{P}\exp\left(\frac{1}{\epsilon}\int_0^T d\tau\,\psi_\mu\dot{x}_\mu\right)\\ =e^{T/\epsilon}\int_{\mathrm{AP}}[d\phi_i]\exp\left[\int_0^T d\tau\left(\phi_i\dot{\phi}_i+C_{ij}\,\phi_i\phi_j\right)\right]\\ =e^{T/\epsilon}\,\det^{1/2}\left(\delta_{ij}\frac{d}{d\tau}+C_{ij}\right)\,.\end{aligned}\tag{32}$$

The quantity $\Phi(C)=\det^{1/2}\left(\delta_{ij}\frac{d}{d\tau}+C_{ij}\right)$, subject to antiperiodic boundary conditions in the interval $[0,T]$, furnishes the so-called spin factor which earmarks the propagation of spin-1/2 modes along the closed path C. It was first introduced by Polyakov [1,2] in connection with the *ab initio* path-integral description of pointlike excitations. A detailed study of $\Phi(C)$ was given subsequently by Korchemsky [23]. A work in which it is shown how the spin factor follows from the action for a spinning particle is Ref. [24]. The extension to higher spins was treated in [25]. In our case, the spin factor emerged naturally during the procedure of translating an original casting of spin-1/2 systems as field theories, with a built-in resolution scale, into (particle) path-integral form.

Let us formulate our final conclusion, which strictly holds in even dimensions, as follows:

$$\mathrm{tr}\mathcal{P}\exp\left(\frac{1}{\epsilon}\int_0^T d\tau\,\dot{x}\cdot\gamma\right)=\exp\left(T/\epsilon\right)\,\mathrm{tr}\Phi(C)\,.\tag{33}$$

For an odd number of dimensions, its validity is contingent upon the insertion of (29) into the path integral. The Gaussian factor $\exp\left[-\frac{1}{2\epsilon}\int_0^T d\tau\,\dot{x}(\tau)^2\right]$ entering (16) and (23) facilitates the elimination of terms with an odd number of γ matrices and the even-dimensional situation is thereby reproduced. In principle, then, the contribution to the path integral from the spin factor in the odd-dimensional case entails the complete tracing of each closed path.

The preceding analysis has led to the following path-integral representation of the full effective action in the presence of an external set of gauge fields:

$$-\ln Z_A = \int_c^\infty \frac{dT}{T}\, e^{-Tm}\mathcal{N}e^{T/2\epsilon}\int_{x(0)=x(T)}[dx(\tau)]\exp\left(-\frac{1}{2\epsilon}\int_0^T d\tau\,\dot{x}^2\right)\exp\left(ig\int_0^T d\tau\,\dot{x}\cdot A\right)\mathrm{tr}\Phi(C)\,. \tag{34}$$

To make final contact with Polyakov's Euclidean path-integral formula [cf. (1)], we simply need to take into account that

$$\begin{aligned}\lim_{\epsilon\to 0}\int_{x(0)=x(\tau)}[dx(\tau)]\,\exp\left\{-\frac{1}{2\epsilon}\int_0^T d\tau\,[\dot{x}^2(\tau)-1]\right\} &= \int_{0^+-i\infty}^{0^++i\infty}[d\alpha(\tau)]\,\exp\left[\int_0^T d\tau\,\alpha(\tau)\right]\int[dx(\tau)]\\ &\quad\times\exp\left[-\int_0^T d\tau\,\alpha(\tau)\,\dot{x}^2(\tau)\right]\\ &= \int[dx(\tau)]\,\delta\left[\dot{x}^2(\tau)-1\right]\,.\end{aligned} \tag{35}$$

In what follows we shall systematically work with (34) and pursue the limit $\epsilon\to 0$ on physical, rather than formal, grounds.

IV. EXTRACTION OF DIRAC DETERMINANT TERMS IN EVEN DIMENSIONS

The fact that (33) holds as an identity for an even number of dimensions implies that the spin-factor contribution can be surmised on the basis of global loop characteristics; it does not require the step-by-step tracing of the closed contour C. Identifying global extrinsic properties of closed curves in two dimensions is a simple task, as self-intersection becomes the only relevant property. Multiple traversals (equivalently, reentries) of simple loops without change of sense (see Fig. 1) define classes characterized by the integer ν (= No. loop repetitions). The spin factor tabulates the motion of the perpendicular "plane" (the normal vector in this case) and provides a contribution of the form $(-1)^{\nu+1}$.

In this section we shall restrict ourselves to situations where classes associated with multiple repetitions of simple curves furnish complete results. Configurations such as figure-8-shaped crossings (see Fig. 2) in two dimensions lead to interesting topological analyses which will be discussed subsequently. In particular, we shall put the spin factor to work by letting it lead us to the free Dirac determinant and to quadratic, with respect to an Abelian gauge potential, effective-action terms for $d=2,4$.

As already mentioned, self-intersection is the only extrinsic feature of closed curves in two dimensions. For multiple traversals of a simple loop C, the spin-factor contribution for $d=2$ becomes

$$\sum_{\{C\}}\mathrm{tr}\Phi(C) = \mathrm{tr}\mathbf{1}\sum_{\nu=0}^{\infty}(-1)^\nu = \frac{1}{2}\mathrm{tr}\mathbf{1} = 1\,. \tag{36}$$

In four dimensions, we expect a similar arrangement to produce a complete result, as long as one is dealing with Abelian gauge potentials (or, of course, the free theory). We shall not attempt to go beyond the Abelian (QED) case in this paper. Repetition of simple closed curves will, therefore, generate the contribution

$$\sum_{\{C\}}\mathrm{tr}\Phi(C) = \mathrm{tr}\mathbf{1}\sum_{\nu=0}^{\infty}(-1)^\nu = \frac{1}{2}\mathrm{tr}\mathbf{1} = 2\,. \tag{37}$$

Let us start with the calculation of the free Dirac determinant. As it turns out, this task is instructive not only because it will identify the impact of the spin-factor contribution on the final result, but also because it will lead us to appreciate the role played by the distribution $\exp\left[-(1/2\epsilon)\int_0^T d\tau\dot{x}(\tau)^2\right]$, which regulates the flow along "proper time." It should be remembered that this factor is a direct consequence of the original regularized casting of the Dirac field system. Our first order of business is to harmonize the velocity-dependent factors entering the integrand in (34) with the integration measure $[dx(\tau)]$. Assuming that we are dealing with, almost everywhere, differentiable curves, we introduce the following change of variables:

$$\begin{aligned}\dot{x}_\mu(\tau) &= e_\mu(\tau)\,,\\ x_\mu(\tau) &= x_\mu(0)+\int_0^\tau dt e_\mu(t)\,.\end{aligned} \tag{38}$$

To effect the utility of this change of variables, we insert into (34), modulo the Wilson factor, the identity (using for the sake of simplicity a vector notation)

$$\int[d\epsilon\, e(t)]\delta(\vec{e}-\dot{\vec{x}}) = 1\,. \tag{39}$$

Once due consideration is payed to normalization factors, and after the periodicity codition $x(0)=x(T)$ is taken into account, we arrive at the following result for the "free effective action":

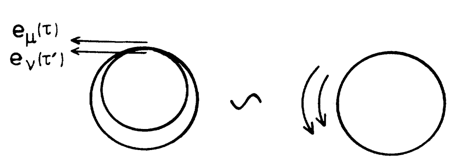

FIG. 1. Self-intersecting curve embedded in $\mathbb{R}^2$ with $e_\mu(t)$ parallel to $e_\mu(t')$ for $t\neq t'$.

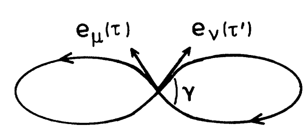

FIG. 2. Figure-8, self-intersecting curve embedded in $\mathbb{R}^2$ as a realization of Eq. (65) in the text.

$$\ln Z^{\epsilon,c}_{\text{free}} = -\int d^d x_0 \int_0^\infty \frac{dT}{T}\, e^{T/2\epsilon} e^{-Td\ln(2\pi)/2\epsilon} \times \int \frac{d^d\lambda}{(2\pi)^d} \int [de(\tau)] \exp\left[-i\lambda_\mu \int_0^T d\tau e_\mu(\tau) - \frac{1}{2\epsilon}\int_0^T d\tau e^2(\tau)\right] \text{tr}\Phi(C)\ , \tag{40}$$

where we have set $x(0) = x_0$. Notice that the periodicity condition is hidden in the factor $\int \frac{d^d\lambda}{(2\pi)^d} \exp\left[-i\lambda_\mu \int_0^T d\tau e_\mu(\tau)\right]$, which furnishes the δ function $\delta\left[\int_0^T d\tau e_\mu(\tau)\right]$.

We now make the transformation $e_\mu(\tau) \to e_\mu(\tau) - i\epsilon\lambda_\mu$, whereupon we get

$$\ln Z^{\epsilon,c}_{\text{free}} = -\int d^d x_0 \int_c^\infty \frac{dT}{T}\, e^{T/2\epsilon} e^{-Td\ln(2\pi)/2\epsilon} \int \frac{d^d\lambda}{(2\pi)^d}\, e^{-\epsilon T\lambda^2/2} \int [de(\tau)] \exp\left[-\frac{1}{2\epsilon}\int_0^T d\tau\, e^2(\tau)\right] \text{tr}\,\Phi(C)\ . \tag{41}$$

Now the calculation goes through with relative ease. The functional integration can be performed first to give

$$\int [de(\tau)] \exp\left\{-\frac{1}{2\epsilon}\int_0^T d\tau\, e^2(\tau)\right\} = \lim_{N\to\infty} \prod_{i=1}^N (2\pi)^{d/2} = \lim_{\epsilon\to 0} \exp\left\{[Td\ln(2\pi)]/2\epsilon\right\}\ . \tag{42}$$

Substituting this in (41), we obtain after reenstating the mass term

$$-\ln Z'_{\text{free}} = \int d^d x_0 \int_c^\infty \frac{dT}{T}\, e^{-T(m-\frac{1}{\epsilon})/2} \int \frac{d^d\lambda}{(2\pi)^d}\, e^{-\epsilon T\lambda^2/2}\ , \tag{43}$$

where the prime serves to remind us that we are, temporarily, ignoring the presence of the spin factor.

We now make the redefinition $T \to (\epsilon T)/2$ by which the new parameter T acquires units of $(\text{mass})^{-2}$. We rename the new cutoff for the T integral as $\Lambda^{-2} \equiv c\epsilon/2$; this will serve as an overall momentum cutoff in the limit of zero resolution scale $\epsilon \to 0$. Then (43) becomes

$$-\ln Z'_{\text{free}} = \int d^d x_0 \int_{\Lambda^{-2}}^\infty \frac{dT}{T}\, e^{-T(m-\bar{m})/\epsilon} \int \frac{d^d\lambda}{(2\pi)^d}\, e^{-T\lambda^2}\ , \tag{44}$$

where $\bar{m} = 1/\epsilon$.

We immediately surmise that the limit $\epsilon \to 0$ entails an adjustment of the "mass" parameter m according to which

$$m(\epsilon) \underset{\epsilon\to 0}{\sim} \bar{m} = \frac{1}{\epsilon}\ . \tag{45}$$

The quantity

$$M^2 = \frac{1}{\epsilon}(m - \bar{m}) \tag{46}$$

defines, in the limit where the resolution scale goes to zero, a physical mass (squared) parameter for the spin-1/2 particle.

For the free Dirac particle, where no topological complications arise during its propagation, the spin-factor contribution reduces to (36). Consequently, (44) becomes

$$\begin{aligned} -\ln Z_{\text{free}} &= \frac{1}{2}\text{tr}\mathbf{1} \int d^d x_0 \int \frac{d^d\lambda}{(2\pi)^d} \int_{\Lambda^{-2}}^\infty \frac{dT}{T}\, e^{-T(M^2+\lambda^2)} \\ &= \frac{1}{2}\text{Tr}\ln\left(-\partial^2 + M^2\right)\ , \end{aligned} \tag{47}$$

where d takes only even values.

We recognize in the above result the (logarithm of) the Dirac determinant in second-order formalism (and even dimensions). Note, moreover, that the crucial factor of 1/2 has been supplied by the spin factor.

In conclusion, not only did our regularization procedure at the field-theory level retain all continuum properties of the free Dirac system, but the limit $\epsilon \to 0$ has given rise to quantities ascribing to spin and physical mass of a free particle. This is very satisfying, indeed, given that the recovery of continuous spacetime structure, as $\epsilon \to 0$, goes hand in hand with the two quantities which correspond to eigenvalues of the Casimir operators of the (continuous) spacetime symmetry group.

We next turn our attention to quadratic, in the external gauge field, effective-action terms. To this end, let us replace $A_\mu(x)$, which enters (10) via the Wilson loop factor, by its Fourier expansion

$$A_\mu[x(\tau)] = \int \frac{d^d k}{(2\pi)^d}\, e^{-ik\cdot x(\tau)} \tilde{A}_\mu(k)\ . \tag{48}$$

Expanding, in turn, the Wilson loop exponential in powers of $\tilde{A}$ and isolating the quadratic term, we end up with the expression

$$I_2 = \frac{1}{2}g^2 \int d^dx \int \frac{d^dk}{(2\pi)^d}\frac{d^dk'}{(2\pi)^d}\, e^{-i(k+k')\cdot x}\tilde{A}_\mu(k)\tilde{A}_\nu(k') \int_c^\infty \frac{dT}{T}\, e^{T/2\epsilon}\, e^{-Td\ln(2\pi)/2\epsilon}$$
$$\times \int \frac{d^d\lambda}{(2\pi)^d}\int_0^T d\tau \int_0^T d\tau' \int [de(\tau)] e_\mu(\tau)e_\nu(\tau')$$
$$\times \exp\left[-\frac{1}{2\epsilon}\int_0^T d\tilde{\tau}\, e(\tilde{\tau})^2 - i\int_0^T d\tilde{\tau}\, \lambda\cdot e(\tilde{\tau}) + i\int_0^{\tau'} d\tilde{\tau}\, k'\cdot e(\tilde{\tau}) + i\int_0^{\tau} d\tilde{\tau}\, k\cdot e(\tilde{\tau})\right] \mathrm{tr}\Phi(C)\ . \tag{49}$$

As with the free-field case, the spin-factor contribution will be based on multiple traversals of simple closed curves. For QED, this is a satisfactory working hypothesis. In even dimensions, the aforementioned contribution factorizes to $\frac{1}{2}\mathbf{1}$.

We now isolate the quantity (after performing the x integration)

$$W_{\mu\nu} = \frac{1}{2}\int \frac{d^d\lambda}{(2\pi)^d}\int_0^T d\tau \int_0^T d\tau' \int [de(\tau)] e_\mu(\tau)e_\nu(\tau')$$
$$\times \exp\left[-\frac{1}{2\epsilon}\int_0^T d\tilde{\tau}\, e(\tilde{\tau})^2 - i\int_0^T d\tilde{\tau}\, \lambda\cdot e(\tilde{\tau}) + i\int_0^{\tau'} d\tilde{\tau}\, k\cdot e(\tilde{\tau}) - i\int_0^{\tau} d\tilde{\tau}\, k\cdot e(\tilde{\tau})\right]\ , \tag{50}$$

which, owing to the periodicity relation $\int_0^T d\tau e_\mu(\tau) = 0$, has the structure

$$W_{\mu\nu} = f(k^2)\left(k^2\delta_{\mu\nu} - k_\mu k_\nu\right)\ . \tag{51}$$

For the same reason, the velocity vectors $e_\mu(\tau)$ can be represented via a Fourier series in the interval $[0,T]$:

$$e_\mu(\tau) = \frac{1}{\sqrt{T}}\sum_{n=-\infty}^{\infty} \alpha_\mu \exp\left[(2\pi i n\tau)/T\right]\ , \qquad \alpha_\mu^*(n) = \alpha_\mu(-n)\ . \tag{52}$$

Once this is done, the calculation of the host of integrals entering I_2 proceeds with relative ease. Relegating the actual task to the Appendix, we here quote the final result:

$$I_2 = -\frac{2}{(4\pi)^{d/2}}\, g^2 \int \frac{d^dk}{(2\pi)^d}\, \tilde{A}_\mu(k)\left(k^2\delta_{\mu\nu} - k_\mu k_\nu\right)\tilde{A}_\nu(-k)$$
$$\times \int_0^1 dx x(1-x)\left[x(1-x)k^2 + M^2\right]^{d/2-2}\Gamma\left(2-\frac{d}{2}, x(1-x)\frac{k^2}{\Lambda^2} + \frac{M^2}{\Lambda^2}\right)\ . \tag{53}$$

Specializing to particular dimensions, we obtain the final results. For $d = 2$ [$\mathrm{tr}\,\Phi(C) = 1$],

$$S_{\text{eff}}[A] = -\frac{g^2}{2\pi}\int \frac{d^2k}{(2\pi)^2}\, \tilde{A}_\mu(k)(k^2\delta_{\mu\nu} - k_\mu k_\nu)\tilde{A}_\nu(-k)\int_0^1 dx\, \frac{x(1-x)}{x(1-x)k^2 + M^2}\ . \tag{54}$$

For $d = 4$ [$\mathrm{tr}\,\Phi(C) = 2$]

$$S_{\text{eff}}[A] = -\frac{g^2}{4\pi^2}\int \frac{d^4k}{(2\pi)^4}\, \tilde{A}_\mu(k)\left(k^2\delta_{\mu\nu} - k_\mu k_\nu\right)\tilde{A}_\nu(-k)\int_0^1 dx x(1-x)\Gamma\left(0, x(1-x)\frac{k^2}{\Lambda^2} + \frac{M^2}{\Lambda^2}\right)\ . \tag{55}$$

For $d = 2$ and $M = 0$, one readily recovers the exact and well-known result (in Minkowski version)

$$S_{\text{eff}}[A] = \frac{g^2}{2\pi}\int \frac{d^2k}{(2\pi)^2}\, \tilde{A}_\mu(k)\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\tilde{A}_\nu(-k)\ . \tag{56}$$

For $d = 4$, (55) reveals the need for wave-function renormalization since a logarithmic divergence is lurking in the incomplete gamma function. Specifically, we find

$$S_{\text{eff}}[A] = \frac{g^2}{48\pi^2}\ln\left(\frac{\Lambda^2}{M^2}\right)\int d^4x\, F_{\mu\nu}^2$$
$$= \frac{\alpha}{3\pi}\ln\left(\frac{\Lambda^2}{M^2}\right)\frac{1}{4}\int d^4x\, F_{\mu\nu}^2\ , \tag{57}$$

where α is the fine structure constant.

Assume, for a moment, that our theory included a Maxwell term in the action. Overall, we would then write $(1 - Z_3)\frac{1}{4}\int d^4xF_{\mu\nu}^2$ which identifies, to first order in α, the well-known wave-function renormalization constant for the electromagnetic field in four-dimensional QED.

V. TOPOLOGICAL TERMS IN EFFECTIVE ACTIONS

Remaining faithful to our commitment to an Abelian background gauge field, we shall proceed in this section to study the emergence of topological terms in effective actions. On general grounds, we expect to encounter an instanton and a Chern-Simons term in two and three spacetime dimensions, respectively. Let us first consider the $(1+1)$-dimensional situation. Keeping in (34) only the linear term, we write, after employing the change of variables introduced in (38) and (48),

$$I_1 = ig\int d^2x \int \frac{d^2k}{(2\pi)^2}\, \tilde{A}_\mu(k) e^{-ik\cdot x} \int_c^\infty \frac{dT}{T}\, e^{T[1-2\ln(2\pi)]/2\epsilon} \int \frac{d^2\lambda}{(2\pi)^2}\int [de(\tau)] \times \exp\left[-i\lambda_\mu \int_0^T d\tau e_\mu(\tau) - \frac{1}{2\epsilon}\int_0^T d\tau e^2(\tau)\right] \int_0^T d\tau\, e_\mu(\tau) \exp\left[-ik_\mu \int_0^\tau dt\, e_\mu(t)\right] \mathrm{tr}\Phi(C)\ . \tag{58}$$

We must be careful not to exchange the x integration with the k integration before ascertaining what is the exact nature of surface contributions. In fact, since we are interested in topological terms, we must be particularly cautious as to what happens at the integration limits. We now perform the derivative expansion

$$\int_0^T d\tau\, e_\mu(\tau) \exp\left[-ik_\mu \int_0^\tau dt\, e_\mu(t)\right] = \int_0^T d\tau\, e_\mu(\tau)\left\{1 - ik_\nu \int_0^\tau dt\, e_\nu(t) - k_\nu k_\rho \int_0^\tau dt \int_t^\tau dt'\, e_\nu(t) e_\rho(t') + \dots\right\}. \tag{59}$$

The first, i.e., the zeroth-order term, involves $\int_0^T d\tau e_\mu(\tau)$ and gives a vanishing contribution on account of the δ function in (58) (which results from the λ integration). The second term provides the factor

$$\begin{aligned} A_{\mu\nu} &\equiv \int_0^T d\tau \int_0^\tau dt\, e_\mu(\tau) e_\nu(t) \\ &= \int_0^T d\tau \int_0^T dt\, e_\mu(\tau) e_\nu(t) \theta(\tau - t)\ . \end{aligned} \tag{60}$$

It is easy to show that only the antisymmetric part of $A_{\mu\nu}$ contributes to the integral. Indeed,

$$A_{\mu\nu} + A_{\nu\mu} = \int_0^T d\tau\, e_\mu(\tau) \int_0^T dt\, e_\nu(t)\ , \tag{61}$$

which vanishes upon insertion into (58).

Designating the contribution of the second term to I_1 by $I_{1.2}$, we write

$$I_{1.2} = \frac{g}{2}\int d^2x \int \frac{d^2k}{(2\pi)^2}\, k_\nu\, \tilde{A}_\mu(k)\, e^{-ik\cdot x} \int_c^\infty \frac{dT}{T}\, e^{T/2\epsilon}\, e^{-T\ln(2\pi)/\epsilon} \int_0^T d\tau \int_\tau^T dt\, \langle e_\mu(\tau) e_\nu(t) - e_\nu(\tau) e_\mu(t)\rangle\ , \tag{62}$$

where we have explicitly kept the antisymmetric part of $A_{\mu\nu}$, and we have introduced

$$\langle\,\cdot\,\rangle \equiv \int \frac{d^2\lambda}{(2\pi)^2}\int [de(\tau)]\,(\,\cdot\,) \exp\left[-i\lambda_\mu \int_0^T d\tau\, e_\mu(\tau) - \frac{1}{2\epsilon}\int_0^T d\tau\, e^2\right] \mathrm{tr}\Phi(C)\ . \tag{63}$$

Our calculation will proceed by omitting the spin factor since its contribution factorizes in two dimensions. We first make the transformation $e_\mu(\tau) \to e_\mu(\tau) - i\epsilon\lambda_\mu$ which considerably simplifies matters, since

$$-i\lambda_\mu \int_0^T d\tau\, e_\mu(\tau) - \frac{1}{2\epsilon}\int_0^T d\tau\, e^2(\tau) \to -\frac{\epsilon T \lambda^2}{2} - \frac{1}{2\epsilon}\int_0^T d\tau\, e^2(\tau)\ . \tag{64}$$

On the other hand, the change which is induced on $e_{[\mu}(\tau) e_{\nu]}(t)$ by the above transformation amounts to linear e terms which give zero contribution on account of the Gaussian. All in all, we are faced with the calculation of the average $\langle e_{[\mu}(\tau) e_{\nu]}(t)\rangle$, with respect to the Gaussian $\exp\left[\frac{1}{2\epsilon}\int_0^T dt\, e^2(t)\right]$. It is straightforward to see that the only nonzero contribution to this average comes from self-intersecting junctions in the sense of figure-8 paths (see Fig. 2), described by the relation

$$e_\mu(\tau) = \epsilon_{\mu\nu} e_\nu(t)\, \sin\gamma + e_\mu(t)\, \cos\gamma\ , \qquad t \neq \tau, \tag{65}$$

with a subsequent averaging over the angle γ. For future purposes, we remark that the figure-8 path configuration that has been just singled out furnishes a zero overall rotation of the normal (or tangent) vector when traversed completely and uniformly.

We proceed to compute the "expectation value" $\langle e_{[\mu}(\tau)e_{\nu]}(t)\rangle$ ignoring the presence of the factor $\mathrm{tr}\Phi(C)$. All the contribution comes from the vicinity of the point of self-intersection p, defined by the spacetime resolution scale ϵ. By (65), we determine that

$$\langle e_{[\mu}(\tau)e_{\nu]}(t)\rangle' = \epsilon_{\nu\mu}\langle e^2(\tau)\rangle' = \epsilon_{\nu\mu}\exp\left[T\ln(2\pi)/\epsilon\right] , \tag{66}$$

where the prime signifies absence of the spin-factor contribution. Consequently, (62) becomes

$$I'_{1.2} = g\,\epsilon\int d^2x\int\frac{d^2k}{(2\pi)^2}\,\epsilon_{\nu\mu}\,k_\mu\,\tilde{A}_\nu(k)\,e^{-ik\cdot x}\int_c^\infty dT\,e^{T/\epsilon}\int\frac{d^2\lambda}{(2\pi)^2}\,e^{-\epsilon T\lambda^2/2} , \tag{67}$$

where we have included the contribution from integrations over parameter integrals

$$\int_0^T d\tau\int_{p-\epsilon}^{p+\epsilon} dt = 2T\epsilon . \tag{68}$$

Inserting a mass term and making the redefinition $T\to T\epsilon/2$, we arrive at the result

$$I'_{1.2} = -\frac{g}{2\pi}\,\Gamma\left(0,\frac{M^2}{\Lambda^2}\right)\int d^2x\int\frac{d^2k}{(2\pi)^2}\,\epsilon_{\mu\nu}\,k_\mu\,\tilde{A}_\nu(k)e^{-ik\cdot x} , \tag{69}$$

where M is the physical mass [cf. (46)] and $1/\Lambda^2$ is the new cutoff for the T integral. The above expression can be recast into the more suggestive form

$$I'_{1.2} = \frac{\theta}{2\pi}\int d^2x\,\epsilon_{\mu\nu}\,\partial_\mu A_\nu , \tag{70}$$

which has the familiar structure of an Abelian instanton θ term in $1+1$ dimensions.

The fact that up to this point the spin factor has been left out of the calculation assures us that the derived result applies, certainly, to theories with scalar matter fields. The Abelian Higgs model, in particular, serves as a good playground for studying instanton effects. In this connection, recall that θ parametrizes the vacua of distinct Fock-space structures which cannot communicate with each other through gauge-invariant operators. Only statistical mixing of these sectors makes sense.

Restoring our attention to fermions, we now consider the spin-factor contribution. As already mentioned, the perpendicular vector to the curve at each point undergoes zero total rotation as the figure-8 closed path is completely traversed, whether one or more times. Moreover, no additional contributions to $\langle e_{[\mu}(\tau)e_{\nu]}(t)\rangle$ come from multiple crossing of the self-intersection point p as they cancel in pairs, save for the final unsaturated crossing, as a result of opposite orientations of the two tangent vectors. All in all, the spin factor contribution from this set of closed diagrams is $\mathrm{tr}\mathbf{1} = 2$; i.e., (70) should be multiplied by a factor of 2.

Combinations of orientation-preserving and orientation-changing crossings lead to new situations. Suppose that the figure-8 configuration is multiply traversed, but in such a way that the number of repetitions, n_+, of the right branch differs from the corresponding number n_- for the left branch. Notice that n_+ and n_- refer to rotations with opposite helicities. The spin factor for a given type of traversal is of the form $\mathrm{tr}(-1)^{n_+-n_-}$. This implies that the complete contribution to $I_{1.2}$ for a fermionic theory is multiplied by the vanishing factor $\mathrm{tr}\sum_{\nu=-\infty}^{\infty}(-1)^\nu$. In conclusion, if all configurations are included, we obtain a manifestly gauge-invaniant result according to which the θ term is rigorously zero.

One final remark should be made regarding higher-order terms in the derivative expansion contributing to I_1. One easily verifies that these terms vanish in the limit $\epsilon\to 0$, so that there is no need to be considered.

We now turn our attention to three spacetime dimensions in an attempt to unearth the Chern-Simons term. Closed paths in $\mathbb{R}^3$ cannot be tabulated on the basis of a global geometric embedding feature. The phase-space path integral presents a convenient alternative. Indeed, nothing is missed as long as the various integrations entering (27) are carried through, irrespective of whether one works in even or odd dimensions.

Turning to (27), we consider its expansion in powers of the gauge potential. We shall go a certain distance keeping the spacetime dimensions unspecified before we specialize to $d=3$. Upon making the variable changes (38), (48), the quadratic term of the effective action according to (27) is

$$F_2 = g^2 \,\mathrm{tr} \int \frac{d^dk}{(2\pi)^d}\, \tilde{A}_\mu(k)\tilde{A}_\nu(-k) \int_c^\infty \frac{dT}{T} \int_0^T d\tau \int_\tau^T d\tau' \int \frac{d^d\lambda}{(2\pi)^d} \int [de(\tau)][dp(\tau)] e_\mu(\tau) e_\nu(\tau')$$
$$\times \exp\left[i\int_0^T d\tilde{\tau}\, p(\tilde{\tau})\cdot e(\tilde{\tau}) - i\int_0^T d\tilde{\tau}\, \lambda\cdot e(\tilde{\tau}) + i\int_\tau^{\tau'} d\tilde{\tau}\, k\cdot e(\tilde{\tau})\right] \mathcal{P}\exp\left[-i\int_0^T d\tilde{\tau}\,\gamma\cdot p(\tilde{\tau})\right] . \quad (71)$$

Consider now the quantity

$$W_{\mu\nu}[p] = \int [de(\tau)] \exp\left[i\int_0^T d\tilde{\tau}\, p(\tilde{\tau})\cdot e(\tilde{\tau}) - i\int_0^T d\tilde{\tau}\, \lambda\cdot e(\tilde{\tau}) + i\int_0^{\tau'} d\tilde{\tau}\, k\cdot e(\tilde{\tau})\right] e_\mu(\tau) e_\nu(\tau')$$
$$\equiv -\frac{\delta}{\delta p_\mu(\tau)} \frac{\delta}{\delta p_\nu(\tau')}\, \Delta[p] , \quad (72)$$

where we have defined

$$\Delta[p] \equiv \int [de(\tau)] \exp\left[i\int_0^T d\tilde{\tau}\, p(\tilde{\tau})\cdot e(\tilde{\tau}) - i\int_0^T d\tilde{\tau}\, \lambda\cdot e(\tilde{\tau}) + i\int_\tau^{\tau'} d\tilde{\tau}\, k\cdot e(\tilde{\tau})\right]$$
$$= \prod_{(0,\tau]} (2\pi)^d \delta\left[p(\tilde{\tau}) - \lambda\right] \prod_{(\tau,\tau']} (2\pi)^d \delta\left[p(\tilde{\tau}) - \lambda + k\right] \prod_{(\tau',T]} (2\pi)^d \delta\left[p(\tilde{\tau}) - \lambda\right] . \quad (73)$$

The comprehensive notation in the last line, though formal in appearance, is self-apparent. More generally, the foregoing formulas, as well as the ones that follow, are well defined as long as the resolution scale is not set equal to zero.

The latter equation along with (71) leads to

$$F_2 = -g^2 \mathrm{tr} \int \frac{d^dk}{(2\pi)^d}\, \tilde{A}_\mu(k)\tilde{A}_\nu(-k) \int_c^\infty \frac{dT}{T} \int_0^T d\tau \int_\tau^T d\tau' \int \frac{d^d\lambda}{(2\pi)^d}$$
$$\times \int [dp(\tau)] \mathcal{P}\exp\left[-i\int_0^T d\tilde{\tau}\,\gamma\cdot p(\tilde{\tau})\right] \frac{\delta}{\delta p_\mu(\tau)} \frac{\delta}{\delta p_\nu(\tau')} \prod_{(0,\tau]} (2\pi)^d \delta\left[p(\tilde{\tau}) + \lambda\right]$$
$$\times \prod_{(\tau,\tau']} (2\pi)^d \delta\left[p(\tilde{\tau}) + \lambda - k\right] \prod_{(\tau',T]} (2\pi)^d \delta\left[p(\tilde{\tau}) + \lambda\right] . \quad (74)$$

The functional integration can be easily performed to give

$$F_2 = g^2 \int \frac{d^dk}{(2\pi)^d}\, \tilde{A}_\mu(k)\tilde{A}_\nu(-k) \int_c^\infty \frac{dT}{T} \int_0^T d\tau \int_\tau^T d\tau' \int \frac{d^d\lambda}{(2\pi)^d} \mathrm{tr}\left[e^{-i\tau \not{\lambda}} \gamma_\mu\, e^{-i(\tau-\tau')(\not{\lambda}-\not{k})}\, \gamma_\nu\, e^{-i(T-\tau')\not{\lambda}}\right] . \quad (75)$$

Upon making the variable redefinitions $\tau = Tz_1$ and $\tau' = Tz_2$, as well as inserting a mass term, we find

$$F_2 = g^2 \int \frac{d^dk}{(2\pi)^d}\, \tilde{A}_\mu(k)\tilde{A}_\nu(-k) \int_c^\infty dT\,T \int_0^1 dz_1 \int_{z_1}^1 dz_2 \mathrm{tr}\left[\gamma_\mu\, e^{-T(z_2-z_1)[i(\not{\lambda}-\not{k})+m]}\, \gamma_\nu\, e^{-T[1-(z_2-z_1)](i\not{\lambda}+m)}\right] . \quad (76)$$

Note that m has the meaning of a physical mass because the resolution scale has been set equal to zero.

A suitable redefinition of T by which it acquires units of $(\text{mass})^{-2}$ along with integration over one of the z parameters leads to

$$F_2 = \frac{g^2}{2} \int \frac{d^dk}{(2\pi)^d}\, \tilde{A}_\mu(k)\tilde{A}_\nu(-k) \int_{\Lambda^{-2}}^\infty dT\,T \int_0^1 dx \int \frac{d^d\lambda}{(2\pi)^d}$$
$$\times \mathrm{tr}\left\{\gamma_\mu\left[-i(\not{\lambda}-\not{k}) + m\right]\gamma_\nu\left(-i\not{\lambda}+m\right)\right\} e^{-Tx\left[(\lambda-k)^2+m^2\right] - T(1-x)\left(\lambda^2+m^2\right)} . \quad (77)$$

We recognize in the above expression the one-loop contribution to the effective action in the Feynman-diagram sense. Accordingly, once we set $d = 3$, the calculation will be reduced to the one performed by Deser, Jackiw, and Templeton [26] where the Chern-Simons term was first identified. Indeed, one routinely determines

$$\mathrm{tr}\left\{\gamma_\mu\left[-i(\not{\lambda}-\not{k}) + m\right]\gamma_\nu(-i\not{\lambda}+m)\right\} = -2\left[(\lambda-k)_\mu\lambda_\nu + (\lambda-k)_\nu\lambda_\mu - (\lambda-k)\cdot\lambda\,\delta_{\mu\nu} - m^2\delta_{\mu\nu}\right] + 2mk_\rho\epsilon_{\rho\mu\nu} \quad (78)$$

by which F_2 naturally splits into two parts: $F_2 = F_2^{(1)} + F_2^{(2)}$.

Straightforward manipulations give

$$F_2^{(1)} = -\frac{2g^2}{(4\pi)^{3/2}} \int \frac{d^3k}{(2\pi)^3} \tilde{A}_\mu(k)\tilde{A}_\nu(-k) \int_{\Lambda^{-2}}^{\infty} dT T^{-1/2} \int_0^1 dx\, x(1-x)\, e^{-T\left[x(1-x)k^2+m^2\right]} \left(k^2\delta_{\mu\nu} - k_\mu k_\nu\right) \tag{79}$$

and

$$F_2^{(2)} = \frac{mg^2}{(4\pi)^{3/2}} \int \frac{d^3k}{(2\pi)^3} \tilde{A}_\mu(k)\tilde{A}_\nu(-k)\, k_\rho\, \epsilon_{\rho\mu\nu} \int_0^1 dx \int_{\Lambda^{-2}}^{\infty} dT T^{-1/2}\, e^{-T\left[x(1-x)k^2+m^2\right]} . \tag{80}$$

Each T integration yields the factor $\left[x(1-x)k^2+m^2\right]^{-1/2}\Gamma\left(\frac{1}{2}, x(1-x)\frac{k^2}{\Lambda^2}+\frac{m^2}{\Lambda^2}\right)$, so that in the limit $\Lambda \to \infty$, we obtain

$$F_2^{(1)} = -\frac{g^2}{4\pi} \int \frac{d^3k}{(2\pi)^3} \tilde{A}_\mu(k)\tilde{A}_\nu(-k)(k^2\delta_{\mu\nu} - k_\mu k_\nu) \int_0^1 dx \frac{x(1-x)}{\left[x(1-x)k^2+m^2\right]^{1/2}} \tag{81}$$

and

$$F_2^{(2)} = \frac{mg^2}{8\pi} \int \frac{d^3k}{(2\pi)^3} \tilde{A}_\mu(k)\tilde{A}_\nu(-k)\, k_\rho\, \epsilon_{\rho\mu\nu} \int_0^1 dx \frac{1}{\left[x(1-x)k^2+m^2\right]^{1/2}} . \tag{82}$$

We remark that (81) would result had we applied the methodology of the preceding section (multiple traversals of simple loops). The Chern-Simons term, on the other hand, would have been completely missed. In a derivative expansion the leading terms furnish the result

$$F_2 = -\frac{mg^2}{8\pi|m|} \int \frac{d^3k}{(2\pi)^3} \tilde{A}_\mu(k)\tilde{A}_\nu(-k)\, k_\rho\, \epsilon_{\mu\nu\rho} - \frac{g^2}{24\pi|m|} \int \frac{d^3k}{(2\pi)^3} \tilde{A}_\mu(k)\tilde{A}_\nu(-k) \left(k^2\delta_{\mu\nu} - k_\mu k_\nu\right) + O\left(\frac{1}{m^2}\right) . \tag{83}$$

We recognize in the first term the Chern-Simons action originally discovered by Redlich [27] via a different method.

VI. CONCLUSIONS

In this paper we have developed an approach to spin-1/2 systems in Euclidean spacetime which reformulates their original field-theoretical representation into a particle-based one. In this way the propagation of field quanta attains a geometric mode of description in terms of intrinsic and extrinsic properties of paths, identical to that postulated by Polyakov [1,2] for particlelike excitations.

An important aspect of our methodology is that we employed a two-step procedure before arriving at the final path-integral casting of the systems under study. First, the field-theoretical action was so defined as to embody *ab initio* a spacetime resolution scale. Second, a "proper-time" parameter was carefully employed in our scheme, which brings with it an independent cutoff scale. The latter acquires the meaning of an ultraviolet cutoff in the same sense as it enters calculations in field theory in the limit of a vanishing resolution scale. Such a decoupling of the roles between resolution scale and ultraviolet cutoff (which controls high-frequency fluctuations) offers a novel perspective on manipulations involving spin-1/2 quanta. We hope that our assortment of derivations of effective-action terms, which includes topological ones, provides sufficient evidence for the applicability of our approach to field theory. In the following paper [28] we extend the computational capabilities of our scheme to quantities which are directly associated with physical processes, e.g., Green functions and form factors.

ACKNOWLEDGMENTS

The authors wish to acknowledge the valuable input from N. G. Stefanis during numerous conversations that transpired in the course of the present work. C.N.K. wishes to thank E. Kiritsis for a number of critical comments as well as Professor A. V. Efremov who brought to his attention the work in Ref. [23].

APPENDIX

Our present concern is to deal with the quantity

$$I_2 = \frac{1}{2}g^2 \int \frac{d^dk}{(2\pi)^d} \tilde{A}_\mu(k)\tilde{A}_\nu(-k) \int_c^{\infty} \frac{dT}{T} e^{-T(m-\frac{1}{\epsilon})/2} \int_0^T d\tau \int_0^{T'} d\tau' G_{\mu\nu}(k;\tau,\tau') , \tag{A1}$$

where

$$G_{\mu\nu}(k;\tau,\tau') = e^{-Td\ln(2\pi)/2\epsilon}\int\frac{d^d\lambda}{(2\pi)^d}\int[de(\tau)]e_\mu(\tau)e_\nu(\tau')\exp\left[-\frac{1}{2\epsilon}\int_0^T d\tilde{\tau}e(\tilde{\tau})^2 - i\int_0^T d\tilde{\tau}\lambda_\mu e_\mu(\tilde{\tau}) - i\int_0^\tau d\tilde{\tau}k_\mu e_\mu(\tilde{\tau}) + i\int_0^{\tau'} d\tilde{\tau}k_\mu e_\mu(\tilde{\tau})\right]. \quad \text{(A2)}$$

Inserting $e_\mu(\tau)$ through its Fourier expansion, given by (52), we obtain

$$G_{\mu\nu} = \frac{C^{-1}}{T}\int\frac{d^d\lambda}{(2\pi)^d}\int\left[\prod_n d^d\alpha(n)\right]\sum_{l,l'}\alpha_\mu(l)\alpha_\nu(l')\exp\left(2\pi i\frac{\tau}{T}l + 2\pi i\frac{\tau'}{T}l'\right) \times\exp\left[-\frac{1}{2\epsilon}\sum_n|\alpha(n)|^2 - i\sqrt{T}\,\lambda_\mu\alpha_\mu(0) + i\sqrt{T}\,\sum_n\alpha_\mu(n)k_\mu(n;\tau,\tau')\right], \quad \text{(A3)}$$

where

$$C \equiv \int\left[\prod_n d^d\alpha(n)\right]\exp\left(-\frac{1}{2\epsilon}\sum_n|\alpha(n)|^2\right)$$

and

$$k_\mu(n;\tau,\tau') \equiv \frac{k_\mu}{2\pi i n}\left(e^{2\pi i n\tau'/T} - e^{2\pi i n\,\tau/T}\right).$$

The λ integration yields

$$\int\frac{d^d\lambda}{(2\pi)^d}e^{-i\sqrt{T}\,\lambda\cdot\alpha(0)} = T^{-d/2}\delta[\alpha(0)]\,, \quad \text{(A4)}$$

whereupon

$$G_{\mu\nu} = \frac{1}{(2\pi\epsilon T)^{d/2}}\frac{1}{T}\sum_{l,l'\neq 0}\exp\left(2\pi i\frac{\tau}{T}l + 2\pi i\frac{\tau'}{T}l'\right) \times\left[\epsilon\delta_{\mu\nu}\delta_{l,-l'} - T\epsilon^2 k_\mu(-l;\tau,\tau')k_\nu(-l';\tau,\tau')\right]\exp\left[-\frac{\epsilon T}{2}\sum_{n\neq 0}|k(n;\tau,\tau')|^2\right]. \quad \text{(A5)}$$

The summations give

$$\sum_{n\neq 0}|k(n;\tau,\tau')|^2 = k^2\frac{|\tau-\tau'|}{T}\left(1 - \frac{|\tau-\tau'|}{T}\right) \quad \text{(A6)}$$

and

$$\sum_{l\neq 0}e^{2\pi i l\tau/T}k_\mu(-l;\tau,\tau') = \theta(\tau-\tau') - (\tau-\tau') - \frac{1}{2}\,. \quad \text{(A7)}$$

Effecting in succession the changes

$$\tau,\tau' \longrightarrow T\tau, T\tau'\,, \qquad T \longrightarrow \frac{\epsilon T}{2}\,, \qquad \tau'-\tau = \lambda\,,$$

we finally get

$$I_2 = -\frac{2g^2}{(2\pi)^{d/2}}\int\frac{d^dk}{(2\pi)^d}\,\tilde{A}_\mu(k)\left(k^2\delta_{\mu\nu} - k_\mu k_\nu\right)\tilde{A}_\nu(k) \times\int_0^1 dx\,x(1-x)\int_{\Lambda^{-2}}^\infty dT\,T^{1-d/2}\exp\left[-Tx(1-x) - TM^2\right], \quad \text{(A8)}$$

where $\Lambda^{-2} = c\epsilon/2$ and $M^2 = (m - 1/\epsilon)/\epsilon$.

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