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Some properties of the Kerr solution to low energy string theory

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The Kerr solution to axidilaton gravity is analyzed in the Debney-Kerr-Schild formalism. It is shown that the Kerr principal null congruence retains its property to be geodesic and shear-free; however, the axidilatonic Kerr solution is not algebraically special; it is of type I in the Petrov-Pirani classification and may not be represented in the Kerr-Schild form. A limiting form of this solution is considered near the ringlike Kerr singularity. This limiting solution is similar to that for the field of fundamental heterotic string obtained by Sen.

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I. INTRODUCTION

Much attention has been paid recently to the connection of black hole physics and string theory. In particular, many important solutions of Einstein gravity have found their analogue among the solutions of low energy string theory, including axion and dilaton corrections. Such classical solutions to axidilaton gravity can be interpreted as stable extended solitonlike states or fundamental strings [1–3]. In this paper we analyze a new rotating and charged solution to axidilaton gravity, which is an analogue of the Kerr solution. This solution was obtained by Sen [2] and, in a more general form [including the Newman-Unti-Tamborino (NUT) parameter], by Gal'tsov and Kechkin [4]. The rather complicated character of the Kerr solution puts obstacles in the way of directly obtaining this rotating solution from the field equations; so this solution was obtained by a method for generating new solutions from the known ones [5, 2, 4]. However, by using this method some important characteristics of the new solutions remain unknown. For example, there was no information concerning the type of the new Kerr-like solution in the Petrov-Pirani classification.[1] We partially compensate for this deficiency.

By using the Kerr coordinates [7] we analyze this solution near the singular ring and find the limiting form of the solution to be remarkably similar to the solution constructed by Sen [2, 3] for the field around a fundamental heterotic string.

II. SOME ALGEBRAIC PROPERTIES OF THE KERR SOLUTION TO AXIDILATON GRAVITY

We will restrict ourselves in this paper to the case of an electric charged Kerr solution (without the NUT parameter and magnetic charge). We are going to use the algorithm for obtaining this solution from the original Kerr solution given by Gal'tsov and Kechkin [4]. According to Ref. [4], starting with the vacuum Kerr solution

$$ds^2 = \frac{\Delta - a^2 \sin^2\theta}{\Sigma}(dt - \omega d\varphi)^2 - \Sigma\left(\frac{dr^2}{\Delta} + d\theta^2 + \frac{\Delta \sin^2\theta}{\Delta - a^2\sin^2\theta} d\varphi^2\right), \qquad (1)$$

where

$$\Delta = r(r-2M) + a^2, \quad \Sigma = r^2 + a^2\cos^2\theta, \qquad (2)$$

$$\omega = 2Mra\sin^2\theta/(a^2\sin^2\theta - \Delta) \qquad (3)$$

(a is the Kerr rotation parameter; M is the mass), one can write the transformed metric corresponding to the axidilaton gravity in the same form, where the substitutions

$$\Delta_d \to \Delta, \Sigma_d \to \Sigma \qquad (4)$$

are to be done, where[2]

$$\Delta_d = r(r + r_-) - 2Mr + a^2, \qquad (5)$$

$$\Sigma_d = r(r + r_-) + a^2\cos^2\theta, \qquad (6)$$

$$r_- = Q^2/M. \qquad (7)$$

Q is the electric charge.

It will be convenient for our analysis to represent the Kerr solution to axidilaton gravity in the Kerr coordinates [7]. We will do it in two steps by representing the charged Kerr solution (the Kerr-Newman solution) at the first step in the Boyer-Lindquist form [8] in terms of parameters Δ and Σ:

$$ds^2 = -\frac{\Delta}{\Sigma}\left[dt - a\sin^2\theta d\varphi\right]^2 + \frac{\sin^2\theta}{\Sigma}\left[(r^2+a^2)d\varphi - adt\right]^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2. \qquad (8)$$

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[1] It was known only that this solution does not belong to type D in contrast with the Kerr solution of Einstein gravity [6].

[2] The coordinate r used here corresponds to r_0 in the definition of Ref. [4].

The corresponding electromagnetic field is given by the vector potential[3]

$$A = 2^{3/2} Q \frac{r}{\Sigma}(dt - a\sin^2\theta d\varphi). \tag{9}$$

Next we rewrite the Kerr-Newman solution in the Kerr coordinates by using the relations [8]

$$d\tilde{V} = dt - \frac{\Sigma + a^2\sin^2\theta}{\Delta}dr, \quad d\tilde{\varphi} = d\varphi + \frac{a}{\Delta}dr, \tag{10}$$

and express it again in terms of the parameters Δ and Σ:

$$ds^2 = \Sigma(d\theta^2 + \sin^2\theta d\tilde{\varphi}^2) + 2K(dr - a\sin^2\theta d\tilde{\varphi}) - (1-2H)K^2. \tag{11}$$

Here

$$H = (\Sigma + a^2\sin^2\theta - \Delta)/\Sigma \tag{12}$$

and K is a vector field tangent to the one of two principal null directions of the Kerr solution:

$$K = d\tilde{V} - a\sin^2\theta d\tilde{\varphi}. \tag{13}$$

The electromagnetic field for the electric charged Kerr solution is given by the vector potential

$$A = 2^{3/2} Q(r/\Sigma)K. \tag{14}$$

After substituting $\Delta_d \to \Delta$, $\Sigma_d \to \Sigma$, expressions (11)–(14) yield, according to the Gal'tsov-Kechkin algorithm, the transformed Kerr solution to the axidilaton gravity in the Kerr coordinates.[4] Now the gauge field is given by

$$A = 2^{3/2} Q e^{\Phi_0}(r/\Sigma_d)K, \tag{15}$$

where Φ_0 is the asymptotic value of the dilaton field. The axion field Ψ and the dilaton field Φ are joined in the complex axidilaton field

$$\lambda = \Psi + ie^{-2\Phi} = \lambda_0 + ir_- e^{-2\Phi_0}/(r + ia\cos\theta), \tag{16}$$

where

$$\lambda_0 = \Psi_0 + ie^{-2\Phi_0} \tag{17}$$

is an asymptotic value of the axidilaton.

This form allows us to use the Debney-Kerr-Schild (DKS) formalism [7] to analyze the solution. We represent the metric of the transformed solution in tetrad form,

$$ds_d^2 = 2\tilde{e}^3\tilde{e}^4 + 2\tilde{e}^1\tilde{e}^2, \tag{18}$$

and express it via the original DKS tetrad $e^a, a = 1,2,3,4$, as a deformation of the Kerr solution by the dilaton factor

$$ds_d^2 = 2e^3e^4 + 2e^1e^2e^{-2(\Phi-\Phi_0)}, \tag{19}$$

where

$$e^{-2(\Phi-\Phi_0)} = \Sigma_d/\Sigma. \tag{20}$$

Thus we have a new tetrad,

$$\tilde{e}^1 = e^{-(\Phi-\Phi_0)}e^1, \quad \tilde{e}^2 = e^{-(\Phi-\Phi_0)}e^2, \tag{21}$$

$$\tilde{e}^3 = e^3, \quad \tilde{e}^4 = e^4(\text{with substitution } H_d \to H), \tag{22}$$

where the original DKS tetrad is the following:[5] The tetrad null vectors e^1 and e^2 are complex conjugate,

$$e^1 = 2^{-1/2}Z^{-1}(d\theta + i\sin\theta d\tilde{\varphi}) = (PZ)^{-1}dY, \quad e^2 = \bar{e}^1; \tag{23}$$

e^3 and e^4 are real null vectors,

$$e^3 = K, \quad e^4 = dr + iaP^{-2}(\bar{Y}dY - Yd\bar{Y}) + 2^{-1}(H-1)e^3. \tag{24}$$

From (12) we obtain the function H_d:

$$H_d = 2Mr/\Sigma_d. \tag{25}$$

The functions P, Z, and Y are

$$P = 2^{-1/2}(1 + Y\bar{Y}), \quad Z = (r + ia\cos\theta)^{-1}, \quad Y = e^{i\tilde{\varphi}}\tan\theta/2. \tag{26}$$

Now we would like to get some algebraic characteristics of the new solution in comparison with the corresponding characteristics of the original Kerr solution. In the Kerr solution the vector $e^3 = K$ is the multiple Debever-Penrose vector tangent to a geodesic and shear-free null congruence; thus, the Kerr solution is algebraically special of type D in the Petrov-Pirani classification. The condition for e^3 to be a Debever-Penrose null vector is expressed via the component of Weyl's conformal curvature tensor [7]

$$C^{(5)} = 2R_{4242} = 0. \tag{27}$$

The condition for e^3 to be a double Debever-Penrose vector (or solution to be algebraically special) is

$$C^{(4)} = R_{1242} + R_{3442} = 0. \tag{28}$$

The geodesic and shear-free condition for e^3 is

$$\Gamma_{424} = \Gamma_{422} = 0. \tag{29}$$

Checking these conditions for the axidilatonic Kerr solution we obtain[6]

[3] The extra factor $2^{3/2}$ in the definition of the electric charge has been introduced to match the definitions of Refs. [2, 4] and Ref. [8].

[4] Equivalence of these forms for Δ_d and Σ_d may also be verified by direct calculations by using the relations given in Appendix A.

[5] The DKS-tetrad suffixes are raised or lowered by performing the permutation $1,2,3,4 \to 2,1,4,3$.

[6] In Appendix B the expressions for the Ricci rotation coefficients $\widetilde{\Gamma^a_{bc}}$ are given via the known values of the coefficients for the original Kerr solution Γ^a_{bc}; some necessary tetrad components of the curvature tensor are also given.

(i)

$$\widetilde{C^{(5)}} = 2\widetilde{R_{4242}} = 0, \quad (30)$$

or $\tilde{e}^3$ is a Debever-Penrose null vector forming the principal null congruence,

(ii)

$$\widetilde{\Gamma_{424}} = \widetilde{\Gamma_{422}} = 0, \quad (31)$$

and the principal null congruence of $\tilde{e}^3$ is geodesic and shear free,

(iii)

$$\widetilde{C^{(4)}} = \widetilde{R_{1242}} + \widetilde{R_{3442}} \neq 0; \quad (32)$$

therefore, the new axidilatonic Kerr solution is not algebraically special; it is of type I in the Petrov-Pirani classification.

III. LIMITING FORM OF THE AXIDILATONIC KERR SOLUTION NEAR THE SINGULAR RING

The Kerr singular ring is one of the remarkable peculiarities of the Kerr solution. It is a branch line of space on two sheets, "negative" and "positive," where the fields change their signs and directions. There exist the Newton and Coulomb analogues of the Kerr solution possessing the Kerr singular ring. The corresponding Coulomb solution was obtained by Appel in 1887 by a method of complex shift [9].

A pointlike charge q, placed on the complex Z axis $(x_0, y_0, z_0) = (0, 0, ia)$, gives the real Appel potential

$$\phi_a = q/\tilde{r} + \bar{q}/\bar{\tilde{r}}. \quad (33)$$

Here $\tilde{r}$ is in fact the Kerr complex radial coordinate $Z^{-1} = r + ia\cos\theta$. It may be expressed in the usual rectangular Cartesian coordinates x, y, z, t as

$$\begin{aligned}\tilde{r} \equiv Z^{-1} &= [(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2]^{1/2} \\ &= [x^2 + y^2 + (z-ia)^2]^{1/2}. \quad (34)\end{aligned}$$

It is not difficult to see that the Appel potential ϕ_a is singular at the ring $z = 0$, $x^2 + y^2 = a^2$, or by $r = \cos\theta = 0$.

We would like to consider the axidilaton corrections to the original Kerr field near the singular ring and will consider the radius of the ring a to be much larger than the distance δ from the singular line. Thus, the parameter δ/a will be used as a small parameter to get an approximate limiting form of the metric near the Kerr singularity.

Formulas for the connection of the Cartesian and the Kerr angular coordinates are

$$x + iy = (r + ia)e^{i\tilde{\varphi}} \sin\theta, \quad (35)$$

$$z = r\cos\theta, \quad (36)$$

$$t = \tilde{V} - r. \quad (37)$$

By using these coordinates the Kerr metric may be expressed in the Kerr-Schild form [7] $g_{\mu\nu} = \eta_{\mu\nu} + 2hK_\mu K_\nu$, where η is the metric of the auxiliary Minkowski space. The coordinates $r, \theta, \tilde{\varphi}$ cover the Minkowski space twice, by positive and by negative values of r with a branch line along the singular ring $r = \cos\theta = 0$; so the coordinate r will be two valued near the Kerr singular filament.

Near the point of singularity $(x, y, z) = (a, 0, 0)$, in the orthogonal to the filament two-plane $y = 0$, we introduce coordinates with their origin on the filament;

$$u = z, \quad v = x - a, \quad (38)$$

and obtain,[7] from (34), keeping the leading term in δ/a,

$$\tilde{r} = Z^{-1} \simeq a[2(v + iu)/a]^{1/2}, \quad (39)$$

$$d\tilde{r} \simeq (dv + idu)/[2(v + iu)/a]^{1/2}. \quad (40)$$

The function Y in Cartesian coordinates may be extracted from Eq. (5.72) of [7],

$$Y = (z - ia - \tilde{r})/(x - iy), \quad (41)$$

which yields

$$dY \simeq (dz - d\tilde{r})/a. \quad (42)$$

By using the coordinate transformations (35)–(37) and relations (39)–(42) one finds the limiting form of the tetrad (21)–(24) near the singular filament, up to leading terms in δ/a:

$$\begin{aligned}\tilde{e}^1 &= -e^{-(\Phi-\Phi_0)} 2^{-1/2}(dv + idu), \\ \tilde{e}^2 &= -e^{-(\Phi-\Phi_0)} 2^{-1/2}(dv - idu), \quad (43)\end{aligned}$$

$$\begin{aligned}\tilde{e}^3 &= 2^{-1/2}(dt - dy), \\ \tilde{e}^4 &= 2^{-1/2}(dt + dy) + H_d 2^{-1/2}(dt - dy), \quad (44)\end{aligned}$$

where dy is directed along the singular filament. The functions Σ, Σ_d, H_d, and $e^{-(\Phi-\Phi_0)}$ are given by

$$\Sigma \simeq 2a(v^2 + u^2)^{1/2}, \quad (45)$$

$$\begin{aligned}\Sigma_d \simeq 2a(v^2 + u^2)^{1/2} &+ ar_-\{[2(v + iu)/a]^{1/2} \\ &+ [2(v - iu)/a]^{1/2}\}, \quad (46)\end{aligned}$$

$$H_d = 2Mr/\Sigma_d, \quad (47)$$

$$e^{-2(\Phi-\Phi_0)} = \Sigma_d/\Sigma = 1 + r_-(Z + \bar{Z})/2. \quad (48)$$

The limiting form of the metric is

$$\begin{aligned}ds_d^2 = e^{-2(\Phi-\Phi_0)}(dv^2 + du^2) &+ dy^2 - dt^2 \\ &+ (2Mr/\Sigma_d)(dy - dt)^2. \quad (49)\end{aligned}$$

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[7]Our approximation will be the most effective for the case of a large $|a|$. The Kerr solution with $|a| \gg m$ has attracted special attention because it displays some relationships with the spinning elementary particles [7,13,10,11]. For example, the corresponding parameters of the electron will be $a \approx 10^{22}$, $m \approx 10^{-22}$, in units $\hbar = c = 1$. In this case all the fields concentrate very close to the singular filament.

The gauge field is given by the vector potential

$$A = 2Q(r/\Sigma_d)(dt - dy). \tag{50}$$

By introducing an electric charge per unit length of the Kerr ring $q = 2^{(3/2)}Q/(2\pi a)$ and a two-dimensional (two-valued) Green's function $G_a^{(2)}$ in the (u,v) complex plane near the Kerr singularity,

$$G_a^{(2)} = 2\pi ar/\Sigma \simeq \pi\left\{\left[\frac{a}{2(u+iv)}\right]^{1/2} + \left[\frac{a}{2(u-iv)}\right]^{1/2}\right\}, \tag{51}$$

the dilaton factor may be represented as

$$e^{-2(\Phi-\Phi_0)} = 1 + NG_a^{(2)}, \tag{52}$$

where

$$N = r_-/2\pi a. \tag{53}$$

Then the rescaled σ-model metric $ds^2_{\rm str} = e^{2(\Phi-\Phi_0)}ds^2_d$, used in string theory, may be written in the form

$$ds^2_{\rm str} = (dv^2 + du^2) + \frac{1}{1+NG_a^{(2)}}(dy^2 - dt^2) + \frac{2MG_a^{(2)}}{2\pi a(1+NG_a^{(2)})^2}(dy - dt)^2. \tag{54}$$

This metric is remarkably similar to the form of the metric obtained by Sen for a field around a fundamental heterotic string[8] [3, 2].

However, the structure of axidilaton field λ and the form of two-dimensional Green's function $G_a^{(2)}$ differ from those of the Sen solution. These differences are very natural and they are connected to the two valuedness of the fields near the Kerr singularity and with the known twofoldedness of the Kerr space.

This two valuedness was an object of the special consideration in the old problem of the source of the Kerr solution [11]. One of the traditional solutions of this problem is cutting off the negative sheet of the Kerr space by introducing a disklike source spanned by the Kerr singular ring. The analysis shows [11] that this disk has to be in a rigid relativistic rotation and consists of an exotic material with superconducting properties. Thus, the Kerr singular string is placed at the board of the superconducting disk. The superconducting nature of the heterotic strings was also mentioned before in Refs. [3, 12]. Some earlier presumptions concerning the Kerr singular ring to be a string may be found in Ref. [13].

Further, it would be interesting to consider electromagnetic and axidilatonic excitations of the Kerr string in the form of traveling waves[9] [14] and the case of massive dilaton.

There is one more stringlike structure in the Kerr geometry which is connected to the above representation of the Kerr source as an object propagating along a complex world line and based on the fact that the complex world line is really a world sheet [16]. The physical role of these strings and their interaction are still unclear.

Note added. After this paper was written I was informed that the Petrov-Pirani-type Kerr solution in axidilaton gravity was also determined by Gal'tsov and Lunin (unpublished).

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APPENDIX A

To match the notations of Refs. [2] and [4] we will add subscripts s for the Sen parameters and g for the Gal'tsov-Kechkin parameters. Then we have

$$Q_s = Q_g = Q, \quad M = M_s = M_g = m_s \cosh^2\frac{\alpha_s}{2}, \tag{A1}$$

$$q = 2\sqrt{2}Q, \tag{A2}$$

$$m_s = M - \frac{r_-}{2}, \tag{A3}$$

$$r_- = Q^2/M = 2m_s \sinh^2\frac{\alpha_s}{2} = 2(M_s - m_s). \tag{A4}$$

The following relations are useful when deriving the transformed solution in the Kerr coordinates:

$$(\Sigma_d + a^2\sin^2\theta)^2 - \Delta_d a^2 \sin^2\theta = (\Sigma_d + a^2\sin^2\theta)\Sigma_d + 2Mra^2\sin^2\theta, \tag{A5}$$

$$dt - a\sin^2\theta d\varphi = K - \frac{\Sigma_d}{\Delta_d}dr, \tag{A6}$$

where the vector K is given by

$$K = d\tilde{V} - a\sin^2 d\tilde{\varphi} \tag{A7}$$

and points in the principal null direction. In the Kerr coordinates

[8]Sen has constructed this solution by the method for generating new solutions from the fundamental string solution of Ref. [1].

[9]Similar model for the Kerr solution in Einstein gravity was suggested in Ref. [15].

$$\tilde{V} = dt + dr, \tag{A8}$$

if the principal null congruence is directed "inside."

In the expression for vector potential (15) the term $\frac{q}{\Sigma_d}\frac{\Sigma_d}{\Delta_d}dr$ is omitted since it is full differential.

APPENDIX B

We use a freedom of tetrad transformations [Eqs. (2.21) of Ref. [7]] to adopt the tetrad (23),(24) to the DKS form of Sec. 3 of Ref. [7]:

$$e'^1 = e^1, \quad e'^2 = e^2, \quad e'^3 = Pe^3, \quad e'^4 = P^{-1}e^4, \quad Z' = PZ. \tag{B1}$$

Dropping primes, we calculate the Ricci rotation coefficients to the axidilaton solution expressed via the coefficients of the original Kerr solution Γ_{abc}. For example, we extract $\widetilde{\Gamma_{2bc}}$ from the relations

$$\tilde{e}^1 = e^{-(\Phi-\Phi_0)}e^1, \qquad d\tilde{e}^1 = \widetilde{\Gamma^1_{bc}}\tilde{e}^b \wedge \tilde{e}^c = e^{-(\Phi-\Phi_0)}(de^1 - d\Phi \wedge e^1), \tag{B2}$$

where $de^1 = \Gamma_{2[bc]}e^b \wedge e^c$.

The result is given by

$$\widetilde{\Gamma_{121}} = -e^{(\Phi-\Phi_0)}\bar{Z}(\bar{Z}^{-1}),_1 + e^{(\Phi-\Phi_0)}\Phi,_{\tilde{1}}\,,$$

$$\widetilde{\Gamma_{122}} = e^{(\Phi-\Phi_0)}Z(Z^{-1}),_2 - e^{(\Phi-\Phi_0)}\Phi,_{\tilde{2}}\,,$$

$$\widetilde{\Gamma_{123}} = e^{(\Phi-\Phi_0)}\Gamma_{123} + e^{(\Phi-\Phi_0)}(1-e^{(\Phi-\Phi_0)})\left[(\Gamma_{312}-\Gamma_{321})/2 + (H-H_d)(Z-\bar{Z})/4\right],$$

$$\widetilde{\Gamma_{124}} = e^{(\Phi-\Phi_0)}(1-e^{(\Phi-\Phi_0)})(Z-\bar{Z})/2,$$

$$\widetilde{\Gamma_{311}} = \widetilde{\Gamma_{314}} = 0,$$

$$\widetilde{\Gamma_{312}} = e^{(\Phi-\Phi_0)}(1+e^{(\Phi-\Phi_0)})\Gamma_{312}/2 - e^{(\Phi-\Phi_0)}(1-e^{(\Phi-\Phi_0)})\Gamma_{321}/2 + (H-H_d)(e^{(\Phi-\Phi_0)}+1)(\bar{Z}-Z),$$

$$\widetilde{\Gamma_{313}} = -(H-H_d),_{\tilde{1}} + e^{(\Phi-\Phi_0)}[\Gamma_{313} + (H-H_d)Z(Z^{-1}),_2],$$

$$\widetilde{\Gamma_{341}} = -e^{(\Phi-\Phi_0)}\bar{Z}(\bar{Z}^{-1}),_1\,,$$

$$\widetilde{\Gamma_{342}} = -e^{(\Phi-\Phi_0)}Z(Z^{-1}),_2\,,$$

$$\widetilde{\Gamma_{343}} = \Gamma_{343} + (H-H_d),_{\tilde{4}}/2,$$

$$\widetilde{\Gamma_{344}} = 0,$$

$$\widetilde{\Gamma_{421}} = -Ze^{(\Phi-\Phi_0)}(1+e^{(\Phi-\Phi_0)})/2 - \bar{Z}e^{(\Phi-\Phi_0)}(1-e^{(\Phi-\Phi_0)})/2,$$

$$\widetilde{\Gamma_{422}} = \widetilde{\Gamma_{423}} = \widetilde{\Gamma_{424}} = 0.$$

Directional derivatives along the tetrad vectors are $,_a =,_\mu e^\mu_a$ and $,_{\tilde{a}} =,_\mu \tilde{e}^\mu_a$.

The curvature tensor is defined by the Cartan formula

$$\mathcal{R}^a_b = R^a_{bcd}e^c \wedge e^d = d\Gamma^a_b + \Gamma^a_m \wedge \Gamma^m_b. \tag{B3}$$

Some tetrad components of the curvature tensor for the axidilatonic Kerr solution are

$$\widetilde{R_{4242}} = \widetilde{R_{4234}} = \widetilde{R_{4223}} = 0, \tag{B4}$$

$$\widetilde{R_{4214}} = (\chi^2 - \chi,_{\tilde{4}})/2, \qquad \widetilde{R_{4212}} = \chi(\widetilde{\Gamma_{212}} - 2e^{(\Phi-\Phi_0)}\Phi,_{\tilde{2}}) - 2\chi,_{\tilde{2}}\,, \tag{B5}$$

where

$$\chi = -(1/2)e^{(\Phi-\Phi_0)}[Z(1+e^{(\Phi-\Phi_0)}) + \bar{Z}(1-e^{(\Phi-\Phi_0)})], \tag{B6}$$

and $Z = (r + ia\cos\theta)/P$.

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