Thermodynamic properties of γ fluids and the quantum vacuum

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The thermodynamic behavior of a relativistic perfect simple fluid obeying the equation of state $p = (\gamma - 1)\rho$, where $0 \leq \gamma \leq 2$ is a constant, is investigated. Particular cases include a vacuum ($p = -\rho, \gamma = 0$), a randomly oriented distribution of cosmic strings ($p = -\frac{1}{3}\rho, \gamma = 2/3$), blackbody radiation ($p = \frac{1}{3}\rho, \gamma = 4/3$), and stiff matter ($p = \rho, \gamma = 2$). Fluids with $\gamma < 1$ become hotter when they expand adiabatically ($T \propto V^{1-\gamma}$). By assuming that such fluids may be regarded as a kind of generalized radiation, the general Planck-type form of the spectrum is deduced. As a limiting case, a new Lorentz-invariant spectrum of the vacuum which is compatible with the equation of state and other thermodynamic constraints is proposed. Some possible consequences to early universe physics are also discussed.

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I. INTRODUCTION

The concept of vacuum has pervaded the development of our understanding about space, matter, and forces in the universe since the ancient Greek philosophers [1]. In the same way that quantum mechanics was a major breakthrough for the theories of ordinary matter, so it was for modern physical models of vacuum. The first advance arose already in the years of the old quantum theory. It is closely related to the possible existence of a zero point energy for blackbody radiation. In fact, the random background radiation corresponding to the zero-point field is, presently, the key ingredient of so called stochastic electrodynamics (SED) [2,3]. With the development of quantum electrodynamics (QED) and other quantum field theories a new concept arose; namely, the physical vacuum is the ground state of a system of quantum fields on the space-time manifold. But now, we have to address at least two problems: first, how do we single out the vacuum state? Secondly, what is an intuitive picture of the physical vacuum?

The answer to the first question depends on the quantization method used, as well as on the observer. For instance, in canonical quantization the vacuum state is defined as that which contains no quanta. Technically, this means that the effect of the annihilation operator acting on the state gives zero. On the other hand, when using functional methods in quantum field theory, the vacuum state is defined as the state which realized the minimum of the effective potential [4]. Recently, some authors addressed the issue that both of the above definitions could give different answers for the vacuum energy density [5]. Such drawbacks are present even for quantum field theories (QFT's) formulated in the Minkowski spacetime when a particle detector is uniformly accelerated [6].

For the second question, two different approaches may be found in the literature. Stochastic electrodynamics postulates the vacuum as a random background of real electromagnetic fields endowed with a well-defined frequency spectrum [$\rho(\nu) \propto \nu^3$], whereas for QED the vacuum is filled with so called virtual pairs of particles (electron-positron pair) whose direct detection is not possible. Usually, as a kind of paradigm, QFT takes this last picture for granted.

In the 1960s, it was remarked that Lorentz invariance of the vacuum requires an energy-momentum tensor (EMT) of the form [7]

$$\langle T_{\mu\nu} \rangle = \langle \rho \rangle \eta_{\mu\nu} \ , \tag{1}$$

where ρ is the energy density and $\eta_{\mu\nu}$ is the Minkowski tensor. Therefore, the EMT of the vacuum describes a particular relativistic perfect simple fluid for which the equation of state is $p = -\rho$ [see Eq. (3) below]. Since the energy-momentum tensor (1) is divergenceless, the vacuum energy density is constant in space-time. In addition, performing a change of inertial frame, the energy density of a fluid transforms as [8]

$$\rho' = \frac{\rho + p\frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}} \ , \tag{2}$$

where v is the relative velocity between the frames. Thus it follows from the equation of state that the energy density of the vacuum is a Lorentz-invariant quantity, regardless of the form of its frequency spectrum. In other words, all inertial observers are comoving with the vacuum background.

In this work we are mainly interested in the above macroscopic point of view. It will be assumed that the vacuum state of any bosonic or fermionic field is the less

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rigid state of matter compatible with the relativity theory [9]. As we will see, regarding the vacuum as an unusual substance described by the equation $p = -\rho$, the overall thermodynamic properties of it can be easily deduced. As in the case of blackbody radiation, such properties shed light on the underlying nature of the quantum vacuum, determining, for instance, the general form of its frequency spectrum. For the sake of generality and also to simplify the comparison between the vacuum and the blackbody radiation properties, we will consider a monoparametric class of γ fluids for which radiation and vacuum are two important particular cases.

The paper is organized as follows. In Sec. II we present the general thermodynamical properties of a γ fluid. In Sec. III we deduce the spectrum of a γ fluid assuming as a natural ansatz that such spectrum is a Wien's type. In Sec. IV we present a formal deduction of such spectrum without assuming any ansatz while, in Sec. V, we specialize to the case ($\gamma = 0$) and some striking consequences are found for the vacuum case, which are opposite to what happens for ordinary matter. In Sec. VI, Einstein's derivation of the blackbody radiation spectrum is generalized in order to include the family of γ fluids. In Sec. VII, some consequences of our approach to early universe physics are discussed. Finally, in Sec. VIII we conclude with some comments.

II. THERMODYNAMICS OF A γ FLUID

The thermodynamic states of a relativistic simple fluid are characterized by an energy-momentum tensor $T^{\alpha\beta}$, a particle current N^α, and an entropy current S^α. For a perfect fluid such quantities are defined by [10]

$$T^{\alpha\beta} = (\rho + p)u^\alpha u^\beta - pg^{\alpha\beta} \ , \tag{3}$$

$$N^\alpha = nu^\alpha \ , \tag{4}$$

$$S^\alpha = n\sigma u^\alpha \ , \tag{5}$$

where ρ is the energy density, p is the pressure, n is the particle number density, and σ is the specific entropy (per particle). The quantities p, ρ, n, and σ are related with the temperature by the Gibbs law

$$nT\,d\sigma = d\rho - \frac{\rho + p}{n} dn \ . \tag{6}$$

The basic quantities are constrained by the relations

$$T^{\alpha\beta}{}_{;\beta} = 0 \ , \tag{7}$$

$$N^\alpha{}_{;\alpha} = 0 \ , \tag{8}$$

$$S^\alpha{}_{;\alpha} = 0 \ , \tag{9}$$

where a semicolon denotes a covariant derivative.

Equations (7) and (8) express, respectively, the energy momentum and the number of particle conservation laws whereas (9) is the thermodynamic second law restricted to an adiabatic flow ("equation of continuity" for entropy).

Bearing in mind the applications discussed ahead, first, the temperature evolution equation will be obtained. From (7) and (8) it follows that

$$\dot{\rho} + (\rho + p)\theta = 0 \ , \tag{10}$$

$$\dot{n} + n\theta = 0 \ , \tag{11}$$

where an overdot means a comoving time derivative (for instance, $\dot{\rho} = u^\alpha \rho_{;\alpha}$) and $\theta = u^\alpha_{;\alpha}$ is the scalar of expansion. Further, using (11) and taking n and T as independent thermodynamic variables, Eq. (10) can be rewritten as

$$\left(\frac{\partial \rho}{\partial T}\right)_n \dot{T} = \left[n\left(\frac{\partial \rho}{\partial n}\right)_T - \rho - p\right]\theta \ , \tag{12}$$

and since $d\sigma$ is an exact differential, the Gibbs law (6) yields the well known thermodynamic identity

$$T\left(\frac{\partial p}{\partial T}\right)_n = \rho + p - n\left(\frac{\partial \rho}{\partial n}\right)_T \ . \tag{13}$$

Finally, replacing (13) into (12) and using (11) again one has

$$\frac{\dot{T}}{T} = \left(\frac{\partial p}{\partial \rho}\right)_n \frac{\dot{n}}{n} \ . \tag{14}$$

In what follows, we consider the class of fluids described by the "γ law" equation of state:

$$p = (\gamma - 1)\rho \ , \tag{15}$$

where the "adiabatic index" γ lies on the interval [0,2]. This generalized equation of state accounts for a one-parametric family of fluid systems, including a subclass with negative pressure. The limit cases of vacuum ($\gamma = 0$) and stiff matter ($\gamma = 2$) are determined from causality requirements [9], whereas $\gamma = \frac{4}{3}$ describe the photon fluid. With this choice, a straightforward integration of (14) furnishes

$$Tn^{1-\gamma} = \text{const} \ , \tag{16}$$

and since n scales with V^{-1}, where V is the volume of the considered portion within the fluid, Eq. (16) assumes the form (from now on only the case $\gamma \neq 1$ will be considered)

$$T^{1/(\gamma-1)}V = \text{const} \ , \tag{17}$$

which is the usual adiabatic law for fluids with conserved net number of particles [11].

One additional thermodynamic constraint obeyed by the γ fluid is the generalized Stefan-Boltzmann law: namely,

$$\rho(T) = \eta T^{\gamma/(\gamma-1)} \ , \tag{18}$$

where η is a γ-dependent constant. As shown in Ref. [12],

the above expression may be derived by at least three different methods, among them that one applied by Boltzmann (Carnot cycle). It also follows naturally from the adiabatic condition ($dS = 0$) when one considers the γ-law equation of state. Note that for $\gamma = \frac{4}{3}$ it reduces to $\rho = \eta T^4$ and for the vacuum case ($\gamma = 0$) it reduces to $\rho =$ const as one should expect. As we shall see, the interesting point here is that the relation (18) together with Eq. (16), rewritten as

$$n(T) = \text{const} \times T^{1/(\gamma-1)} , \tag{19}$$

are the thermodynamic constraints fixing the general form of the spectrum for a γ fluid, including, of course, the vacuum spectrum itself.

III. THE γ-FLUID SPECTRUM

In order to see how the thermodynamic constraints work to fix the general form of the spectrum, we make, initially, a simple and natural ansatz, which is nothing more than a generalization of Wien's law obeyed by the radiation fluid. In fact, the approach used below was earlier applied to the case $\gamma = 0$ in Ref. [13]. In the next section a formal deduction of such a spectrum will be presented.

First, it will be assumed that the γ fluid is a kind of radiation (such as blackbody radiation), whose spectrum is a Wien's-type one; that is,

$$\rho_T(\nu) = \text{const} \times \nu^\beta \phi(\nu^\beta T^\lambda) , \tag{20}$$

where ϕ is an arbitrary function and β and λ are constants to be determined by using the constraint equations (18) and (19): namely,

$$\rho(T) = \int_0^\infty \rho_T(\nu) d\nu = \text{const} \times T^{\gamma/(\gamma-1)} , \tag{21}$$

and

$$n(T) = \int_0^\infty \frac{\rho_T(\nu)}{h\nu} d\nu = \text{const} \times T^{-1/(1-\gamma)} . \tag{22}$$

Substituting the spectrum (20) into the constraint equations and defining a new variable $u = \nu^\beta T^\lambda$, we can write

$$\rho(T) = T^{-\lambda(1+\beta)/\beta} \int_0^\infty f(u) du = \text{const} \times T^{\gamma/(\gamma-1)} \tag{23}$$

and

$$n(T) = T^{-\lambda} \int_0^\infty g(u) du = \text{const} \times T^{1/(\gamma-1)} , \tag{24}$$

where $f(u)$ and $g(u)$ are functions related with the original arbitrary function $\phi(u)$.

Comparing the powers of T in Eqs. (23) and (24) we get

$$\lambda = \frac{1}{1-\gamma} = -\beta . \tag{25}$$

Further, replacing (25) into (20) we obtain the spectral function for the γ fluid:

$$\rho_T(\nu) = \alpha \nu^{1/(\gamma-1)} \phi\left(\frac{T}{\nu}\right) , \tag{26}$$

where α is a dimensional constant and $\phi(T/\nu)$ an arbitrary function of its argument. As expected, if $\gamma = \frac{4}{3}$, Eq. (26) reduces to the well known Wien's law for blackbody radiation. Note also that, although assuming a Wien-type spectrum [Eq. (20)] for the γ fluid, this is not a completely arbitrary assumption. We have simply assumed a more general law, in analogy with Wien's, which satisfies the constraints (18) and (19) for $\gamma = \frac{4}{3}$.

IV. WIEN-TYPE LAW FOR γ FLUIDS: A FORMAL DEDUCTION

As a preliminary point of principle, we recall that if a hollow cavity containing blackbody radiation changes its volume, adiabatically, the ratio between the energy and the corresponding frequency of each component remains constant: namely,

$$\frac{E_\nu}{\nu} = \text{const} , \tag{27}$$

for any "proper oscillation."

This result, usually called the theorem of adiabatic invariance, holds indeed for an arbitrary oscillating system when one of its parameters is slowly modified by some external effect. For blackbody radiation the constancy of the above quantity also means that for each band the mean number of quanta is unaltered by reflection from the moving walls. In what follows, since the overall existence of this adiabatic invariant can be proved regardless of the nature of the oscillating system (see, for instance, Ref. [14]), its validity will be assumed for the whole family of radiative γ fluid with $\gamma \neq 1$.

Now, if $\rho_T(\nu)$ is the spectral energy density inside an enclosure with volume V, Eq. (27) may be rewritten as

$$\frac{\rho_T(\nu) d\nu\, V}{\nu} = \text{const} . \tag{28}$$

Note also that due to the thermal equilibrium state, the energy density in the band $d\nu$ varies with the temperature in the same manner as the total energy density (in principle, only this band could be present in the cavity). Hence from the generalized Stefan-Boltzmann law, the above adiabatic invariant takes the form

$$\frac{T^{\gamma/(\gamma-1)} V}{\nu} = \text{const} , \tag{29}$$

and since $T^{1/(\gamma-1)} V =$ const [see Eq. (17)], it follows that T/ν is invariant. Thus, whether one compresses or expands adiabatically a hollow cavity containing a radiation γ fluid, then

$$\lambda T = \text{const} . \tag{30}$$

The above result means that the displacement Wien law,

which is valid for photons ($\gamma = \frac{4}{3}$), holds in fact for the entire one-parametric family of γ fluids.

Before discussing the remarkable physical consequences of Eq. (30) on the vacuum state, we proceed to determine its effects on the general form of the spectral distribution. To that end, we consider an enclosure containing γ fluid at temperature T_1 and focus our attention on the band $\Delta\lambda_1$ centered on the wavelength λ_1 whose energy density is $\rho_{T_1}(\lambda_1)\Delta\lambda_1$.

If the temperature T_1 changes to T_2 due to an adiabatic expansion (note that T_1 does not necessarily decrease), the energy of the band changes to $\rho_{T_2}(\lambda_2)\Delta\lambda_2$ and according to Eq. (30) $\Delta\lambda_1$ and $\Delta\lambda_2$ are related by

$$\frac{\Delta\lambda_2}{\Delta\lambda_1} = \frac{T_1}{T_2} \ . \tag{31}$$

Now, since one can assume that distinct bands do not interact, it follows that

$$\frac{\rho_{T_2}(\lambda_2)\Delta\lambda_2}{\rho_{T_1}(\lambda_1)\Delta\lambda_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)} \ . \tag{32}$$

By combining the above result with (31) we conclude that

$$\frac{\rho_{T_2}(\lambda_2)}{\rho_{T_1}(\lambda_1)} = \left(\frac{T_2}{T_1}\right)^{(2\gamma-1)/(\gamma-1)} \ , \tag{33}$$

and using again the displacement law given by (30), we obtain for an arbitrary component

$$\rho_T(\lambda)\lambda^{(2\gamma-1)/(\gamma-1)} = \text{const} \ . \tag{34}$$

Note that in the case of blackbody radiation the above expression reduces to $\rho_T(\lambda)\lambda^5 = \text{const}$, as it should. Of course, due to Eq. (30) the above result takes the form

$$\rho_T(\lambda) = \text{const} \times \lambda^{(1-2\gamma)/(\gamma-1)}\phi(\lambda T) \ , \tag{35}$$

where ϕ is an arbitrary function of its arguments. Now in terms of the frequency, since $\rho_T(\nu)d\nu = \rho_T(\lambda)|\frac{d\nu}{d\lambda}|d\lambda$ it is easy to see that (35) can be rewritten as

$$\rho_T(\nu) = \alpha\nu^{1/(\gamma-1)}\phi\left(\frac{T}{\nu}\right) \ , \tag{36}$$

as obtained in the previous section [see Eq. (26)] .

V. THERMODYNAMICS AND THE VACUUM SPECTRUM

In the case of blackbody radiation ($\gamma = \frac{4}{3}$) Eq. (17) reduces to $T^3V = \text{const}$, a well known result, while for the vacuum state ($\gamma = 0$) we obtain

$$T = \text{const} \times V \ . \tag{37}$$

We have therefore reached the conclusion that the vacuum becomes hotter if it undergoes an adiabatic expansion. Such a result may be compared with those ones of the usual theory of fluids for which $\gamma > 1$ ($p > 0$).

It should be emphasized that in the derivation of (17) the conservation of the number of particles was explicitly used. However, the meaning of such an assumption needs to be clarified. For $p = \frac{1}{3}\rho$ we see from (16) that n scales with T^3. As we know, since the chemical potential of photons is zero its total number is indefinite so that n must be interpreted as the average number density of photons. As is well known, such an interpretation is in agreement with the Planck distribution which furnishes $n = \int_0^\infty \frac{\rho_T(\nu)}{h\nu}d\nu = bT^3$, where b is a constant [15].

In what follows we assume that similar considerations hold for the vacuum state ($\gamma = 0$), for which Eq. (16) yields

$$n = \frac{\text{const}}{T} \ . \tag{38}$$

Hence we see that for the vacuum state, the average density of particles decreases with increasing T. In the limit $T \to \infty$, n goes to zero, being infinite in the opposite extremum ($T = 0$). It should be noticed that both results are consistent with Eq. (37).

Let us now consider the vacuum spectrum itself. From the above results we can say that the energy spectrum $\rho_T(\nu)$ must satisfy two thermodynamic constraints:

$$\rho = \int_0^\infty \rho_T(\nu)d\nu = \text{const} \ , \tag{39}$$

and

$$n = \int_0^\infty \frac{\rho_T(\nu)}{h\nu}d\nu = \frac{\text{const}}{T} \ , \tag{40}$$

which are just the constraints (18) and (19) for $\gamma = 0$.

Taking $\gamma = 0$ in Eq. (36), instead of the result $\rho_T(\nu) = \text{const}\times\nu^3$, claimed by the proponents of SED, we find that the only Wien-type spectrum for the vacuum state compatible with the thermodynamic constraints is given by

$$\rho_T(\nu) = \text{const} \times \nu^{-1}\phi\left(\frac{T}{\nu}\right) \ , \tag{41}$$

a result obtained earlier in Ref. [13].

It should be noticed that even in the limit $T \to 0$, $\rho_T(\nu)$ scales with ν^{-1} instead of ν^3, as usually inferred from the blackbody radiation spectrum [2,3]. In fact, the later results follows from our Eq. (36) by choosing $T = 0$ and $\gamma = \frac{4}{3}$, since zeropoint radiation in the context of SED satisfies $p = \frac{\rho}{3}$. Of course, this kind of vacuum is rather different from the one considered here. In the present case, we remark that the existence of a temperature-dependent spectrum for the vacuum state is not forbidden by the relativity principle, as long as the vacuum fluid is described by the equation of state $p = -\rho$.

Finally, we would like to stress some physical consequences of the displacement Wien's law to the case of the vacuum state or, in general, for γ-radiation fluids with $\gamma < 1$.

First, it should be recalled that if a blackbody radia-

tion fluid expands adiabatically its temperature is lowered ($T \propto V^{-1/3}$) and since $\lambda T = \text{const}$, the wavelength of each band increases, thereby lowering the total energy density in accordance with the Stefan-Boltzmann law. This is the typical behavior for fluids with $\gamma > 1$. For $\gamma < 1$, however, the temperature grows if the fluid undergoes an adiabatic expansion ($T^{1/(\gamma-1)}V = \text{const}$). The vacuum state behaves like a limiting case of this subclass, the one for which the energy density remains constant in the course of expansion.

VI. PLANCK-TYPE SPECTRUM OF γ RADIATION

In this section we will derive, up to a constant, a formula giving the spectral distribution for generalized γ radiation. As a limiting case, a new Lorentz invariant spectrum for the vacuum state will be presented. Our derivation will be carried out through a slight modification of the arguments used by Einstein [16] in his original deduction of the Planck radiation spectrum which was based on Wien's law plus some additional hypotheses concerning the interaction between radiation and matter.

Let us consider an atomic or molecular gas, the particles of which can exist in a number of discrete energy levels $E_n = 1, 2, \ldots$, in thermal equilibrium with the γ radiation at temperature T. The probability that an atom is in the energy level E_n is given by the Boltzmann factor

$$W_n = p_n e^{-E_n/kT} , \tag{42}$$

where p_n, the statistical weight of the nth quantum state, is independent of the temperature.

Of course, transitions happen by emission or absorption of quanta of the γ radiation which satisfies the following hypotheses.

(H1) The γ-radiation spectrum is Wien's type, as deduced earlier: namely,

$$\rho_T(\nu) = \alpha \nu^{1/(\gamma-1)} \phi\left(\frac{T}{\nu}\right) , \tag{43}$$

where α is a dimensional γ-dependent constant.

(H2) Bohr's postulate for atomic emission or absorption remains valid for quanta of γ radiation: that is,

$$E_m - E_n = h\nu . \tag{44}$$

Following Einstein, in such a system there exist three types of transition processes by which equilibrium is established. The first one is due to absorption of γ radiation, with the atom making an upward transition from E_n to the level E_m according to the probability, per unit time,

$$\dot{W}_{nm} = B_n^m \rho_T(\nu) , \tag{45}$$

where B_n^m is a constant characterizing the specific transition.

The second one is spontaneous emission, which happens in the absence of any γ radiation and is determined by the coefficient A_m^n, and, finally, the stimulated emission characterized by B_m^n. For these processes the net transition probability per unit time is

$$\dot{W}_{mn} = A_m^n + B_m^n \rho_T(\nu) . \tag{46}$$

Hence, from Eq. (42) the equilibrium condition can be written as

$$p_n e^{-E_n/kT} B_n^m \rho_T(\nu) = p_m e^{-E_m/kT} [B_m^n \rho_T(\nu) + A_m^n] , \tag{47}$$

and solving for the energy density one obtains

$$\rho_T(\nu) = \frac{\frac{p_m}{p_n}\frac{A_m^n}{B_n^m}}{e^{(E_m-E_n)/kT} - \frac{p_m}{p_n}\frac{B_m^n}{B_n^m}} . \tag{48}$$

Now, at very high temperatures, it will be assumed that stimulated emission is much more probable than spontaneous emission so that (47) leads to

$$p_n B_n^m = p_m B_m^n , \tag{49}$$

and using H2, Eq. (48) can be recast in the form

$$\rho_T(\nu) = \frac{A_m^n / B_m^n}{e^{h\nu/kT} - 1} . \tag{50}$$

Finally, comparing (50) and (43) it follows that

$$\frac{A_m^n}{B_m^n} = \alpha \nu^{1/(\gamma-1)} , \tag{51}$$

and

$$\phi\left(\frac{T}{\nu}\right) = \frac{1}{e^{h\nu/kT} - 1} , \tag{52}$$

with (50) taking the form

$$\rho_T(\nu) = \frac{\alpha \nu^{1/(\gamma-1)}}{e^{h\nu/kT} - 1} . \tag{53}$$

This is the most natural generalization of Planck's radiation formula for γ radiation [17]. Einstein's result follows for $\gamma = \frac{4}{3}$. However, more interesting for fundamental physics is that the spectrum for the "hot vacuum" ($\gamma = 0$) is given by

$$\rho_T^{\text{vac}}(\nu) = \frac{\alpha \nu^{-1}}{e^{h\nu/kT} - 1} . \tag{54}$$

From Eq. (21) or by straightforward integration of the above equation, it is easy to see that

$$\rho^{\text{vac}} = \int_0^\infty \rho_T^{\text{vac}}(\nu) d\nu = \text{const} , \tag{55}$$

as it should be.

It is worth mentioning that, in the framework of QFT, several attempts have been made to assign a definite spectrum to the vacuum state in connection with the so-called Casimir effect [18]. As we know, this effect is a response of the vacuum structure to constraints im-

posed by spatial boundaries or a nontrivial topology. As a matter of fact, the Casimir spectrum and that one given above are quite different, even considering that both do not have a Planckian form. However, aside from some inevitable ambiguities present in the former (a fact explicitly recognized in the quoted papers), we notice that (54) has been deduced as a direct consequence of the equation of state $p = -\rho$. In particular, this means that such a spectrum describes bulk properties of the vacuum fluid instead of "distortions" effects produced by spatial boundaries as occur, for instance, with blackbody radiation in microcavities or more generally, with confined quantum gases [19]. Naturally, since far from the boundaries the vacuum properties must be the same as what it would present in free space, one may expect to recover (54) as a limiting case of a proper Casimir spectrum. This issue is presently under investigation.

The above results show that, under very reasonable hypotheses, the spectrum of the family of γ fluids is uniquely determined up to the constant scale factor α. In principle, the value of this constant could be determined using the low frequencies limit ($h\nu \ll kT$). However, unlike the blackbody radiation case, an independent derivation of the Rayleigh-Jeans (RJ) limit for arbitrary values of the γ parameter does not exist presently. It should be noticed that for $h\nu$ much smaller than kT, (53) leads to a RJ-type form given by

$$\rho_T(\nu) = \Omega k T \nu^{(2-\gamma)/(\gamma-1)} , \tag{56}$$

where the unknown Ω is a dimensional γ-dependent function such that $\Omega\left(\frac{4}{3}\right) = \frac{8\pi}{c^3}$. The question related with the precise form of the RJ limit for a γ fluid and its influence on the fluctuations of energy will be discussed elsewhere.

In terms of the wavelength, Eq. (53) may be rewritten as

$$\rho_T(\lambda) = \frac{\beta \lambda^{(1-2\gamma)/(\gamma-1)}}{e^{hc/k\lambda T} - 1} , \tag{57}$$

where β is also a γ-dependent constant. As is well known, for blackbody radiation the wavelength λ_m for which $\rho_T(\lambda)$ assumes its maximum value satisfies a displacement law under the form [15]

$$\lambda_m T = 0.289 \text{ cm deg} . \tag{58}$$

It turns out that the above result can be easily generalized for a γ fluid using Eq. (57). In fact, since λ_m is determined by the condition $\frac{\partial \rho_T(\lambda)}{\partial \lambda} = 0$, it follows from (57) that

$$\frac{\partial}{\partial \lambda}\left(\frac{\lambda^{(1-2\gamma)(\gamma-1)}}{e^{hc/kT\lambda} - 1}\right) = 0 , \tag{59}$$

or still

$$x + b e^{-x} - b = 0 , \tag{60}$$

where $x = \frac{hc}{k\lambda T}$ and $b = \frac{2\gamma-1}{\gamma-1}$. Hence, if $p(\gamma)$ denotes the roots of the above equation, then

$$\lambda_m T = \frac{hc}{kp(\gamma)} = \frac{1.438}{p(\gamma)} \text{ cm deg} . \tag{61}$$

Note that $x = 0$, or equivalently $p = 0$, is a trivial solution of (60) regardless of the values of γ. However, for $\gamma > 0$ there always exists another physically meaningful solution. For instance, if $\gamma = \frac{4}{3}$ one has $p(\frac{4}{3}) = 4.965$ so that the result (58) is recovered. Another example is provided by stiff matter ($\gamma = 2$) for which $p(2) = 2.821$ (61), $\lambda_m T = 0.510$ cm deg. For the vacuum ($\gamma = 0$) case, however, it is easy to show that there only exists the trivial solution. In other words, the graph of $\rho_T(\lambda)$ does not exhibit a finite maximum value characteristic of the class of γ fluids. It is infinite for $\lambda = 0$ and decreases monotonically to zero when λ goes to infinity. As the reader may conclude by himself, in fact, this is the only physical possibility allowed by the Lorentz invariance of the vacuum spectrum.

VII. SOME CONSEQUENCES IN COSMOLOGY

The above results may be interesting to early universe physics, mainly to the so called inflationary models. The essential feature of such models is the appearance of an accelerated expansion of the universe driven by the vacuum stress arising, for instance, from a scalar field with a global minimum in its effective potential or some types of phase transition. It turns out that negative pressure is the key condition to generate either exponential or "power law" inflation.

To the best of our knowledge the thermodynamic behavior of the field driving inflation has so far been neglected. By assuming that it behaves like a perfect fluid with $p = (\gamma - 1)\rho$, where $\gamma < \frac{2}{3}$ for power law inflation, the results presented here can be easily adapted. In fact, since our results are generally covariant, we can apply Eq. (17) for a Friedmann-Robertson-Walker metric ($V \propto R^3$) to obtain

$$T = T_* \left(\frac{R_*}{R}\right)^{3(\gamma-1)} , \tag{62}$$

where $R(t)$ is the universal scale function and $T_* = T(R_*)$ is the temperature when the scale factor takes on the value R_*. For $\gamma = \frac{4}{3}$ one finds $T \propto R^{-1}$ as usual for a radiation-dominated phase. Special results are (i) exponential inflation ($\gamma = 0$, vacuum, $T \propto R^3$) and (ii) power-law inflation ($0 < \gamma < \frac{2}{3}, T \propto R^{3(1-\gamma)}$).

It should be noticed that the above result holds regardless of the nature of the γ fluid; that is, it does not matter whether it is regarded as a generalized radiation. As a matter of fact, the temperature law given by (62) is a consequence of the "γ-law" equation of state. For instance, it can be applied even for dust ($\gamma = 1$) furnishing $T = T_* =$ const in accordance with (16). Another interesting example is provided by a randomly oriented distribution of infinitely thin straight strings averaged over all directions. As shown by Vilenkin [20], such a system behaves like a perfect fluid with $p = -\frac{1}{3}\rho$ ($\gamma = \frac{2}{3}$) and from (62) we obtain $T = \text{const} \times R$ [12].

An additional point, supporting the present treatment of the vacuum as a fluid endowed with a definite temperature-dependent spectrum, appears at the interface of particle physics and cosmology in the so-called cosmological constant problem. In fact, as is well known, the vacuum energy density due to the zeropoint energy of all normal modes of a field contributes to the cosmological Λ term which behaves like a fluid with $p = -\rho$. However, the cosmological estimates of such a term ($\Lambda/8\pi G \lesssim 10^{-47}$ GeV^4) is smaller than the estimates which follow from field theories by at least forty orders of magnitude. This puzzle which makes up the essence of the problem has been the subject of numerous papers [21–23].

A possible approach to circumvent this problem, which has been investigated in the recent literature (see Ref. [23] and references therein), is to assume that the effective Λ term is a fluid interacting with the other matter fields of the universe (as in a multifluid model). In this case, the vacuum energy density is not constant since the energy momentum tensor of the mixture must be conserved in the course of the expansion. Thus a slow decaying of the vacuum energy density may provide the source term for material particles or radiation, thereby suggesting a natural solution to this puzzle: namely, the cosmological constant is very small today because the universe is very old.

VIII. CONCLUSION

In this paper we have attempted to give a systematic treatment of how thermodynamics and semiclassical considerations can be used to determine the spectrum of a γ fluid, including the vacuum spectrum as a particular case. The physical motivation of such a study is based on two different, although closely related features: namely, the Lorentz invariance of the vacuum state which requires that its energy-momentum tensor is proportional to the Minkowski tensor, that is, a perfect fluid obeying the equation of state $p = -\rho$ and the probable existence of a universal Λ term which is also equivalent, in the cosmological domain, to a vacuum fluid satisfying the same equation of state.

For the sake of generality, several thermodynamic properties of γ fluids with positive and negative pressures have been investigated. In this connection, we remark that thermodynamic states with negative pressures are metastable but they are not forbidden by any law of nature. These states are also hydrodynamically unstable for bubbles and cavity formation and a spontaneous collapse could also be expected [15]. As remarked in Ref. [13] one may speculate whether such collapse may be answerable for the matter creation process from "nothing" with the particles being ultimately described as a kind of vacuum condensation.

By regarding the class of γ fluids as radiation with different equations of state, a formal deduction for Wien's law has been presented and such a result allows us to derive, up to a dimensional constant, the generalized Planckian type form of the spectrum. Probably, only using QFT or statistical methods, such a constant will be determined. It was also shown that in the limit of low frequencies the spectrum scales with $kT\nu^{(2-\gamma)/(\gamma-1)}$. For comparison, the usual blackbody expressions have systematically been recovered by taking $\gamma = \frac{4}{3}$. Further, as a special case, the thermodynamic behavior and the vacuum spectrum satisfying the equation of state $p = -\rho$ were obtained and its unusual features discussed in detail. The vacuum temperature, or more generally, the temperature of a γ fluid (for $\gamma < 1$) increases in the course of an adiabatic expansion and, unlike blackbody radiation, their wavelength decreases as required by Wien's law. In particular, this explains why the energy density of a pure vacuum (cosmological constant) remains constant if a hollow cavity (universe) undergoes an adiabatic expansion.

We also argued that the unsettled situation arising from the overall existence of the vacuum and its consequences on the interface uniting QFT, general relativity, and cosmology may be circumvented by a more comprehensive picture of the vacuum itself. Theoretically, as happens when one includes quantum corrections to the general relativity, the treatment of the vacuum as a fluid also suggests a cosmological scenario where the evolution may initially be supported by a pure vacuum state. By virtue of the expansion, the vacuum decays generating all matter and entropy existing in the universe, thereby explaining naturally the small value of the Λ term presently observed (see Ref. [23] for a cosmology satisfying such conditions).

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